Process Capability raising standards worldwide™

Learning Objectives

At the end of this section delegates will be able to:

- Recognise the difference between process control and process capability
- Carry out capability studies on variable and attribute data
- Assess and interpret the data from capability studies
- Convert data from capability studies into DPM or Cpk values

1

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Process Capability Agenda

- · Process Control v Process Capability
- Process Capability Variable Data
- Capability Study 7 steps
- · Capability Study Minitab
- Workshop
- · Capability Study Issues
- Process Capability Non-normal Data
- Summary

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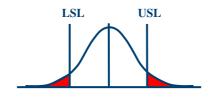
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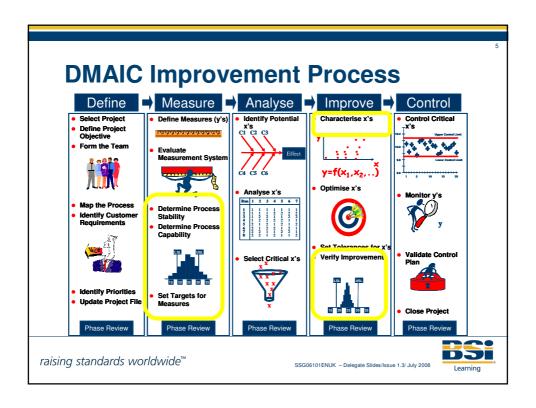
Process Control

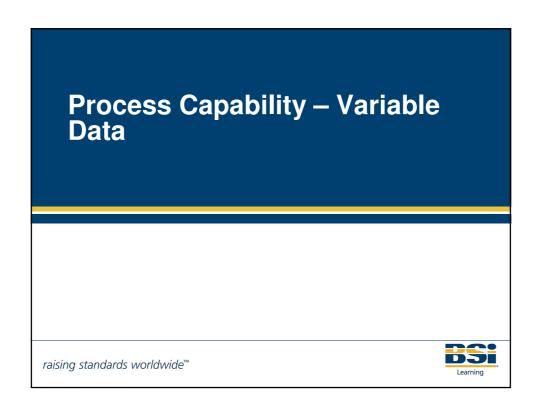
- Process Control refers to the evaluation of process stability over time
- Process Capability refers to the evaluation of how well a process meets specifications

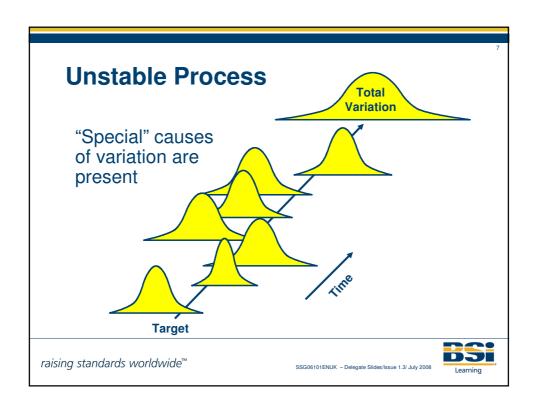
0 5 10 15 20 25 **Time**

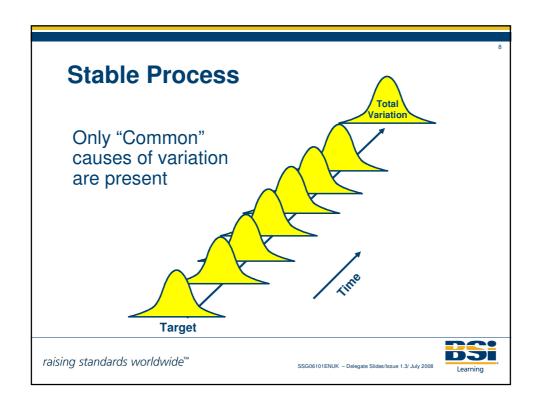


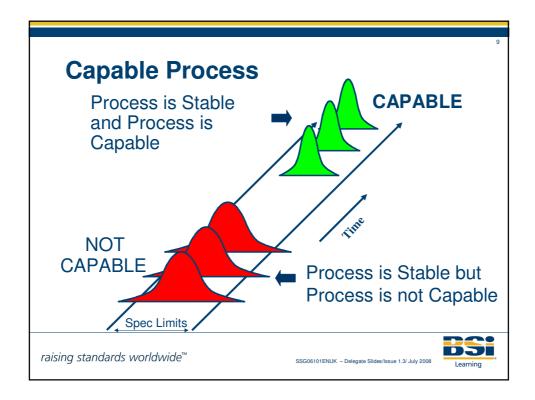
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Process Capability, C_p Index

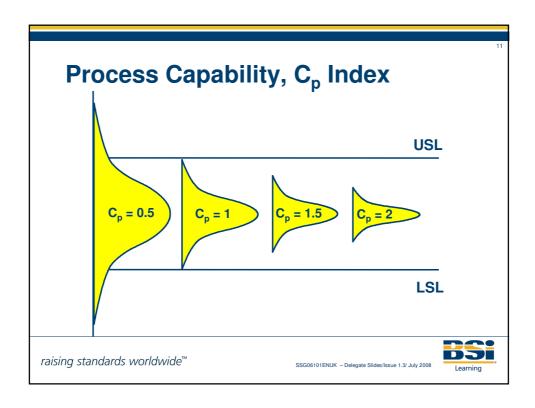
· Defined as:

$$C_{p} = \frac{USL - LSL}{6\sigma_{Within}} \approx \frac{USL - LSL}{6\left(\overline{R}/d_{2}\right)}$$

