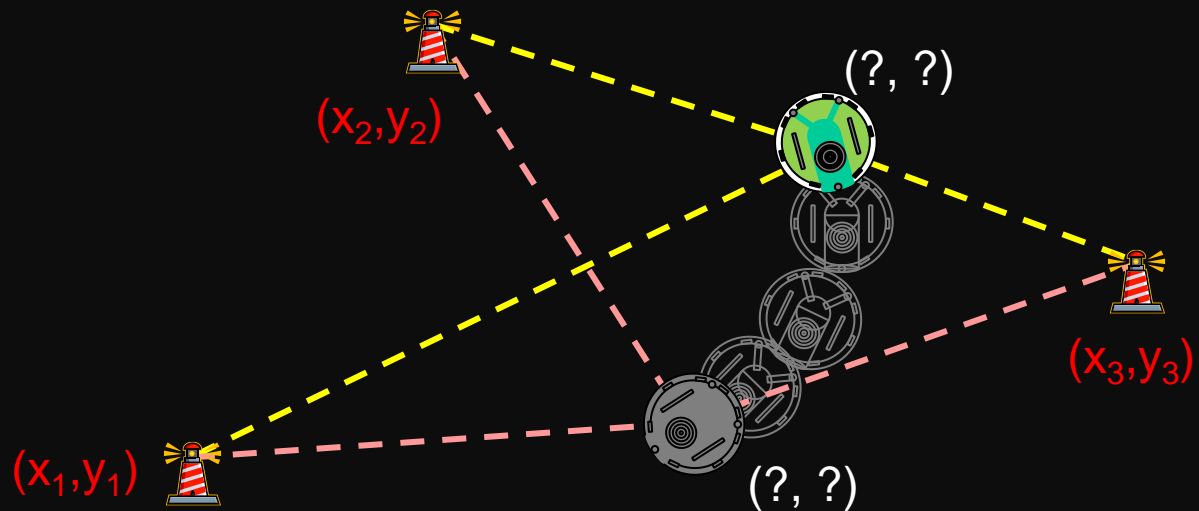


Beacon Positioning (Triangulation)

Beacon Positioning

- A **beacon** is a detectable device that is placed at a fixed (i.e., known) position in the environment.
 - An **active beacon** transmits and/or receives signals
- A robot can estimate its “absolute” position and orientation by determining the **distance** and/or **angle** to three or more beacons.



Beacon Systems

- Active Beacon systems ...



- + can produce high accuracy in position estimation



- can be expensive to install and maintain

- require alterations to environment (i.e., installation)

- There are many commercially available indoor beacon systems:

- Active Badge

- Active Bat

- RADAR

- RICE project

- E911

- Cricket

- MotionStar Magnetic Tracker

- Easy Living

- Smart Floor



Image from
<http://cricket.csail.mit.edu/>

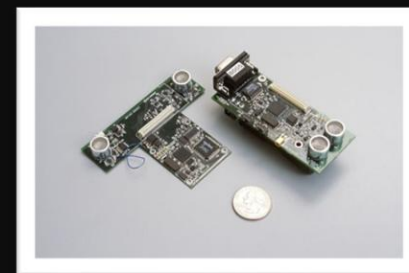
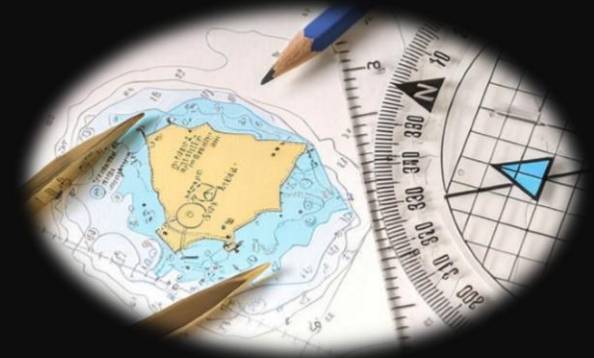


