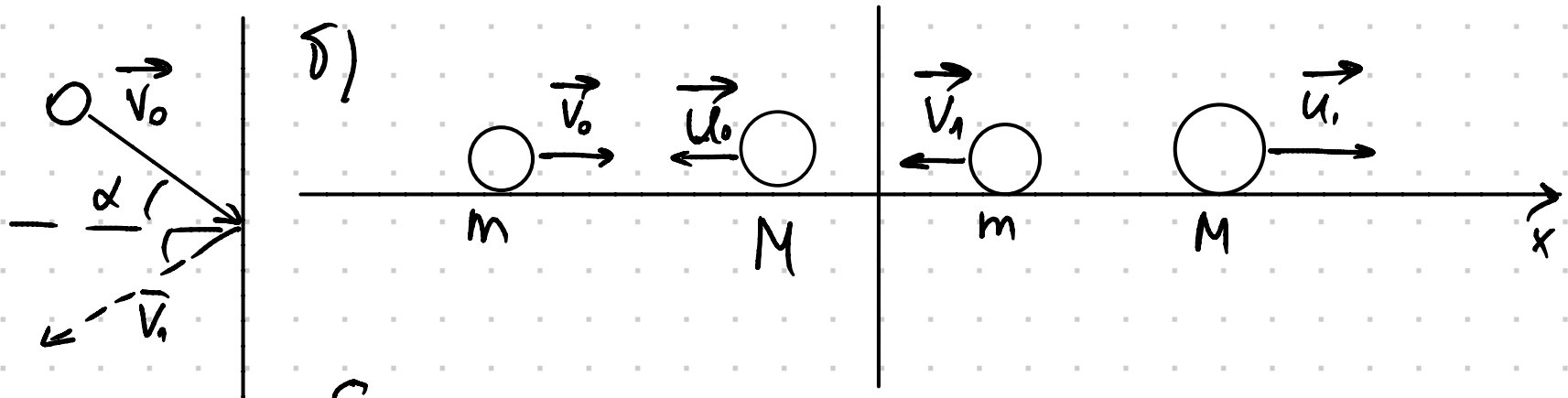
M2 m, V – шар

а) об стенку (α)

б) об шар(M, u_0)

1) Абсолютно упругий удар



$$\begin{cases} V_{0x} = -V_{1x} \\ V_{0y} = V_{1y} \end{cases}$$

$$Ox: \begin{cases} mV_0 - Mu_0 = Mu_1 - mV_1 \\ \frac{mV_0^2}{2} + \frac{Mu_0^2}{2} = \frac{Mu_1^2}{2} + \frac{mV_1^2}{2} \end{cases} \Rightarrow$$

$mV_0 - Mu_0 = Mu_1 - mV_1 \Rightarrow m(V_0+V_1) = M(u_1+u_0)$

$mV_0^2 - mV_1^2 = Mu_1^2 - Mu_0^2$

$m(V_0-V_1)(V_0+V_1) = M(u_1-u_0)(u_1+u_0)$

$V_0 - V_1 = u_1 - u_0$

$u_1 = (V_0+u_0) - V_1$

$mV_0 - Mu_0 = M(V_0+u_0) - mV_1 - MV_1$

$$\boxed{V_1 = \frac{(M-m)V_0 + 2Mu_0}{m+M}}$$

$$\boxed{u_1 = \frac{mV_0 + MV_0 + Mu_0 + Mu_0 - (M-m)V_0 - 2Mu_0}{m+M} = \frac{(M-m)u_0 + 2mV_0}{m+M}}$$