- USL-LSL represents the width of the spec
- $6\sigma_{Within}$ represents width of process ($\pm 3\sigma$)

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Process Capability, C_{pk} Index

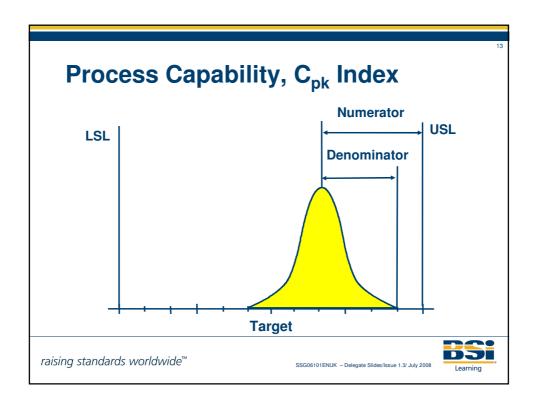
Defined as:

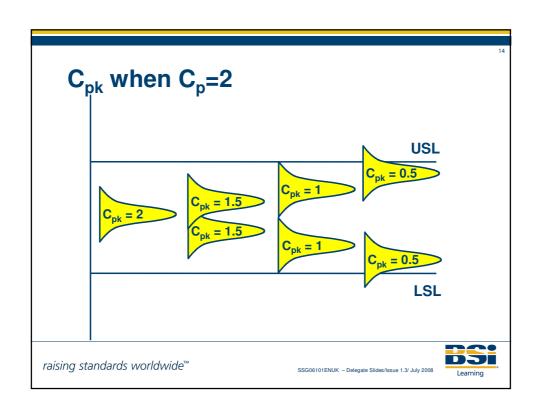
$$C_{pk} = \frac{\min(USL - \overline{\overline{X}}, \overline{\overline{X}} - LSL)}{3\sigma_{Within}} \qquad \sigma_{Within} \approx \frac{\overline{R}}{d_2}$$

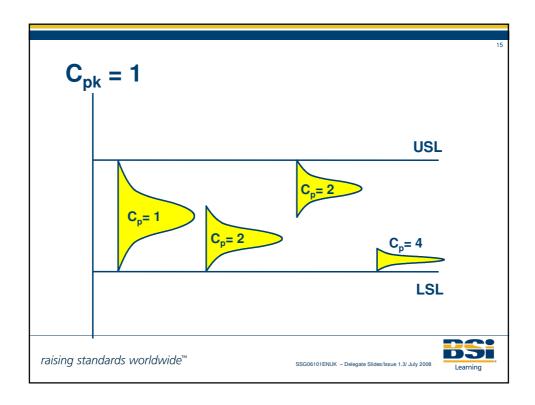
- Numerator represents the distance to the nearest spec
- Denominator represents the amount of this distance consumed by variation

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\mathbf{C}_{pk} and \mathbf{C}_{p}

- $C_{pk} \le C_{p}$
- $C_{pk} = C_p$ only when the process is perfectly centred
- C_p represents the highest possible value for C_{pk}

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One Spec Limit

- C_p is not defined requires 2 spec limits
- C_{pk} can still be calculated requires only one spec limit

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Six Sigma Quality - Measurable Characteristics

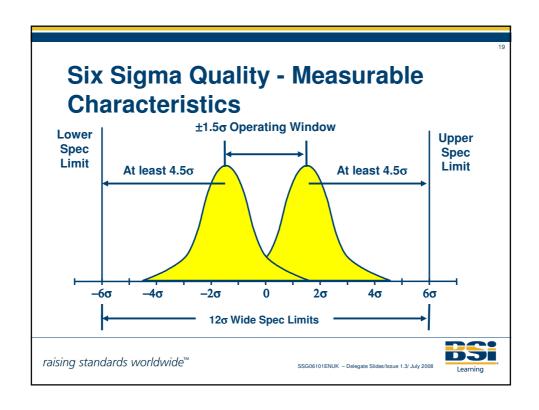
- Requires no more than 3.4 defects per million for each measurable characteristic
- Producing no more than 3.4 defects per million requires keeping the process average at least 4.5σ from the nearest spec

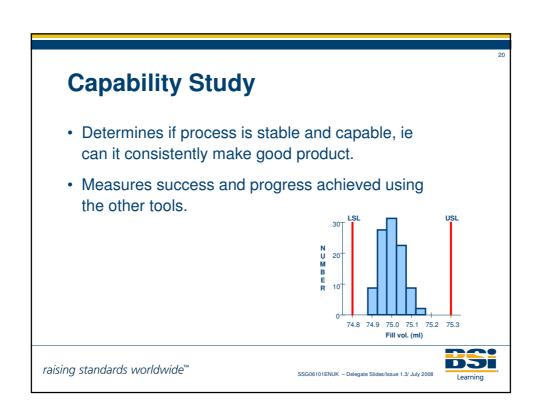
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18





Capability Study

- 1. Collect data
- 2. Calculate required statistics
- 3. Draw charts
- 4. Plot data
- 5. Interpret charts and draw conclusions
- 6. Draw histogram
- 7. Calculate C_p and C_{pk} and draw conclusions