Image from
<http://cricket.csail.mit.edu/>

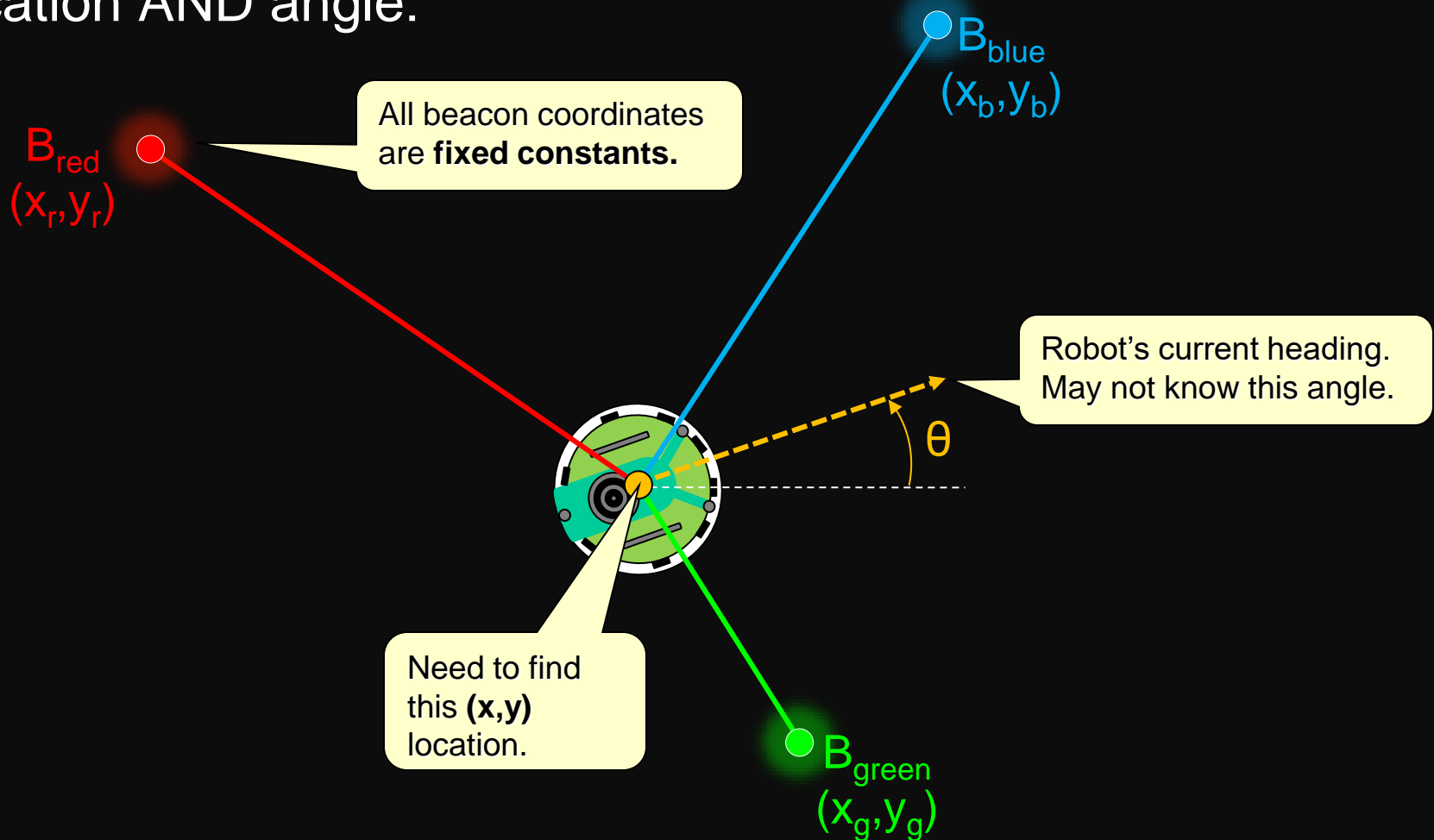
Triangulating a Robot's Position

- Based on triangulation principle
 - uses geometric properties of triangles to compute position.
- Two types of triangulation techniques:
 - **Tri-Angulation**
 - determine robot position and angle based on **angle** to beacons
 - 2D requires 2 angles and 1 known distance, or 3 angles.
 - 3D requires 1 additional azimuth measurement
 - **Tri-Lateration**
 - determine robot position based on **distance** from beacons
 - 2D requires 3 **non-collinear** points
 - 3D requires 4 non-coplanar points



