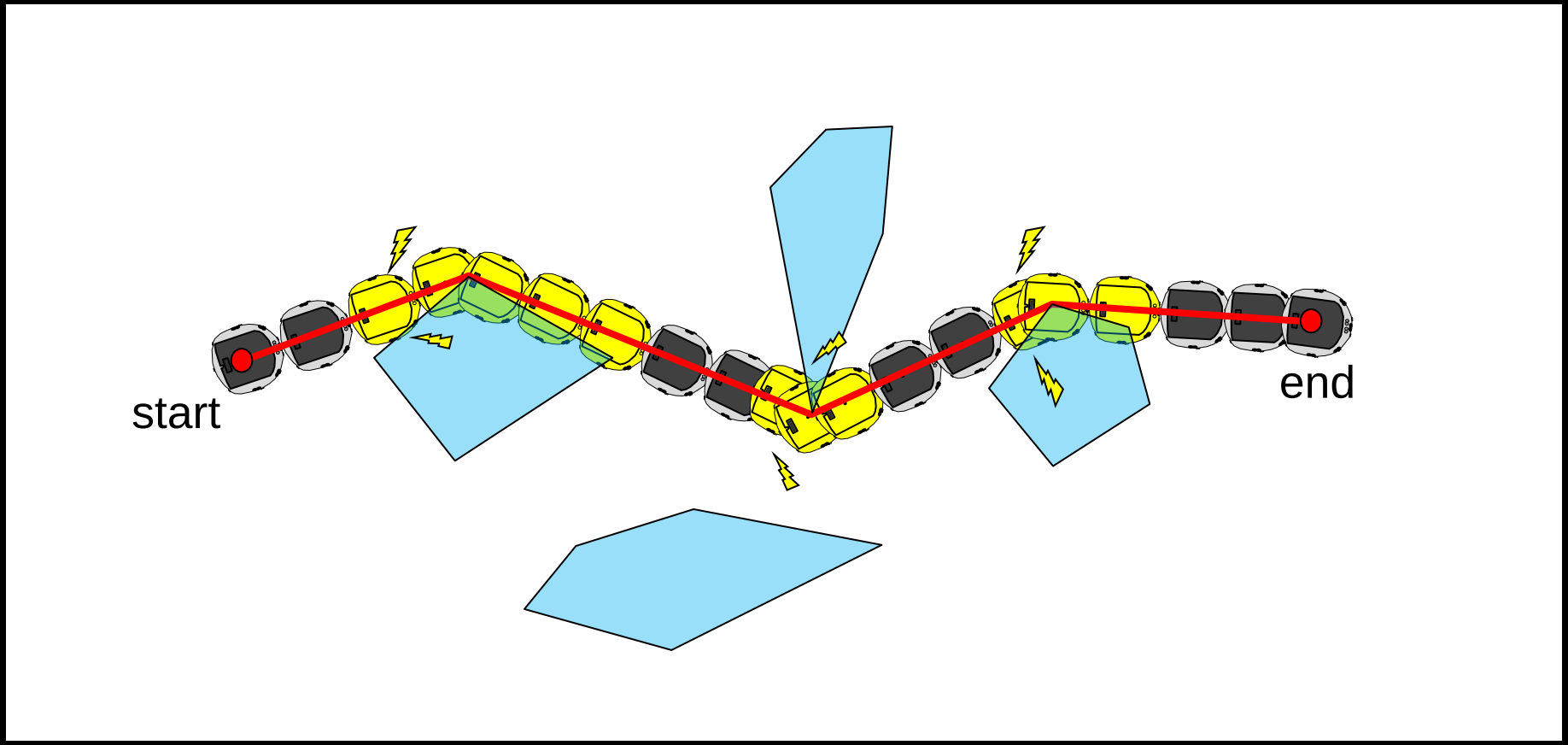
Grown Obstacle Space

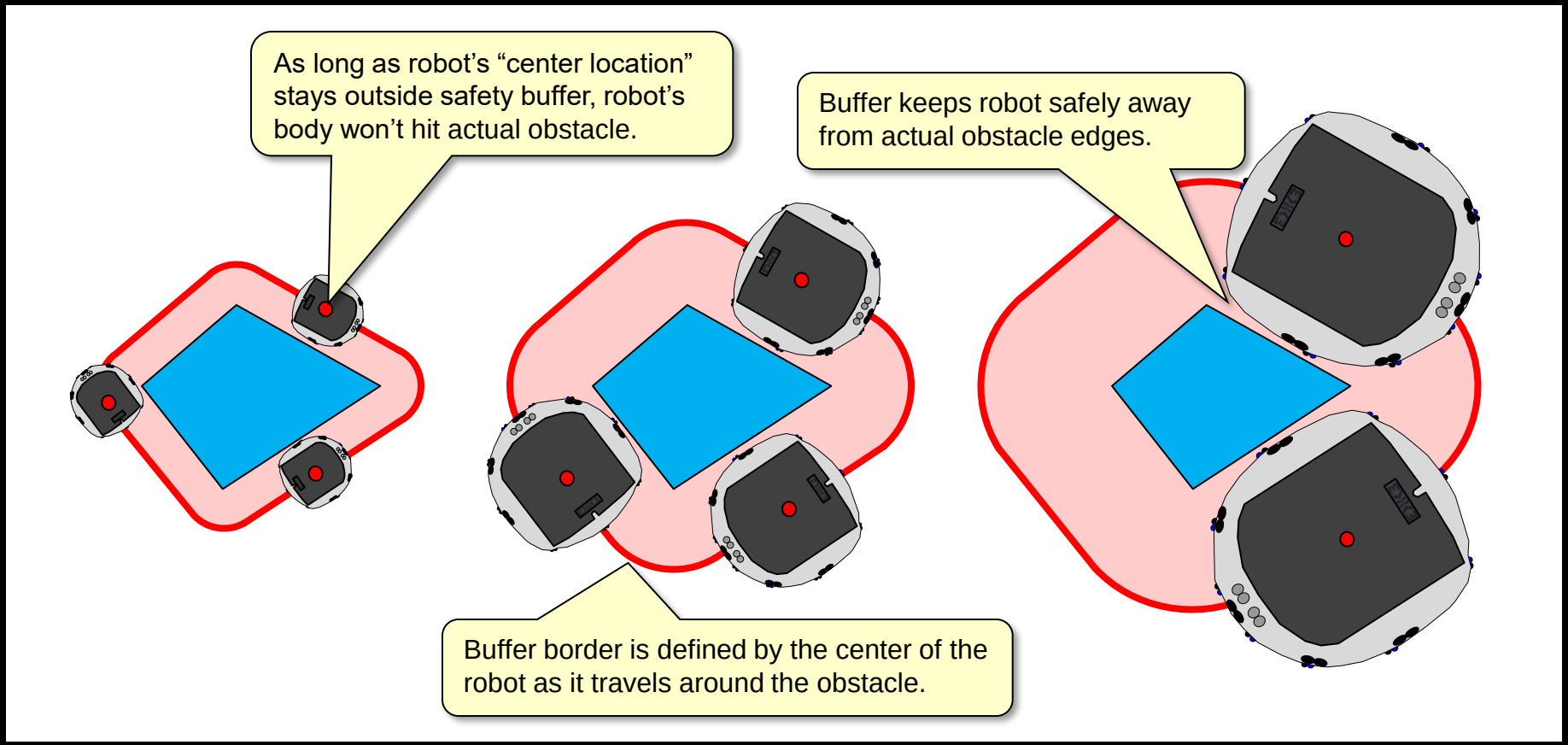
Shortest Path Problems

- Any real robot will collide with obstacles if it travels along our computed shortest path because we assumed that the robot was just a point, but a robot has a shape:



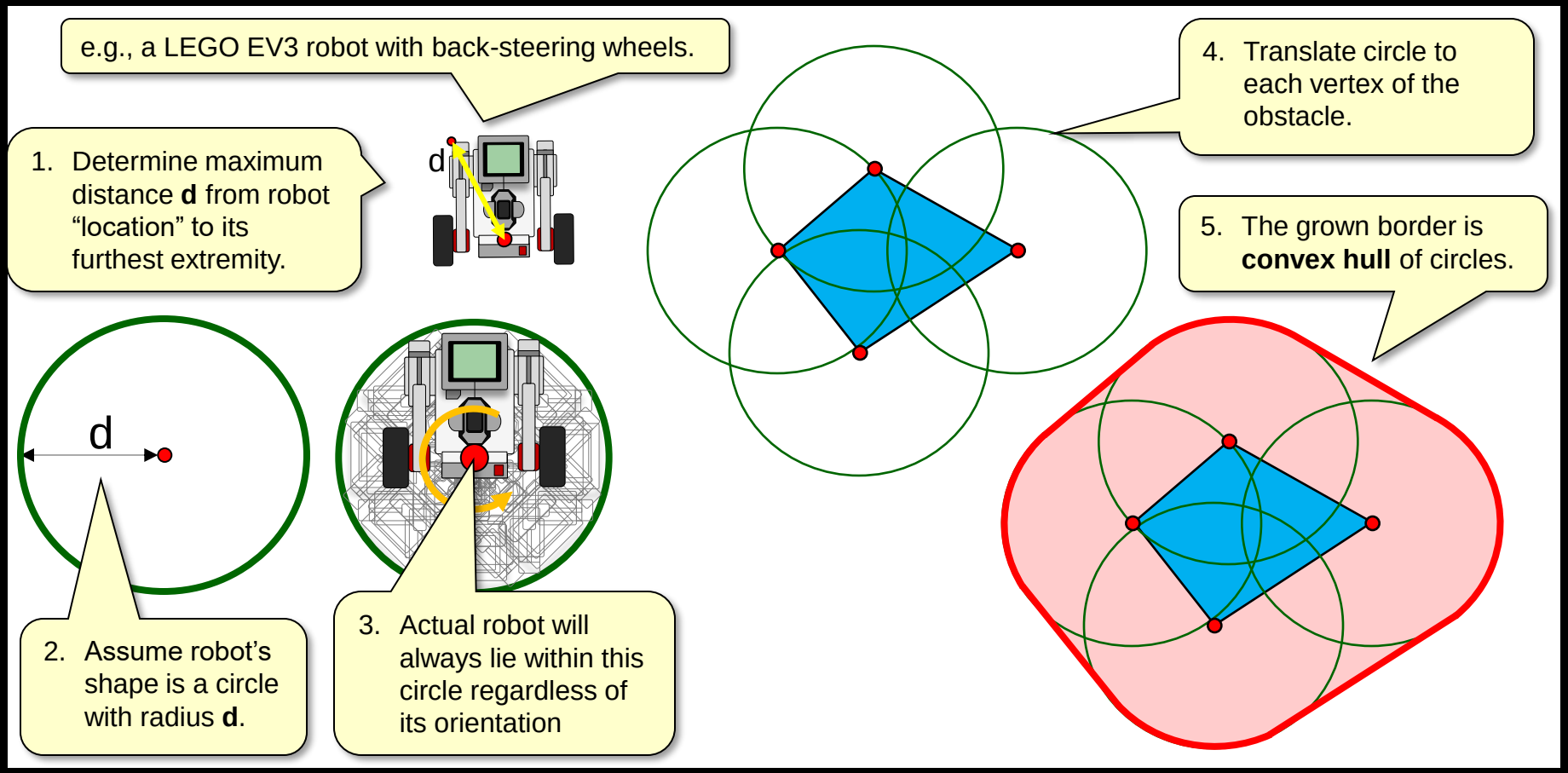
Safety Buffers

- Need to have a “safety buffer” around the obstacles which takes into account robot’s size and shape.



Grown Obstacle Space

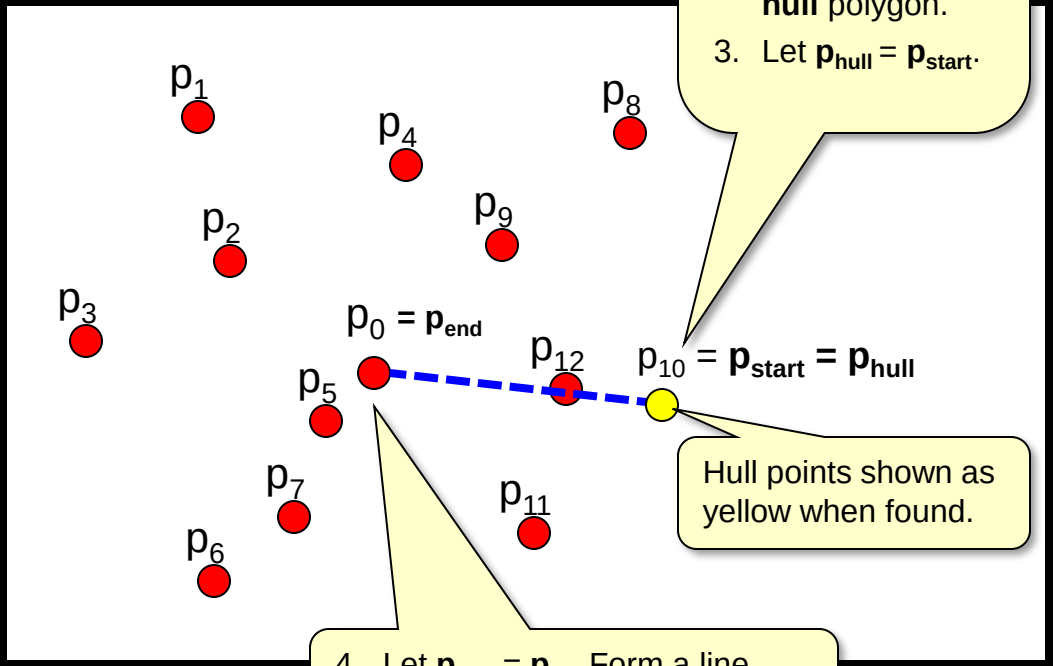
- Must "grow" obstacles by amount that considers any orientation of robot at any point on obstacle border:



Computing the Convex Hull

- Algorithms exist to find convex hull of a set of points
  - Graham Scan (a.k.a. “Gift-Wrapping” method) is a simple one:

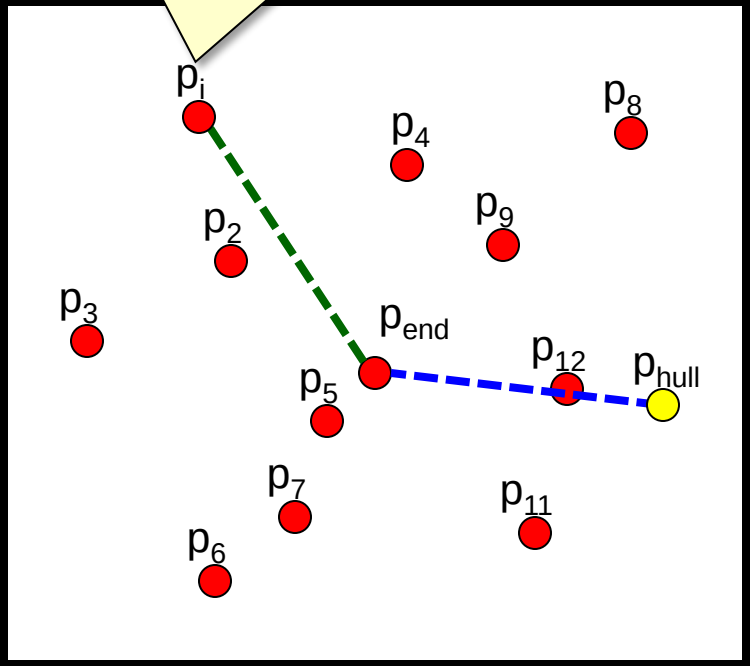
1. Start with rightmost point, call it $\mathbf{p_{start}}$.
2. Add $\mathbf{p_{start}}$ to the **hull** polygon.
3. Let $\mathbf{p_{hull}} = \mathbf{p_{start}}$.



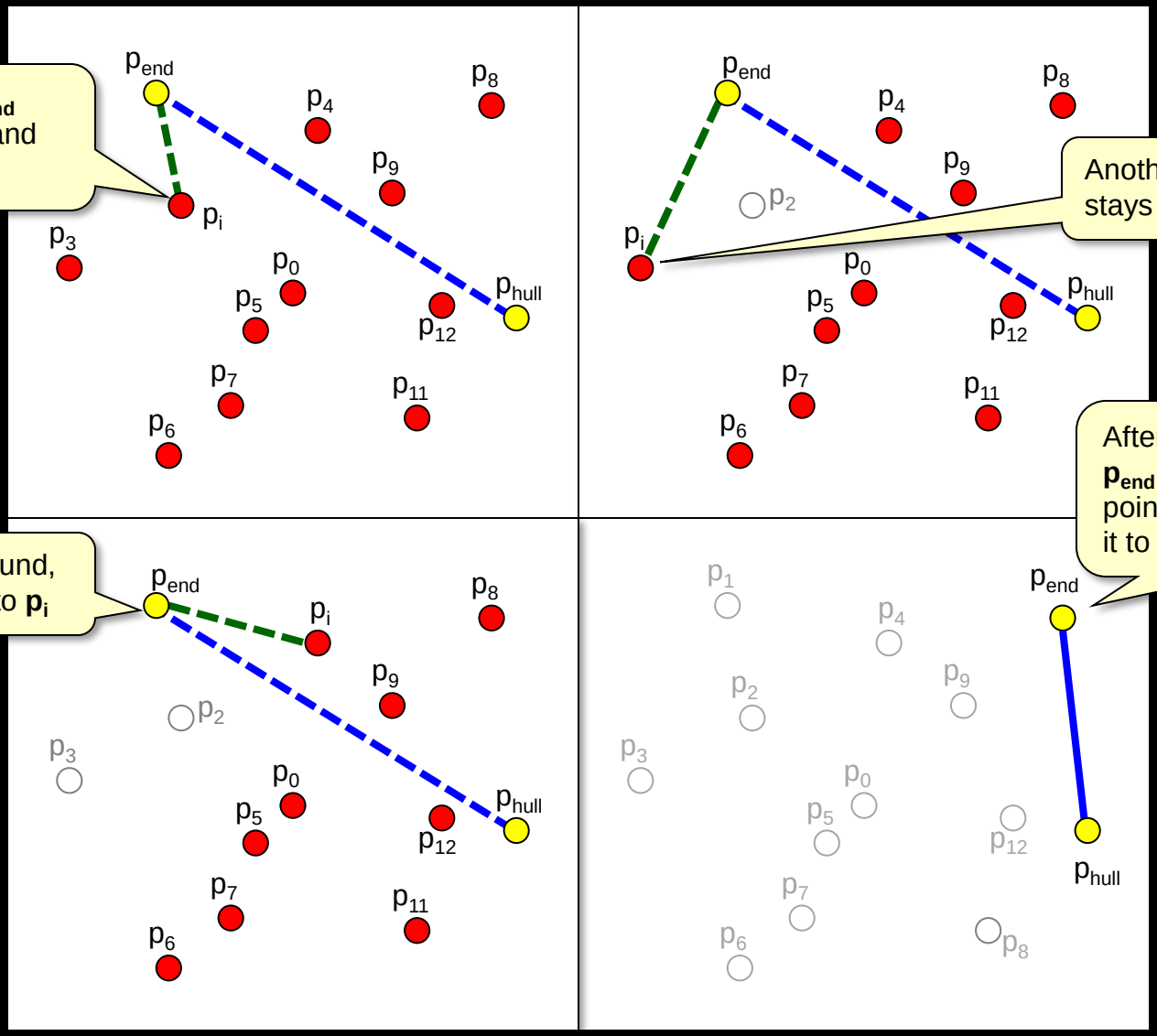
Hull points shown as yellow when found.

4. Let $\mathbf{p_{end}} = \mathbf{p_0}$. Form a line segment from $\mathbf{p_{hull}}$ to $\mathbf{p_{end}}$.

5. Check each point $\mathbf{p_i}$ to see if $\overrightarrow{\mathbf{p_{hull}}\,\mathbf{p_{end}}\mathbf{p_i}}$ is a right turn. If so, then set $\mathbf{p_{end}} = \mathbf{p_i}$ and continue checking the remaining points.