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(1) Collect Data

The data below refers to a filling operation (target fill = 75.0 ml)

1	2	3	4	5	6	7	8	9	10
75.02	75.03	75.03	74.98	75.03	75.03	75.02	75.03	75.05	75.02
75.03	75.06	75.05	75.01	75.02	75.06	74.99	74.98	75.00	75.01
75.05	75.06	75.03	75.03	75.03	75.01	75.02	75.03	75.03	75.00
75.01	75.02	75.04	75.04	75.04	75.04	75.04	75.04	75.01	75.07
75.02	75.09	75.06	75.02	75.02	75.00	75.07	75.05	75.07	75.00

11	12	13	14	15	16	17	18	19	20
75.04	75.08	75.00	75.07	75.02	75.04	75.05	75.05	75.04	75.06
75.05	74.99	75.04	75.05	75.01	75.02	75.03	75.04	75.02	75.04
74.97	75.02	75.01	75.06	75.04	75.04	75.06	75.03	75.04	75.02
75.00	75.05	75.02	75.03	75.03	75.06	75.03	75.01	75.01	75.05
75.01	74.99	75.04	75.01	75.02	75.05	75.08	74.99	75.03	75.00

 Collect 20 subgroups of 5 data points each over several hours, where the 5 data points in each subgroup are collected at the same time

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(1) Collect Data

- Most capability studies are carried out over short time periods
- Typically, high volume production should be sampled at least hourly
- Lower volume production should be adjusted accordingly a rule of thumb would be one subgroup of 5 every 50 components
- Practical considerations may dictate the time period selected, but subgroups should be taken at regular intervals
- Consider extending the time period to cover process changes (eg 2nd shift, material changes etc)

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(2) Calculate Required Statistics

	1	2	3	4	5	6	7	8	9	10
	75.02	75.03	75.03	74.98	75.03	75.03	75.02	75.03	75.05	75.02
	75.03	75.06	75.05	75.01	75.02	75.06	74.99	74.98	75.00	75.01
	75.05	75.06	75.03	75.03	75.03	75.01	75.02	75.03	75.03	75.00
	75.01	75.02	75.04	75.04	75.04	75.04	75.04	75.04	75.01	75.07
	75.02	75.09	75.06	75.02	75.02	75.00	75.07	75.05	75.07	75.00
average	75.026	75.052	75.042	75.016	75.028	75.028	75.028	75.026	75.032	75.020
range	0.04	0.07	0.03	0.06	0.02	0.06	0.08	0.07	0.07	0.07

			.0	1.7		.0		.0	.0	
	75.04	75.08	75.00	75.07	75.02	75.04	75.05	75.05	75.04	75.06
	75.05	74.99	75.04	75.05	75.01	75.02	75.03	75.04	75.02	75.04
	74.97	75.02	75.01	75.06	75.04	75.04	75.06	75.03	75.04	75.02
	75.00	75.05	75.02	75.03	75.03	75.06	75.03	75.01	75.01	75.05
	75.01	74.99	75.04	75.01	75.02	75.05	75.08	74.99	75.03	75.00
average	75.014	75.026	75.022	75.044	75.024	75.042	75.050	75.024	75.028	75.034
range	0.08	0.09	0.04	0.06	0.03	0.04	0.05	0.06	0.03	0.06

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12

(2) Calculate Required Statistics

· Controls Limits are always:

Average ± 3 Standard Deviations

Where the average and standard deviation are the average and standard deviation of whatever is plotted: subgroup averages, subgroup ranges, ...

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(2) Calculate Required Statistics

The control limits can be calculated directly as follows:
 UCL = X + A₂R

$$\frac{1}{2}$$
 $\frac{1}{2}$ $\frac{1}{2}$

$$LCL_{\bar{x}} = \overline{\overline{X}} - A_2 \overline{R}$$

$$UCL_{\overline{R}} = D_4 \overline{R}$$

$$LCL_{\overline{R}} = D_3 \overline{R}$$

- · Using these constants simplifies the calculations
- Minitab allows us to choose whether to use R-Bar or the within subgroup variation

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(2) Calculate Required Statistics

Sample size	A 2	D 3	D 4	d ₂
2	1.880	0	3.267	1.128
3	1.023	0	2.574	1.693
4	0.729	0	2.282	2.059
5	0.577	0	2.114	2.326
6	0.483	0	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078

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(2) Calculate Required Statistics

 The upper and lower control limits for the average & range are:

$$\overline{X} = 75.0303, \quad \overline{R} = 0.0555$$

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_{2}\overline{R} = \overline{\overline{X}} + 0.577\overline{R} = 75.0303 + (0.577 \times 0.0555) = 75.06$$

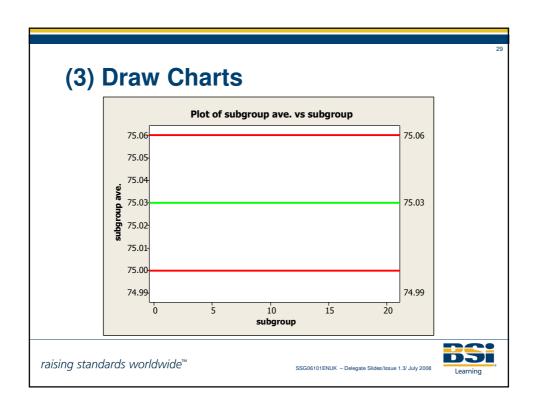
$$LCL_{\overline{X}} = \overline{\overline{X}} - A_{2}\overline{R} = \overline{\overline{X}} - 0.577\overline{R} = 75.0303 - (0.577 \times 0.0555) = 75.00$$

