# Non-linear Value Function Approximation: Deep Q-Network

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CSE4/510 Reinforcement Learning
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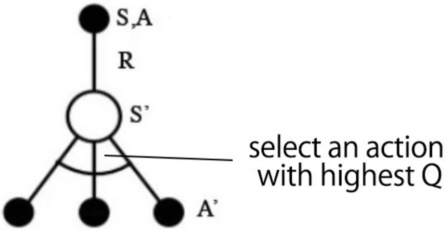
*Slides are based on David Silver's Deep Learning Tutorial, ICML 2016
& Mnih, Volodymyr, et al. "Human-level control through deep reinforcement learning."

# Overview

# Table of Contents

# Recap: Q-Learning Algorithm

- Q-learning learns the action-value function $Q(s, a)$: how good to take an action at a particular state.
- From the memory table, we determine the next action $a'$ to take which has the maximum $Q(s', a')$.



Loop for each step of episode:

Choose $A$ from $S$ using policy derived from $Q$ (e.g., $\varepsilon$-greedy)

Take action $A$, observe $R$, $S'$

$$Q(S,A) \leftarrow Q(S,A) + \alpha\Big[\underbrace{\overbrace{\boxed{\underbrace{R}_{\text{Immediate Reward}} + \gamma \max_a Q(S',a)}}^{\text{Target}} - \overbrace{\boxed{Q(S,A)}}^{\text{Prediction}}}_{\text{loss}}\Big]$$

$S \leftarrow S'$

until $S$ is terminal

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- Optimize objective end-to-end by SGD, using $\frac{\partial L(w)}{\partial w}$

# Supervised SGD vs Q-Learning SGD

- SGD update assuming supervision

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[ (q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))^2 \right]$$

$$\Delta \mathbf{w} = \alpha (q_\pi(S, A) - \hat{q}(S, A, \mathbf{w})) \nabla_{\mathbf{w}} \hat{q}(S, A, \mathbf{w})$$

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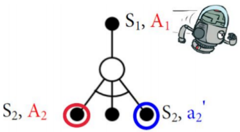


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$$Q(S_1, A_1) := Q(S_1, A_1) + \alpha\left(R_2 + \gamma \max_{a_2'} Q(S_2, a_2') - Q(S_1, A_1)\right)$$

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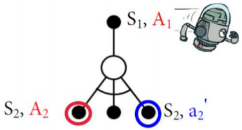


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# Stability issues with Deep RL

Naive Q-learning oscillates or diverge with neural nets

1. Data is sequential
   - Successive samples are correlated, non-iid
2. Policy changes rapidly with slight changes to Q-values
   - Policy can oscillate
   - Disctribution of data can swing from one extreme to another
3. Scale of reqards and Q-values is unknown
   - Naive Q-learning gradients can be large unstable when backpropagated

# Deep Q-Networks

DQN provides a stable solution to deep value-based RL

1. Use experience replay
   - Break correlations in data, bring us back to iid setting
   - Learn from all past policies
2. Freeze target Q-network
   - Avoid oscillations
   - Break correlations between Q-network and target
3. Clip rewards or normalize network adaptive to sensible range
   - Robust gradients

# Stable Deep RL (1): Experience Replay

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- Optimize MSE between Q-network and Q-learning targets, e.g.

$$\mathcal{L}(w) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}} \left[ \left( r + \gamma \max_{a'} Q(s', a', w) - Q(s, a, w) \right)^2 \right]$$

This breaks the similarity of subsequent training samples, which otherwise might drive the network into a local minimum.