Triangulation

- Problem: Given 3 beacons at fixed positions, compute robot location AND angle:



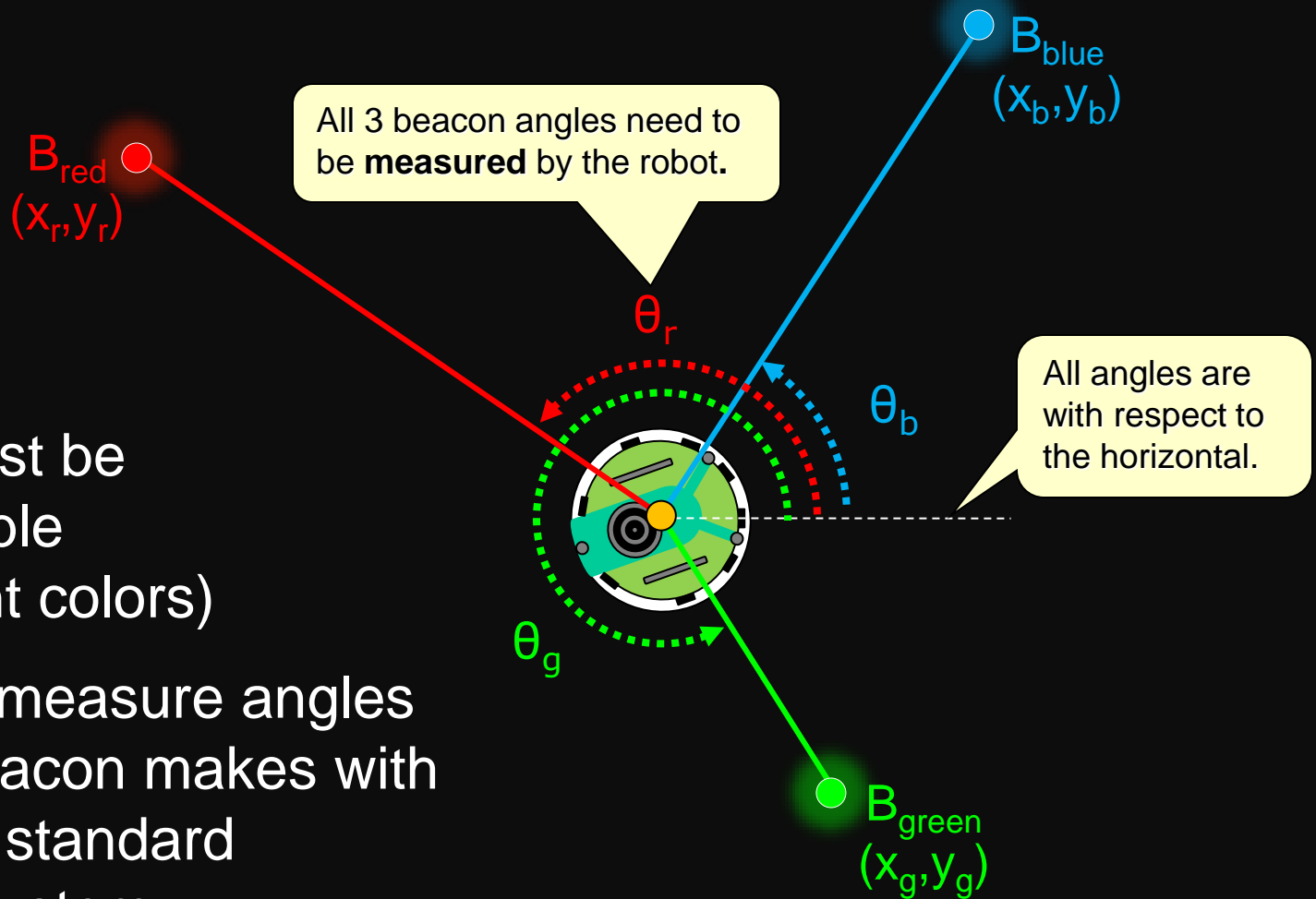
Triangulation - Techniques

- There are many triangulation techniques for mobile robot positioning.
 - some require a particular beacon ordering
 - some have “blind spots”
 - some only work within the triangle defined by the three beacons
 - more reliable methods exist but are more complex or require the handling of certain spatial arrangements separately.
- We will use the “To Tal” algorithm:
 - V. PIERLOT, M. VANDROOGENBROECK, and M. Urbin-Choffray. **A new three object triangulation algorithm based on the power center of three circles**. In Research and Education in Robotics (EUROBOT), volume 161 of Communications in Computer and Information Science, pages 248-262, 2011. Springer.



