Exercise sheet Nr. 2

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(G 1) Line diagram

a) **Define:** What is a lattice?
A lattice is a graphical representation which is used to represent hierarchical relationships between formal concepts.

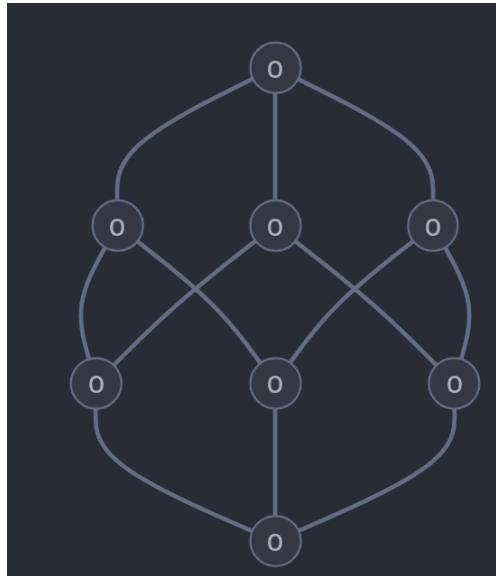b) Find a preferably small lattice and draw its diagram.

Figure 1: Lattice

c) Which of the following line diagrams (see Figure 2) does not represent a lattice? Why?
Diagram (v) lacks the supremum and infimum which are crucial characteristics of concept lattices.
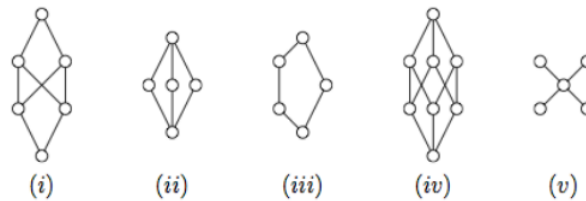

Figure 2: Order diagrams

There are no clear top and bottom elements, and the property that each pair of elements must have

a unique supremum and unique infimum isn't met. This can be seen if we take any outer node in consideration.

(G 2) Complete lattice

a) **Define:** What is a complete lattice?
A complete lattice is a lattice where every subset of elements (elements at the same level in the hierarchy) has a supremum and infimum.

b) Can you find a complete lattice among the lattices of Exercise 1c?
Diagrams (i) to (iii) are complete lattices. We can find a supremum and infimum for each subset of nodes at the same level

c) Can you give an example of a lattice which is not a complete lattice?
Diagram (iv) is an incomplete lattice. If we look at the second row from top to bottom, if we take the whole subset of nodes, there is no common infimum, and if we look at the third row from top to bottom, if we take the whole subset of nodes again, there is no common supremum.

(G 3) Formal Concepts
Try to compute all formal concepts of the formal context shown in Figure 3

	generation			sex		financial status		
	older	middle	younger	male	female	rich	carefree	indebted
Tick			×	×			×	
Trick			×	×			×	
Track			×	×			×	
Donald		×		×				×
Daisy		×			×		×	
Gustav		×		×			×	
Dagobert	×			×		×		
Annette	×				×		×	
Primus v. Quack	×			×			×	

Figure 3: Grobian Gans: Die Ducks. Pshycogramm einer Sippe. Rowohlt, Reinbek bei Hamburg 1972, ISBN 3-499-11481-X

- $(\{Tick, Trick, Track\}, \{younger, male, carefree\})$
- $(\{Donald\}, \{middle, male, indebted\})$
- $(\{Daisy\}, \{middle, female, carefree\})$
- $(\{Gustav\}, \{middle, male, rich\})$
- $(\{Dagobert\}, \{older, male, rich\})$
- $(\{Annette\}, \{older, female, carefree\})$
- $(\{Primus v.Quack\}, \{older, male, carefree\})$
- $(\{Tick, Trick, Track, Donald, Gustav, Dagobert, Primus v.Quack\}, \{male\})$
- $(\{Tick, Trick, Track, Daisy, Annette\}, \{carefree\})$
- $(\{Donald, Daisy, Gustav\}, \{middle\})$
- $(\{Dagobert, Annette, Primus v.Quack\}, \{older\})$

- $(\{Tick, Trick, Track\}, \{younger\})$
- $(\{Daisy, Annette\}, \{female\})$
- $(\{Dagobert\}, \{rich\})$
- $(\{Donald\}, \{indebted\})$
- $(\{Tick, Trick, Track, Primusv.Quack, Annette\}, \{carefree\})$
- $(\{\}, \{older, middle, younger, male, female, rich, carefree, indebted\})$
- $(\{Tick, Trick, Track, Donald, Daisy, Gustav, Dagobert, Annette, Primusv.Quack\}, \{\})$