CS885 Reinforcement Learning Lecture 7b: May 23, 2018

Actor Critic Algorithms
[SutBar] Sec. 13.4-13.5,
[Sze] Sec. 4.4, [SigBuf] Sec. 5.3

Outline

- Policy gradient with a baseline
- Actor Critic algorithms
- Deterministic policy gradient

Actor Critic

- Q-learning
 - Model-free value-based method
 - No explicit policy representation
- Policy gradient
 - Model-free policy-based method
 - No explicit value function representation
- Actor Critic
 - Model-free policy and value based method

Stochastic Gradient Policy Theorem

Stochastic Gradient Policy Theorem

$$\nabla V_{\theta}(s_0) \propto \sum_{s} \mu_{\theta}(s) \sum_{a} \nabla \pi_{\theta}(a|s) Q_{\theta}(s,a)$$

• Equivalent Stochastic Gradient Policy Theorem with a baseline b(s)

$$\nabla V_{\theta}(s_0) \propto \sum_{s} \mu_{\theta}(s) \sum_{a} \nabla \pi_{\theta}(a|s) \left[Q_{\theta}(s,a) - b(s) \right]$$

since
$$\sum_{a} \nabla \pi_{\theta}(a|s) b(s) = b(s) \nabla \sum_{a} \pi_{\theta}(a|s) = b(s) \nabla 1 = 0$$

Baseline

• Baseline often chosen to be $b(s) \approx V^{\pi}(s)$

• Advantage function: $A(s,a) = Q(s,a) - V^{\pi}(s)$

Gradient update:

$$\theta \leftarrow \theta + \alpha \gamma^n A(s_n, a_n) \nabla \log \pi_{\theta}(a_n | s_n)$$

Benefit: faster empirical convergence

REINFORCE Algorithm with a baseline

REINFORCEwithBaseline(s_0, π_θ)

Initialize π_{θ} to anything

Initialize V_w to anything

Loop forever (for each episode)

Generate episode s_0 , a_0 , r_0 , s_1 , a_1 , r_1 , ..., s_T , a_T , r_T with π_{θ}

Loop for each step of the episode n = 0, 1, ..., T

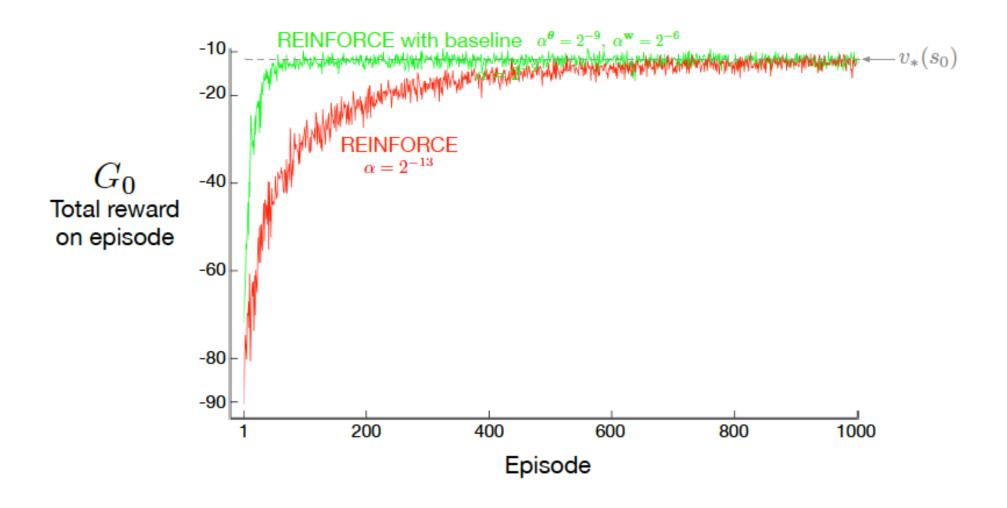
$$G_n \leftarrow \sum_{t=0}^{T-n} \gamma^t r_{n+t}$$
$$\delta \leftarrow G_n - V_w(s_n)$$

Update value function: $w \leftarrow w + \alpha_w \gamma^n \delta \nabla V_w(s_n)$

Update policy: $\theta \leftarrow \theta + \alpha_{\theta} \gamma^{n} \delta \nabla \log \pi_{\theta}(a_{n} | s_{n})$