Triangulation – The *ToTal* Algorithm

- Assumes 3 fixed beacon locations:



- beacons must be distinguishable (e.g. different colors)
- Robot must measure angles that each beacon makes with reference to standard coordinate system

Triangulation – The *ToTal* Algorithm

- Don't worry about the math. Just need to plug in formulas:
 - STEP 1: compute the modified beacon coordinates ...

$$x'_1 = x_r - x_g$$

$$y'_1 = y_r - y_g$$

$$x'_3 = x_b - x_g$$

$$y'_3 = y_b - y_g$$

- STEP 2: compute the three cotangents ...

$$T_{12} = 1 / \tan(\theta_g - \theta_r)$$

$$T_{23} = 1 / \tan(\theta_b - \theta_g)$$

$$T_{31} = \frac{1 - T_{12} * T_{23}}{T_{12} + T_{23}}$$

Triangulation – The *ToTal* Algorithm

- STEP 3: compute the modified circle center coordinates...

$$x'_{12} = x'_1 + T_{12} * y'_1$$

$$y'_{12} = y'_1 - T_{12} * x'_1$$

$$x'_{23} = x'_3 - T_{23} * y'_3$$

$$y'_{23} = y'_3 + T_{23} * x'_3$$

$$x'_{31} = (x'_3 + x'_1) + T_{31} * (y'_3 - y'_1)$$

$$y'_{31} = (y'_3 + y'_1) - T_{31} * (x'_3 - x'_1)$$

- STEP 4: compute k'_{31} ...

$$k'_{31} = x'_1 * x'_3 + y'_1 * y'_3 + T_{31} * (x'_1 * y'_3 - x'_3 * y'_1)$$

- STEP 5: compute d ...

$$d = (x'_{12} - x'_{23}) * (y'_{23} - y'_{31}) - (y'_{12} - y'_{23}) * (x'_{23} - x'_{31})$$

Triangulation – The *ToTal* Algorithm

- if $d = 0$, then there is no solution
 - corresponds to situation when robot is on perimeter of circle defined by beacons
 - no algorithm can handle this case
 - can detect when this happens by examining d .
 - As d gets smaller, error grows ... so if, for example, $|d| < 100$ or so ... then the calculation may be inaccurate.



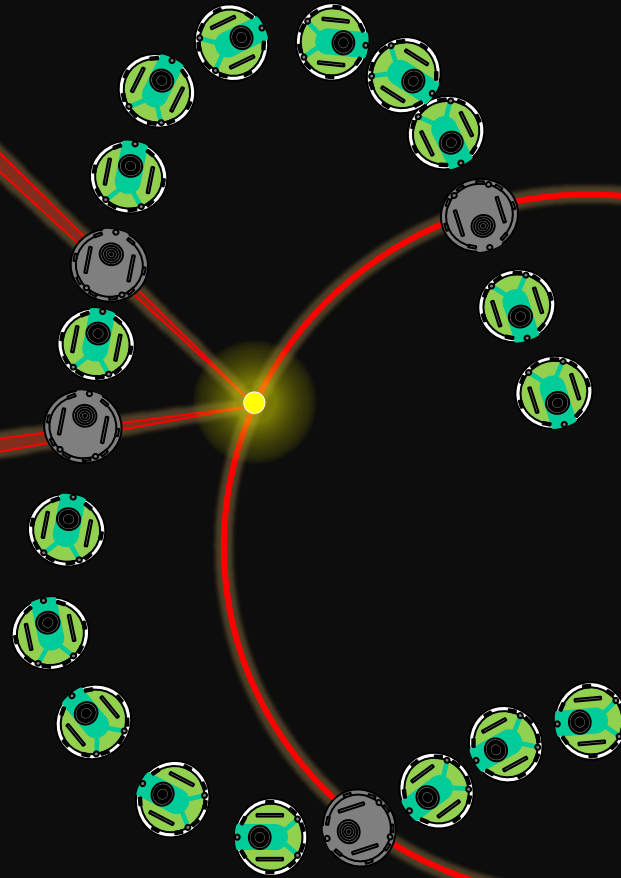
```
if (Math.abs(d) < 100)
    ...
```

- STEP 6: compute the robot position (x , y) ...

