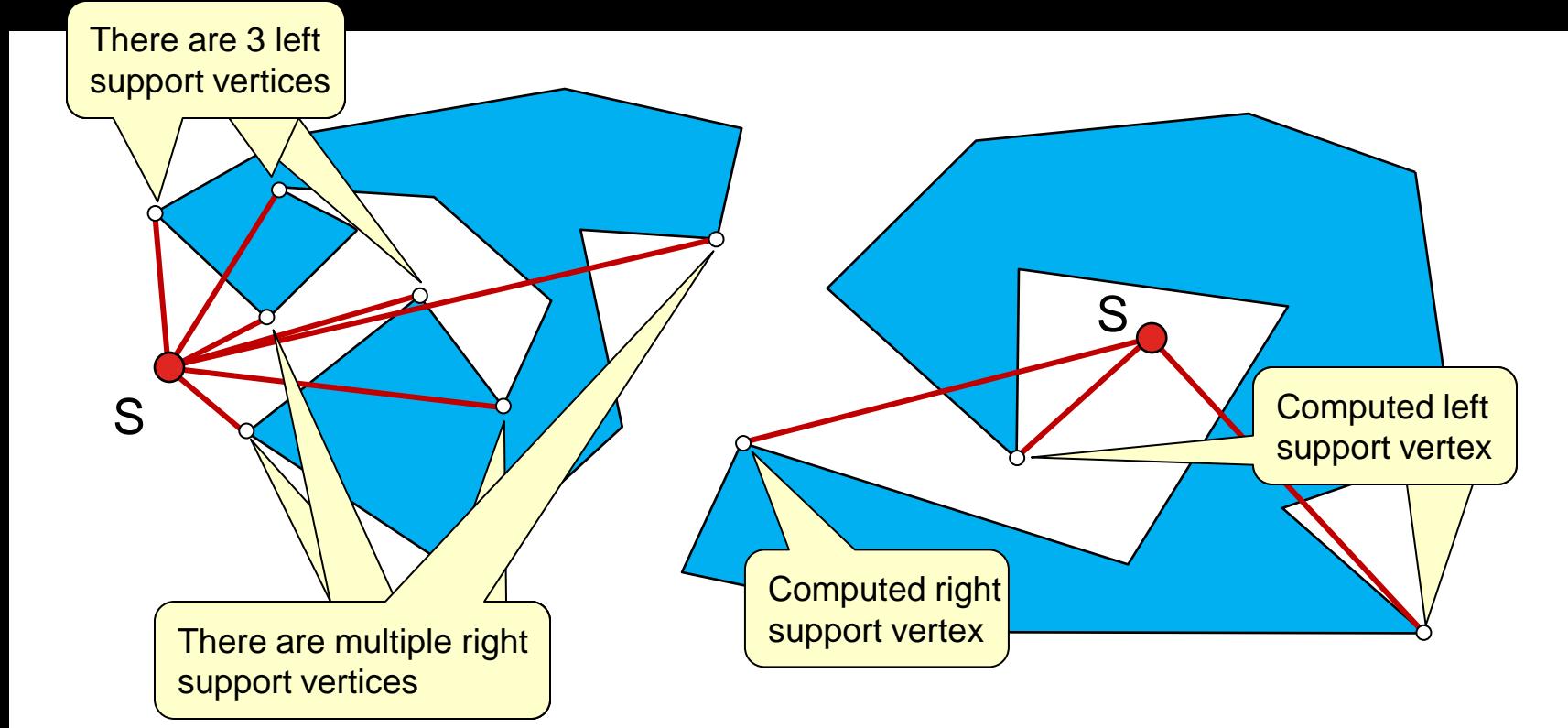


# Path Planning (Non-Convex Obstacles)

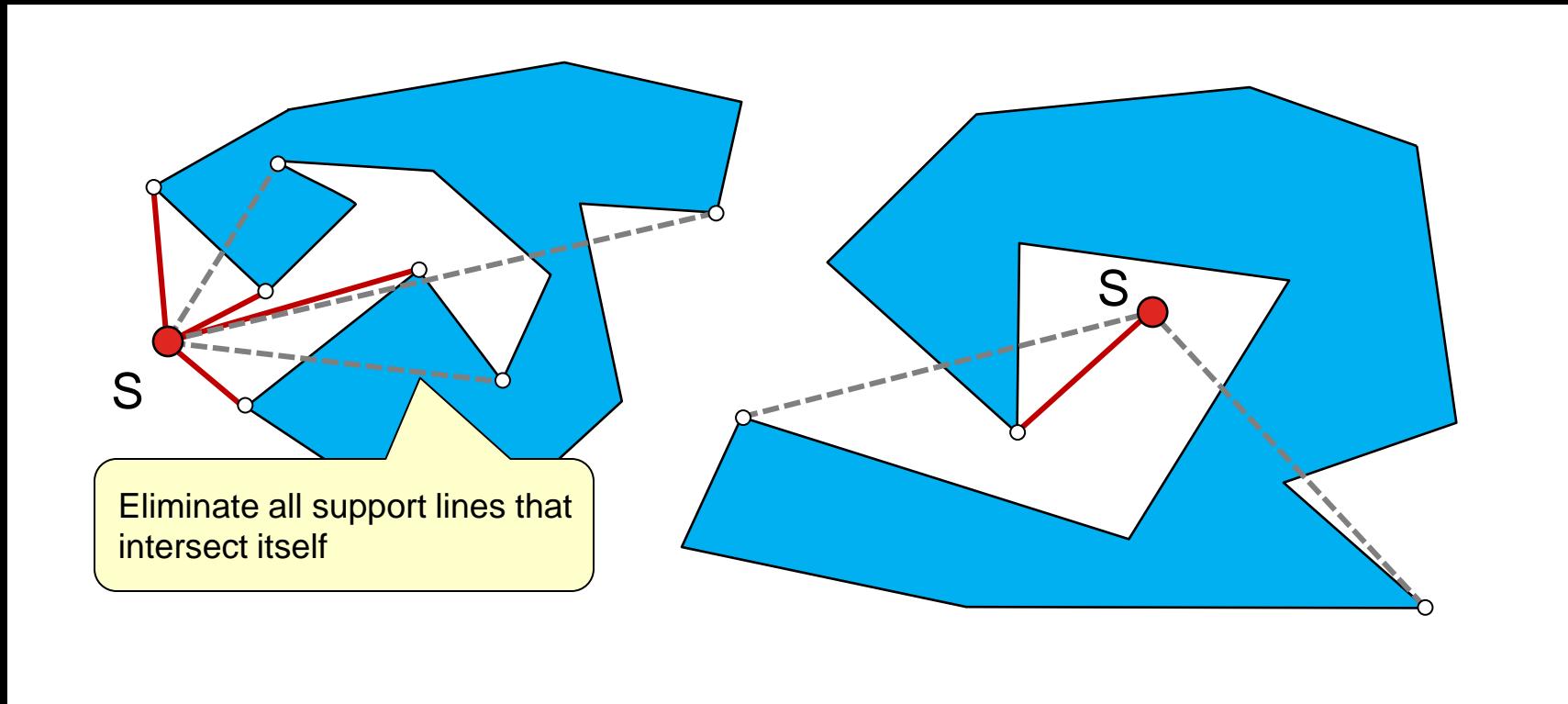
# Non-Convex Obstacle Supports

- Our support-line algorithm can produce multiple support