Automat finit (AF)

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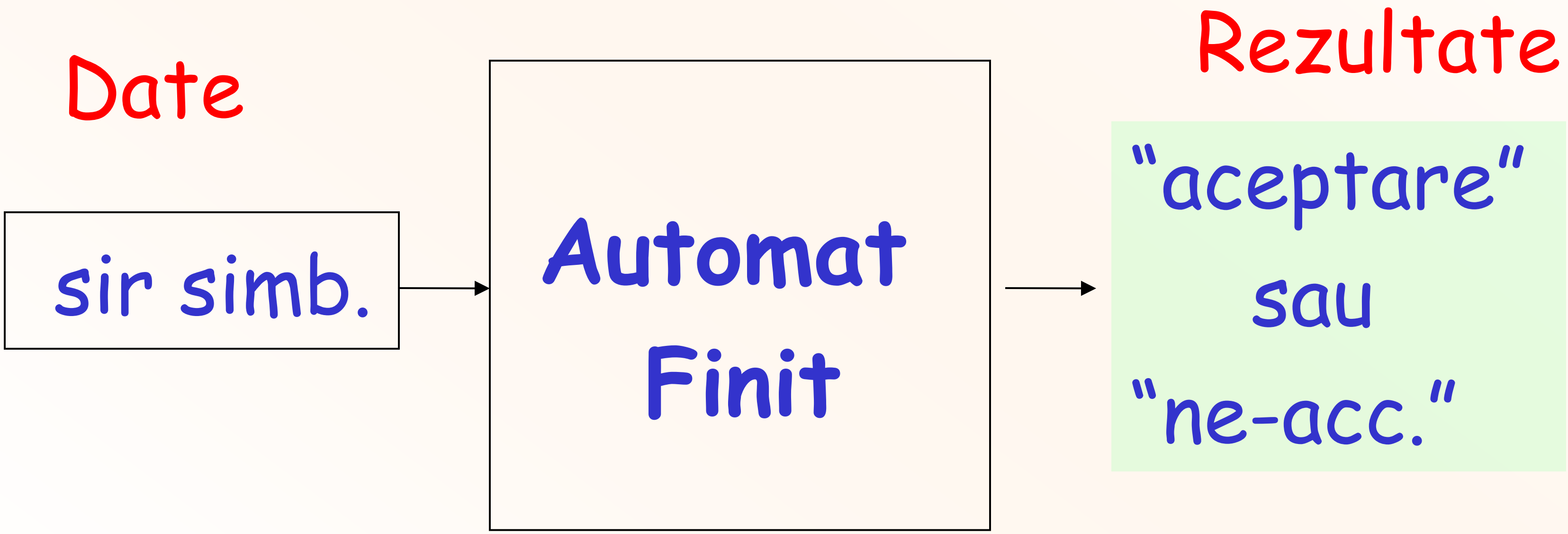
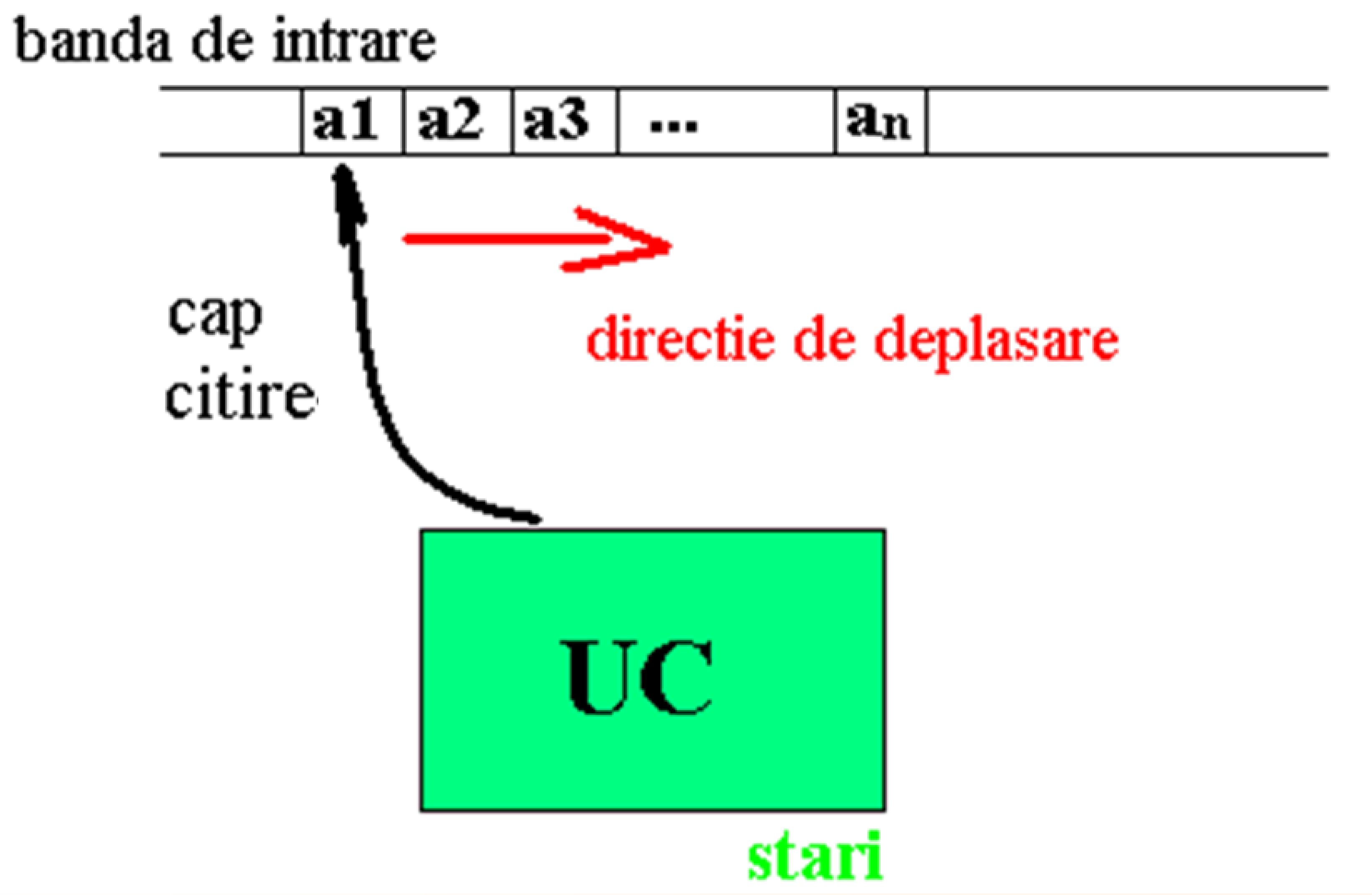