$$x = x_g + k'_{31} * (y'_{12} - y'_{23}) / d$$

$$y = y_g + k'_{31} * (x'_{23} - x'_{12}) / d$$

Triangulation - Issues



Robot cannot determine its location within these wedges.

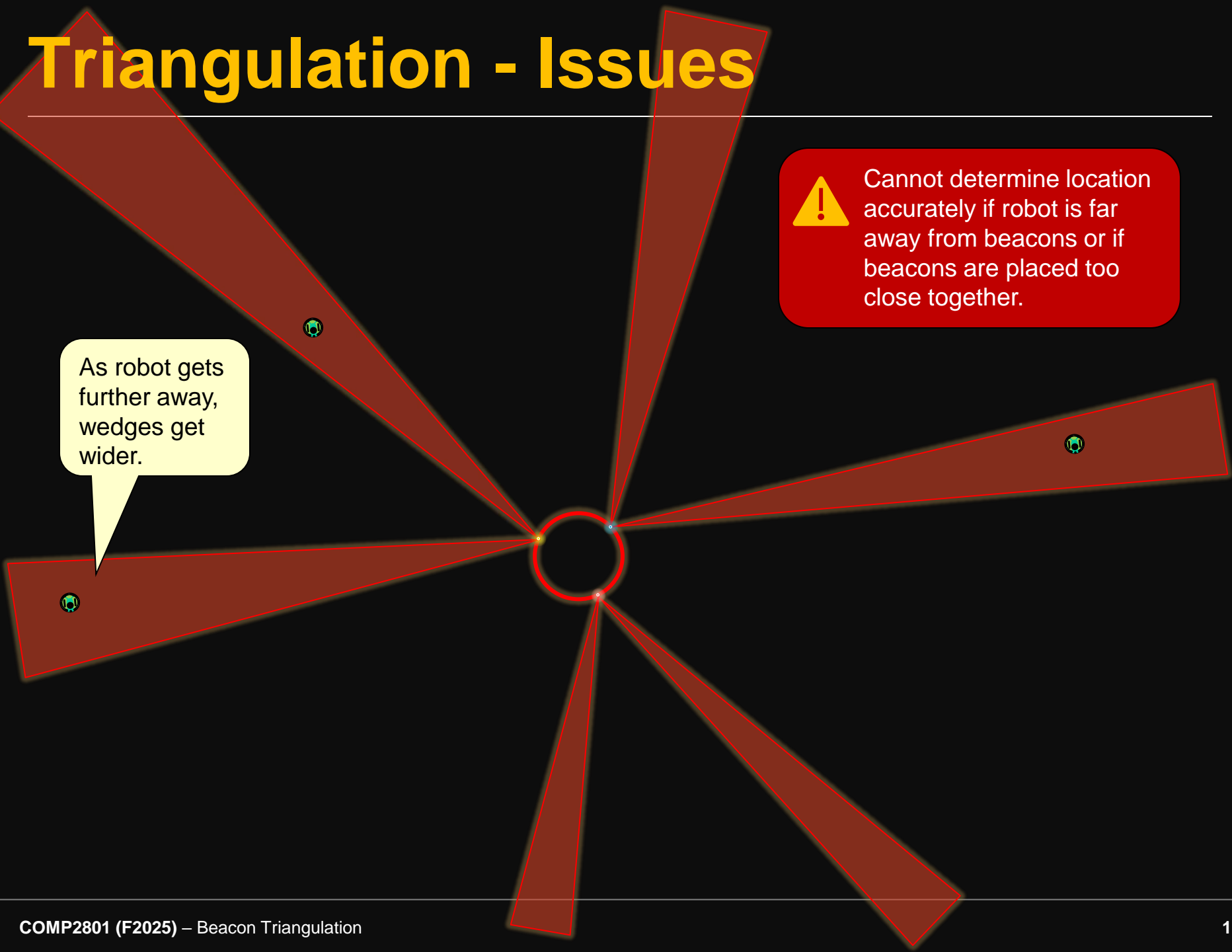
Robot cannot determine its location when on or near this circle.