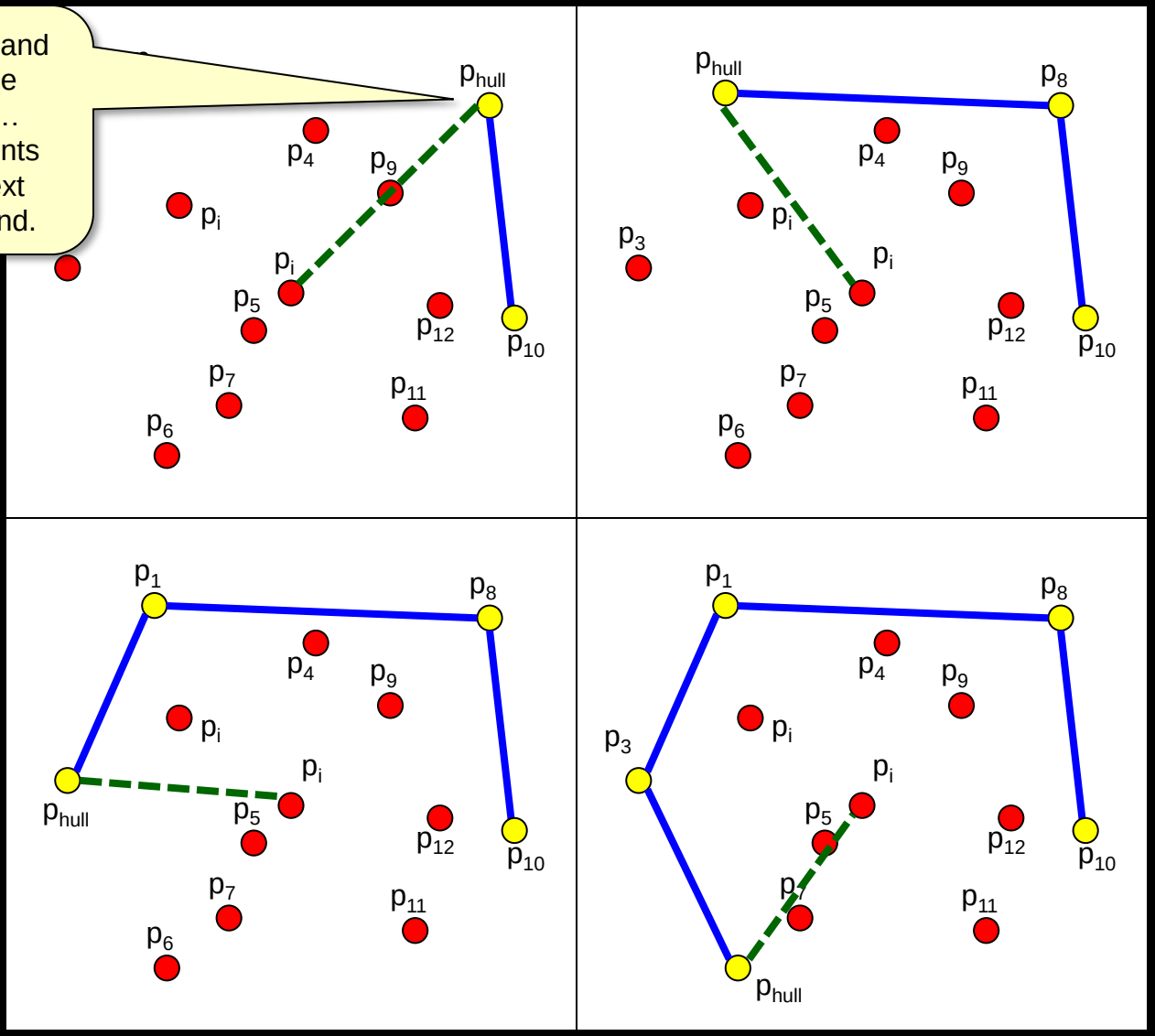


Computing the Convex Hull

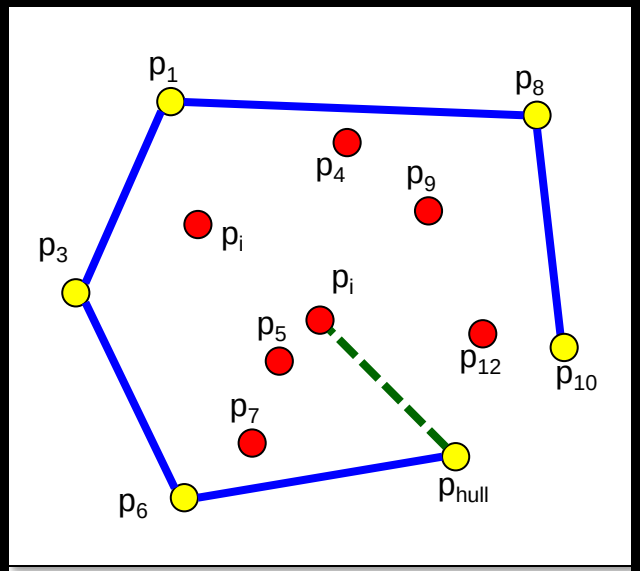


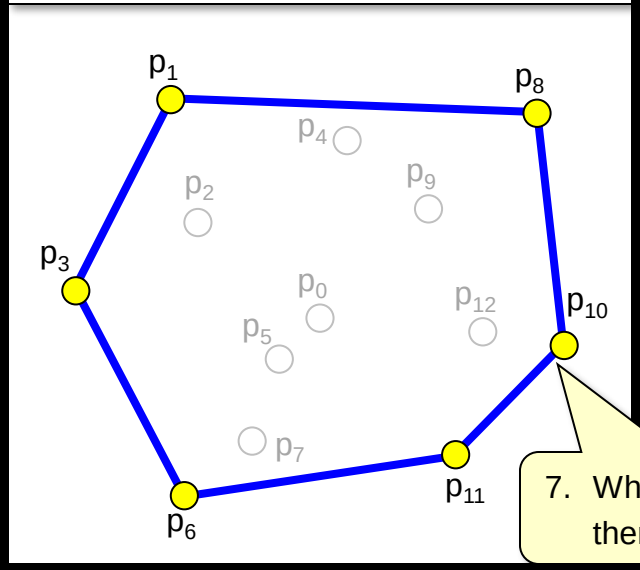
Computing the Convex Hull

6. Set $\mathbf{p_{hull}} = \mathbf{p_{end}}$ and repeat the whole process again … checking all points in order until next hull point is found.



Computing the Convex Hull





7. When $\mathbf{p_{end}} = \mathbf{p_{start}}$ then we are done.

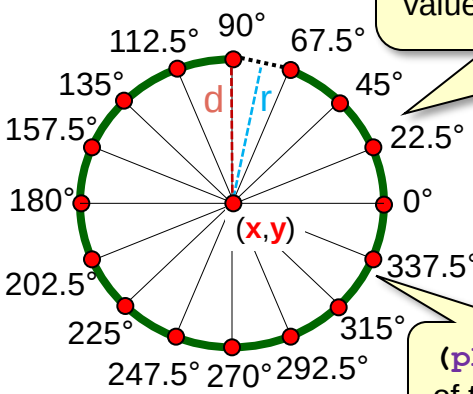
```
1   FUNCTION ConvexHull(PointList points)
2       hull = an empty list
3       pHull = the rightmost point in points
4       pStart = pHull
5       pEnd = NULL
6       WHILE (pEnd is not equal to pStart) DO
7           Add pHull to hull
8           pEnd = the first point in points
9           FOR (each point pi in points) DO
10              IF (pHull is the same as pEnd) THEN
11                  pEnd = pi
12              IF (angle pHull to pEnd to pi is a right turn) THEN
13                  pEnd = pi
14          pHull = pEnd
15      RETURN hull
```

Special case

it is a right turn only if
$((pEnd_x - pHull_x)*(pi_y - pHull_y) - (pEnd_y - pHull_y)*(pi_x - pHull_x)) < 0$

Convex Hulls Around Obstacles

- Find Convex Hull of "points circles" around obstacle vertices:

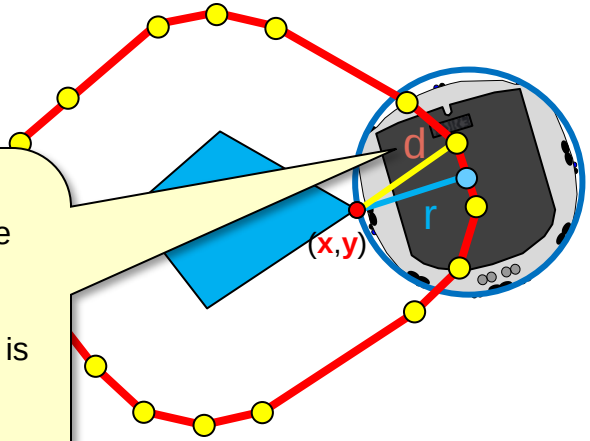


Decide upon how many points you want on the circle by setting a **DEGREE_UNIT** value (e.g., 22.5° shown here).