Automat finit: model fizic



Automat finit: model matematic

- Un ***automat finit*** este un ansamblu

  $M = (Q, \Sigma, \delta, q_0, F)$:
- Q – alfabetul starilor
- Σ – alfabet de intrare
- $\delta : Q x \Sigma \to \mathcal{P}(Q)$ functie de tranzitie
- $q_0 \in Q$ - stare initialã
- $F \subseteq Q$ multimea stãrilor finale

AF – reprezentare tabelara

δ		a_j		

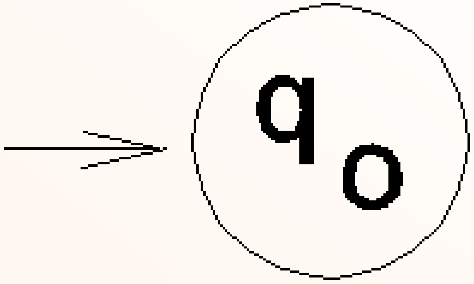
$z_i =$ 0 daca q_i nu e stare finala
1 daca q_i este stare finala

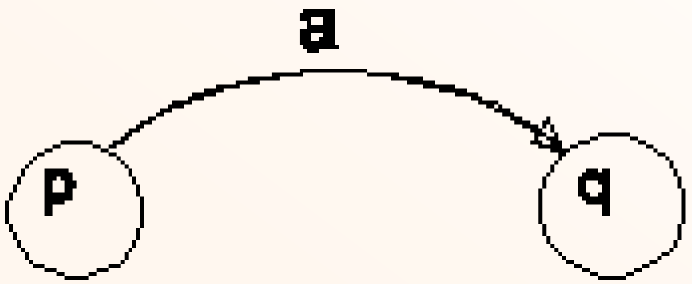
AF reprezentat tabelar; exemplu

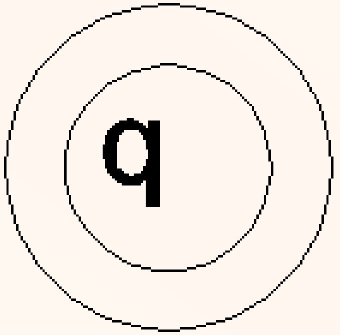
δ	**0**	**1**	
p	q	p	0
q	r	p	0
r	r	r	1

AF – reprezentare sub forma de graf

- graf orientat
- cu noduri si arce etichetate
- (graf de tranzitii)







Configuratii si relatii de tranzitie

$M = (Q, \Sigma, \delta, q_0, F)$.

configuratie: $(q,x) \in \mathbf{Q x \Sigma^*}$

tranzitie: element din $\mathbf{(Q x \Sigma^*) \, x \, (Q x \Sigma^*)}$

- $\vdash$ tranzitie directa
- $\vdash^k$ k-tranzitie
- $\vdash^+$ +-tranzitie
- $\vdash^*$ *-tranzitie