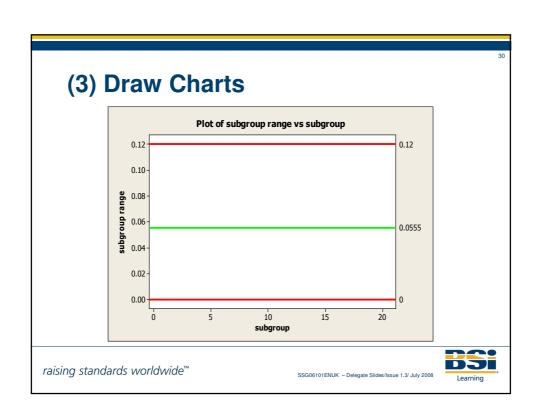
$$UCL_{\overline{R}} = D_4 \overline{R} = 2.114 \text{ x } 0.0555 = 0.12$$

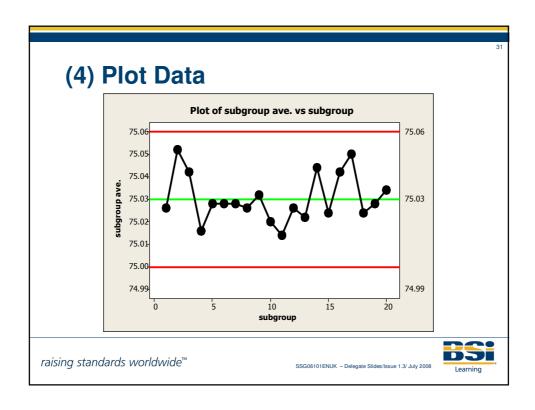
 $LCL_{\overline{R}} = D_3 \overline{R} = 0$

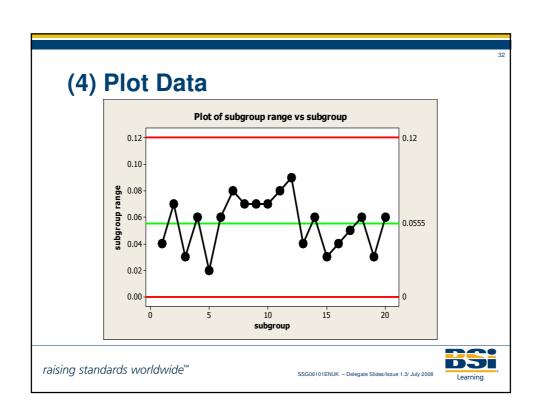
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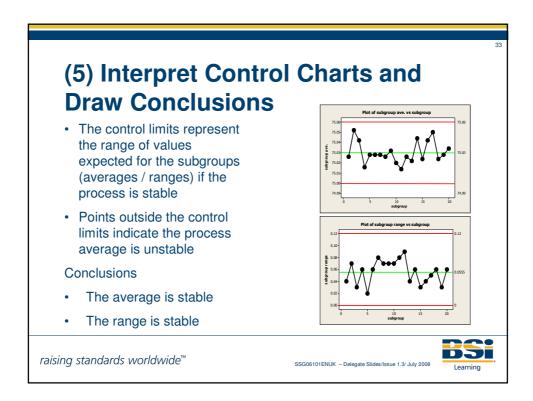


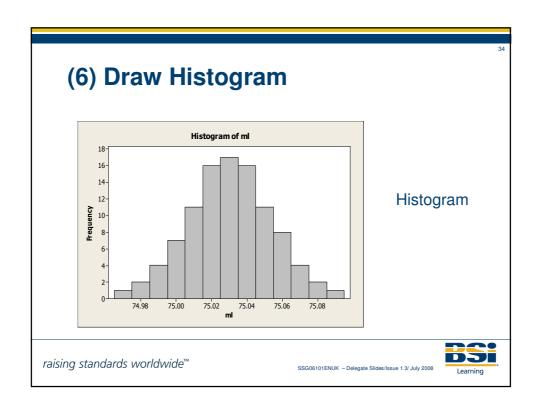


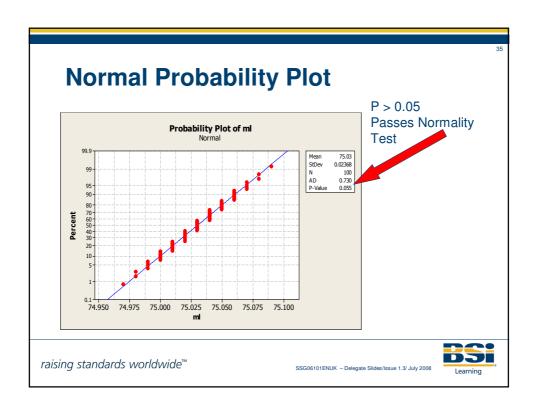












Reasons for Failing a Normality Test

- · A shift occurred in the middle of the data
- · Mixed populations
- · Truncated data
- · Rounding to a small number of values
- Outliers
- Too much data
- · The underlying distribution is not normal
- With this data set, we have just passed the test for normality.
- · What do you think are the reasons for this situation?