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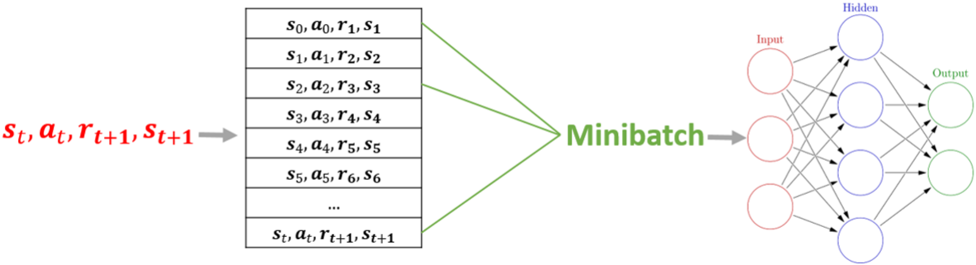

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- Create two deep networks $w^-$ and $w$
- Use the first one to retrieve $Q$ values while the second one includes all updates in the training. After $C$ updates synchronize $w^- \leftarrow w$.

  **Motivation:** Fix the Q-value targets temporarily so we don't have a moving target.

# Stable Deep RL (2): Fixed Target Q-Network

To avoid oscillations, fix parameters used in Q-learning target

- Compute Q-learning targets w.r.t. old, fixed parameters $w^-$

$$r + \gamma \max_{a'} Q(s', a', w^-)$$

- Optimize MSE between Q-network and Q-learning targets

$$\mathcal{L}(w) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}} \left[ \left( r + \gamma \max_{a'} Q(s', a', w^-) - Q(s, a, w) \right)^2 \right]$$

- Periodically update fixed parameters $w^- \leftarrow w$

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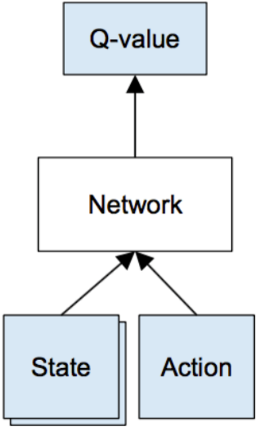
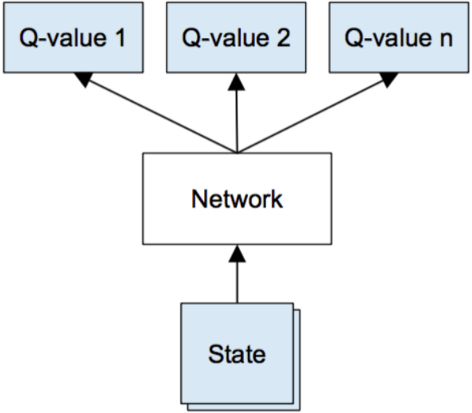
- DQN clips the reward $[-1, +1]$
- This prevents Q-values from becoming too large
- Ensures gradients are well-conditioned
- Can't tell difference between small and large rewards

# Deep Q-Network (DQN) Architecture

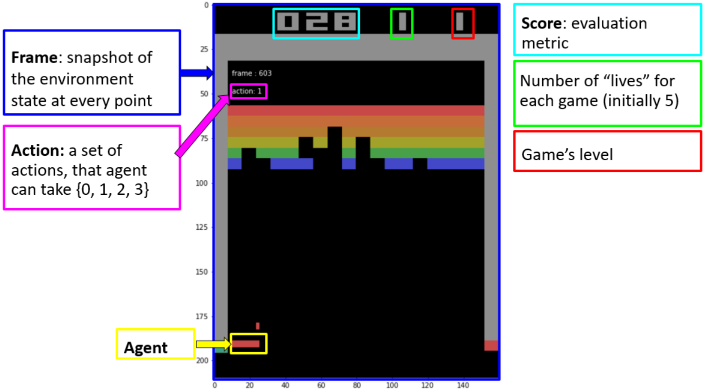
Naive DQN




Optimized DQN used by DeepMind



# Reinforcement Learning in Atari



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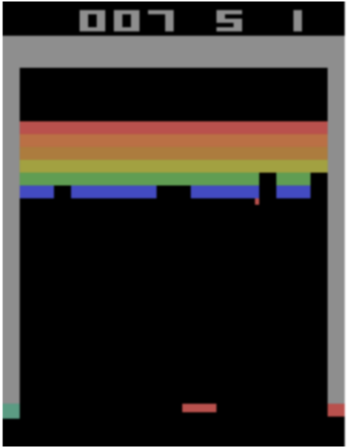