lines per polygon if it is non-convex:



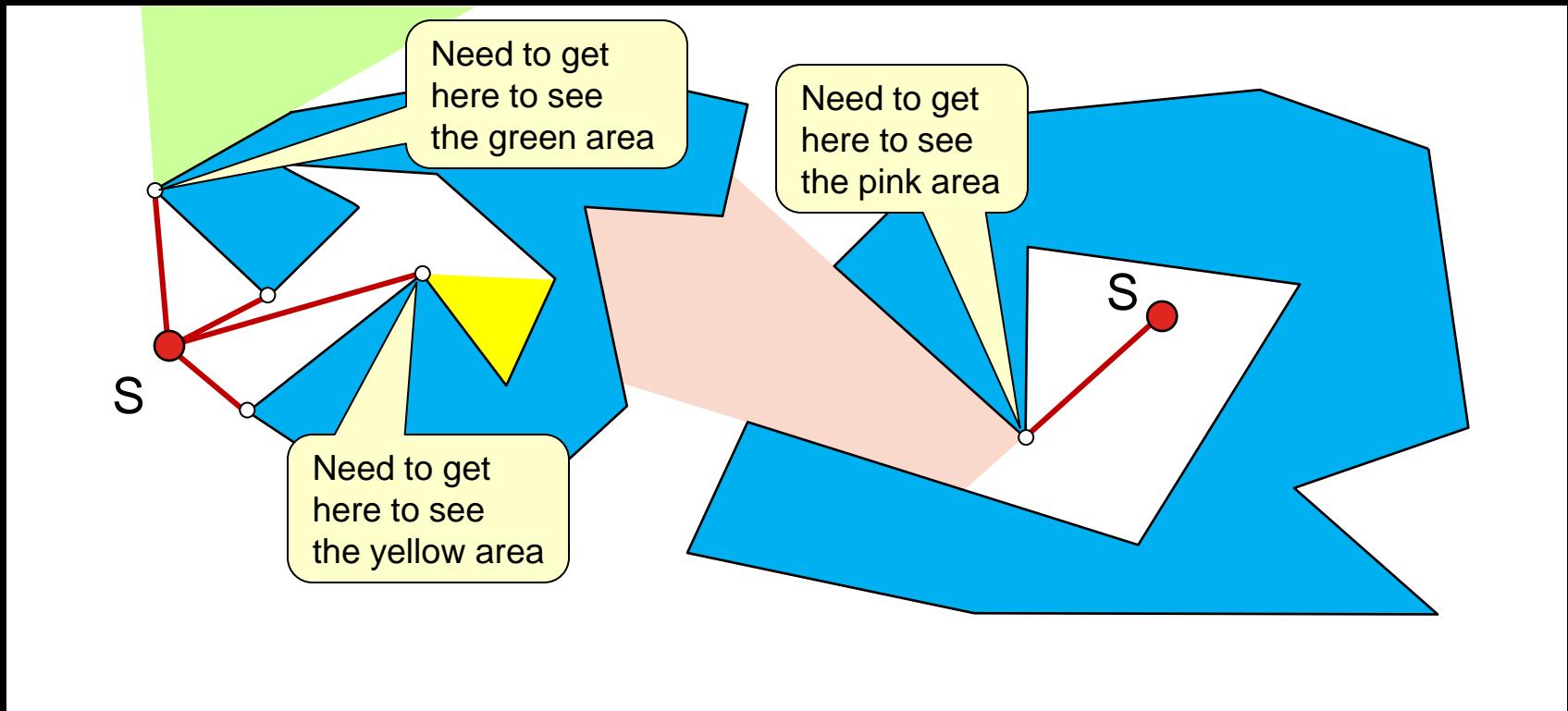
# Non-Convex Obstacle Supports

- We can eliminate those that intersect itself as well as those that intersect other polygons.



