

# Fitting Linear Models

## Description

lm is used to fit linear models. It can be used to carry out regression, single stratum analysis of variance and analysis of covariance (although [aov](#) may provide a more convenient interface for these).

## Usage

```
lm(formula, data, subset, weights, na.action,
   method = "qr", model = TRUE, x = FALSE, y = FALSE, qr = TRUE,
   singular.ok = TRUE, contrasts = NULL, offset, ...)
```

## Arguments

formula	an object of class " <a href="#">formula</a> " (or one that can be coerced to that class): a symbolic description of the model to be fitted. The details of model specification are given under ‘Details’.
data	an optional data frame, list or environment (or object coercible by <a href="#">as.data.frame</a> to a data frame) containing the variables in the model. If not found in data, the variables are taken from <code>environment(formula)</code> , typically the environment from which <code>lm</code> is called.
subset	an optional vector specifying a subset of observations to be used in the fitting process.
weights	an optional vector of weights to be used in the fitting process. Should be NULL or a numeric vector. If non-NULL, weighted least squares is used with weights <code>weights</code> (that is, minimizing $\sum(w_i e^2)$ ); otherwise ordinary least squares is used. See also ‘Details’.
na.action	a function which indicates what should happen when the data contain NAs. The default is set by the <code>na.action</code> setting of <a href="#">options</a> , and is <a href="#">na.fail</a> if that is unset. The ‘factory-fresh’ default is <a href="#">na.omit</a> . Another possible value is NULL, no action. Value <a href="#">na.exclude</a> can be useful.
method	the method to be used; for fitting, currently only <code>method = "qr"</code> is supported; <code>method = "model.frame"</code> returns the model frame (the same as with <code>model = TRUE</code> , see below).
model, x, y, qr	logicals. If TRUE the corresponding components of the fit (the model frame, the model matrix, the response, the QR decomposition) are returned.
singular.ok	logical. If FALSE (the default in S but not in R) a singular fit is an error.
contrasts	an optional list. See the <code>contrasts.arg</code> of <a href="#">model.matrix.default</a> .
offset	this can be used to specify an <i>a priori</i> known component to be included in the linear predictor during fitting. This should be NULL or a numeric vector of length equal to the number of cases. One or more <a href="#">offset</a> terms can be included in the formula instead or as well, and if more than one are specified their sum is used. See <a href="#">model.offset</a> .
...	additional arguments to be passed to the low level regression fitting functions (see below).

## Details

Models for `lm` are specified symbolically. A typical model has the form `response ~ terms` where `response` is the (numeric) response vector and `terms` is a series of terms which specifies a linear predictor for `response`. A terms specification of the form `first + second` indicates all the terms in `first` together with all the terms in `second` with duplicates removed. A specification of the form `first:second` indicates the set of terms obtained by taking the interactions of all terms in `first` with all terms in `second`. The specification `first*second` indicates the *cross* of `first` and `second`. This is the same as `first + second + first:second`.

If the formula includes an [offset](#), this is evaluated and subtracted from the response.

If `response` is a matrix a linear model is fitted separately by least-squares to each column of the matrix.

See [model.matrix](#) for some further details. The terms in the formula will be re-ordered so that main effects come first, followed by the interactions, all second-order, all third-order and so on: to avoid this pass a terms object as the formula (see [aov](#) and `demo(glm.vr)` for an example).

A formula has an implied intercept term. To remove this use either `y ~ x - 1` or `y ~ 0 + x`. See [formula](#) for more details of allowed formulae.

Non-NULL `weights` can be used to indicate that different observations have different variances (with the values in `weights` being inversely proportional to the variances); or equivalently, when the elements of `weights` are positive integers  $w_i$ , that each response  $y_i$  is the mean of  $w_i$  unit-weight observations (including the case that there are  $w_i$  observations equal to  $y_i$  and the data have been summarized).

`lm` calls the lower level functions [lm.fit](#), etc, see below, for the actual numerical computations. For programming only, you may consider doing likewise.

All of `weights`, `subset` and `offset` are evaluated in the same way as variables in `formula`, that is first in `data` and then in the environment of `formula`.

## Value

`lm` returns an object of [class](#) "lm" or for multiple responses of class `c("mlm", "lm")`.