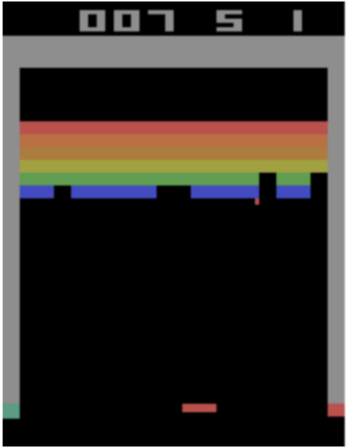


Do we have all the information to start training?

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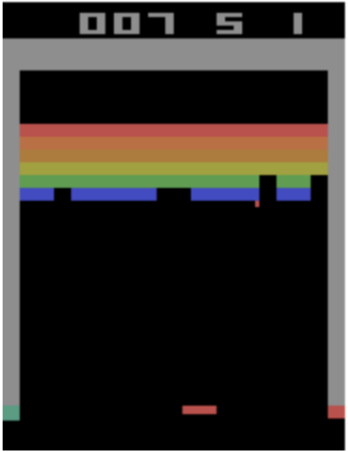


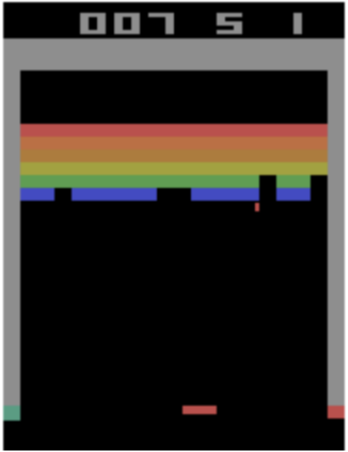

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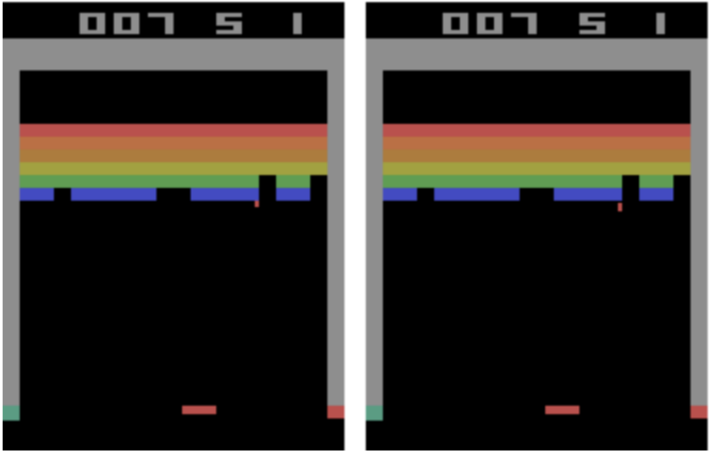


We know the direction and the velocity of the ball. But do we know its acceleration?

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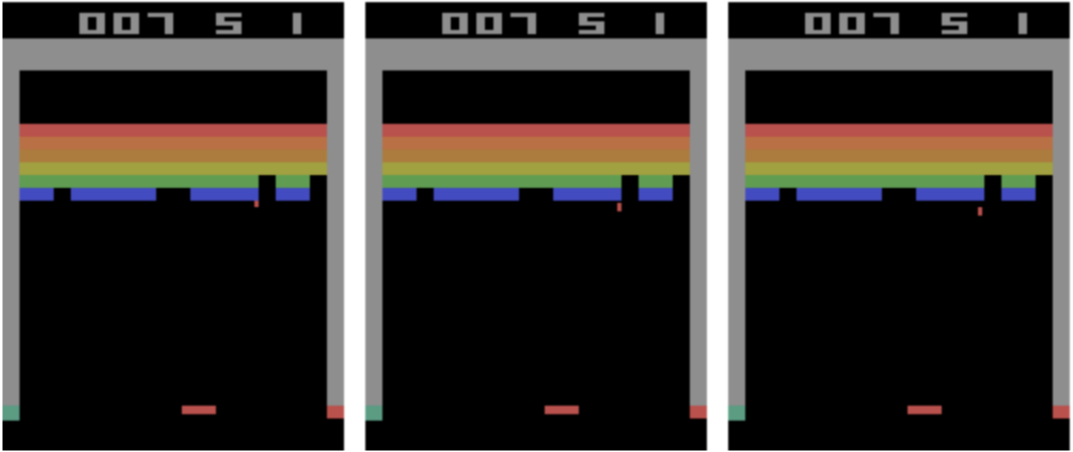