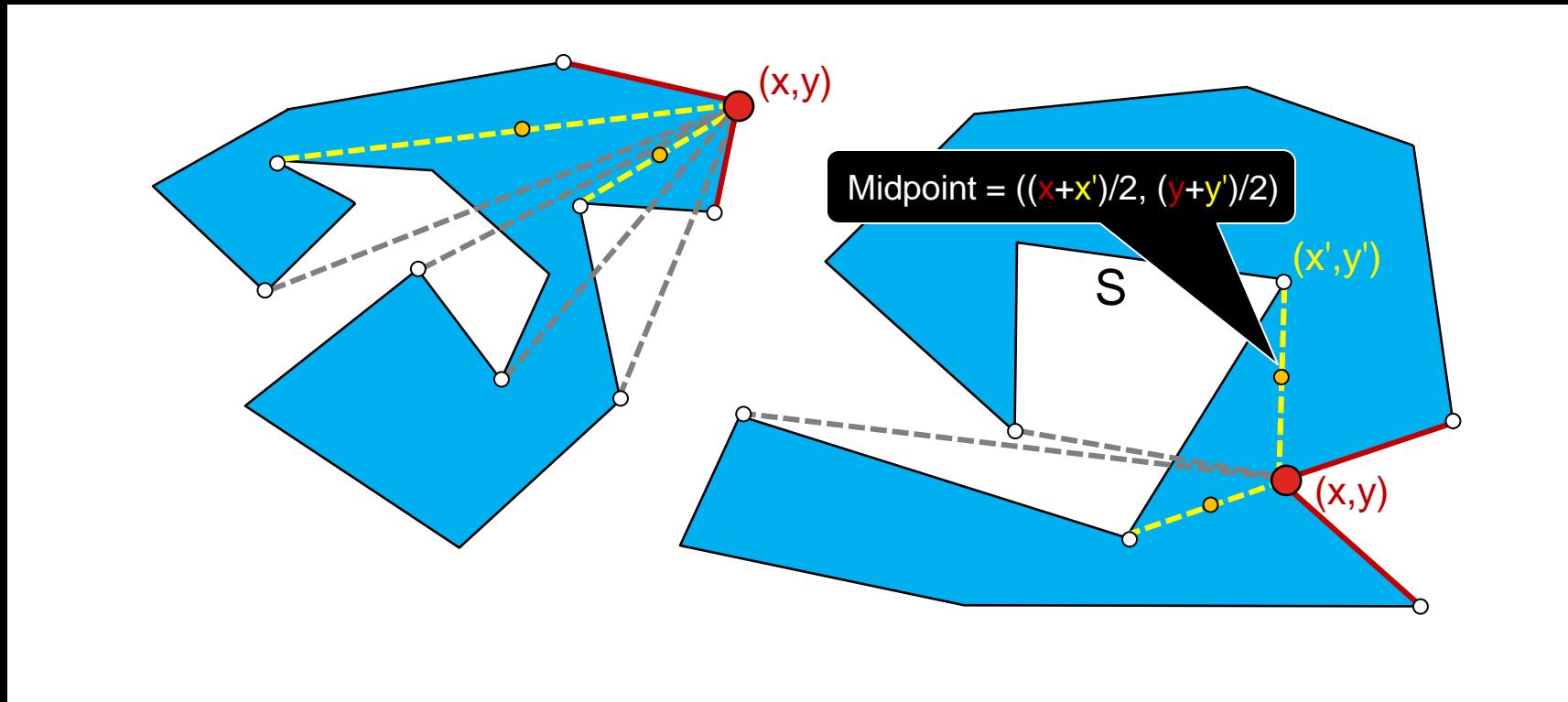
# Non-Convex Visibility

- A support vertex represents an “observation point” that robot must be at in order to “see” (i.e., have visibility) around a corner. They are all necessary.