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(7) Calculate C_p and C_{pk} and Draw Conclusions

• C_p is defined as:

$$C_{p} = \frac{USL - LSL}{6\sigma_{Within}} \approx \frac{USL - LSL}{6\left(\overline{R}/d_{2}\right)}$$

- · USL-LSL represents the width of the spec
- $6\sigma_{Within}$ represents width of process (±3 σ)

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(7) Calculate C_p and C_{pk} and Draw Conclusions

• C_p is calculated as follows:

$$C_p = \frac{\text{USL-LSL}}{6\left(\frac{\overline{R}}{d_2}\right)} = \frac{75.1 - 74.9}{6\left(\frac{0.0555}{2.326}\right)} = 1.40$$

- C_p is a measure of whether the variation is acceptable
- A value ≥ 1.5 is often acceptable

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(7) Calculate C_p and C_{pk} and Draw Conclusions

• C_{pk} is defined as:

$$C_{pk} = \frac{\min(USL - \overline{X}, \overline{X} - LSL)}{3\sigma_{Within}} \quad \sigma_{Within} \approx \frac{\overline{R}}{d_2}$$

- Numerator represents the distance to the nearest spec
- Denominator represents the amount of this distance consumed by variation

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(7) Calculate C_p and C_{pk} and Draw Conclusions

· Cpk is calculated as follows:

$$C_{pk} = \frac{\min(USL - \overline{X}, \overline{X} - LSL)}{3\left(\frac{\overline{R}}{d_2}\right)}$$

$$C_{pk} = \frac{\min(75.1 - 75.0303, 75.0303 - 74.9)}{3\left(\frac{0.0555}{2.326}\right)}$$

$$C_{pk} = \min(0.97, 1.82) = 0.97$$

- $C_{\mbox{\scriptsize pk}}$ jointly measures centring and variation
- A value ≥ 1.5 is required for Six Sigma

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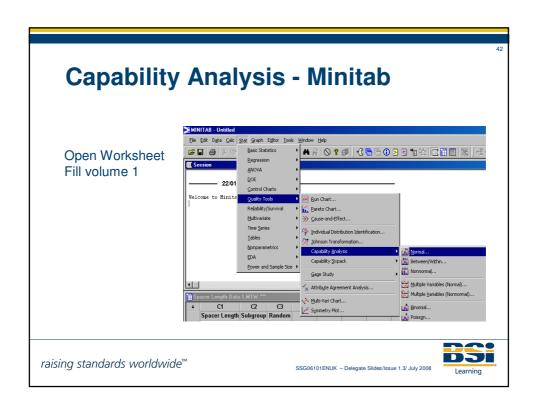


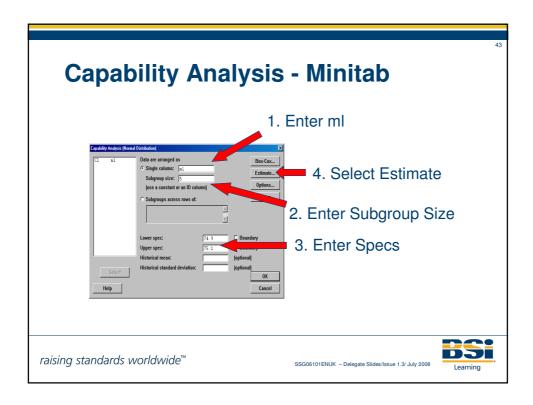
Final Conclusion

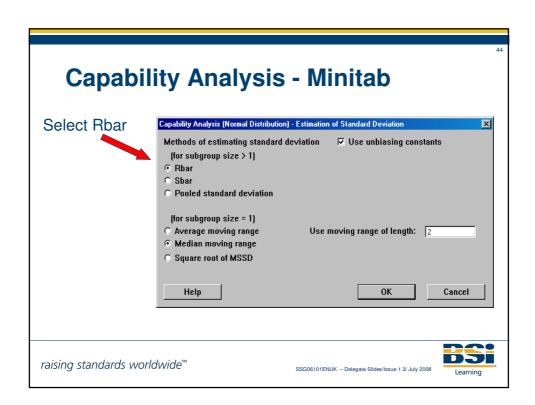
- · Both the average and variation are stable
- ullet Centring the process will improve $C_{\rm pk}$
- Variation must be reduced in order to achieve a C_{pk} of ≥ 1.5

