Learning and Planning with Tabular Methods

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Overview

1. Definitions
2. Model
3. Recap: Dynamic Programming

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Definitions

Learning: ?

Planning: ?

Definitions

Learning: the acquisition of knowledge or skills through experience, study, or by being taught.

e.g., we learn value functions from real experience (action/state trajectories) using Monte Carlo methods, or we learn a model (transition function)

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Learning: the acquisition of knowledge or skills through experience, study, or by being taught.

e.g., we learn value functions from real experience (action/state trajectories) using Monte Carlo methods, or we learn a model (transition function)

Planning: any computational process that uses a model to create or improve a policy

e.g., we compute value functions from simulated experience (action/state trajectories)

$$\text{Model} \xrightarrow{\text{Planning}} \text{Policy}$$

Planning Examples

- Value iteration
- Policy iteration
- TD-gammon (look-ahead search)
- Alpha-Go (Monte Carlo Tree Search)
- Chess (heuristic search)

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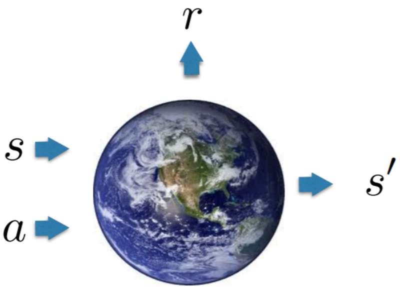
Model

Anything the agent can use to predict how the environment will respond to its actions, concretely, the state transition $T(s'|s, a)$ and reward $R(s, a)$.



Ground-truth

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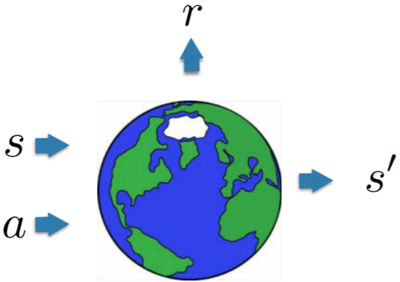

Learnt

- Distribution model: ?
- Sample model ?

Distribution VS Sample Models

- **Distribution model:** lists all possible outcomes and their probabilities, $T(s'|s,a)$ for all (s,a,s') . (We used those in DP)