# Non-Convex Problems

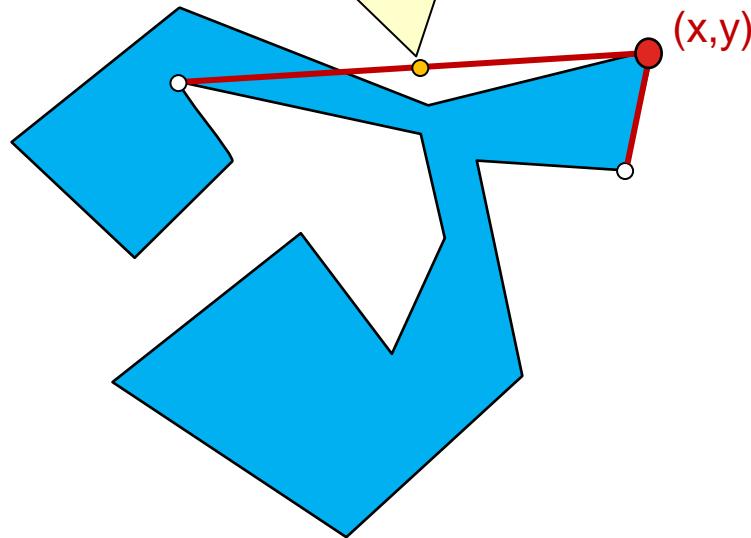
- Some “inner” support lines intersect edges only at vertices (see yellow below) and these are invalid.
- Perhaps we could check if midpoint of line lies within obstacle



# Non-Convex Problems

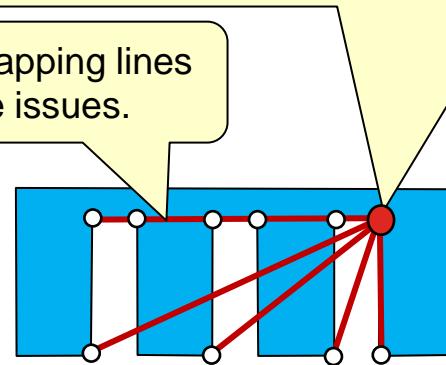
- But there are cases where this will not work:

Midpoint of this inner support line lies outside of the polygon, so it cannot be detected as invalid.



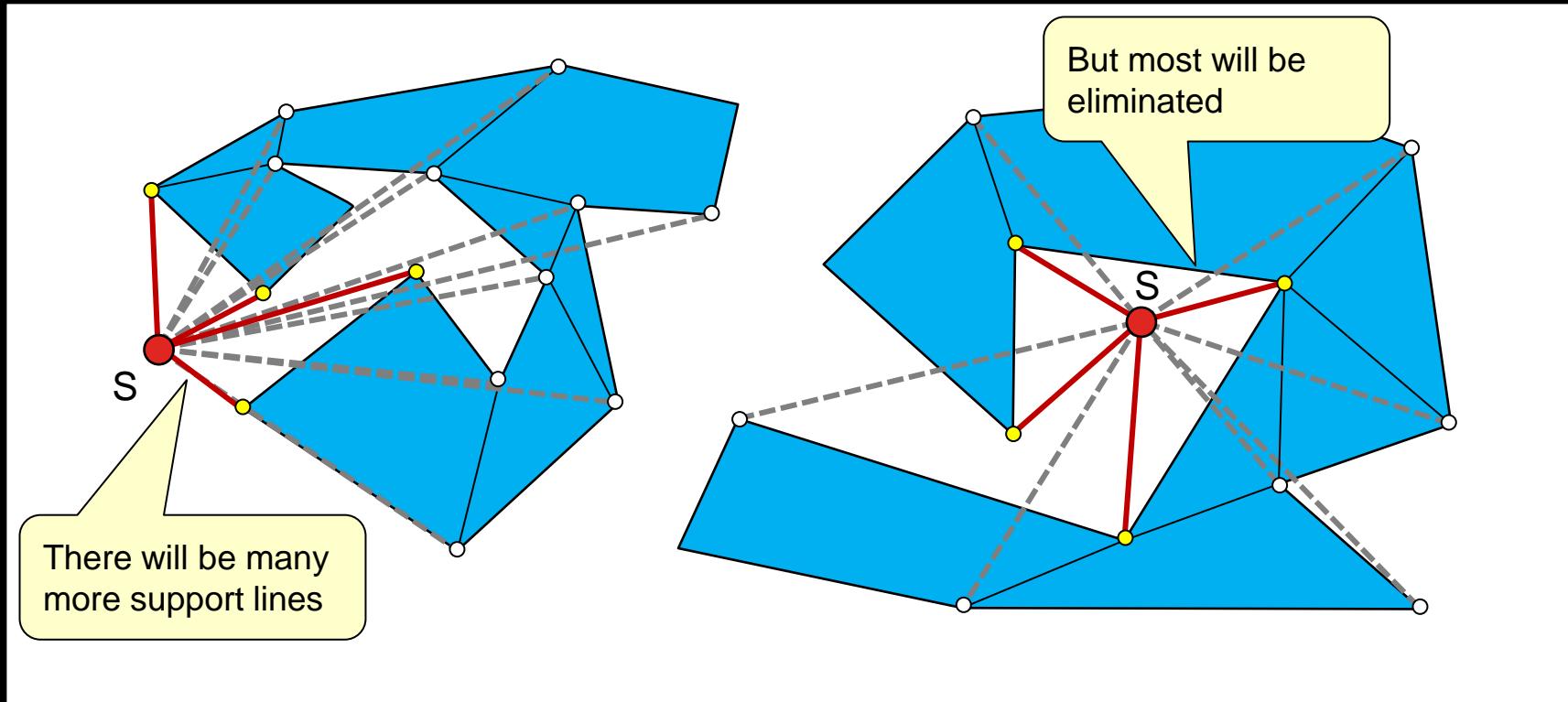
From this vantage point, there are multiple support lines ... many of them overlap due to the collinearity.

