Assignment 1 - due March 30th

Answer 1. bla bla bla Examples:

(a) We will prove that $U^*U = I$. We know U is diagonalizable, so U can be written as

$$U = \sum_{i=0}^{n} \lambda_i |v_i\rangle\langle v_i|$$

where all λ_i are eigenvalues and $|v_i\rangle$ are there respective eigenvectors. Then,

$$U^* = \sum_{i=0}^{n} \overline{\lambda_i} \langle v_i|v_i\rangle$$

Note that all eigenvectors are orthogonal, each means that, $\langle v_i|v_i\rangle|v_j\rangle\langle v_j| = 0$ when $i \neq j$, and $\langle v_i|v_i\rangle|v_j\rangle\langle v_j| = 1$ when $i = j$. So we have,

$$U^*U = \sum_{i=0}^{n} \overline{\lambda_i}\lambda_i$$

and by the hypothesis $\overline{\lambda_i}\lambda_i = |\lambda_i| = 1$. Therefore $U^*U = I$.

(b) Take λ a eigenvalue of U unitary, and $|v\rangle$ his associated eigenvector, so the equity bellow is true,

$$U|v\rangle = \lambda|v\rangle$$

Note that, $= \overline{\lambda}\langle v|$. Then, when we multiply the first equation for $\langle v|U^*$ on the left, we get

$$\langle v|U^*U|v\rangle = \langle v|U^*\lambda|v\rangle$$
$$\langle v|v\rangle = \langle v|\overline{\lambda}\lambda|v\rangle$$
$$\langle v|v\rangle = \overline{\lambda}\lambda\langle v|v\rangle$$

By the property of the inner product, we have that $\langle v|v\rangle \neq 0$, so we can divide the last equation on both sides for $\langle v|v\rangle$. And then,

$$1 = \overline{\lambda}\lambda = |\lambda|$$

(c) $\langle u|M|v\rangle$

Answer 2. bla bla bla

Answer 3. bla bla bla

Answer 4. bla bla bla

Answer 5. bla bla bla

Answer 6. bla bla bla

Answer 7. bla bla bla