$(p,aw) \vdash (q,w) \iff \delta(p,a) \ni q;$

$p,q \in Q,\ a \in \Sigma,\ w \in \Sigma^*$

Limbaj acceptat; autom. echivalente

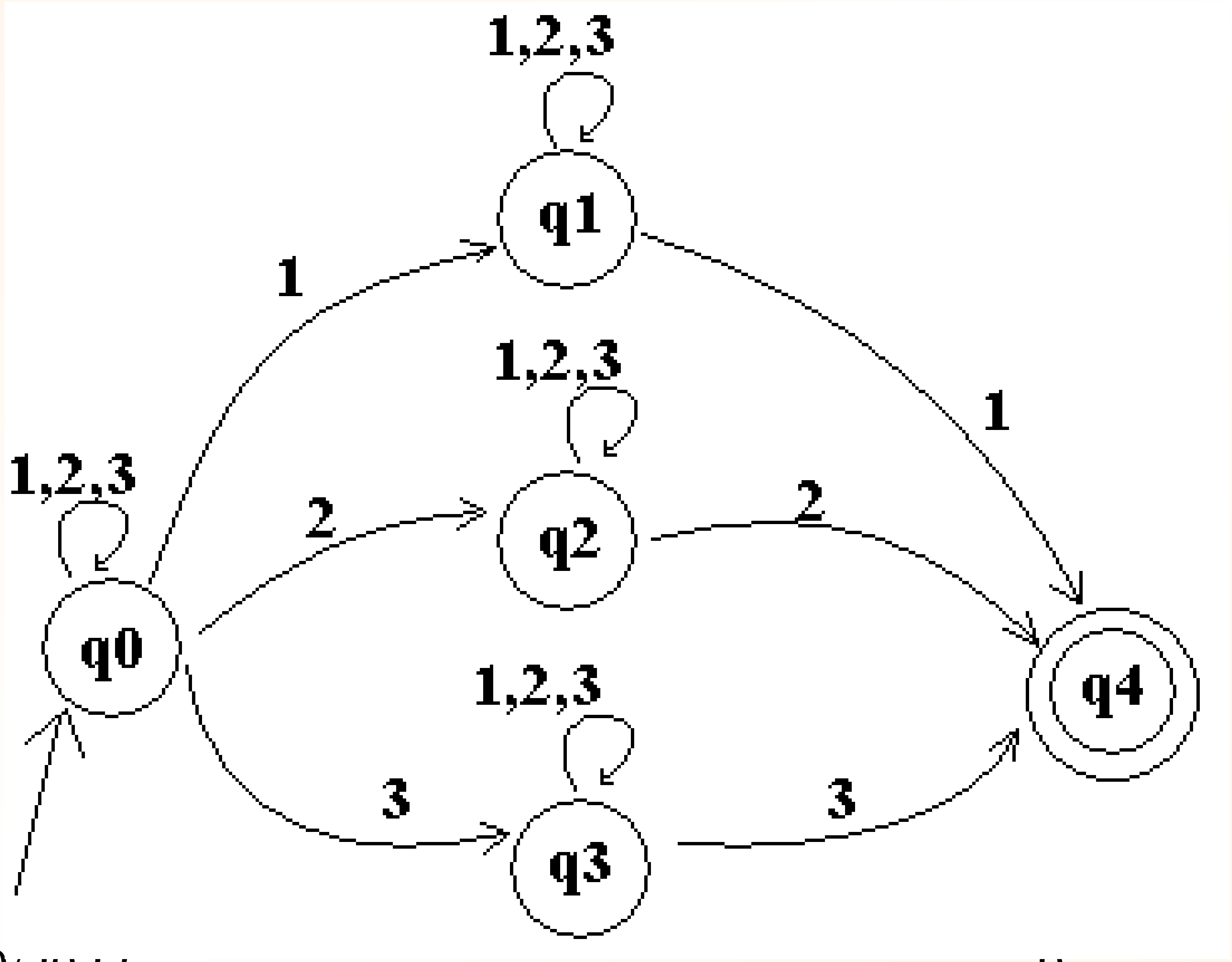
- Limbaj acceptat de automat

  $\mathbf{L(M) = \{w \mid w \in \Sigma^*, (q_0,w) \vdash^* (q_f,\varepsilon), q_f \in F\}}$

- Automate echivalente

  M_1 echivalent cu M_2 daca: $L(M_1) = L(M_2)$

Automat finit - exemplu



Determinism

- Automat finit determinist (AFD)

  $|\delta(q,a)| <= 1 \quad \forall\ q \in Q,\ a \in \Sigma$
- Automat finit nedeterminist (AFN)