Overlapping lines cause issues.



# Non-Convex Easy Solution

- Easiest solution is to break up the non-convex polygons into convex pieces. Then the algorithm from before should work.

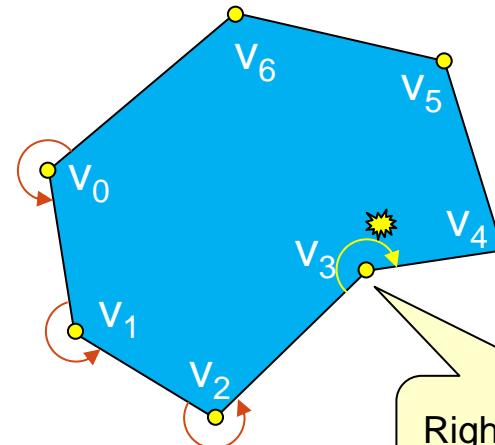
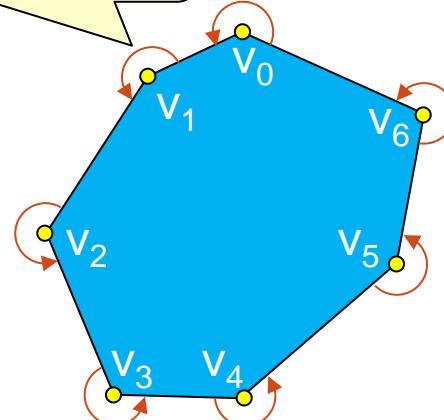


# Convexity Checking

- How do we know if a polygon is convex or not ?
  - traverse vertices CCW ... all must make **left** turns:

All **left** turns indicates that polygon is convex.

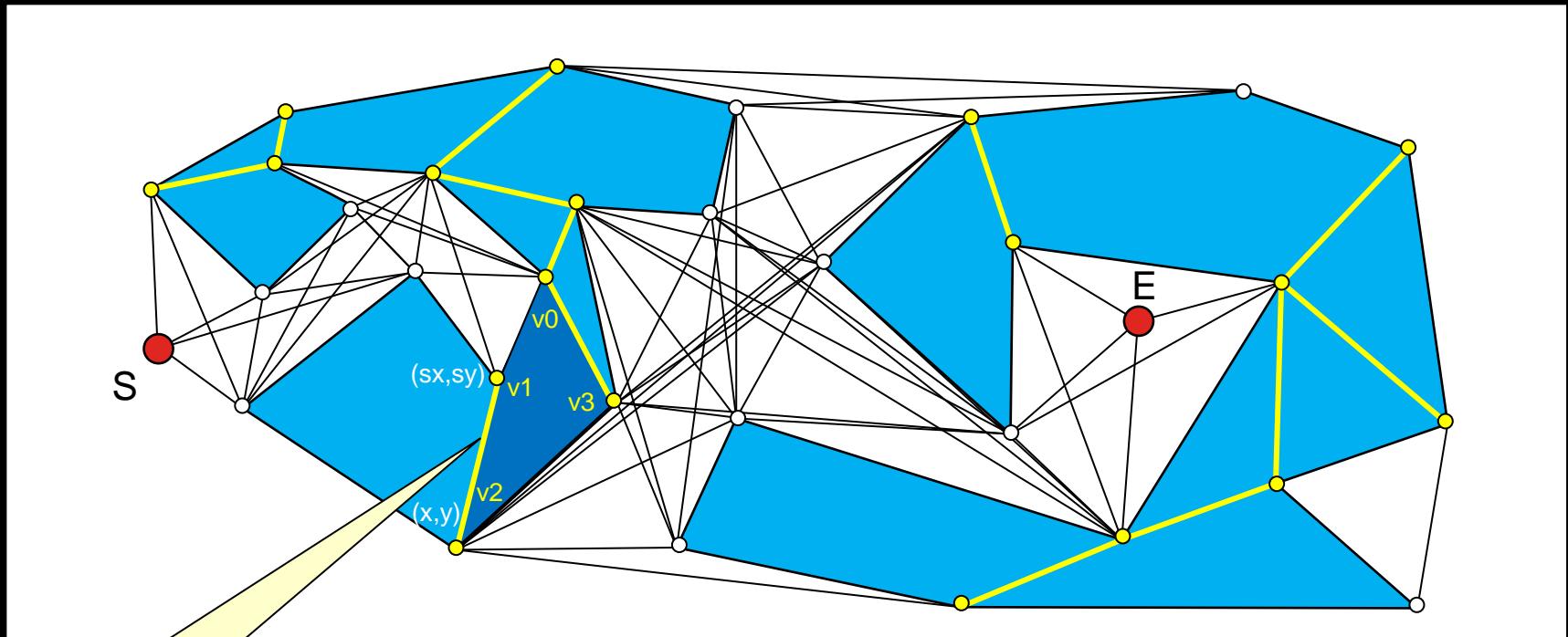
$v_{i-1} \rightarrow v_i \rightarrow v_{i+1}$  is a left turn if  
 $((x_i - x_{i-1}) * (y_{i+1} - y_{i-1}) - (y_i - y_{i-1}) * (x_{i+1} - x_{i-1})) > 0$



Right Turn! Polygon is not convex.

