

# Title

Yashas.N

## 1 Section1

### Point1

for  $\{1, 2, \dots, n\} = A_n \subset \mathbb{Z}^+$

$$\begin{aligned} \blacksquare \sum_{a \in A_n} a &= \frac{n(n+1)}{2}. \\ \blacksquare \sum_{a \in A_n} a^2 &= \frac{n(n+1)(2n+1)}{6}. \\ \blacksquare \sum_{a \in A_n} a^3 &= \left( \frac{n(n+1)}{2} \right)^2. \end{aligned}$$

### 1.1 subsection1

#### Theorem

$$\sum_{a \in A_n} a^k = \frac{(n+1)^{k+1} - 1 - \sum_{i=0}^{k-1} \left( \binom{k+1}{i} \sum_{a \in A_n} a^i \right)}{k+1}.$$

*Proof.* consider the following

$$2^{k+1} = (1+1)^{k+1} = \sum_{i=1}^{k+1} \binom{k+1}{i}. \quad (1)$$

$$\begin{aligned} 3^{k+1} &= (2+1)^{k+1} = \sum_{i=1}^{k+1} \binom{k+1}{i} 2^i, \\ &= 2^{k+1} + \sum_{i=0}^k \binom{k+1}{i} 2^i. \end{aligned} \quad (2)$$

Substituting (1) in (2) we get

$$\begin{aligned} (2+1)^{k+1} &= \sum_{i=0}^{k+1} \binom{k+1}{i} + \sum_{i=1}^{k+1} \binom{k+1}{i} 2^i, \\ &= 1 + \sum_{i=0}^k \binom{k+1}{i} (2^i + 1^i). \end{aligned} \quad (3)$$

similarly we get

$$\begin{aligned} 4^{k+1} &= (3+1)^{k+1} = \sum_{i=1}^{k+1} \binom{k+1}{i} 3^i, \\ &= 3^{k+1} + \sum_{i=0}^k \binom{k+1}{i} 3^i, \\ &= 1 + \sum_{i=1}^k \binom{k+1}{i} (3^i + 2^i + 1^i). \text{ (from (3))} \end{aligned}$$

continuing in the same way until  $(n+1)^{k+1}$  we get

$$\begin{aligned} (n+1)^{k+1} &= n^{k+1} + \sum_{i=1}^k \binom{k+1}{i} n^i, \\ &= 1 + \sum_{i=1}^k \binom{k+1}{i} (n^i + (n-1)^i + \dots + 2^i + 1^i), \\ &= 1 + \sum_{i=1}^k \binom{k+1}{i} \left( \sum_{a=1}^n a^i \right). \end{aligned} \quad (4)$$

now rewriting equation (4) to get the term  $\sum_{a=1}^n a^k$  we get the theorem.  $\square$