(pX,pY) is one of the points.

Given a robot radius of **r**, then points must be at least distance **d** from (**x**,**y**) so that robot does not collide when travelling from one vertex to another, where **d** is

$$\frac{r}{\cos\left(\frac{\text{DEGREE\_UNIT}}{2}\right)}$$

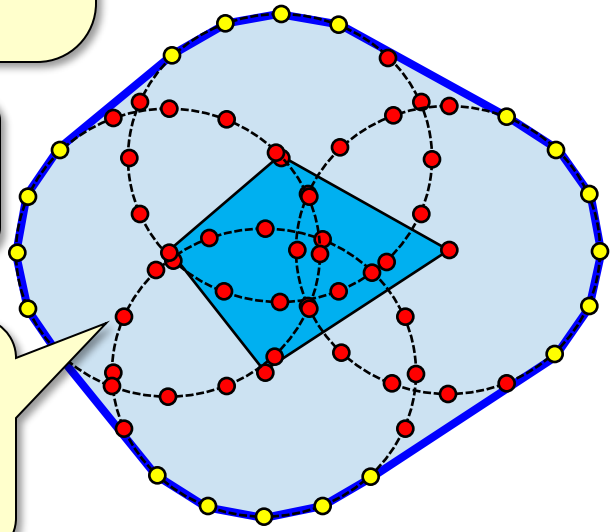



```
circlePoints = empty list

FOR (a=0; a<360; a+=DEGREE_UNIT) DO {
  xoff = d*cos(a)
  yoff = d*sin(a)
  if (xoff > 0) pX = x + ceil(xoff)
  otherwise pX = x + floor(xoff)
  if (yoff > 0) pY = y + ceil(yoff)
  otherwise pY = y + floor(yoff)
  add point (pX, pY) to circlePoints
}
```

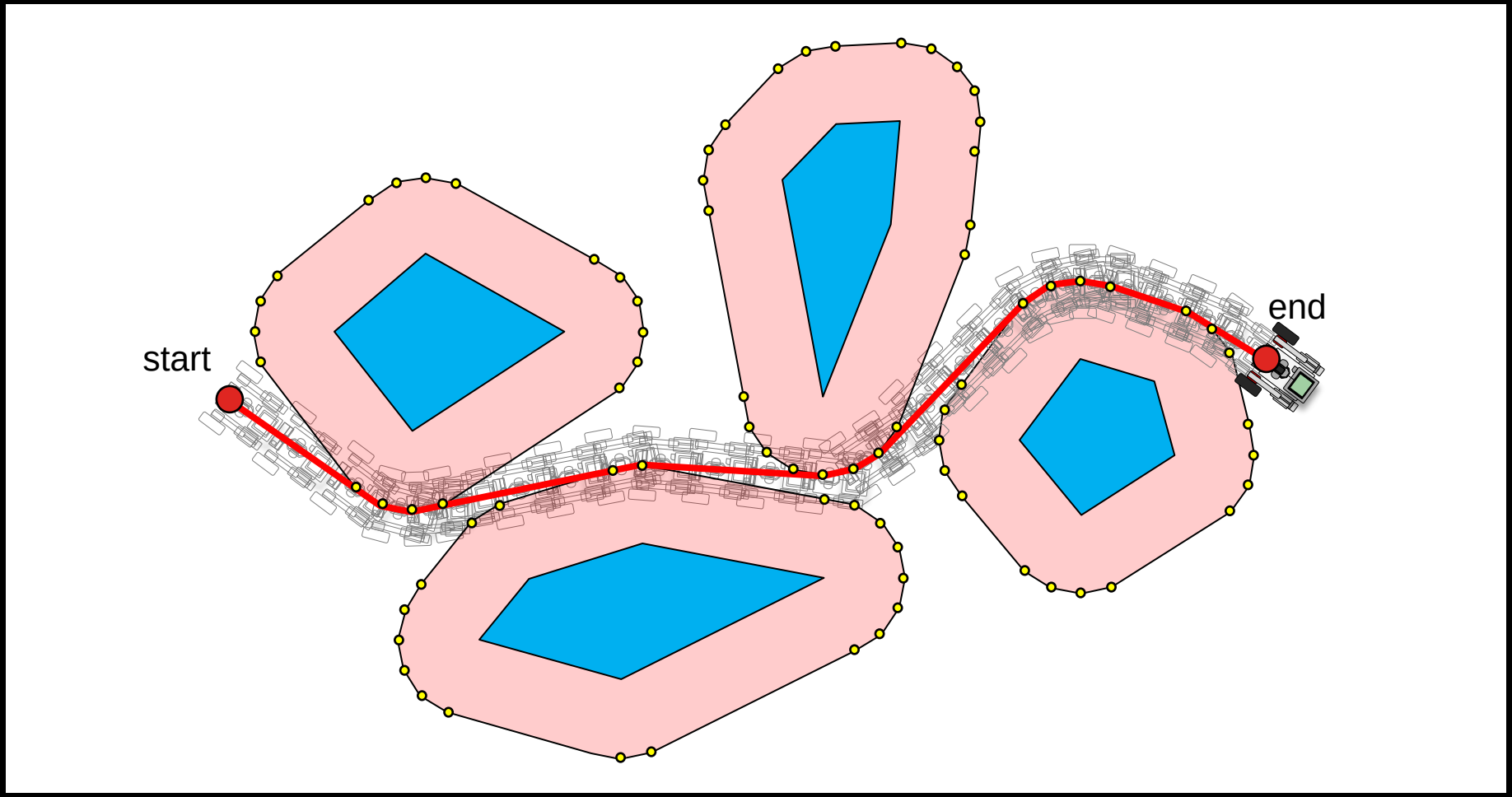
Create a list of **circlePoints** centered around an obstacle vertex (**x**, **y**) as shown here.

Merge all **circlePoints** lists from each vertex (**x**, **y**) of the obstacle. Using this "merged" list of points … find the convex hull.