2) $F \sim -\Delta X \qquad F = -k\Delta X$

a) Удар об стенку $\qquad m\ddot{x} + kx = 0$

$x(t) = \frac{V_0}{\omega}\sin(\omega t) \qquad \omega = \sqrt{\frac{k}{m}}$

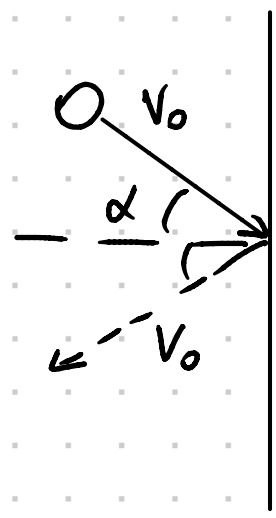
$X_{max} = V\sqrt{\frac{m}{k}}$

$F(t) = -kx(t) = -k\frac{V}{\omega}\sin(\omega t)$

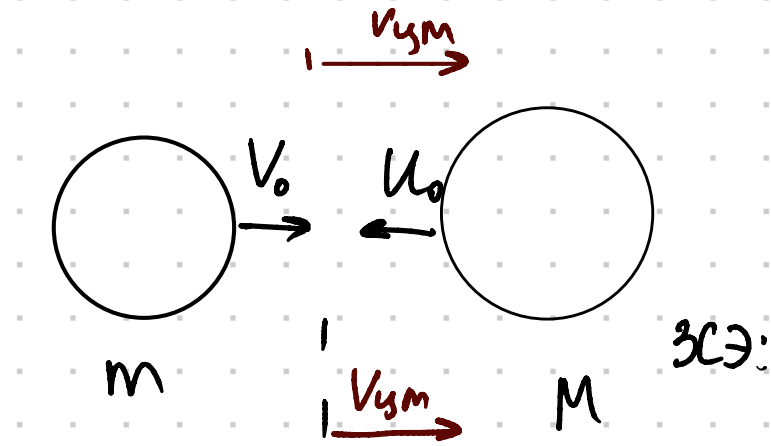
$t_c = \frac{\pi}{\omega} = \pi\sqrt{\frac{m}{k}} \qquad J = \int_0^{t_c} F(t)dt = -2mV_0 \Rightarrow$

$V_1 = V_0 + \frac{J}{m} = V_0 - 2V_0 = -V_0$

$\Rightarrow$ Аналогично ситуации в 1.а)



б)



сохранение скорости ц.м. $V_{цм0} = V_{цм1}$

ЗСЭ: $\frac{mV_0^2}{2} + \frac{MU_0^2}{2} = E_{деф} + \frac{(m+M)V_{цм}^2}{2}$

$V_{цм_x} = \frac{mV_0 - MU_0}{m+M}$

$E_{деф} = \int_0^{x_{max}} F_{упр} dx = \int_0^{x_{max}} kx dx = \frac{kx_{max}^2}{2}$

$mV_0^2 + MU_0^2 = kx_{max}^2 + \frac{(mV_0 - MU_0)^2}{(m+M)}$

$x_{max}^2 = \frac{(m+M)(mV_0^2 + MU_0^2) - (mV_0 - MU_0)^2}{k(m+M)} =$

$= \frac{mMU_0^2 + MmV_0^2 + 2mV_0MU_0}{2k(m+M)} = \frac{Mm}{m+M}\frac{(U_0+V_0)^2}{k}$

$x_{max} = \sqrt{\frac{Mm}{m+M}}\sqrt{\frac{1}{k}}(U_0+V_0)$

2ой закон Ньютона: $\begin{cases} m\ddot{x}_1 = -F \\ M\ddot{x}_2 = F \end{cases}$ $x = x_1 - x_2$

$\ddot{x} = \ddot{x}_1 - \ddot{x}_2 = -\frac{F}{m_1} - \frac{F}{m_2} = -F\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$

$\Rightarrow \mu\ddot{x} = -F_{упр}$ $\mu = \frac{mM}{m+M}$ приведенная масса

$\mu\ddot{x} = -kx$

$\mu\ddot{x} + kx = 0$ $\quad x(t) = \frac{V_{отн}}{\omega}\sin(\omega t)$ $\quad V_{отн} = V_0 + U_0$

$\omega = \sqrt{\frac{k}{\mu}}$

$x_{max} = \frac{V_0+U_0}{\omega} = \sqrt{\frac{Mm}{m+M}}\sqrt{\frac{1}{k}}(U_0+V_0)$

$t_c = \frac{\pi}{\omega} = \pi\sqrt{\frac{\mu}{k}}$ (время столкновения)

$$J = \int_0^{t_c} F_{упр}\,dt = \int_0^{t_c} kx\,dt = \int_0^{\frac{\pi}{\omega}} -k\,\frac{v_{0mH}}{\omega}\sin(\omega t) = -k\,\frac{v_0+u_0}{\omega}\int_0^{\frac{\pi}{\omega}} \sin(\omega t)\,dt =$$

$$= -\frac{k(v_0+u_0)}{\omega^2}\,2 = -2\mu(v_0+u_0)$$

$$v_1 = v_0 + \frac{J}{m} = v_0 - \frac{2\mu(v_0+u_0)}{m}$$

$$u_1 = u_0 - \frac{J}{M} = u_0 + \frac{2\mu(v_0+u_0)}{M}$$