Now we can extract all the information about the state.

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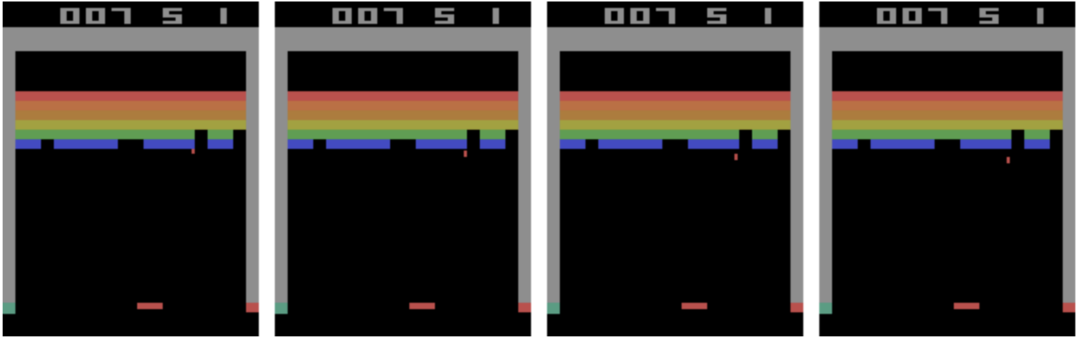
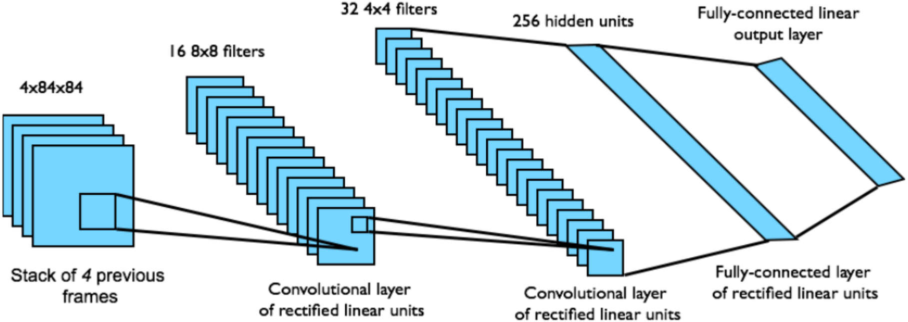



To make sure we can generalize for other games as well, we keep 4 frames as an input.

# DQN in Atari

- End-to-end learning of values $Q(s, a)$ from pixels $s$
- Input state $s$ is stack of raw pixels from last 4 frames
- Output is $Q(s, a)$ for 18 joystick/button positions
- Reward is change in score for that step