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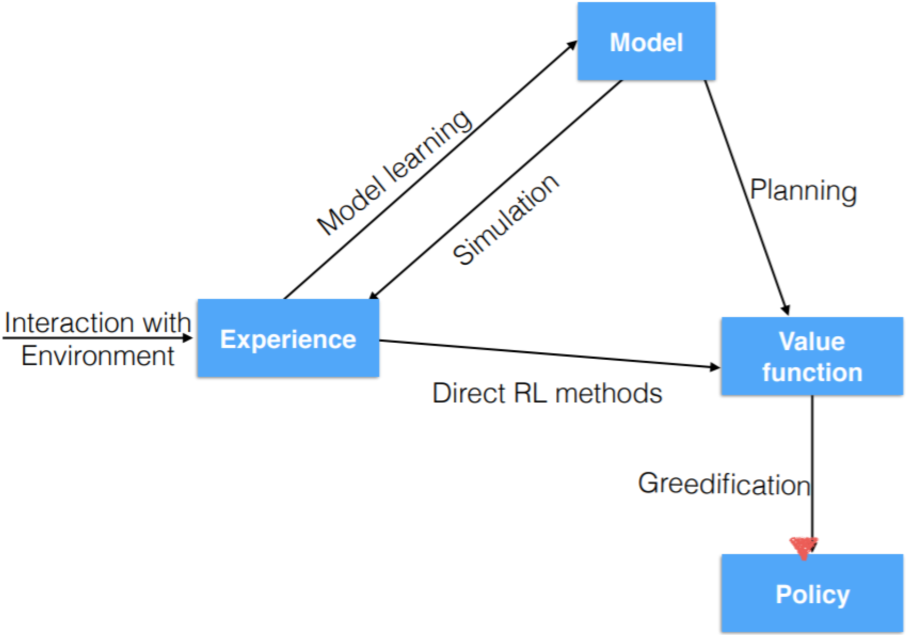
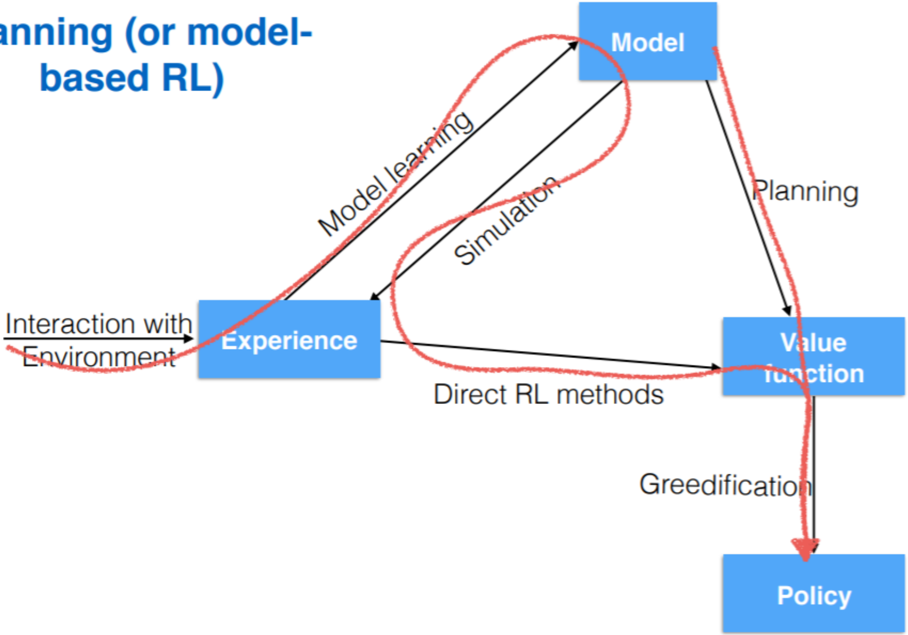
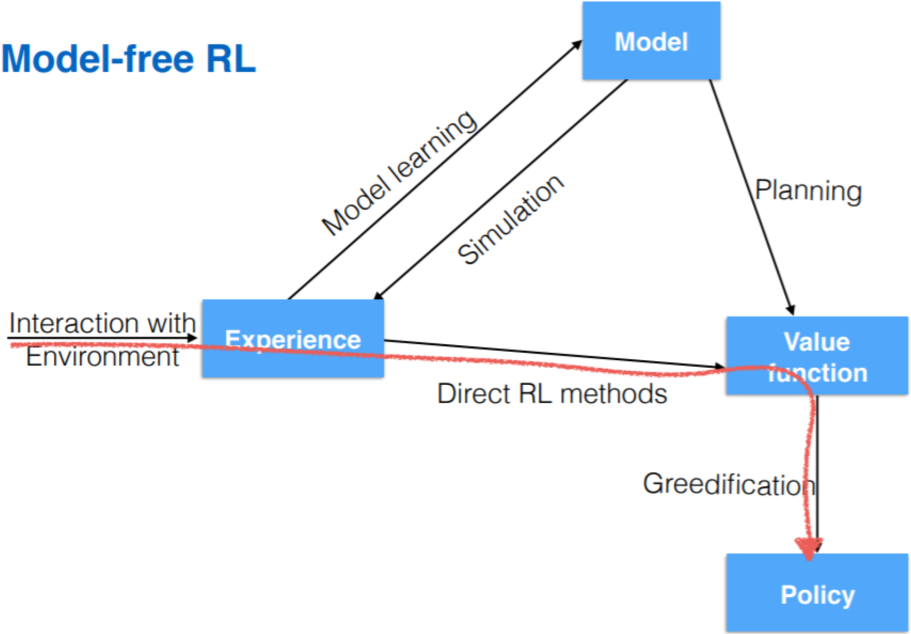
Q: which one is more powerful? Which one is easier to obtain/learn?


Model
Model learning
Simulation
Planning
Interaction with
Environment
Experience
Direct RL methods
Value
function
Greedification
Policy

Planning (or model-based RL)



Model-free RL



Advantages of Planning (Model-based RL)

Advantages:

- Model learning transfers across tasks and environment configurations (learning physics)
- Better exploits experience in case of sparse rewards
- Helps exploration: Can reason about model uncertainty

