Monte Carlo

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Monte Carlo

- Monte Carlo methods are learning methods
  - Experience $\rightarrow$ values, policy
- Monte Carlo uses the simplest possible idea: value = mean return
- Monte Carlo methods can be used in two ways:
  - Model-free: No model necessary and still attains optimality
  - Simulated: Needs only a simulation, not a full model
- Monte Carlo methods learn from complete sample returns
  - Only defined for episodic tasks (this class)
  - All episodes must terminate (no bootstrapping)

Monte-Carlo Policy Evaluation

- Goal: learn $v_\pi(s)$ from episodes of experience under policy π

$$S_1, A_1, R_2, ..., S_k \sim \pi$$

- Remember that the return is the total discounted reward:

$$G_t = R_{t+1} + \gamma R_{t+2} + ... + \gamma^{T-1} R_T$$

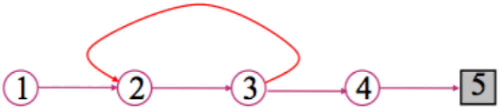
- Remember that the value function is the expected return:

$$v_\pi(s) = \mathbb{E}_\pi [G_t \mid S_t = s]$$

- Monte-Carlo policy evaluation uses empirical mean return instead of expected return

Monte-Carlo Policy Evaluation

- Goal: learn $v_\pi(s)$ from episodes of experience under policy π
- Idea: Average returns observed after visits to s:



- Every-Visit MC: average returns for every time s is visited in an episode
- First-visit MC: average returns only for first time s is visited in an episode
- Both converge asymptotically

First-Visit MC Policy Evaluation

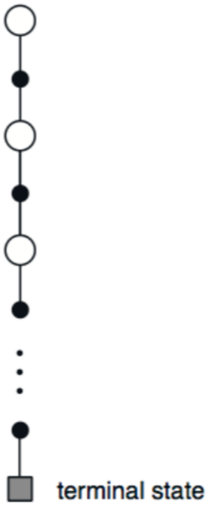
- To evaluate state s
- The first time-step t that state s is visited in an episode,
- Increment counter: $N(s) \leftarrow N(s) + 1$
- Increment total return: $S(s) \leftarrow S(s) + G_t$
- Value is estimated by mean return $V(s) = S(s)/N(s)$
- By law of large numbers $V(s) \rightarrow v_\pi(s)$ as $N(s) \rightarrow \infty$

Every-Visit MC Policy Evaluation

- To evaluate state s
- Every time-step t that state s is visited in an episode,
- Increment counter: $N(s) \leftarrow N(s) + 1$
- Increment total return: $S(s) \leftarrow S(s) + G_t$
- Value is estimated by mean return $V(s) = S(s)/N(s)$
- By law of large numbers $V(s) \rightarrow v_\pi(s)$ as $N(s) \rightarrow \infty$

Backup Diagram for Monte Carlo

- Entire rest of episode included
- Only one choice considered at each state (unlike DP)
  - thus, there will be an explore/exploit dilemma
- Does not bootstrap from successor state's values (unlike DP)
- Value is estimated by mean return
- Time required to estimate one state does not depend on the total number of states



Incremental Mean

- The mean $\mu_1, \mu_2, \ldots$ of a sequence $x_1, x_2, \ldots$ can be computed incrementally:

$$\begin{aligned}
\mu_k &= \frac{1}{k}\sum_{j=1}^{k} x_j \\
&= \frac{1}{k}\left(x_k + \sum_{j=1}^{k-1} x_j\right) \\
&= \frac{1}{k}\left(x_k + (k-1)\mu_{k-1}\right) \\
&= \mu_{k-1} + \frac{1}{k}\left(x_k - \mu_{k-1}\right)
\end{aligned}$$

Incremental Monte Carlo Updates

- Update V(s) incrementally after episode $S_1, A_1, R_2, ..., S_T$
- For each state S_t with return G_t

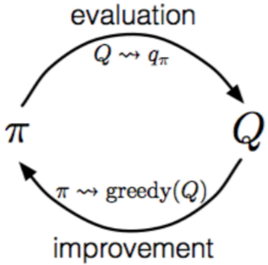
$$N(S_t) \leftarrow N(S_t) + 1$$
$$V(S_t) \leftarrow V(S_t) + \frac{1}{N(S_t)}(G_t - V(S_t))$$

- In non-stationary problems, it can be useful to track a running mean, i.e. forget old episodes.

$$V(S_t) \leftarrow V(S_t) + \alpha(G_t - V(S_t))$$

Monte-Carlo Control

$$\pi_0 \xrightarrow{\text{E}} q_{\pi_0} \xrightarrow{\text{I}} \pi_1 \xrightarrow{\text{E}} q_{\pi_1} \xrightarrow{\text{I}} \pi_2 \xrightarrow{\text{E}} \cdots \xrightarrow{\text{I}} \pi_* \xrightarrow{\text{E}} q_*$$



- MC policy iteration step: Policy evaluation using MC methods followed by policy improvement
- Policy improvement step: greedify with respect to value (or action-value) function

Monte-Carlo Algorithm

Initialize, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$:
- $Q(s, a) \leftarrow$ arbitrary
- $\pi(s) \leftarrow$ arbitrary
- $Returns(s, a) \leftarrow$ empty list

Fixed point is optimal policy π^*

Repeat forever:
- Choose $S_0 \in \mathcal{S}$ and $A_0 \in \mathcal{A}(S_0)$ s.t. all pairs have probability > 0
- Generate an episode starting from S_0, A_0, following π
- For each pair s, a appearing in the episode:
  - $G \leftarrow$ return following the first occurrence of s, a
  - Append G to $Returns(s, a)$
  - $Q(s, a) \leftarrow \text{average}(Returns(s, a))$
- For each s in the episode:
  - $\pi(s) \leftarrow \arg\max_a Q(s, a)$

Monte Carlo

Advantages

- MC can be used to learn optimal behavior directly from interaction with the environment. It does not require a model of the environment's dynamics.
- MC can be used with simulation or sample models.
- MC can be used to focus on one region of special interest and be accurately evaluated without having to evaluate the rest of the state set.

Disadvantages

- MC only works for episodic (terminating) environments. It does not work with environment with no terminating states.
- MC must wait until the end of an episode before return is known. For problems with very long episodes this will become too slow.