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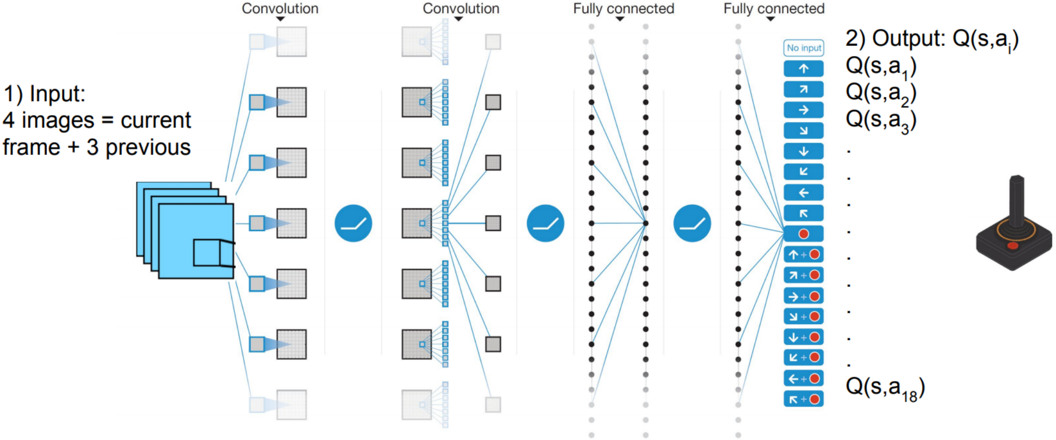
1) Input:
4 images = current
frame + 3 previous
Convolution
Convolution
Fully connected
Fully connected
No input
2) Output: Q(s,a_i)
Q(s,a_1)
Q(s,a_2)
Q(s,a_3)
Q(s,a_18)

# DQN Algorithm

**Algorithm 1: deep Q-learning with experience replay.**

Initialize replay memory $D$ to capacity $N$

Initialize action-value function $\boxed{Q}$ with random weights $\theta$

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    Execute action $a_t$ in emulator and observe reward $r_t$ and image $x_{t+1}$

    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

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    $$\text{Set } y_j = \begin{cases} r_j & \text{if episode terminates at step } j+1 \\ \boxed{r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^{-}\right)} & \text{otherwise} \end{cases}$$

    Perform a gradient descent step on $\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$ with respect to the network parameters $\theta$

    Every $C$ steps reset $\hat{Q} = Q$

  **End For**

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# DQN Example

Demo for Project 4, CSE 574 Machine Learning, Fall 2018 [Instructor: Dr. Sargur N. Srihari]
Authors: Alina Vereshchaka and Nathan Margaglio

Tom & Jerry

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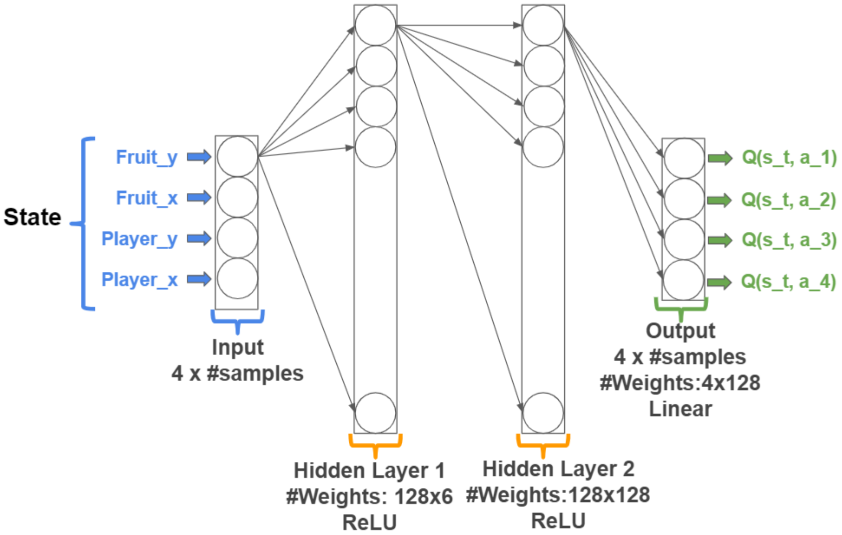
State
Fruit_y
Fruit_x
Player_y
Player_x
Input
4 x #samples
Hidden Layer 1
#Weights: 128x6
ReLU
Hidden Layer 2
#Weights:128x128
ReLU
Output
4 x #samples
#Weights:4x128
Linear
Q(s_t, a_1)
Q(s_t, a_2)
Q(s_t, a_3)
Q(s_t, a_4)

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- By using a Convolutional Neural Network as the function approximator on raw pixels of Atari games where the score is the reward we can learn to play many of those games at human-like performance.