  $\exists\ q \in Q,\ a \in \Sigma$ astfel incat $|\delta(q,a)| > 1$
- Automat finit determinist complet definit

  $|\delta(q,a)| = 1 \quad \forall\ q \in Q,\ a \in \Sigma$

Echivalenta dintre AFD si AFN

Teorema:

- $\forall\ M_1$ – AFN $\quad \exists\ M_2$ – AFD echivalent

Constructie (nu demonstratie!):

- Pornim cu: $M_1=(Q_1,\Sigma_1,\delta_1,q_{01},F_1)$ – AFN oarecare
- Construim: $M_2=(Q_2,\Sigma_2,\delta_2,q_{02},F_2)$ – AFD

  pe baza lui M_1

  a.i. $L(M_1) = L(M_2)$

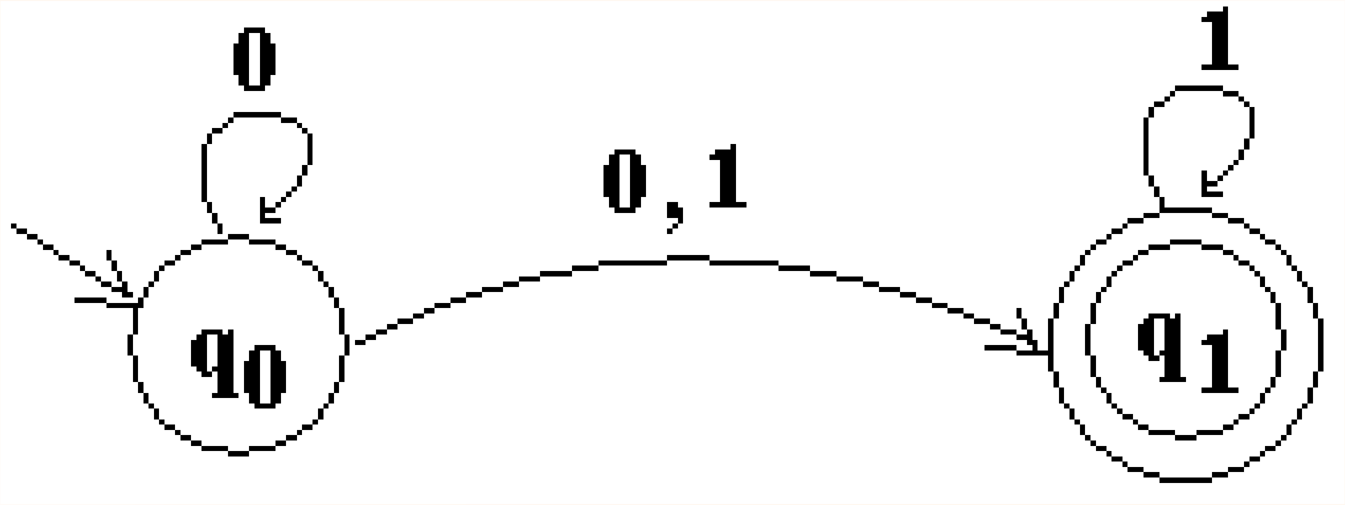
Teor: $\forall\ M_1$ – AFN $\quad \exists\ M_2$ – AFD echivalent