The functions `summary` and [anova](#) are used to obtain and print a summary and analysis of variance table of the results. The generic accessor functions `coefficients`, `effects`, `fitted.values` and `residuals` extract various useful features of the value returned by `lm`.

An object of class "lm" is a list containing at least the following components:

coefficients	a named vector of coefficients
residuals	the residuals, that is response minus fitted values.
fitted.values	the fitted mean values.
rank	the numeric rank of the fitted linear model.
weights	(only for weighted fits) the specified weights.

df.residual	the residual degrees of freedom.
call	the matched call.
terms	the <a href="#">terms</a> object used.
contrasts	(only where relevant) the contrasts used.
xlevels	(only where relevant) a record of the levels of the factors used in fitting.
offset	the offset used (missing if none were used).
y	if requested, the response used.
x	if requested, the model matrix used.
model	if requested (the default), the model frame used.
na.action	(where relevant) information returned by <a href="#">model.frame</a> on the special handling of NAs.

In addition, non-null fits will have components `assign`, `effects` and (unless not requested) `qr` relating to the linear fit, for use by extractor functions such as `summary` and [effects](#).

## Using time series

Considerable care is needed when using `lm` with time series.

Unless `na.action = NULL`, the time series attributes are stripped from the variables before the regression is done. (This is necessary as omitting NAs would invalidate the time series attributes, and if NAs are omitted in the middle of the series the result would no longer be a regular time series.)

Even if the time series attributes are retained, they are not used to line up series, so that the time shift of a lagged or differenced regressor would be ignored. It is good practice to prepare a data argument by [ts.intersect](#)(..., `dframe = TRUE`), then apply a suitable `na.action` to that data frame and call `lm` with `na.action = NULL` so that residuals and fitted values are time series.

## Note

Offsets specified by `offset` will not be included in predictions by [predict.lm](#), whereas those specified by an offset term in the formula will be.

## Author(s)

The design was inspired by the S function of the same name described in Chambers (1992). The implementation of model formula by Ross Ihaka was based on Wilkinson & Rogers (1973).

## References

Chambers, J. M. (1992) *Linear models*. Chapter 4 of *Statistical Models in S* eds J. M. Chambers and T. J. Hastie, Wadsworth & Brooks/Cole.

Wilkinson, G. N. and Rogers, C. E. (1973) Symbolic descriptions of factorial models for analysis of variance. *Applied Statistics*, **22**, 392–9.

## See Also

[summary.lm](#) for summaries and [anova.lm](#) for the ANOVA table; [aov](#) for a different interface.

The generic functions [coef](#), [effects](#), [residuals](#), [fitted](#), [vcov](#).

[predict.lm](#) (via [predict](#)) for prediction, including confidence and prediction intervals; [confint](#) for confidence intervals of *parameters*.

[lm.influence](#) for regression diagnostics, and [glm](#) for **generalized** linear models.

The underlying low level functions, [lm.fit](#) for plain, and [lm.wfit](#) for weighted regression fitting.

More `lm()` examples are available e.g., in [anscombe](#), [attitude](#), [freeny](#), [LifeCycleSavings](#), [longley](#), [stackloss](#), [swiss](#).

`biglm` in package [biglm](#) for an alternative way to fit linear models to large datasets (especially those with many cases).

## Examples

```
require(graphics)

## Annette Dobson (1990) "An Introduction to Generalized Linear Models".
## Page 9: Plant Weight Data.
ctl <- c(4.17,5.58,5.18,6.11,4.50,4.61,5.17,4.53,5.33,5.14)
trt <- c(4.81,4.17,4.41,3.59,5.87,3.83,6.03,4.89,4.32,4.69)
group <- gl(2, 10, 20, labels = c("Ctl","Trt"))
weight <- c(ctl, trt)
lm.D9 <- lm(weight ~ group)
lm.D90 <- lm(weight ~ group - 1) # omitting intercept

anova(lm.D9)
summary(lm.D90)

opar <- par(mfrow = c(2,2), oma = c(0, 0, 1.1, 0))
plot(lm.D9, las = 1)      # Residuals, Fitted, ...
par(opar)

### less simple examples in "See Also" above
```