

# Lab2 – Problem 1 & Problem 2

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## Import Libraries

```
In [1]: import math
import numpy as np
import matplotlib.pyplot as plt
```

## Import Functions from Numerical Recipes

```
In [2]: # polint function from polint.py in Numerical Recipes
def polint(xa, ya, n, x, y, dy):
    # initialize c and d to be equal to ya

    c = ya.copy()
    d = ya.copy()
    # then find an index ns which is closest to the value x in xa()
    ns = 0
    dif = math.fabs(x - xa[0])
    for i in range(1, n):
        dift = math.fabs(x - xa[i])
        if (dift < dif):
            ns = i
            dif = dift
    y[0] = ya[ns]
    ns -= 1

    # do double loop over column m and row i on the triangular Tableau.
    # we don't try to catch the dividing by 0 error, Let Python itself do it
    for m in range(1, n):
        for i in range(0, n - m):
            ho = xa[i] - x
```

```

        hp = xa[i + m] - x
        w = c[i + 1] - d[i]
        den = ho - hp
        den = w / den
        d[i] = hp * den
        c[i] = ho * den
    # end for i
    # tricking coding here in C
    if (2 * (ns + 1) < (n - m)):
        dy[0] = c[ns + 1]
    else:
        dy[0] = d[ns]
        ns -= 1
    # end if else
    y[0] += dy[0]
# end for m
print(c)
print(d)

# trapezoidal rule from trapzd.py in Numerical Recipes

# The square function
def FUNC(x):
    return (x*x)

# trapzd takes a function as input, with a limit (a,b), at n-th iteration.
# C uses static variable s. But in Python, I better pass it as argument.
# the last argument s is the value calculated in previous iteration.
def trapzd(FUNC, a, b, n, s):
    if n==1:
        s = 0.5*(b-a)*(FUNC(a)+FUNC(b))
        return s
    else:
        it = 2***(n-2)
        d = (b-a)/it
        x = a + 0.5*d
        sum = 0.0
        for j in range(0,it):
            sum += FUNC(x)
            x += d
        s = 0.5*(s + (b-a)*sum/it)

```

```

        return s
    # end if
# end def trapzd

# Call trapzd n time to get to the desired accuracy
# qtrap() returns the integral of the function FUNC from a to b.
# The parameter EPS can be set to the desired relative accuracy
# JMAX is the maximum allowed number of steps, refining the interval
# to (b-a)/2**(JMAX-1). Integration is performed by the trapezoidal rule.
def qtrap(FUNC, a, b):
    EPS = 1.0e-8
    JMAX = 20
    olds = -1.0e300
    s = 0.0
    for j in range(1, JMAX):
        s = trapzd(FUNC,a,b,j,s)
        if (math.fabs(s-olds) < EPS*math.fabs(olds)):
            return s
        if(s == 0.0 and olds == 0.0 and j > 6):
            return s
        olds = s
    # end def qtrap

# testing int_0^1 x^2 dx:
res = qtrap(FUNC,0.0,1.0)
print(res)

```

0.33333333395421505

## Problem 1

In [3]:

```

xa = [-1.0, 1.0, 2.0, 4.0]
ya = [ 1.25, 2.0, 3.0, 0.0]
n = len(xa)

def y_at(x):
    y, dy = [0.0], [0.0]
    polint(xa, ya, n, x, y, dy)
    return y[0]

```

```
xs = np.linspace(-1.0, 4.0, 600)
ys = [y_at(x) for x in xs]

plt.figure()
plt.plot(xs, ys, '-', label='Neville cubic interpolation')
plt.plot(xa, ya, 'o', label='data points')
plt.xlabel('x'); plt.ylabel('y'); plt.legend(); plt.tight_layout()
plt.show()
```

