Process Capability Measures and Their Estimation

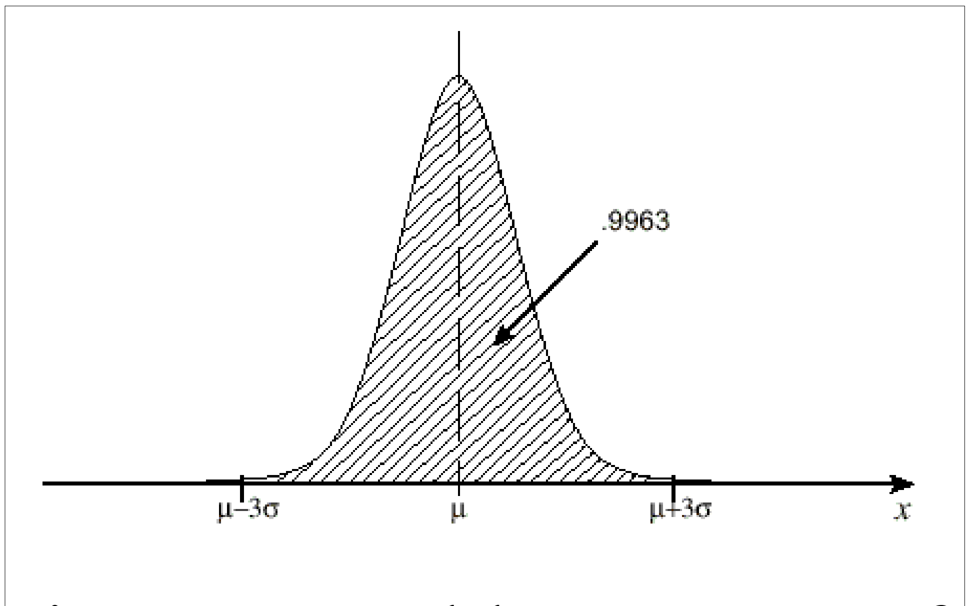
(Section 5.2 of Vardeman and Jobe)

“Capability” Measures

- Where a process is *stable and normal,* what it will do can be described by two numbers, m and s
- Where in addition, there are engineering specifications, L and U, (i.e. I want $L<x<U$ in order to have product functionality) there is sometimes pressure to invent further one-number summaries

"Process Capability"

- Most of a normal distribution is within 3 standard deviations of its mean



- So $6s$ is a reasonable measure of (normal) process capability

Process Capability Ratio (C_p)

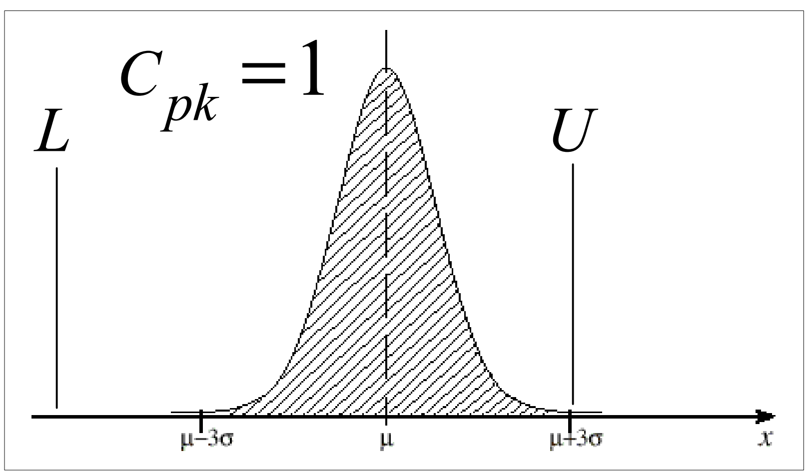
- With (two-sided) specifications, it is perhaps sensible to compare their spread to $6s$ and to create a measure of "process potential" to meet them, $C_p = \dfrac{U - L}{6s}$
  - $C_p = 1$ means that (with perfect aim!) most of process output will meet spec.s
  - $C_p < 1$ means that (even with perfect aim) a substantial part of process output will be out of spec.
  - $C_p > 1$ means there is some room for mis-aim