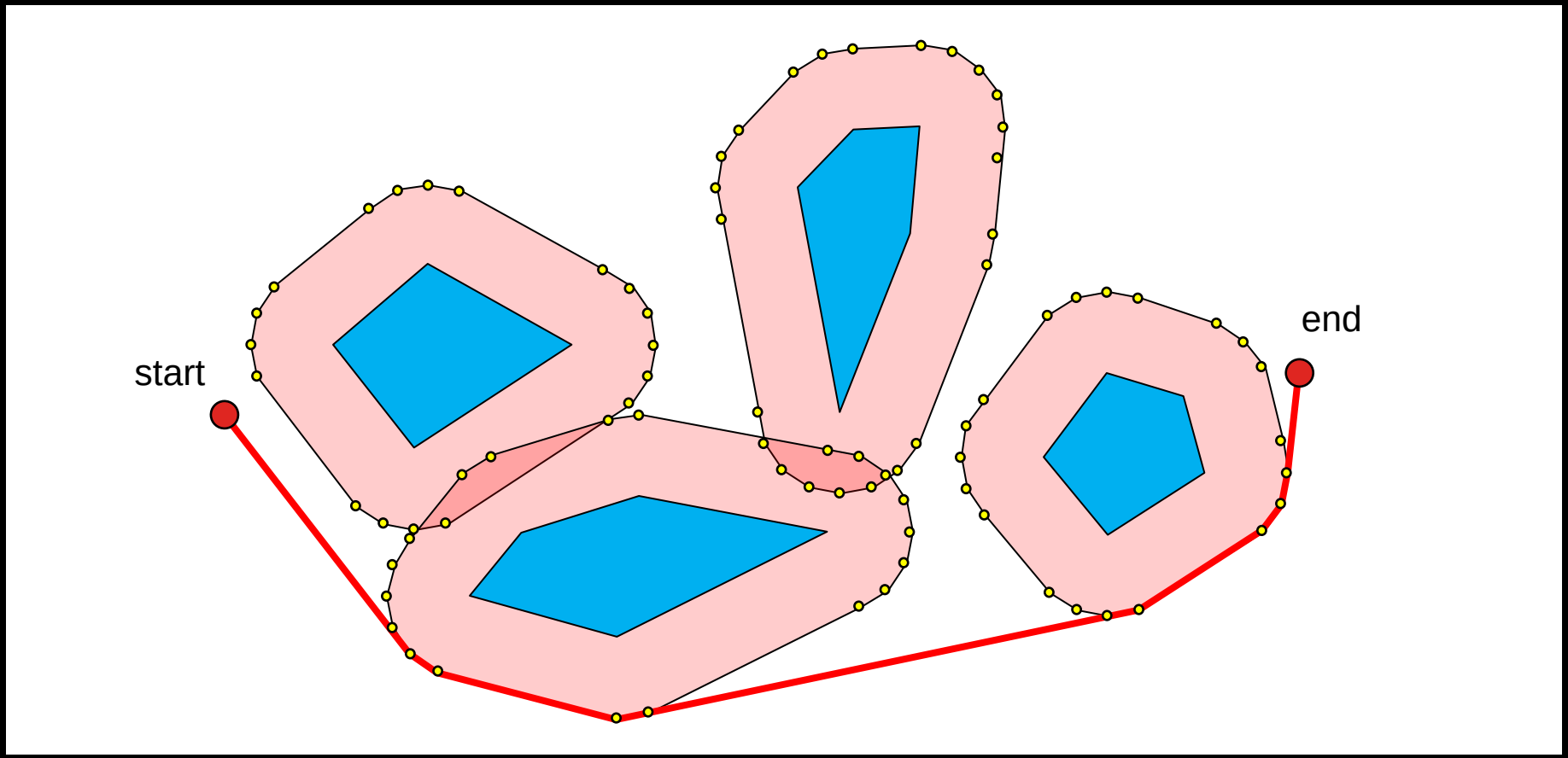
Real Robot Shortest Path

- Now the robot has collision-free travel



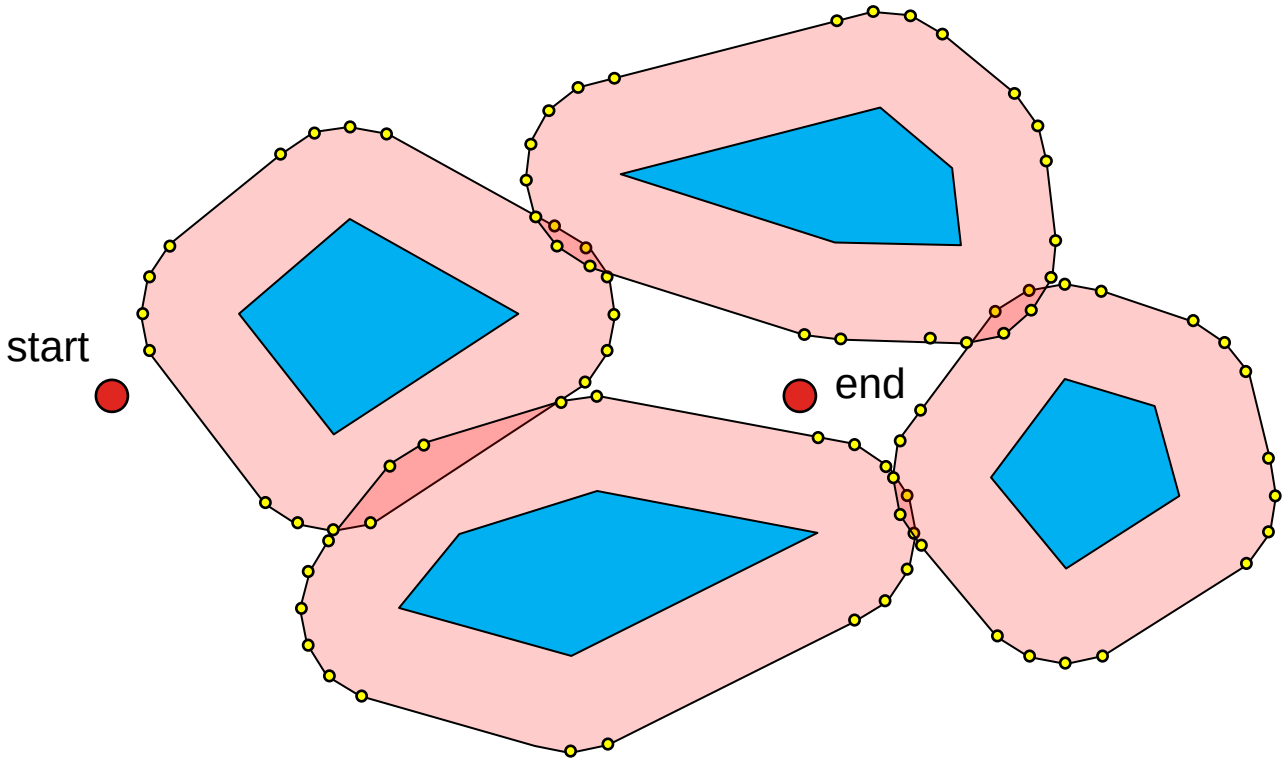
Real Robot Shortest Path

- In some cases, the grown obstacles will intersect, resulting in a different solution path.