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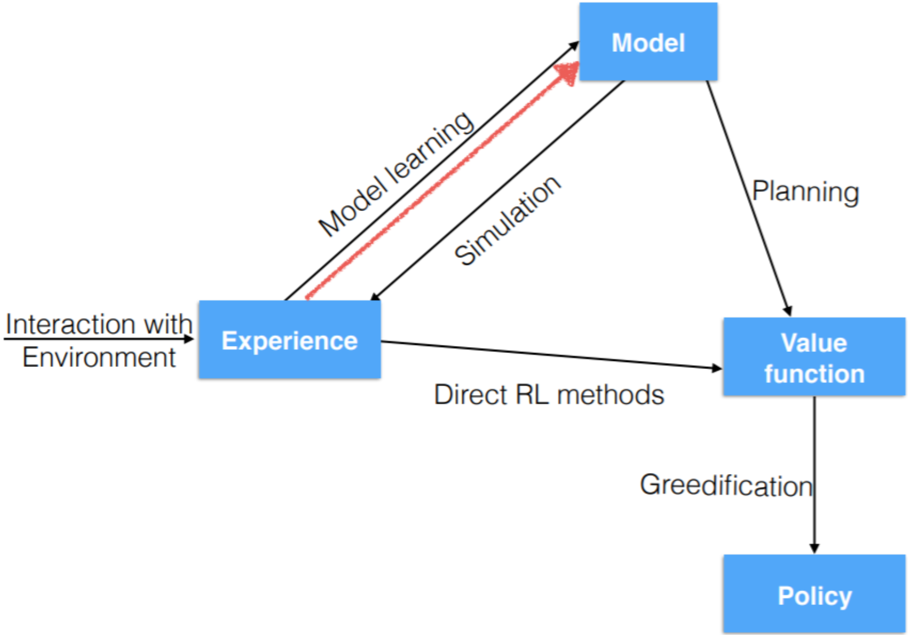
Disadvantages:

- First learn model, then construct a value function: Two sources of approximation error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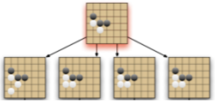
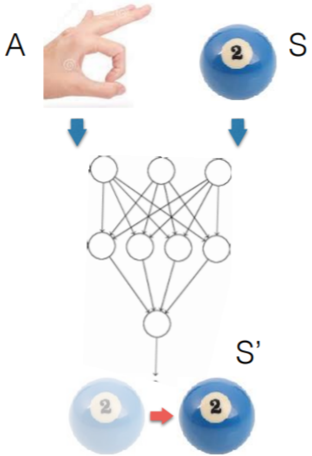
Examples of Models for $T(s'|s,a)$

Table lookup model (tabular): bookkeeping a probability of occurrence for each transition (s,a,s')



Transition function is approximated through some function approximator



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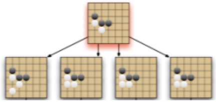
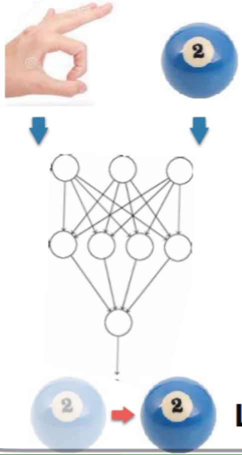


This Lecture

Transition function is approximated through some function approximator



Later..

Table Lookup Model

- Model is an explicit MDP (T, R)
- Count visits $N(s, a)$ to each state-action pair:

$$\hat{T}(s'|s, a) = \frac{1}{N(s, a)} \sum_{t=1}^{\tau} 1(S_t, A_t, S_{t+1} = s, a, s')$$

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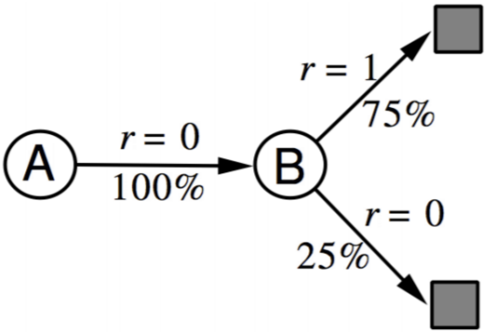
- **Alternatively**
  - At each timestep t, record experience tuple $(S_t, A_t, R_{t+1}, S_{t+1})$
  - To sample model, randomly pick tuple matching $(s, a, ., .)$