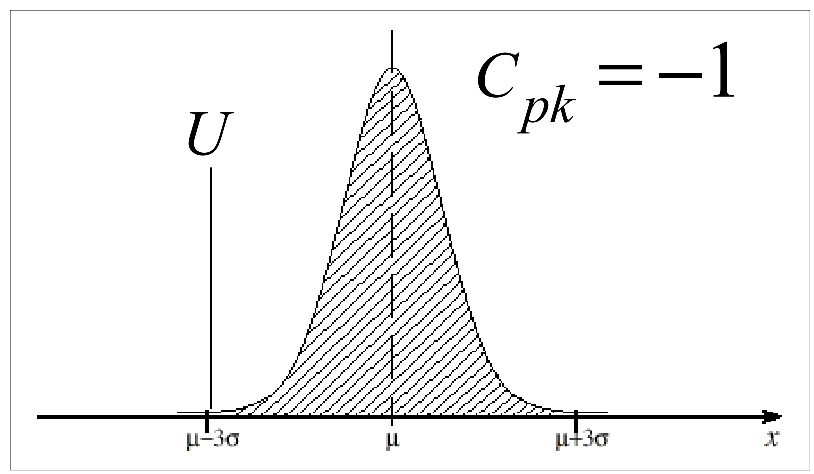
C_{pk}

- This is a measure, not of process potential, but of current performance (it involves m)

$$C_{pk} = \min\left\{\frac{U-m}{3s}, \frac{m-L}{3s}\right\}$$

- This is “the number of $3s$'s that m is to the good side of the nearest spec.”



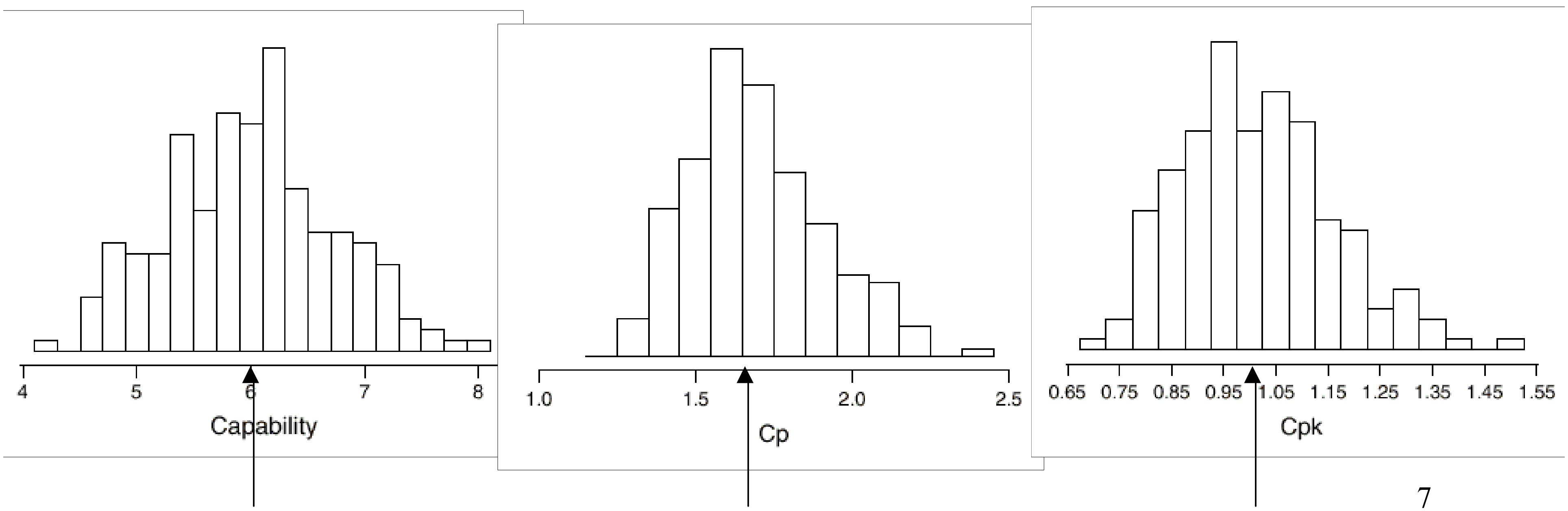


How to Evaluate These?

