Title

Yashas.N

1 Section1

Point1

for $\{1, 2..n\} = A_n \subset \mathbb{Z}^+$

- $\sum_{a \in A_n} a = \frac{n(n+1)}{2}$.
- $\sum_{a \in A_n} a^2 = \frac{n(n+1)(2n+1)}{6}$.
- $\sum_{a \in A_n} a^3 = \left(\frac{n(n+1)}{2}\right)^2$.

1.1 subsection1

Theorem

$$\sum_{a \in A_n} a^k = \frac{(n+1)^{k+1} - 1 - \sum_{i=0}^{k-1} \left(\binom{k+1}{i} \sum_{a \in A_n} a^i \right)}{k+1}.$$

Proof. consider the following

$$2^{k+1} = (1+1)^{k+1} = \sum_{i=1}^{k+1} \binom{n+1}{i}. \tag{1}$$

$$\begin{aligned} 3^{k+1} = (2+1)^{k+1} &= \sum_{i=1}^{k+1} \binom{k+1}{i} 2^i, \\ &= 2^{k+1} + \sum_{i=0}^{k} \binom{k+1}{i} 2^i. \end{aligned} \tag{2}$$

Substituting (1) in (2) we get

$$\begin{aligned} (2+1)^{k+1} &= \sum_{i=0}^{k+1} \binom{k+1}{i} + \sum_{i=1}^{k+1} \binom{k+1}{i} 2^i, \\ &= 1 + \sum_{i=0}^{k} \binom{k+1}{i} (2^i + 1^i). \end{aligned} \tag{3}$$

similarly we get

$$\begin{aligned} 4^{k+1} = (3+1)^{k+1} &= \sum_{i=1}^{k+1} \binom{k+1}{i} 3^i, \\ &= 3^{k+1} + \sum_{i=0}^{k} \binom{k+1}{i} 3^i, \\ &= 1 + \sum_{i=1}^{k} \binom{k+1}{i} (3^i + 2^i + 1^i). \text{ (from (3))} \end{aligned}$$

continuing in the same way until $(n+1)^{k+1}$ we get

$$\begin{aligned} (n+1)^{k+1} &= n^{k+1} + \sum_{i=1}^{k} \binom{k+1}{i} n^i, \\ &= 1 + \sum_{i=1}^{k} \binom{k+1}{i} (n^i + (n-1)^i .. + 2^i + 1^i), \\ &= 1 + \sum_{i=1}^{k} \binom{k+1}{i} \left(\sum_{a=1}^{n} a^i \right). \end{aligned} \tag{4}$$

now rewriting equation (4) to get the term $\sum_{a=1}^{n} a^k$ we get the theorem. □