Here, model learning means save the experience, memorization == learning

Example

Two states A, B; no discounting; 8 episodes of experience

A, 0, B, 0
B, 1
B, 1
B, 1
B, 1
B, 1
B, 1
B, 0



We have constructed a table lookup model from the experience

Planning with a Model

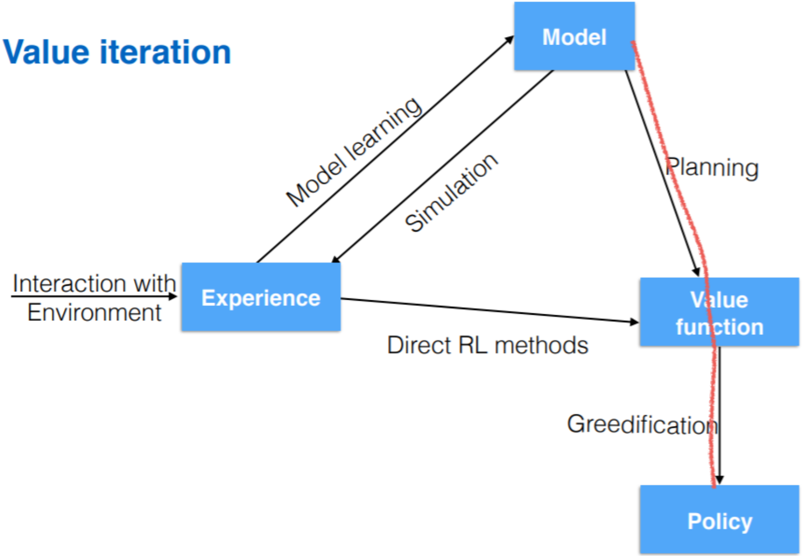
Given a model: $M_\eta = (T_\eta, R_\eta)$

Solve the MDP: (S, A, T_η, R_η)

Using favorite planning algorythm:

- Value iteration
- Policy iteration

Value iteration



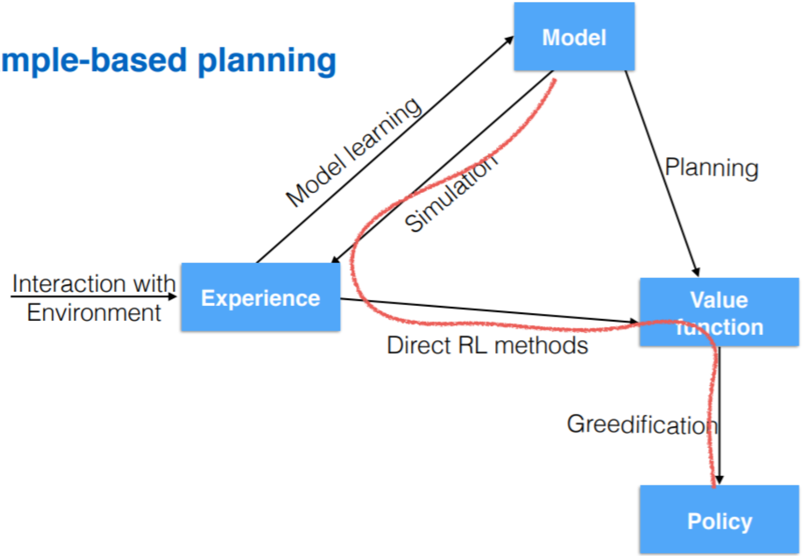
Sample-based Planning

- Use the model only to generate samples, not using its transition probabilities and expected immediate rewards
- Sample experience from model

$$S_{t+1} \sim T_\eta(S_{t+1} | S_t, A_t)$$
$$R_{t+1} = \mathcal{R}_\eta(R_{t+1} | S_t, A_t)$$

- Apply model-free RL to samples, e.g.:
  - Monte-Carlo control
  - Sarsa $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t) \right]$
  - Q-learning $Q(s, a) \leftarrow Q(s, a) + \alpha \left[R + \gamma \max_{a'} Q(S', a') - Q(s, a) \right]$
- Sample-based planning methods are often more efficient: rather than exhaustive state sweeps we focus on what is likely to happen

Sample-based planning



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Optimal Value Functions

Solving the Bellman equation is to find the optimal policy:

State value function:

$$
\begin{aligned}
V^*(s) &= \max_a \mathbb{E}[R_{t+1} + \gamma V^*(S_{t+1}) | S_t = s, A_t = a] \\
&= \max_a \sum_{s',r} p(s', r | s, a) \big[r + \gamma V^*(s') \big]
\end{aligned}
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\end{aligned}
$$