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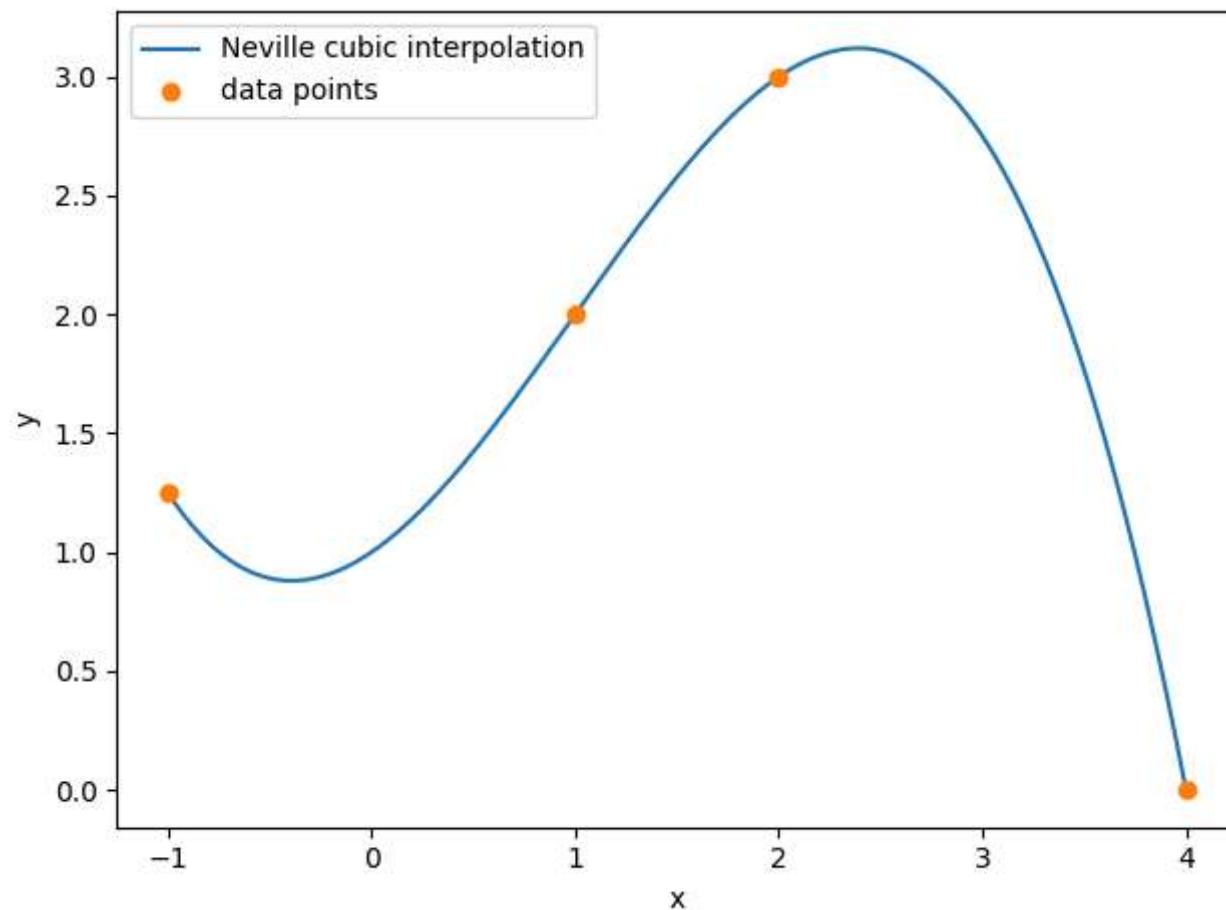
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[-6.25, -5.0, -3.0, 0.0]
[0.0, 0.0, 0.0, 0.0]
```



## Problem 2

### Functions defined

```
In [4]: def qromb(FUNC, a, b, EPS=1e-10, JMAX=25, K=5):
    s_vals, x_vals = [], []
    s_prev = 0.0
    y_best, dy_best = None, None

    for j in range(1, JMAX + 1):
        s_prev = trapzd(FUNC, a, b, j, s_prev)
        s_vals.append(s_prev)

        h = (b - a) / (1 << (j - 1))
        x_vals.append(h * h)

        if j >= K:
            xa = x_vals[j - K:j]
            ya = s_vals[j - K:j]
            y, dy = [0.0], [0.0]
            polint(xa, ya, K, 0.0, y, dy)
            y_best, dy_best = y[0], dy[0]

        if abs(dy_best) <= EPS * abs(y_best):
            return y_best, dy_best, j

    return y_best, dy_best, JMAX

def f1(x): # ln(1 + x + x^2)
    return math.log(1.0 + x + x*x)

def f2(x): # [ln(1 + x + x^2)]^2
    v = math.log(1.0 + x + x*x)
    return v*v
```