- Clearly it must be done on the basis of process data … point (single number) estimates are:
  - *6s* for process capability
  - $\hat{C}_p = \dfrac{U-L}{6s}$
  - $\hat{C}_{pk} = \min\left\{\dfrac{U-\bar{x}}{3s}, \dfrac{\bar{x}-L}{3s}\right\}$
- We had better keep in mind that $\bar{x}$ and s vary

Simulated Example

- Suppose *L=0*, *U=10*, $m = 7$ and $s = 1$ so that $6s = 6, C_p = 1.67$ and $C_{pk} = 1.00$
- (*200*) samples of size ***n=30*** produce estimates:



"Confidence Limits" Take Account of This Variation

- Limits on $6s$ are (display (5.5))

$$6s\sqrt{\frac{n-1}{c^2_{\text{upper}}}} \quad \text{and/or} \quad 6s\sqrt{\frac{n-1}{c^2_{\text{lower}}}}$$

(the c^2 quantiles come from Table A.7 or a statistical package)

- Corresponding limits on C_p are

$$\frac{U-L}{6s}\sqrt{\frac{c^2_{\text{lower}}}{n-1}} \quad \text{and/or} \quad \frac{U-L}{6s}\sqrt{\frac{c^2_{\text{upper}}}{n-1}}$$

More Confidence Limits

- An approximate lower confidence limit on C_{pk} is (display (5.10))

$$\hat{C}_{pk} - z\sqrt{\frac{1}{9n} + \frac{\hat{C}_{pk}^2}{2n-2}}$$

(actually, an upper limit can also be had by replacing the minus sign with a plus above)

  – e.g. use $z=1.645$ to be approximately 95% sure that the real C_{pk} is above the limit

Example 5.5

- Angles of $n=50$ holes drilled by EDM had $\bar{x} = 44.117$ and $s = .983$
- Spec.s were 45 ± 2 so
  - $6s=5.898$
  - $\hat{C}_p = \dfrac{4}{5.898} = .678$
  - $\hat{C}_{pk} = .38$

    but how good are these values?
- Find confidence limits

Example 5.5 continued

- Minitab was used to find c^2 quantiles for $n = n - 1 = 49$ d.f. and then 90% limits for $6s$ are (from display (5.5))

$$6(.983)\sqrt{\frac{50-1}{66.34}} \text{ and } 6(.983)\sqrt{\frac{50-1}{33.93}}\text{, i.e.}$$

$$5.07 \text{ and } 7.09$$

- Corresponding limits for C_p are

$$\frac{4}{7.09} \text{ and } \frac{4}{5.07}\text{, i.e. } .56 \text{ and } .79$$

Example 5.5 continued

- A 95% lower confidence bound for C_{pk} is

$$.38 - 1.645\sqrt{\frac{1}{9(50)} + \frac{(.38)^2}{2(50)-2}} = .28$$

- Notice that even a sample size of *n=50* leaves a fair amount of uncertainty in the estimation of capability figures

Workshop Exercise

- For $L=0$, $U=10$, $\boldsymbol{n=30}$, find $6s$, $\hat{C}_p$ and $\hat{C}_{pk}$ for one of the simulated $(\bar{x}, s)$ pairs below (that Vardeman generated in the making of the figures on slide 7)

7.30687	1.09556
7.05134	0.98880
7.10610	1.01570
6.78466	0.97536
6.94911	1.04932
7.20956	1.07948
7.02438	1.00874
6.58039	1.02006
7.15414	0.99741
6.89519	1.03046

Workshop Exercise

- For your $(\bar{x}, s)$ pair from the previous slide, plug into limits

$$6s\sqrt{\frac{30-1}{42.557}} \text{ and } 6s\sqrt{\frac{30-1}{17.708}}$$

  to get a 90% confidence interval for $6s$
- Find corresponding limits for C_p
- For your $(\bar{x}, s)$ pair, find a 95% lower confidence limit for C_{pk}