Action-state value function:

$$
\begin{aligned}
Q^*(s) &= \mathbb{E}[R_{t+1} + \gamma \max_{a'} Q^*(S_{t+1}, a') | S_t = s, A_t = a] \\
&= \sum_{s',r} p(s', r | s, a) \big[r + \gamma \max_{a'} Q^*(s', a') \big]
\end{aligned}
$$

Recap:Dynamic Programming - Evaluation

Iterative Policy Evaluation, for estimating $V \approx v_\pi$

Input π, the policy to be evaluated
Algorithm parameter: a small threshold $\theta > 0$ determining accuracy of estimation
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(terminal) = 0$

Loop:
    $\Delta \leftarrow 0$
    Loop for each $s \in \mathcal{S}$:
        $v \leftarrow V(s)$
        $V(s) \leftarrow \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a)\big[r + \gamma V(s')\big]$
        $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
until $\Delta < \theta$

New greedy policy π'

$$\begin{aligned}
\pi'(s) &\doteq \arg\max_a Q_\pi(s,a) \\
&= \arg\max_a \mathbb{E}\left[r_{t+1} + \gamma V_\pi(S_{t+1}) | S_t = s, A_t = a\right] \\
&= \arg\max_a \sum_{s',r} p(s', r|s,a)\left[r + \gamma V_\pi(s')\right]
\end{aligned}$$

Recap: Dynamic Programming

Policy Iteration (using iterative policy evaluation) for estimating $\pi \approx \pi_*$

1. Initialization
   $V(s) \in \mathbb{R}$ and $\pi(s) \in \mathcal{A}(s)$ arbitrarily for all $s \in \mathcal{S}$

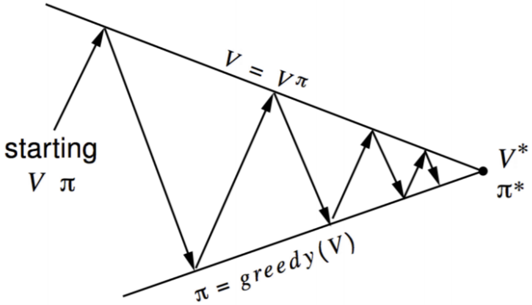
2. Policy Evaluation
   Loop:
   - $\Delta \leftarrow 0$
   - Loop for each $s \in \mathcal{S}$:
     - $v \leftarrow V(s)$
     - $V(s) \leftarrow \sum_{s',r} p(s',r|s,\pi(s))\big[r + \gamma V(s')\big]$
     - $\Delta \leftarrow \max(\Delta, |v - V(s)|)$

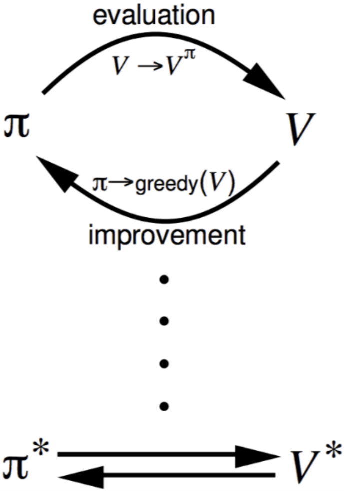
   until $\Delta < \theta$ (a small positive number determining the accuracy of estimation)

3. Policy Improvement
   policy-stable $\leftarrow$ *true*
   For each $s \in \mathcal{S}$:
   - *old-action* $\leftarrow \pi(s)$
   - $\pi(s) \leftarrow \arg\max_a \sum_{s',r} p(s',r|s,a)\big[r + \gamma V(s')\big]$
   - If *old-action* $\neq \pi(s)$, then *policy-stable* $\leftarrow$ *false*

   If *policy-stable*, then stop and return $V \approx v_*$ and $\pi \approx \pi_*$; else go to 2

Policy Improvement





Policy evaluation Estimate v_π

Iterative policy evaluation

Policy improvement Generate $\pi' \geq \pi$

Greedy policy improvement

Recap: Dynamic Programming

Advantages:

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- DP is efficient, it finds optimal policies in polynomial time for most cases
- DP is guaranteed to find optimal policy

Disadvantages:

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- DP is guaranteed to find optimal policy

Disadvantages:

- DP is not suitable for large problems, with millions or more of states
- DP requires the knowledge of the transition probability matrix, however this is an unrealistic requirement for many problems