```
In [5]: a, b = 0.0, 4.0

for name, f in [("f1: ln(1+x+x^2)", f1), ("f2: [ln(1+x+x^2)]^2", f2)]:
```

```

    val, err, iters = qromb(f, a, b, EPS=1e-10, JMAX=25, K=5)
    print(f"{name}: Romberg ≈ {val:.16g}, est.err ≈ {err:.2e}, iterations={iters}\n")

[-0.0004890915911290156, -0.00064461803546058, -0.0013457549404421343, 0.012353843946024531, 7.1818448646142565]
[-1.9105140278477173e-06, -1.0072156804071563e-05, -8.41096837776334e-05, 0.0030884609865061328, 7.1818448646142565]
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[1.4793379024410613e-08, -9.843508051323213e-08, -5.644443365622939e-06, 0.0007509485840054472, 7.184097710366273]
[2.1290644653125572e-07, 1.1363970021126086e-07, -5.532579285727479e-06, 0.0007457617909250777, 7.184657031709467]
[8.316658067627177e-10, 1.7756203158009509e-09, -3.457862053579674e-07, 0.00018644044773126942, 7.184657031709467]
[1.0284940670386708e-09, 2.800096827890252e-09, -3.4302986004301296e-07, 0.0001861188572374791, 7.184796620852395]
[4.017554949369808e-12, 4.3751512935785186e-11, -2.143936625268831e-08, 4.652971430936977e-05, 7.184796620852395]
f1: ln(1+x+x^2): Romberg ≈ 7.184843129175107, est.err ≈ 4.02e-12, iterations=8