Triangulation - Issues



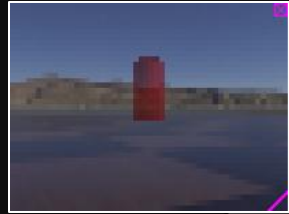
Cannot determine location accurately if robot is far away from beacons or if beacons are placed too close together.

As robot gets further away, wedges get wider.



E-Puck - Measuring Angles

- How does robot measure θ_r , θ_g and θ_b ?

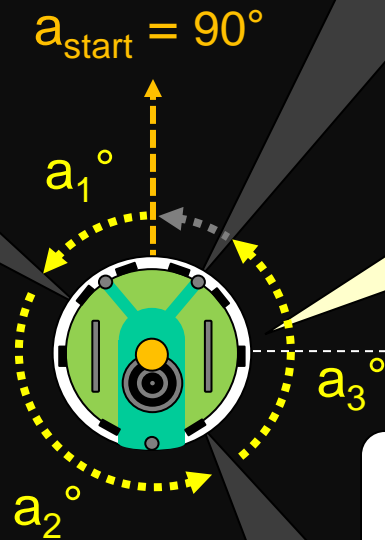


B_{red}

store **red** one
at **angles[0]**

Initialize an array for the 3 angles:
`double angles[] =
{-999, -999, -999};`

- Spin robot around
- Look for beacons centered
- Keep track of rotation angle
- Counter-clockwise direction ensures positive angles