# Non-Convex Easy Solution

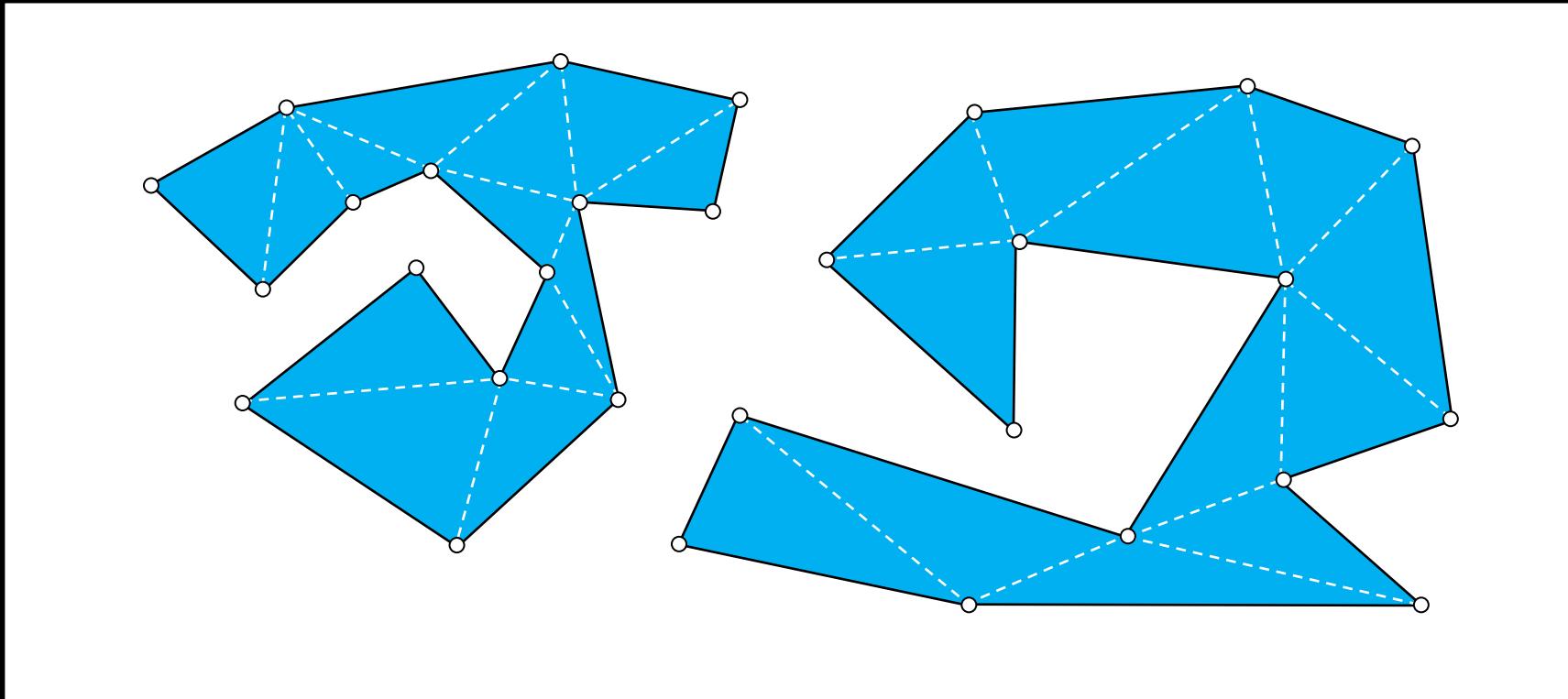
- There will be special cases for shared edges between convex pieces that need to be eliminated when computing the visibility graph:



Must make sure that none of these (yellow) *interior bordering support lines* are in the graph. Just need to check if support line  $(x,y) \rightarrow (sx,sy)$  has same vertex coordinates as edge  $(v_1, v_2) \dots$  or edge  $(v_2, v_1)$ .