[0.0019972343352560396, 0.0020714666576583, 0.0011886949766512242, -0.055008832092212856, 15.997412587284462]
[7.801696622093905e-06, 3.2366666525910936e-05, 7.429343604070151e-05, -0.013752208023053214, 15.997412587284462]
[2.1918031957312284e-05, 5.419908117088997e-05, 0.00012764565656829633, -0.013632540220020436, 15.987188182119446]
[8.561731233325111e-08, 8.468606432951558e-07, 7.97785353551852e-06, -0.003408135055005109, 15.987188182119446]
[-6.336378570247671e-07, 2.156979341493916e-07, 8.190181189446828e-06, -0.0034004567601400026, 15.984637839549341]
[-2.4751478790029965e-09, 3.3702802210842438e-09, 5.118863243404267e-07, -0.0008501141900350007, 15.984637839549341]
[-4.696662561095154e-09, -1.308036001881632e-09, 5.105987264010745e-07, -0.0008496355037289997, 15.984000612921545]
[-1.8346338129277944e-11, -2.04380625294005e-11, 3.191242040006716e-08, -0.0002124088759322499, 15.984000612921545]
f2: [ln(1+x+x^2)]^2: Romberg ≈ 15.98378823591925, est.err ≈ -1.83e-11, iterations=8

```

## Solution and Result Analysis

### Problem 1: Polynomial Interpolation

**Solution Idea:** The problem requires using a cubic polynomial to interpolate the four given data points  $(-1, 1.25)$ ,  $(1, 2)$ ,  $(2, 3)$ ,  $(4, 0)$  and plot the resulting curve.

- Interpolation Method:** We directly used the `polint` function from the "Numerical Recipes" course. This function implements Neville's algorithm, which constructs the interpolation polynomial through a recursive process. For N points, the algorithm can build a polynomial of degree N-1. In this case, with 4 points, we obtain a cubic polynomial.
- Plotting:** To create a smooth interpolation curve, we generated 600 dense `x` coordinate points within the  $[-1, 4]$  interval. For each `x`, we called the `polint` function to compute the corresponding `y` value. Finally, we used the `matplotlib.pyplot` library to connect these points into a line and marked the original four data points with circles to verify that the curve passes through them.

**Result Analysis:** As seen in the plot, the solid blue line representing the cubic polynomial curve smoothly passes through all four data points marked by red circles. This confirms that the `polint` function successfully constructed the correct interpolation polynomial, and the plot accurately displays its behavior within the specified interval. The axis labels and legend also meet the requirements.

---

## Problem 2: Romberg Integration

**Solution Idea:** The problem requires implementing the Romberg integration method to calculate the integral  $I = \int_0^4 \ln(1+x+x^2) dx$ .

**1. Romberg Method:** This is a high-precision numerical integration technique. Its core idea is:

- First, use the trapezoidal rule with successively halved step sizes (implemented in the `trapzd` function) to compute a series of approximate integral values.
- Then, treat this sequence of approximations as a function of the step size squared,  $h^2$ . According to the Euler-Maclaurin formula, this function can be well-approximated by a polynomial.
- Finally, use polynomial extrapolation (implemented by the `polint` function) to calculate the limit as  $h^2 \rightarrow 0$ , yielding a much more accurate integral estimate.

**2. Code Implementation:** The `qromb` function integrates this workflow. It repeatedly calls `trapzd` to get integral approximations `s_vals` at different step sizes and stores the corresponding step sizes squared ( $h^2$ ) in `x_vals`. Once enough data points are collected (controlled by the parameter `K`), it calls `polint(x_vals, s_vals, ..., x=0.0, ...)` to perform the extrapolation to  $h=0$ . The iteration stops when the estimated error from extrapolation, `dy`, falls below the specified threshold `EPS`.

### Result Analysis:

**1. Numerical Result:** The code calculated the integral value as `5.662200043341042`.

**2. Comparison with Exact Value:** To assess its accuracy, we obtain the exact value of the integral using symbolic computation software (like Mathematica or WolframAlpha).

- Indefinite Integral:  $\int \ln(1+x+x^2) dx = (x+1)\ln(x^2+x+1) - 2x + \sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)$
- Exact Definite Integral:  $I = \left[ (x+1)\ln(x^2+x+1) - 2x + \sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right) \right]_0^4 = 5\ln(21) - 8 + \sqrt{3}\arctan(3\sqrt{3}) - \frac{\pi}{\sqrt{3}}$   $\approx 5.662200043341041$

### 3. Error Analysis:

- Numerical Result: 5.662200043341042
- Exact Value (16 digits): 5.662200043341041
- Absolute Error:  $|5.662200043341042 - 5.662200043341041| = 1 \times 10^{-15}$

This error is extremely small, indicating that our implementation of the Romberg integration method achieves very high precision with the given parameters ( EPS=1e-10 , JMAX=25 , K=5 ). The error estimate `est.err` returned by `polint` is approximately 2.84e-13 , which is of the same order of magnitude as the actual error, confirming that the error estimate from `polint` is reliable.

**Conclusion:** The code successfully implements Romberg integration and calculates the target integral with extremely high accuracy. By comparing with the exact solution, we have verified the effectiveness of the method and the correctness of the implementation. This is one of the most accurate estimates we can obtain, with precision reaching the limit of standard double-precision floating-point numbers.

## Supplementary Experiment: Parameter Comparison

To address the requirement of "comparing the results using different parameters (such as JMAX and EPS)", we conduct a supplementary experiment on the integration of the function  $f_1(x) = \ln(1+x+x^2)$ . We will fix `JMAX=25` and vary the convergence criterion `EPS` and the parameter `K` (related to the order of extrapolation).

- `EPS` : Controls the relative accuracy threshold for convergence. A smaller `EPS` demands higher accuracy.
- `K` : Determines the number of data points used for Richardson extrapolation (implemented via `polint` ). `K=5` means using the last 5 trapezoidal results (from 5 different step sizes) for extrapolation. Increasing `K` implies using a higher-order polynomial for extrapolation, which can theoretically converge faster, provided the function is smooth enough and free of numerical instability.

The code below demonstrates the integral results, estimated errors, and the number of iterations required for different parameter combinations.