B_{blue}

store **blue** one
at **angles[2]**

Rotate 360°
looking for
beacons
using camera

store **green** one
at **angles[1]**

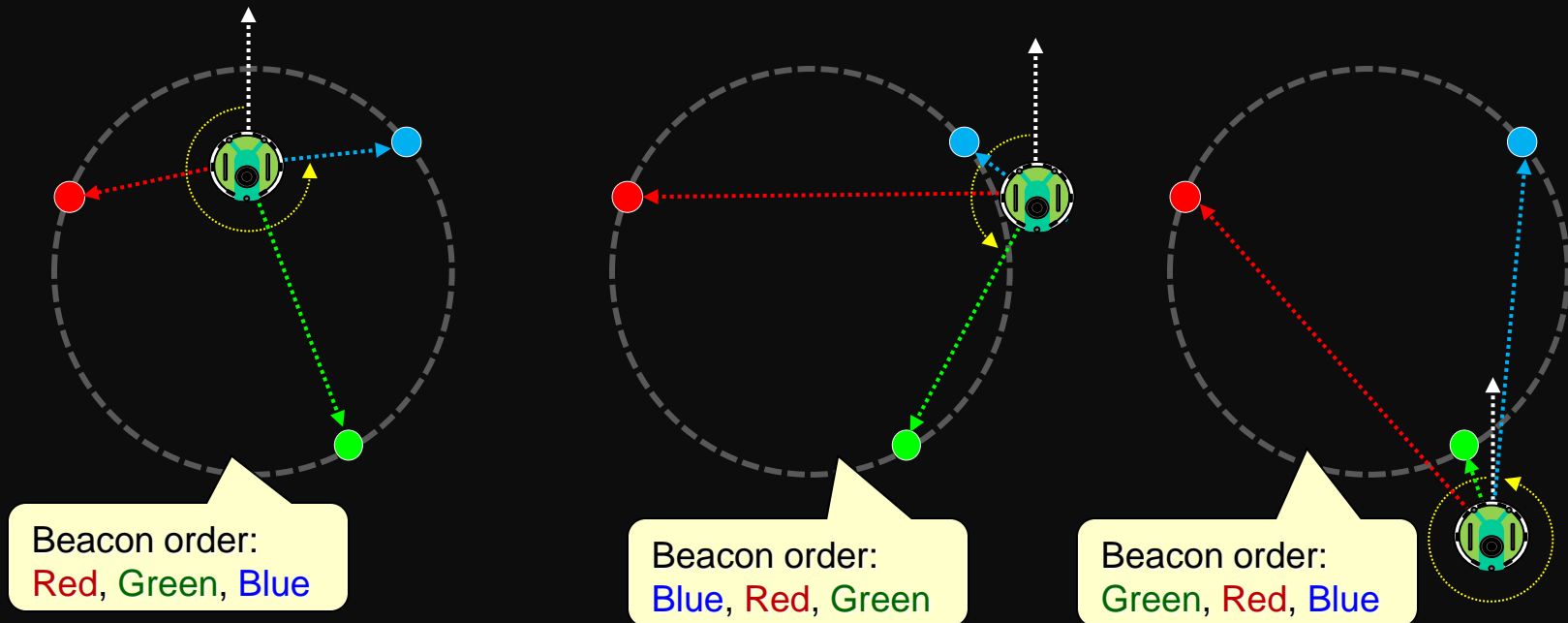
B_{green}



After a full rotation ...
if any angle
is still -999
then a
beacon was
not found ...
there is no
solution.

E-Puck – Beacon Ordering

- Keep in mind, the beacons may be encountered in a different order each time, depending on robot's location:



- In the array, just make sure that **red** goes at position **0**, **green** at position **1** and **blue** at position **2**.

E-Puck - Measuring Angles

- Use wheel position sensors to measure angle while spinning
- Need to rotate 360°
- While rotating, look for the beacons

spinRadians is the number of radians that the wheels should turn in order for the robot to spin a full circle.