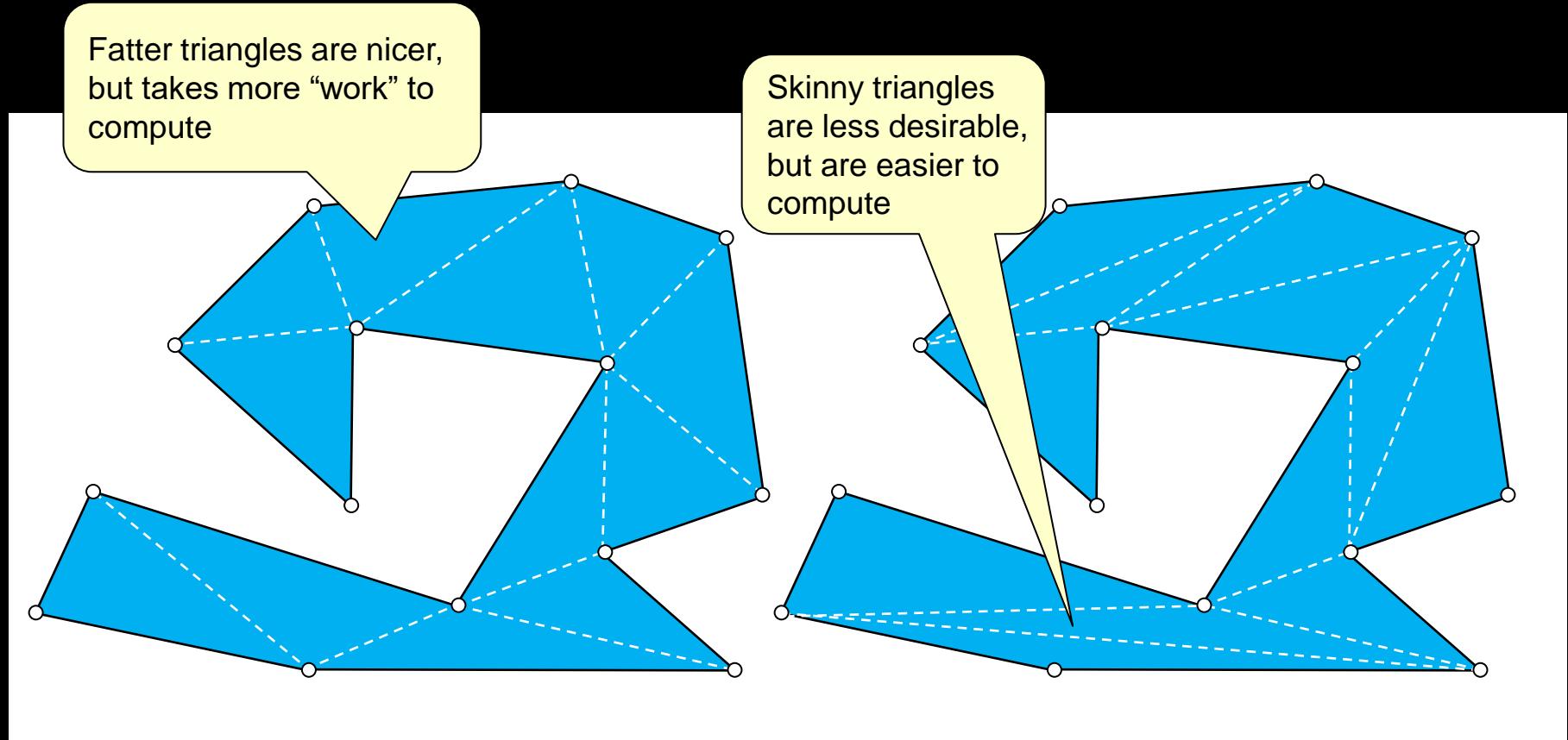
# Convert Non-Convex To Convex

- How do we break a polygon into convex pieces ?
  - There are many algorithms. The most popular is to break into triangles. This is known as *triangulation*.



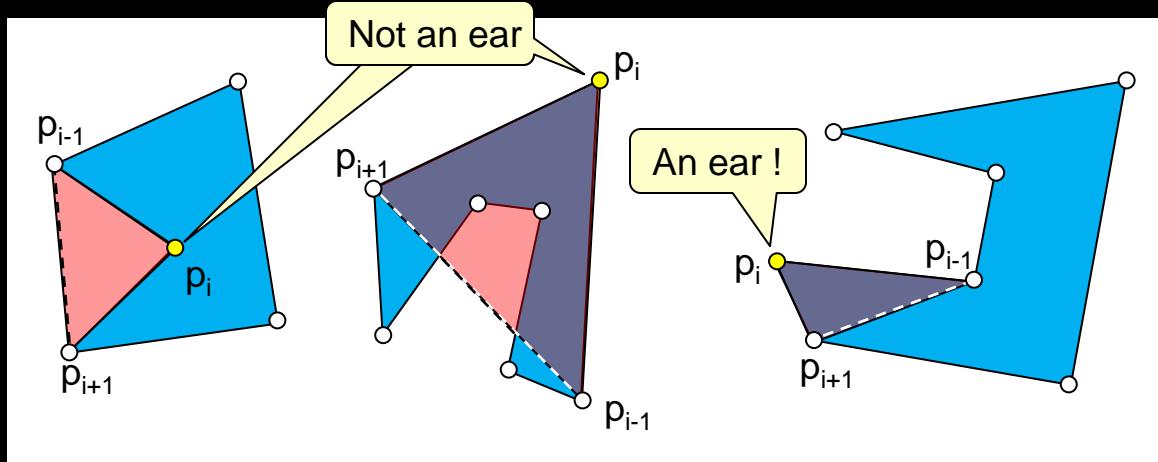
# Triangulation

- How do we triangulate a polygon ?
  - There are many ways/algorithms to do this as well.