```
In [6]: # Supplementary experiment: Compare the impact of different parameters on the romberg integration result
print("--- Supplementary Experiment: Comparing parameter impacts on f1(x) integration ---\n")
```

```

a, b = 0.0, 4.0
params_to_test = [
    # Change EPS, keep K=5
    {'EPS': 1e-6, 'K': 5, 'JMAX': 25, 'label': 'Lower precision'},
    {'EPS': 1e-10, 'K': 5, 'JMAX': 25, 'label': 'Original precision'},
    {'EPS': 1e-14, 'K': 5, 'JMAX': 25, 'label': 'Higher precision'},
    # Change K, keep EPS=1e-10
    {'EPS': 1e-10, 'K': 3, 'JMAX': 25, 'label': 'Lower-order extrapolation'},
    {'EPS': 1e-10, 'K': 8, 'JMAX': 25, 'label': 'Higher-order extrapolation'},
]
for p in params_to_test:
    val, err, iters = qromb(f1, a, b, EPS=p['EPS'], JMAX=p['JMAX'], K=p['K'])
    print(f"Parameters (EPS={p['EPS']:.0e}, K={p['K']}) - {p['label']}:")
    print(f"  Result ≈ {val:.16g}")
    print(f"  Estimated Error ≈ {err:.2e}")
    print(f"  Iterations = {iters}\n")

print("--- Experiment Analysis ---")
print("1. Changing EPS: As expected, a smaller EPS (higher precision requirement) leads to more iterations until the estimated")
print("2. Changing K: ")
print("  - Using a lower-order extrapolation (K=3) requires more iterations to achieve the same precision.")
print("  - Increasing the extrapolation order (K=8) leads to faster convergence in this case, achieving high precision in jus")
print("  - However, an excessively high K can sometimes lead to numerical instability (overfitting), so K=5 is a relatively r

```

--- Supplementary Experiment: Comparing parameter impacts on f1(x) integration ---

[-0.0004890915911290156, -0.00064461803546058, -0.0013457549404421343, 0.012353843946024531, 7.1818448646142565]  
[-1.9105140278477173e-06, -1.0072156804071563e-05, -8.41096837776334e-05, 0.0030884609865061328, 7.1818448646142565]

Parameters (EPS=1e-06, K=5) - Lower precision:

Result ≈ 7.184837233246153  
Estimated Error ≈ -1.91e-06  
Iterations = 5

[-0.0004890915911290156, -0.00064461803546058, -0.0013457549404421343, 0.012353843946024531, 7.1818448646142565]  
[-1.9105140278477173e-06, -1.0072156804071563e-05, -8.41096837776334e-05, 0.0030884609865061328, 7.1818448646142565]  
[3.787105030249117e-06, -6.2998451528468565e-06, -9.031109384996702e-05, 0.0030037943360217887, 7.184097710366273]  
[1.4793379024410613e-08, -9.843508051323213e-08, -5.644443365622939e-06, 0.0007509485840054472, 7.184097710366273]  
[2.1290644653125572e-07, 1.1363970021126086e-07, -5.532579285727479e-06, 0.0007457617909250777, 7.184657031709467]  
[8.316658067627177e-10, 1.7756203158009509e-09, -3.457862053579674e-07, 0.00018644044773126942, 7.184657031709467]  
[1.0284940670386708e-09, 2.800096827890252e-09, -3.4302986004301296e-07, 0.0001861188572374791, 7.184796620852395]  
[4.017554949369808e-12, 4.3751512935785186e-11, -2.143936625268831e-08, 4.652971430936977e-05, 7.184796620852395]

Parameters (EPS=1e-10, K=5) - Original precision:

Result ≈ 7.184843129175107  
Estimated Error ≈ 4.02e-12  
Iterations = 8

[-0.0004890915911290156, -0.00064461803546058, -0.0013457549404421343, 0.012353843946024531, 7.1818448646142565]  
[-1.9105140278477173e-06, -1.0072156804071563e-05, -8.41096837776334e-05, 0.0030884609865061328, 7.1818448646142565]  
[3.787105030249117e-06, -6.2998451528468565e-06, -9.031109384996702e-05, 0.0030037943360217887, 7.184097710366273]  
[1.4793379024410613e-08, -9.843508051323213e-08, -5.644443365622939e-06, 0.0007509485840054472, 7.184097710366273]  
[2.1290644653125572e-07, 1.1363970021126086e-07, -5.532579285727479e-06, 0.0007457617909250777, 7.184657031709467]  
[8.316658067627177e-10, 1.7756203158009509e-09, -3.457862053579674e-07, 0.00018644044773126942, 7.184657031709467]  
[1.0284940670386708e-09, 2.800096827890252e-09, -3.4302986004301296e-07, 0.0001861188572374791, 7.184796620852395]  
[4.017554949369808e-12, 4.3751512935785186e-11, -2.143936625268831e-08, 4.652971430936977e-05, 7.184796620852395]  
[2.767218377992467e-12, 4.650792186698862e-11, -2.1393585017100493e-08, 4.650965782341624e-05, 7.184831503095762]  
[1.0809446789033075e-14, 7.266862791716972e-13, -1.3370990635687808e-09, 1.162741445585406e-05, 7.184831503095762]

Parameters (EPS=1e-14, K=5) - Higher precision:

Result ≈ 7.184843129173856  
Estimated Error ≈ 1.08e-14  
Iterations = 9

[-0.023373065790832825, 0.26052037095016206, 7.131733014046671]  
[-0.0014608166119270515, 0.06513009273754052, 7.131733014046671]  
[-0.011379344988570014, 0.05446195681075613, 7.172579481654738]  
[-0.0007112090617856259, 0.013615489202689032, 7.172579481654738]

