PC5215, Numerical Recipes with Applications
Lab 2 (Due Thursday, 18 Sep 2025)

1. Using the polynomial interpolation Python code polint.py of "Numerical Recipes in C", compute and then plot a curve for x in the interval [-1, 4], passing through exactly the four points (x,y) = (-1, 1.25), (1, 2), (2,3), (4,0) with a cubic polynomial. For plotting, you can use Python itself (need import matplotlib). Label the axis properly, mark the four points with circles, and the interpolated curve with a solid line.

2. Implement the "Romberg integration" method discussed in NR in C 2nd on page 140 sec. 4.3 in Python [following the "Numerical Recipes" code qromb(), trapzd.py, and polint.py], and then compute the integral:

$$\int_0^2 x^4 \ln\left(x+\sqrt{x^2+1}\right) dx.$$

Here ln means natural logarithm (base e). The Romberg method is more sophisticated, where you need the extrapolation process based on the analytic property of the Euler-Maclaurin formula. Use different parameters (such as the number of iterations JMAX and convergence criteria EPS) to compare the results. What is your most accurate estimation of the integral you can get? Give an error estimate of the answers by comparing with the exact answer. You can obtain the exact answer with a symbolic integration system, such as *Mathematica*, and evaluate to high accuracy (say 16 digits).