- $\Sigma_2 = \Sigma_1$
- $Q_2 = \mathcal{P}(Q_1)$
- $q_{02} = \{q_{01}\}$
- $F_2 = \{S \in \mathcal{P}(Q_1) \mid S \cap F_1 <> \Phi\}$
- $\delta_2(q,a) \quad = \{r \in Q_1 \mid \exists\ q_1 \in q \text{ a.i. } r \in \delta_1(q_1,a)\}$

$$= \bigcup_{q1\, \in\, q} \delta(q_1,a)$$

M_2 – determinist (?)

Problema: determinati AFD echiv. pt.



AF – stari care nu contribuie la acceptarea unui cuvant

- stare neproductiva – (nu e stare productiva)
- stare inaccesibila – (nu e stare accesibila)

- stare productiva: $q \in Q$ a.i.

  $\exists\, w \in \Sigma^*$ si $q_f \in F$ a.i. $(q,w) \vdash^* (q_f,\varepsilon)$
- stare accesibila: $q \in Q$ a.i.

  $\exists\, w \in \Sigma^*$ a.i. $(q_0,w) \vdash^* (q,\varepsilon)$

Algoritm determin. stari accesibile

1. $i := 0$

   $A_0 := \{q_0\}$

2. **Repeta**

   $i := i+1$

   $A_{i+1} := A_i \cup \{ q \in Q \mid \exists p \in A_i, \exists a \in \Sigma \text{ a.i. } q \in \delta(p,a)\}$

   pana cand $A_i = A_{i+1}$

$\{A_i$ – multimea starilor accesibile$\}$

Algoritm determin. stari productive

1. $i:=0$

   $A_0:=F$

2. **Repeta**

   $i:=i+1$

   $A_{i+1}=A_i \bigcup \{ q\in Q \mid \exists p\in A_i, \exists a\in\Sigma \text{ a.i. } p \in \delta(q,a)\}$

   pana cand $A_i=A_{i+1}$

$\{A_i$ – multimea starilor productive$\}$

Teorema:

$\forall\ M_1$ – AF exista M_2 – AF fara st. neproductive **echiv**.

Constructie (nu demonstratie!):

- Pornim cu: $M_1=(Q_1,\Sigma_1,\delta_1,q_{01},F_1)$ – AF oarecare
- determinam A – multimea starilor productive (algoritmul anterior)
- Construim: M_2 pe baza lui M_1 (a.i. $L(M_1) = L(M_2)$)

$$M_2=(A,\Sigma_1,\delta_{1/A},q_{01},F_1)$$

$$L(M_1) = L(M_2)\quad !$$

Teorema:

$\forall$ M_1 – AF exista M_2 – AF fara st. inaccesibile **echiv**.

Constructie (nu demonstratie!):

- … analog …

? alta metoda de determinare a AFD echivalent pentru un AFN dat

- M_1 => ? M_2

Ideea:

1. $\{q_{01}\} \in Q_2$
   pornim cu $Q_2=\{q_{01}\}$
2. adaugam la Q_2 toate submultimile lui Q_1 la care se poate ajunge prin functia de tranzitie, atunci cand se aplica unei stari $q \in Q_2$ deja adaugata

AFD complet definit

$M = (Q, \Sigma, \delta, q_0, F)$ **(AFD)**

- $\delta : Q x \Sigma \to \mathcal{P}(Q)$ functie de tranzitie ; $| \delta(q,a) | <= 1$
- ...