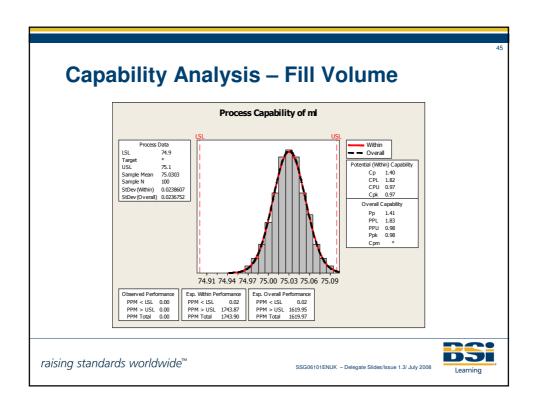
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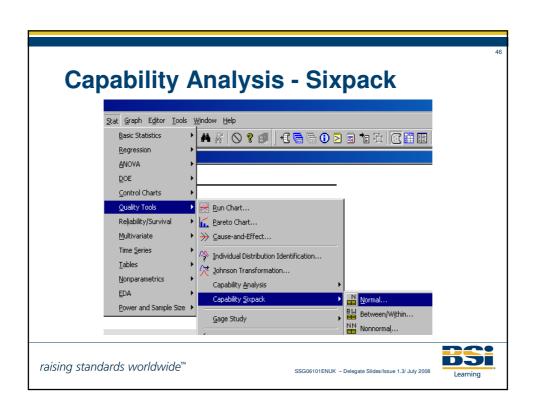


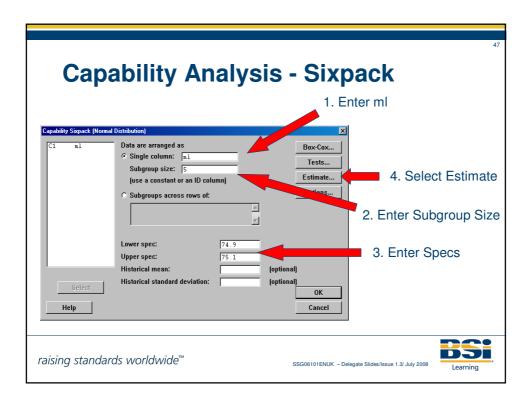


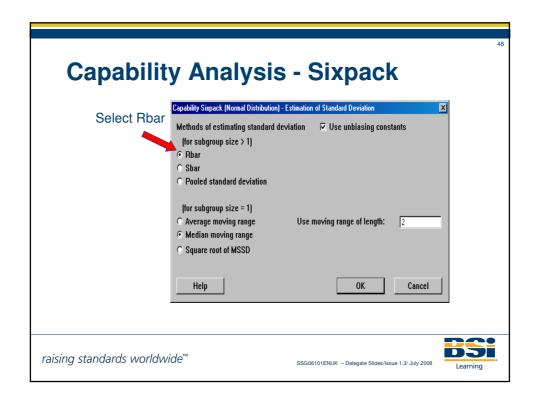


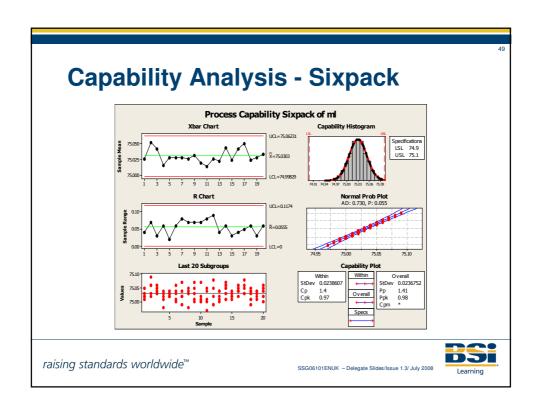










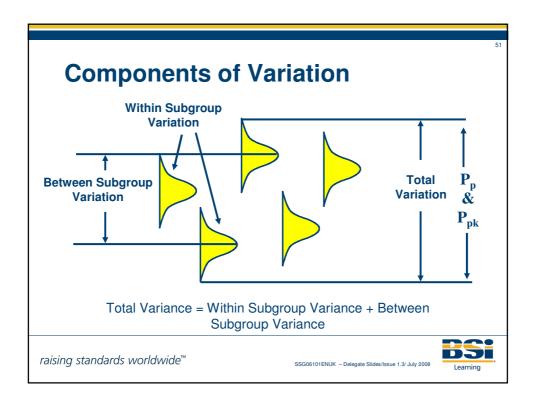


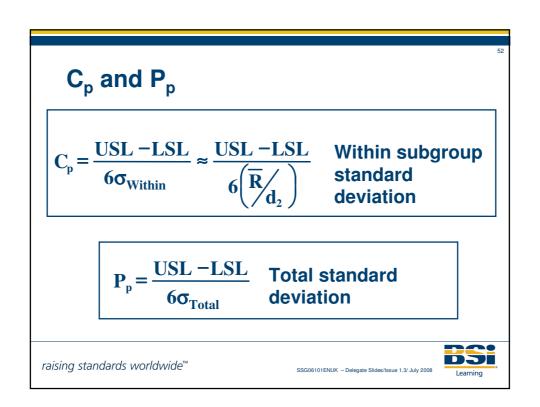
$\boldsymbol{P}_{\boldsymbol{p}}$ and $\boldsymbol{P}_{\boldsymbol{p}\boldsymbol{k}}$

- Minitab, and other statistical packages provide additional measures of process capability called $P_{\rm p}$ and $P_{\rm pk}$
- These measures are the same as C_p and C_{pk} except that P_p and P_{pk} always use the total standard deviation rather than variation within subgroups

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 \mathbf{C}_{pk} and \mathbf{P}_{pk}

$$C_{pk} = \frac{\min(USL - \overline{\overline{X}}, \overline{\overline{X}} - LSL)}{3\sigma_{Within}} \sigma_{Within} \approx \frac{\overline{R}}{d_2}$$

Within subgroup standard deviation

$$P_{pk} = \frac{\min(USL - \overline{\overline{X}}, \overline{\overline{X}} - LSL)}{3\sigma_{Total}}$$
Total standard deviation

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Why measure P_p and P_{pk}?