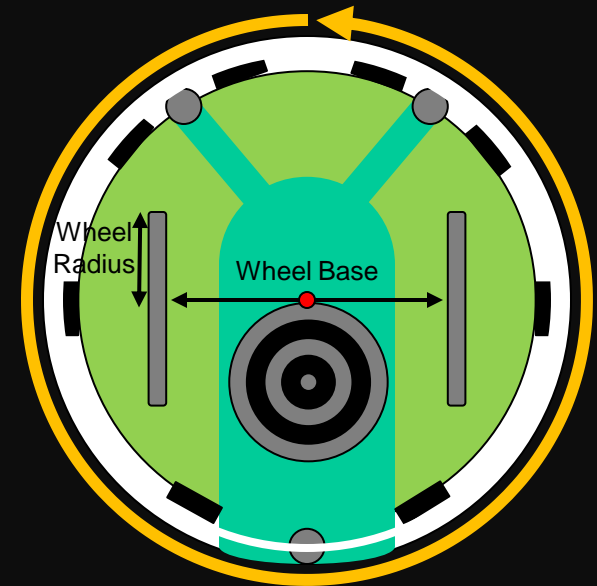
```
spinRadians = PI * WHEEL_BASE / WHEEL_RADIUS
Start rotating robot counter-clockwise
reading = 0;

while (reading < spinRadians) {
    reading = readWheelSensor - previousReading;

    // CODE TO LOOK FOR BEACON GOES HERE
    // STORE THE THREE ANGLES  $\theta_x$ ,  $\theta_g$  and  $\theta_b$ 
}

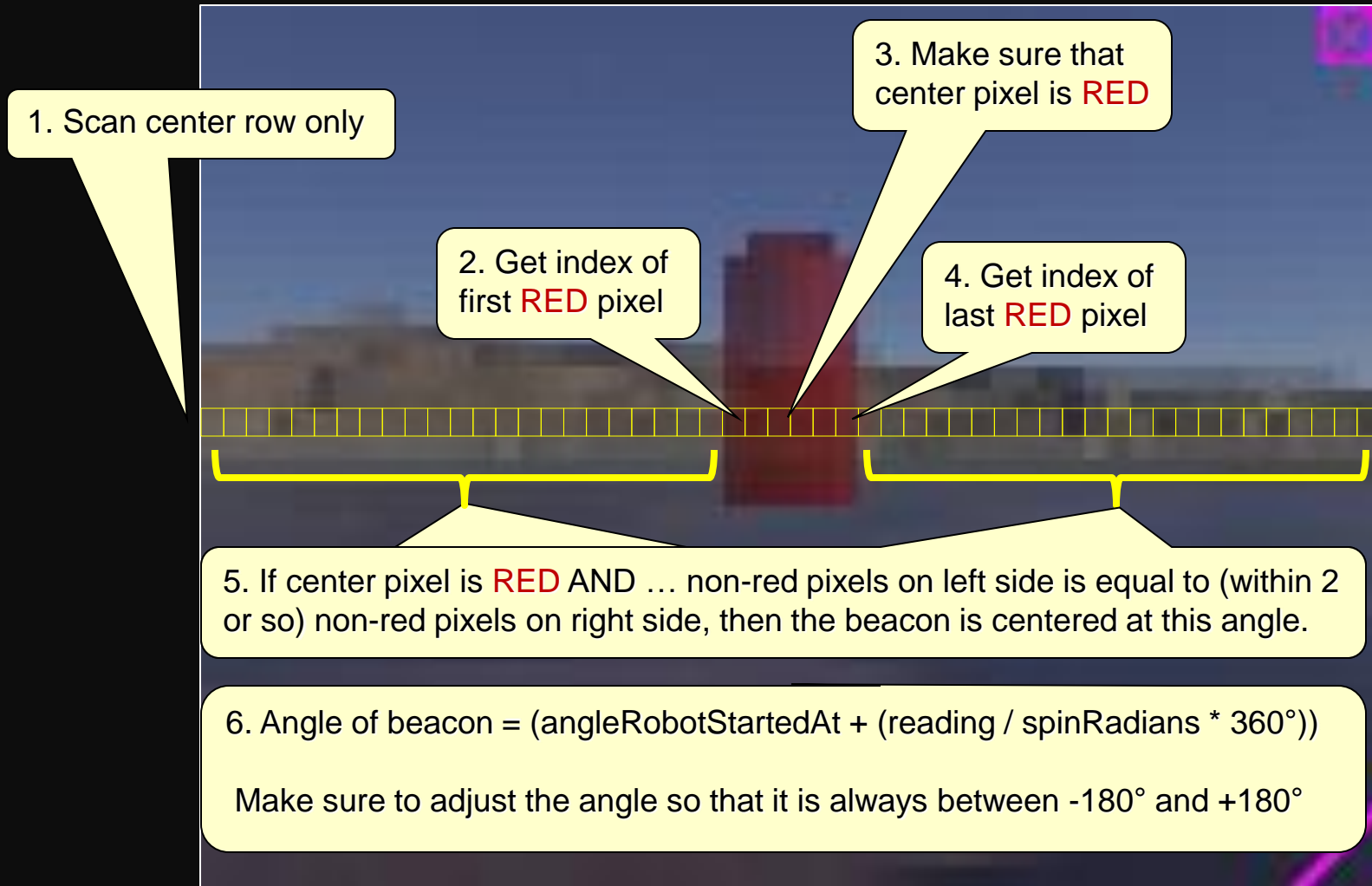
Stop rotating robot
previousReading = readWheelSensor;

Compute the position (x,y) using triangulation method
```



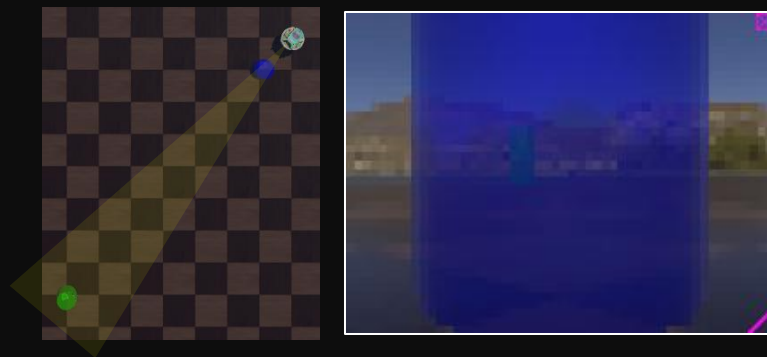
E-Puck - Measuring Angles

- How to check if a beacon is centered at this angle:



E-Puck - Measuring Angles

- You will need to check individually for the green and blue beacons in the same manner.
- It is possible that robot cannot see one of the beacons.
- It is also possible that the robot may see a beacon two or three times ... so just remember the first time it sees it centered (e.g., use a **boolean** flag **foundRed**)



When robot spins CCW, beacon appears to move right





**Start the
Lab ...**