<u>Teor</u>: $\forall$ AFD $\exists$ AFD complet definit echivalent

Constructie:

AFD => AFD complet definit:

- adaugam o stare (neproductiva) *r* si extindem δ astfel:
- $\forall (q,a) \in Q x \Sigma$ a.i. $\delta(q,a)=\phi$ devine: $\delta(q,a)= \{\mathbf{r}\}$
- $\forall\ a \in \Sigma$ $\delta(r,a)= \{\mathbf{r}\}$

Minimizarea automatelor finite

Ce vrem:

Automat determinist cu numar minim de stari !

Automat redus

- AFD
- nu contine stari inaccesibile si neproductive
- nu contine perechi de **stari echivalente**

Minimizarea AFD

- automat cu numar minim de stari

  - fara stari –inaccesibile, neproductive

  - mai putine stari ?

  ideea: relatie de echivalenta; clase de echivalenta
- stari diferentiate
- stari k diferentiate
- stari echivalente
- stari k-echivalente

Automatul redus

Fie M1 – un automat finit oarecare

- Determinam AFD echivalent
- Eliminam starile inaccesibile si neproductive
- Determinam AFD echivalent complet definit

Fie $M = (Q, \Sigma, \delta, q_0, F)$ automatul rezultat.

- Determinam relatia $\equiv$ (stari echivalente, clase de echivalenta)
- Pe baza relatiei $\equiv$ determinam automatul:

  $M_{\equiv} = (Q/{\equiv}, \Sigma, \delta_{\equiv}, [q_0], F_{\equiv})$

  $Q/{\equiv}$ - multimea claselor de echivalenta

  $\delta_{\equiv}([q],a) = [\delta(q,a)]$

  $F_{\equiv} = \{[q] \mid q \in F\}$

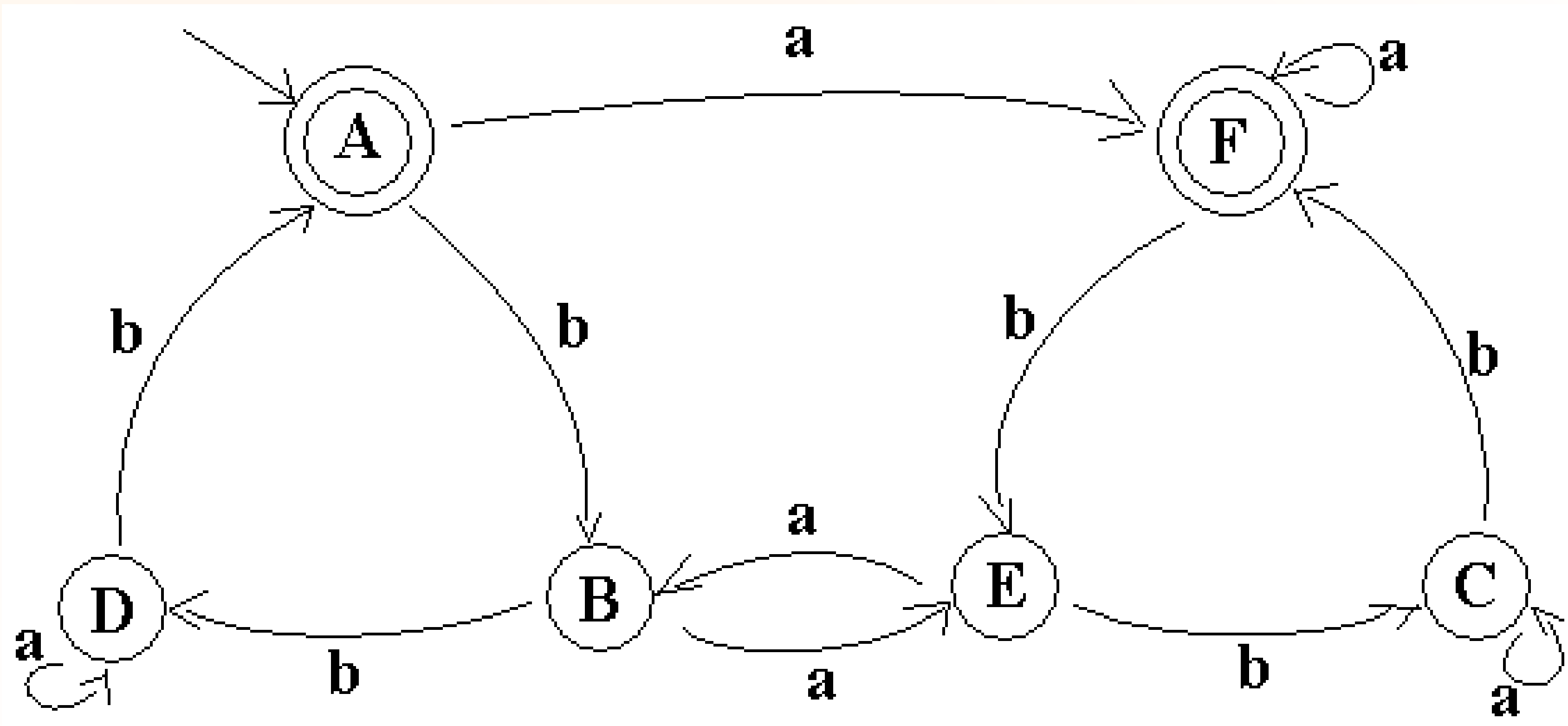
Automat redus

- **Teorema**

  Automatul redus are numar minim de stari dintre toate AFD echivalente
- **Teorema**

  $\forall$ M_1 – AF $\exists$ M2 – automat redus echivalent

Determinati clasele de echivalenta ale starilor automatului de mai jos



determinati automatul redus !

Stari diferentiate

- o alta exprimare

q_1,q_2 sunt *stari diferentiate* de $x \in \Sigma^*$

daca $\exists\ q_f \in F$ a.i. $(q_1,x) \vdash^* (q_f,\varepsilon)$

si nu exista nici un $q \in F$ a.i. $(q_2,x) \vdash^* (q,\varepsilon)$

sau

daca $\exists\ q_f \in F$ a.i. $(q_2,x) \vdash^* (q_f,\varepsilon)$

si nu exista nici un $q \in F$ a.i. $(q_1,x) \vdash^* (q,\varepsilon)$

- x (de mai sus) *diferentiaza* pe q_1 si q_2

Stari diferentiate

- o alta exprimare

PP. AFD complet definit

q_1,q_2 sunt *stari diferentiate* de $x \in \Sigma^*$

daca $\exists\ r_1, r_2$ astfel incat: $(q_1,x) \vdash^* (r_1,\varepsilon)$

si $(q_2,x) \vdash^* (r_2,\varepsilon)$

are loc una dintre:

1. $r_1 \in F$ si $r_2 \in Q\text{-}F$
2. $r_1 \in Q\text{-}F$ si $r_2 \in F$

Relatii intre stari

- q_1,q_2 - stari diferentiate
  – cf. def. de mai sus
  – ($\exists$ x $\in \Sigma^*$ care sa le diferentieze)
- stari k diferentiate
  – daca $\exists$ x $\in \Sigma^*$, $|x|\leq k$ care sa le diferentieze
- stari echivalente ($\equiv$)
  – daca nu exista x $\in \Sigma^*$ care sa le diferentieze
- stari k-echivalente ($\equiv^k$)
  – daca nu exista x $\in \Sigma^*$, $|x|\leq k$, care sa le diferentieze

Proprietati ale rel. de k-echivalenta ($\equiv^k$)

- $q_1 \equiv^0 q_2$ ddaca ($q_1, q_2 \in F$) sau ($q_1, q_2 \in Q\text{-}F$)

- $\equiv^0 \supseteq \equiv^1 \supseteq \equiv^2 \supseteq \cdots \supseteq \equiv^n \supseteq \cdots$
- Daca ($\equiv^k$) = ($\equiv^{k+1}$) atunci ($\equiv^k$) $= \equiv$

Lema:

Pt. orice M exista $n \in \mathbf{N}$ a.i. $q_1 \equiv^n q_2 \Rightarrow q_1 \equiv q_2$

- ideea: pot avea un nr. finit de relatii distincte (max. $|Q|$)