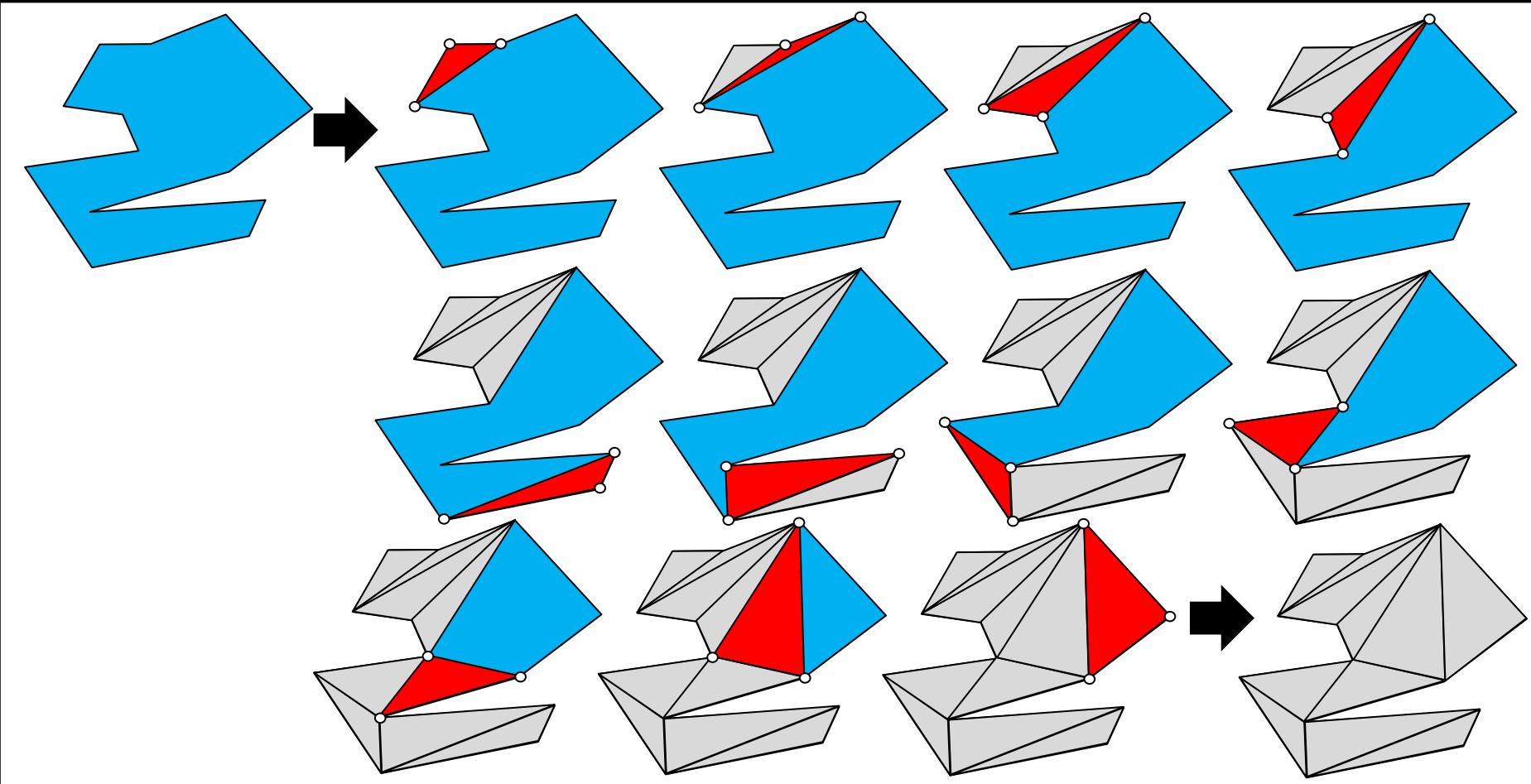
# Ear-Cutting Triangulation

- We will use an “ear-cutting” algorithm since it is easy.
  - Idea is to repeatedly cut off “ears” of the polygon, making a set of triangles along the way.
  - An “ear” is a vertex  $p_i$  such that Line segment  $\overline{p_{i-1}p_{i+1}} \dots$ 
    1. intersects the polygon boundary only at points  $p_{i-1}$  and  $p_{i+1}$  AND...
    2. it lies entirely inside the polygon.



# Ear-Cutting Triangulation

- Repeatedly cut off “ears” of the polygon, making a set of triangles along the way.



# Ear-Cutting Algorithm

```
1 function EarCut(Obstacle P)
2     triangles = an empty list
3     ear = null;
4     if (P has only 3 vertices) then
5         Add P to triangles and return triangles
6     Copy all vertices of P into a list of points Q
7     while (Q has more than 3 points) do
8         earIndex = -1
9         for each point pi in Q do
10            if (pi-1pipi+1 is a left turn) then
11                ear = a new obstacle with vertices pi-1pi and pi+1
12                earIndex = i
13                for each point pk in Q (such that k≠i-1, k≠i, k≠i+1) do
14                    if (point pk lies inside or on boundary of the ear) then
15                        earIndex = -1
16                    if (earIndex != -1) then
17                        Break out of the FOR loop at line 8 ... we found an ear
18                    if (earIndex == -1) quit and return triangles, since no more ears were found
19                    Remove pearIndex from Q
20                    Add ear to triangles
21                    Add (a new obstacle with vertices p0, p1 and p2) to triangles
22    return triangles
```

Nothing to do if only 3 vertices

Copy obstacle vertices into a new list  
that we can add/remove from without  
destroying the original obstacle.

Find  
an  
ear

Left turn if CCW ordering and  
 $((x_i - x_{i-1})(y_{i+1} - y_{i-1}) - (y_i - y_{i-1})(x_{i+1} - x_{i-1})) > 0$

reject p<sub>i</sub> as an ear since it is invalid

Cut the ear off and add to the solution

The last 3 points form an ear, so add it too

Start the  
Lab ...