- If we calculate C_p and C_{pk} using within subgroup variation, we are assessing "best case" performance
- · If the process is stable, then this is generally OK
- If our process is unstable, P_p and P_{pk} provide a more realistic assessment of process capability, because they are calculated using total variation
- However, if the process is unstable, our priority should be to identify and eliminate the factor causing the instability (special cause)

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Behaviour of P_p and P_{pk}

- $P_p < C_p$ until process is stable and under control, then $P_p = C_p$
- $P_{pk} < C_{pk}$ until process is stable and under control, then $P_{pk} = C_{pk}$

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Process Capability Metrics

Metric	Centring?	Stability?
C _p	No	No
C _{pk}	Yes	No
Pp	No	Yes
P _{pk}	Yes	Yes

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57

Components of Variation

- We have talked about total variation in processes, which we have divided into:
 - Within Subgroup Variation
 - Between Subgroup Variation
- · In terms of Variance:

$$\sigma^2_{Total} = \sigma^2_{Within} + \sigma^2_{Between}$$

 Minitab uses the term "overall variation" – this is the same as "total variation"

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5

Process Capability - Workshop

- Open the Worksheet "Fill volume 2". The data has 20 subgroups with n=5
- Perform a complete Capability Study on the data provided (USL=75.1, LSL=74.9)
- Use the Minitab Control Chart option Stat> Control Charts> Variables Charts for Subgroups> Xbar-R
- Use the Minitab Capability option Stat> Quality Tools> Capability Analysis > Normal
- Perform other analyses as required (eg histograms, dot plots, normality tests, Sixpack etc) to aid your interpretation of data
- Make a presentation of your findings to the other groups including recommendations

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Capability Study Issues raising standards worldwide™ Capability Study Issues Learning

Possible Differences

60

- Number of samples per subgroup
- Number of subgroups
- Use of total variation versus within subgroup variation
- Length of time capability study run over (hours, day, week, month)

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61

Sample Size

- The standard procedure uses 100 samples divided into 20 subgroups of 5
- A minimum of 50 samples should be used, 10 subgroups of 5
- Lower sample sizes result in poorer estimates of $C_{\rm p}$ and $C_{\rm pk}$

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Accuracy of C_p and C_{pk}

- Running the standard study of 20 subgroups of 5 results in estimates of Cp and Cpk that are accurate to ±15%
- Reducing this to 10 subgroups of 5 increases the error to ±22%
- Reducing this to 6 subgroups of 5 increases the error to ±28%

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63

Within Versus Total

- Control limits should always be calculated using the within subgroup variation
- C_P and C_{pk} can be calculated using either the within subgroup or total standard deviation:
 - · Which is used changes their interpretation
 - Recommend using within subgroup variation

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Questions to Ask

- Which standard deviation was used to calculate C_{p} and C_{pk} ?
- · Over what period of time was the study run?

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Process Capability – Non-Normal Data

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Non-normal Data Sets

- Some data sets are non-normal because the underlying distribution is non-normal
- Some possible transformations were covered in the Basic Statistics section of this program
- If we have successfully transformed our data then we can use the standard process capability procedure (for normal data) described earlier

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67

Insufficient Discrimination

- The Anderson-Darling normality test can often indicate non-normality when we have insufficient discrimination in our measurement system
- For this reason it is best to ensure that we have sufficient discrimination in our measurement systems before carrying out capability studies
- A good rule of thumb is to have at least 14 levels of discrimination between our lower and upper specifications
- If this is not possible, seek advice from a statistician

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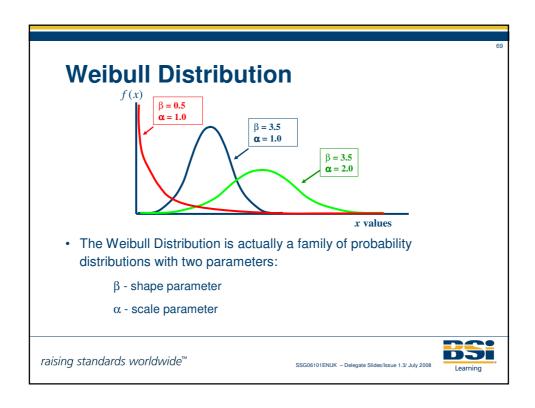