Return π_{θ}

Performance Comparison



Temporal difference update

Instead of updating V(s) by Monte Carlo sampling

$$\delta \leftarrow G_n - V_w(s_n)$$

Bootstrap with temporal difference updates

$$\delta \leftarrow r_n + \gamma V_w(s_{n+1}) - V_w(s_n)$$

Benefit: reduced variance (faster convergence)

Actor Critic Algorithm

```
ActorCritic(s_0, \pi_\theta)
   Initialize \pi_{\theta} to anything
    Initialize Q_w to anything
    Loop forever (for each episode)
        Initialize s_0 and set n \leftarrow 0
        Loop while s is not terminal (for each time step n)
           Sample a_n \sim \pi_{\theta}(a|s_n)
            Execute a_n, observe s_{n+1}, r_n
           \delta \leftarrow r_n + \gamma V_w(s_{n+1}) - V_w(s_n)
            Update value function: w \leftarrow w + \alpha_w \gamma^n \delta \nabla V_w(s_n)
           Update policy: \theta \leftarrow \theta + \alpha_{\theta} \gamma^n \delta \nabla \log \pi_{\theta}(a_n | s_n)
           n \leftarrow n + 1
Return \pi_{\theta}
```

Advantage update

Instead of doing temporal difference updates

$$\delta \leftarrow r_n + \gamma V_w(s_{n+1}) - V_w(s_n)$$

Update with the advantage function

$$A(s_n, a_n) \leftarrow r_n + \gamma \max_{a_{n+1}} Q(s_{n+1}, a_{n+1})$$
$$-\sum_{a} \pi_{\theta}(a|s_n) Q(s_n, a)$$
$$\theta \leftarrow \theta + \alpha_{\theta} \gamma^n A(s_n, a_n) \nabla \log \pi_{\theta}(a_n|s_n)$$

• Benefit: faster convergence

Advantage Actor Critic (A2C)

```
A2C()
    Initialize \pi_{\theta} to anything
    Loop forever (for each episode)
        Initialize s_0 and set n \leftarrow 0
        Loop while s is not terminal (for each time step n)
            Select a_n
            Execute a_n, observe s_{n+1}, r_n
           \delta \leftarrow r_n + \gamma \max Q_w(s_{n+1}, a_{n+1}) - Q_w(s_n, a_n)
            A(s_n, a_n) \leftarrow r_n + \gamma \max Q_w(s_{n+1}, a_{n+1})
                                -\sum_{\alpha}\pi_{\theta}(\alpha|s_n)Q_{w}(s_n,a)
            Update Q: w \leftarrow w + \alpha_w \gamma^n \delta \nabla_w Q_w(s_n, a_n)
            Update \pi: \theta \leftarrow \theta + \alpha_{\theta} \gamma^{n} A(s_{n}, a_{n}) \nabla \log \pi_{\theta}(a_{n} | s_{n})
            n \leftarrow n + 1
```

Continuous Actions

- Consider a deterministic policy $\pi_{\theta}(s) \rightarrow a$
- Deterministic Gradient Policy Theorem

$$\nabla V_{\theta}(s_0) \propto E_{s \sim \mu_{\theta}(s)} \left[\nabla_{\theta} \pi_{\theta}(s) \nabla_{a} Q_{\theta}(s, a) \Big|_{a = \pi_{\theta}(s)} \right]$$

Proof: see Silver et al. 2014

Stochastic Gradient Policy Theorem

$$\nabla V_{\theta}(s_0) \propto \sum_{s} \mu_{\theta}(s) \sum_{a} \nabla_{\theta} \pi_{\theta}(a|s) Q_{\theta}(s,a)$$

Deterministic Policy Gradient (DPG)

```
\mathsf{DPG}(s,\pi_{\theta})
    Initialize \pi_{\theta} to anything
    Loop forever (for each episode)
        Initialize s_0 and set n \leftarrow 0
        Loop while s is not terminal (for each time step n)
           Select a_n = \pi_{\theta}(s_n)
            Execute a_n, observe s_{n+1}, r_n
           \delta \leftarrow r_n + \gamma Q_w(s_{n+1}, \pi_\theta(s_{n+1})) - Q_w(s_n, a_n)
            Update Q: w \leftarrow w + \alpha_w \gamma^n \delta \nabla_w Q_w(s_n, a_n)
            Update \pi: \theta \leftarrow \theta + \alpha_{\theta} \gamma^{n} \nabla_{\theta} \pi_{\theta}(s_{n}) \nabla_{a} Q_{w}(s_{n}, a_{n})|_{a_{n} = \pi_{\theta}(s_{n})}
           n \leftarrow n + 1
Return \pi_{\theta}
```