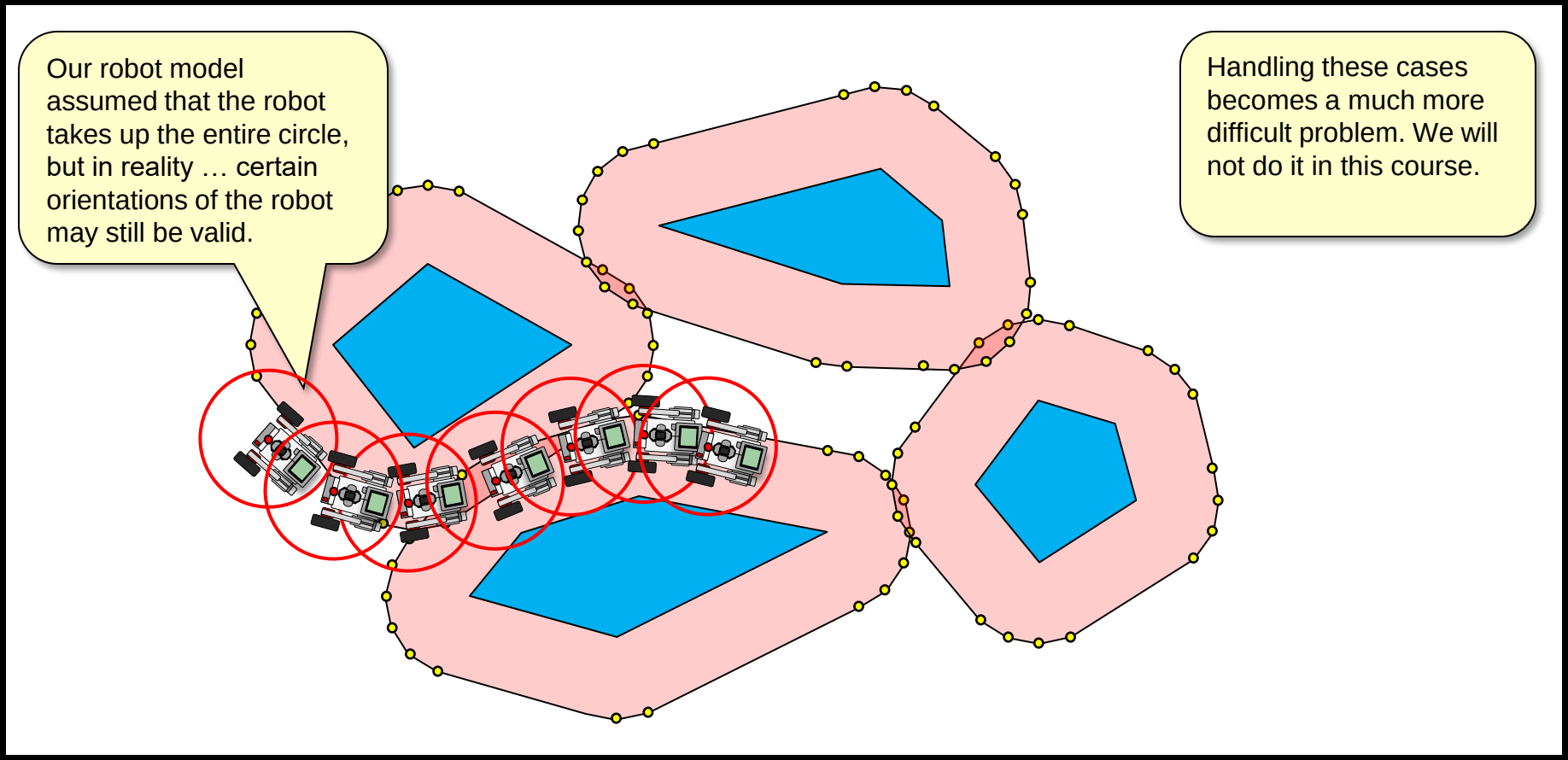
Real Robot Shortest Path

- In some cases, there may even be no solution !



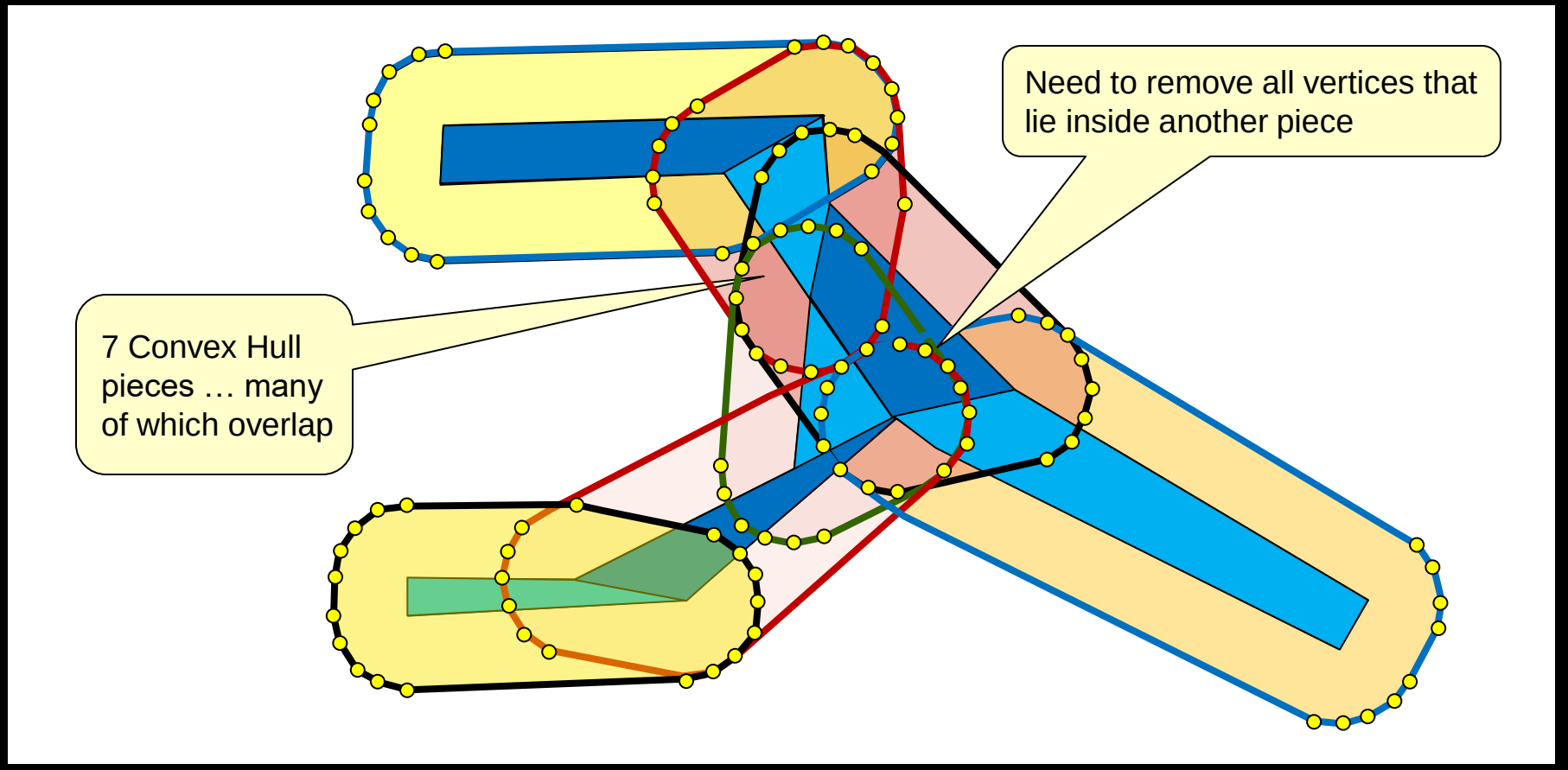
Real Robot Shortest Path

- However, this is a restriction on our robot model and algorithm, as actual robot can “fit”:



Non–Convex Solution

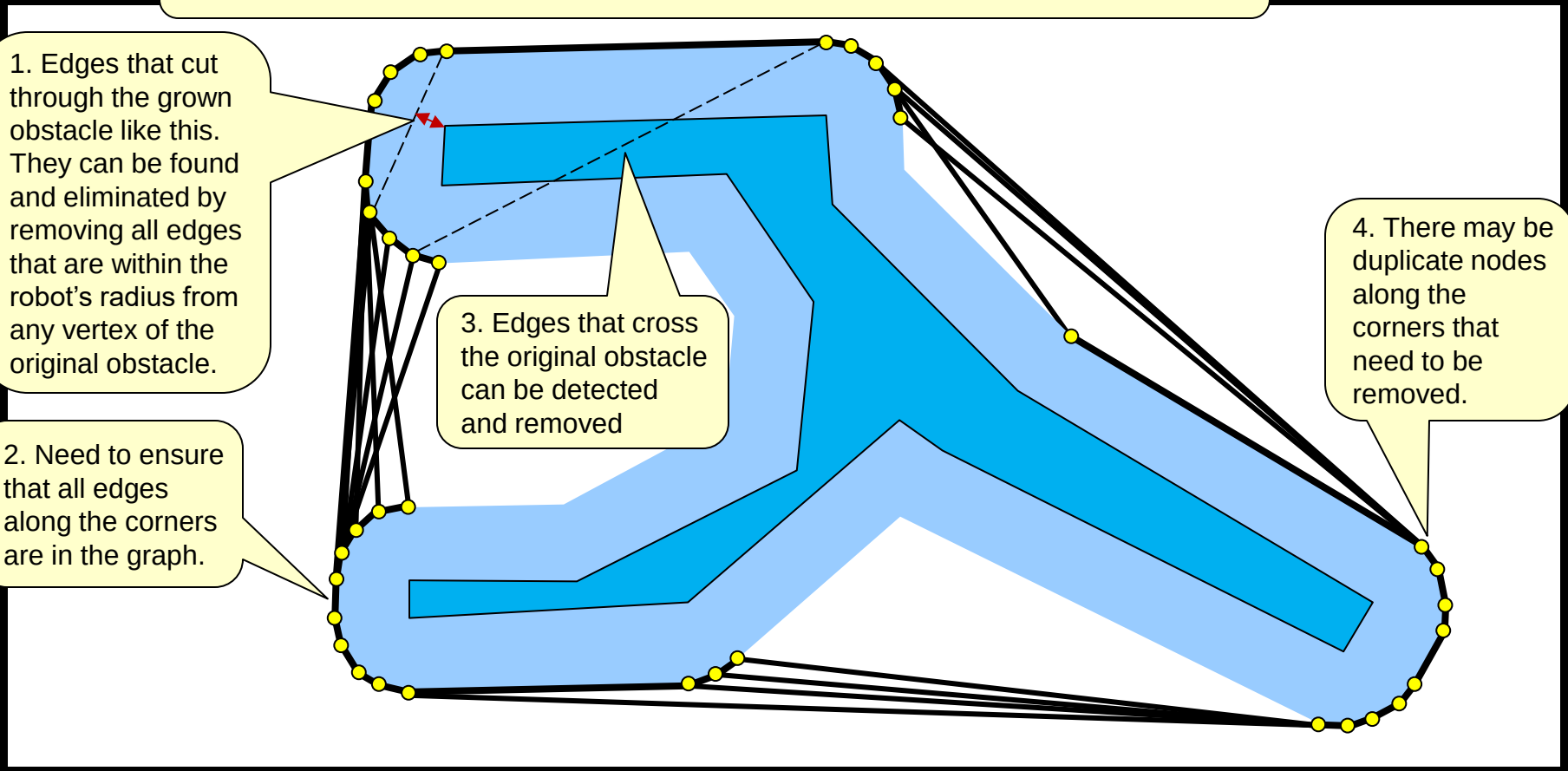
- Compute Convex Hull for each piece:



Non–Convex Challenges

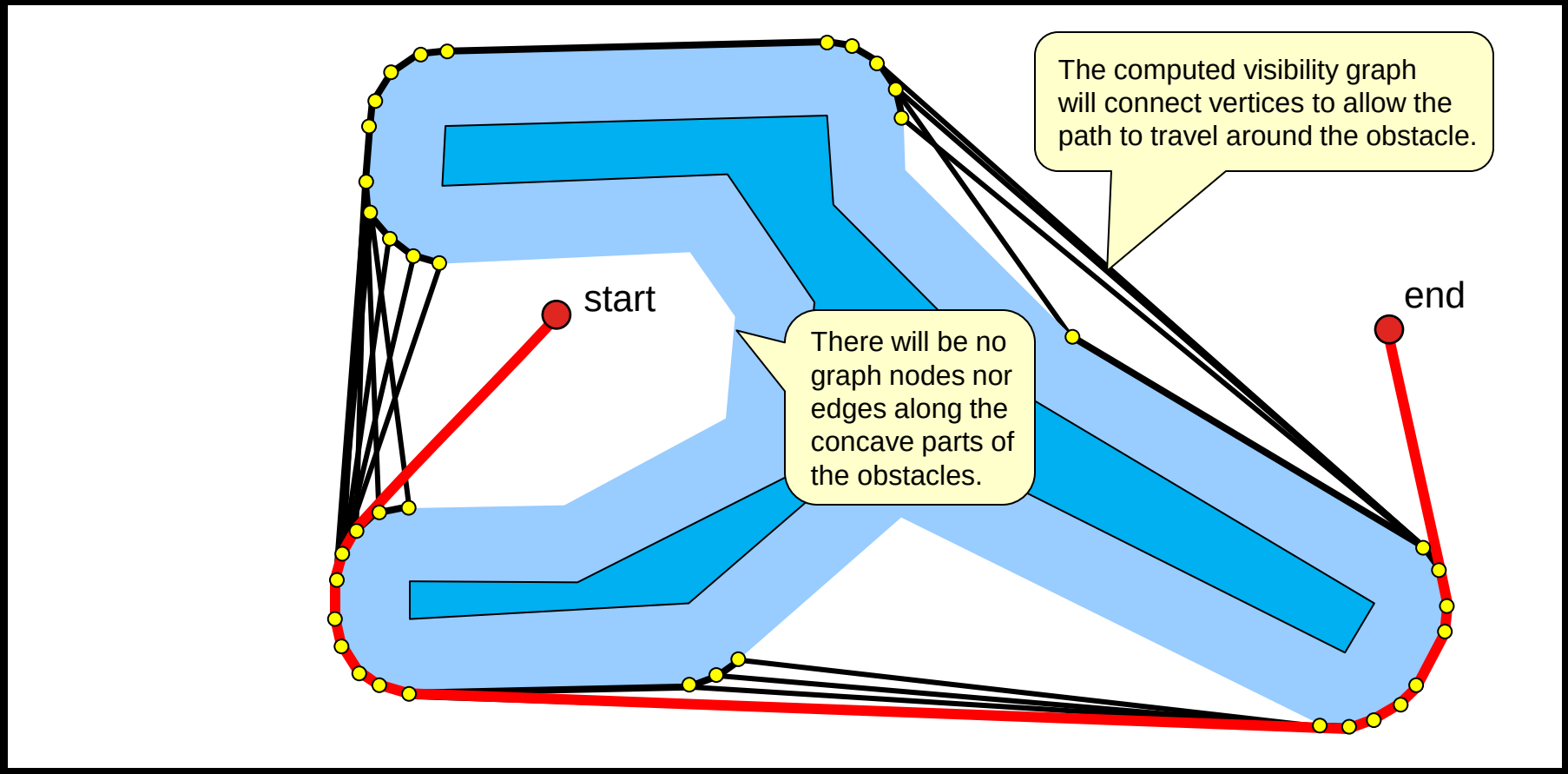
- Compute visibility graph … but it is tricky:

Unfortunately, there are a lot of intersection-related problems that pop up due to the overlapping of the various convex pieces. This will require the coding of special cases.



Non–Convex Shortest Path

- If done correctly, the shortest path can be found:



Start the Lab …