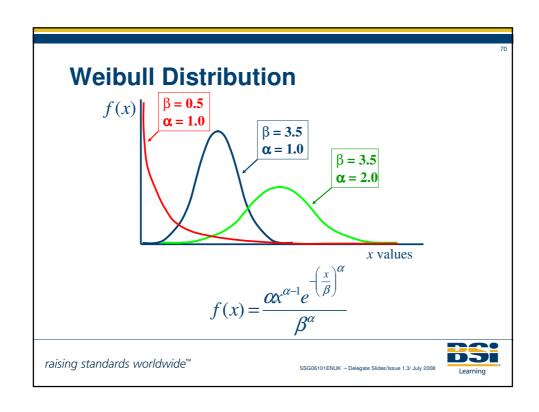
Weibull Distribution

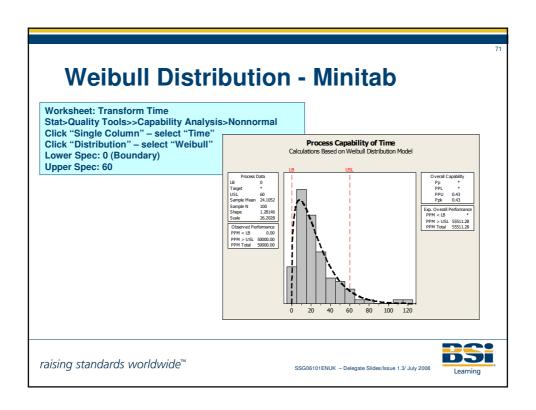
 In some cases, where we have right skewed data sets and cannot successfully transform the data, the Weibull Distribution can be used to good effect

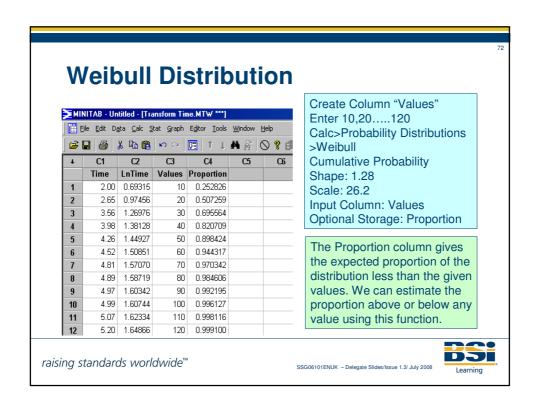
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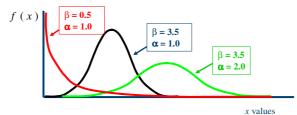








Weibull - Points to Remember



 If our data is non-normal, we should first concentrate on discovering why our data is non-normal

- If it is discovered that our underlying distribution is nonnormal, then we should try some simple transformations
- If a simple transformation is not successful, then the Weibull distribution may be applicable for right skewed data

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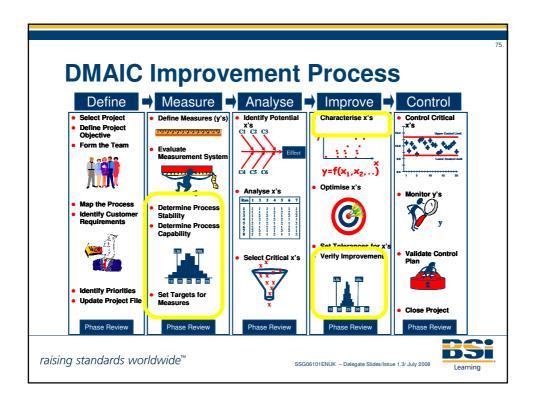


Role of Capability Studies

- Capability Studies are the primary tool for managing variation reduction. They act as the scorecard
- Capability Studies help characterise the type of problem (unstable, off-target, excessive variation) guiding which tools and strategies to apply

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DMAIC - Measure Phase

- · Capability Studies can be used:
 - To determine process stability
 - To determine process capability
 - To determine the nature of the problem (eg unstable process, off-target, excessive variation, outliers)

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77

DMAIC – Improve Phase

- Capability Studies can be used to determine the stability and capability of critical x's
- Capability Studies can be used to verify process improvement
- Capability Studies measure the current state of the processes, track progress and are used to demonstrate improvements
- They measure the success of the other variation reduction tools and processes

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DMAIC - Control Phase

- In conjunction with the use of control charts, process capability can be used during the control phase to:
 - Control critical x's
 - Confirm process capability during Process Validation
 - Ensure effective long-term monitoring of the y's (measures)

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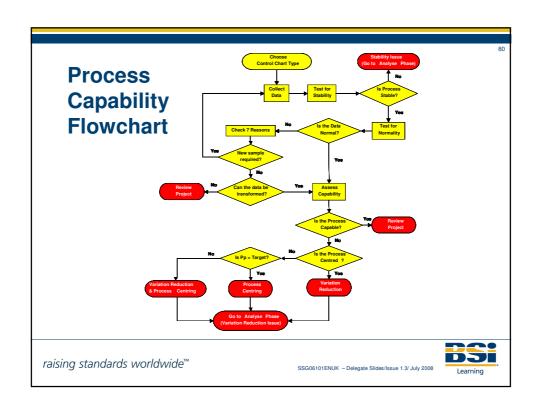


Summary

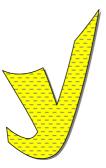
- Long-term studies are more reliable than short-term studies.
 We must ensure that a sufficient time period is sampled
- Always check whether the process is stable before or during a capability analysis. Special causes should be removed, or at least understood, before continuing
- Test for normality. Some types of data will require different capability analyses
- If the process is stable and a sufficient time period is taken, then achieving a C_{pk} / P_pk of 1.5 at the end of your project means that you have achieved the Six Sigma Target!

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DMAIC Measure Phase Summary



- Identification of measures (y's)
- Prioritise y's (not too many!)
- Variable measures if possible
- Evaluate Measurement Systems
- · Establish Current Level of Stability of y's
- Don't characterise unnatural data!
- Investigate & Remove Special Causes
- Assess Process Capability
- Determine the improvement approach required for each measure

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