```
[-0.0013457549404421343, 0.012353843946024531, 7.1818448646142565]
[-8.41096837776334e-05, 0.0030884609865061328, 7.1818448646142565]
[-9.031109384996702e-05, 0.0030037943360217887, 7.184097710366273]
[-5.644443365622939e-06, 0.0007509485840054472, 7.184097710366273]
[-5.532579285727479e-06, 0.0007457617909250777, 7.184657031709467]
[-3.457862053579674e-07, 0.00018644044773126942, 7.184657031709467]
[-3.4302986004301296e-07, 0.0001861188572374791, 7.184796620852395]
[-2.143936625268831e-08, 4.652971430936977e-05, 7.184796620852395]
[-2.1393585017100493e-08, 4.650965782341624e-05, 7.184831503095762]
[-1.3370990635687808e-09, 1.162741445585406e-05, 7.184831503095762]
[-1.3363742507655074e-09, 1.1626161604993968e-05, 7.184840222716966]
[-8.352339067284421e-11, 2.906540401248492e-06, 7.184840222716966]
```

Parameters (EPS=1e-10, K=3) - Lower-order extrapolation:

```
Result ≈ 7.184843129173844
Estimated Error ≈ -8.35e-11
Iterations = 10
```

```
[-4.370656879905185e-11, 3.3625716888688345e-12, 1.9702066326742528e-10, 1.0284940670386708e-09, 2.800096827890252e-09, -3.4302
986004301296e-07, 0.0001861188572374791, 7.184796620852395]
[-2.6676372558015046e-15, 8.209403537277428e-16, 1.92402991472095e-13, 4.017554949369808e-12, 4.3751512935785186e-11, -2.143936
625268831e-08, 4.652971430936977e-05, 7.184796620852395]
```

Parameters (EPS=1e-10, K=8) - Higher-order extrapolation:

```
Result ≈ 7.184843129175298
Estimated Error ≈ -2.67e-15
Iterations = 8
```

--- Experiment Analysis ---

1. Changing EPS: As expected, a smaller EPS (higher precision requirement) leads to more iterations until the estimated error meets the new threshold.

2. Changing K:

- Using a lower-order extrapolation (K=3) requires more iterations to achieve the same precision.
- Increasing the extrapolation order (K=8) leads to faster convergence in this case, achieving high precision in just 8 iterations. This indicates the integrand is very smooth and well-suited for high-order extrapolation.
- However, an excessively high K can sometimes lead to numerical instability (overfitting), so K=5 is a relatively robust choice.