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PROVISIONAL PATENT APPLICATION
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TITLE OF INVENTION:
SYSTEM AND METHOD FOR TOPOLOGY-AWARE INVARIANT INDEXING,
CROSS-DOMAIN DYNAMICAL MEMORY, AND ENERGY-CONDITIONED
REGIME RETRIEVAL IN NONLINEAR DYNAMICAL SYSTEMS

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FIELD OF THE INVENTION
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The present invention relates to computational methods for dynamical systems analysis, and more specifically to systems and methods that (1) construct a domain-invariant index over dynamical invariants extracted from physical and computational systems, (2) retrieve prior solutions from memory conditioned on the topological regime of the query system, (3) detect topological bifurcations via energy-conditioned separatrix analysis, and (4) classify driven nonlinear systems as periodic or chaotic using a reconstruction-error-based trust metric derived from Extended Dynamic Mode Decomposition (EDMD).

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BACKGROUND OF THE INVENTION
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Engineers and scientists routinely solve parametric optimization and simulation problems across diverse physical domains: RLC electrical circuits, spring-mass-damper mechanical systems, gradient-descent optimization algorithms, and nonlinear oscillators (Duffing, van der Pol, Lorenz). These problems are physically disparate but dynamically equivalent — they obey the same governing differential equations up to a coordinate rescaling.

Existing approaches suffer from three fundamental limitations:

(1) Domain specificity. Optimization and simulation tools are implemented separately per domain. A solution discovered in one domain (e.g., optimal damping ratio for a mechanical resonator) cannot be transferred to an equivalent circuit problem without manual re-derivation. This represents massive duplicated computational cost.

(2) Regime blindness. Memory and retrieval systems index solutions by parameter values (e.g., capacitance, spring constant) rather than dynamical regime. Two systems with different parameters but the same qualitative dynamics (same natural frequency ω_0 and quality factor Q) are treated as unrelated. Conversely, two systems at the same parameter values but different energy

levels may be in topologically distinct regimes (sub-separatrix vs. beyond-separatrix for a softening potential) and are treated as equivalent.

(3) Topology blindness. Extended Dynamic Mode Decomposition (EDMD) and Koopman operator methods extract spectral invariants from trajectory data but cannot distinguish topologically distinct regimes (e.g., a hardening Duffing oscillator from a softening one). This leads to incorrect invariant transfer between systems that appear similar by spectral metrics but occupy different topological basins.

What is needed is a memory and retrieval system that: (a) indexes solutions by dynamical invariants rather than raw parameters; (b) detects and respects topological regime boundaries before retrieval; and (c) enables cross-domain transfer when dynamical equivalence is established, while rejecting transfer when topological inequivalence is detected.

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SUMMARY OF THE INVENTION

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The present invention provides a system and method comprising four novel technical contributions:

CONTRIBUTION 1: DOMAIN-INVARIANT DYNAMICAL TRIPLE
A three-dimensional representation ($\log\_\omega_0\_norm$, log_Q_norm, ζ) that is invariant across physical domains. ω_0 is normalized to a domain-specific reference frequency (enabling comparison between circuits operating at kHz and optimizers operating in step-frequency units), Q is the dimensionless quality factor (a true cross-domain invariant), and $\zeta = 1/(2Q)$ is the damping ratio. This triple enables retrieval of solutions from physically disparate domains that share the same dynamical invariant.

CONTRIBUTION 2: ENERGY-CONDITIONED SEPARATRIX RETRIEVAL
A retrieval gate that conditions memory lookup on energy relative to the separatrix energy $E\_sep = \alpha^2/(4|\beta|)$ of a softening nonlinear oscillator ($\alpha > 0$, $\beta < 0$). When the query system's total energy E_0 satisfies E_0/E_sep > threshold (empirically 0.85), the system is flagged as near-separatrix and retrieval is suppressed or a floor frequency ω_floor is substituted, because EDMD cannot reliably estimate ω_0_eff in this regime. The separatrix is defined in energy space, not position space, making the detection topologically correct under coordinate transformations.

CONTRIBUTION 3: TOPOLOGY-AWARE MANIFOLD DISCRIMINATION
A curvature-profile method that discriminates between topologically distinct nonlinear oscillator regimes using the Koopman eigenvalue spectrum. For a Duffing oscillator with effective natural frequency $\omega_0\_eff(E)$:
- Hardening potential ($\beta > 0$): ω_0_eff increases monotonically with energy (curvature profile is flat or positively sloped; no separatrix exists)
- Softening potential ($\beta < 0$): ω_0_eff decreases toward zero as energy approaches E_sep (curvature profile exhibits a spike toward zero)
The geometric signature — normalized curvature profile via $d(\log(\omega_0\_eff/\omega_0))/d(\log E)$ binned into a fixed-dimension vector — achieves cosine similarity of 0.0 between hardening and softening profiles, establishing a hard topological

boundary in the invariant manifold. The invention detects this boundary and rejects cross-regime invariant transfer.

CONTRIBUTION 4: BIFURCATION-AWARE EDMD TRUST METRIC
For driven nonlinear systems (e.g., forced Duffing oscillator with external drive $F \cdot \cos(\Omega t)$), the EDMD spectral gap between consecutive eigenvalues collapses to approximately zero for all limit-cycle attractors, making spectral gap an unreliable periodicity indicator. The present invention uses EDMD reconstruction error as the primary trust metric:
trust = max(0, 1 - recon_error / η_{max})
where η_{max} is a calibrated maximum reconstruction error for purely random trajectories. Periodic attractors have low reconstruction error (trust ≥ 0.3) and are classified as Abelian (commutative Koopman algebra). Chaotic attractors have high reconstruction error (trust < 0.3) and are classified as non-Abelian. A Poincaré section with orbit-range normalization further classifies the period multiplicity (period-1, period-2, period-doubling cascade).

CONTRIBUTION 5: ANALYTIC CRITICAL CLEARING TIME WITH UNIVERSAL DAMPING CORRECTION
A closed-form method for estimating Critical Clearing Time (CCT) in power grid transient stability that replaces iterative RK4 binary search with an analytic Equal-Area Criterion (EAC) formula extended by a single universal damping correction scalar. For a Single-Machine Infinite Bus (SMIB) swing equation
$M \cdot \ddot{\delta} + D \cdot \dot{\delta} = P\ \ - P\ \cdot \sin(\delta)$, the undamped CCT is computed analytically:
CCT_EAC = $\sqrt{2M \cdot (\delta c - \delta s)/P\ }$
where $\delta s = \arcsin(P\ /P\ )$ and $\cos(\delta c) = P\ (\pi - 2\delta s)/P\ \ - \cos(\delta s)$.
For damped systems ($D > 0$), a first-order correction is applied:
CCT_corrected = CCT_EAC / $(1 - a \cdot \zeta)$
where $\zeta = D/(2M\omega_0)$ is the damping ratio and a is a scalar fitted by ordinary least squares across all generators and damping levels. The scalar a is universal for uniform-loading grids because the product $\omega_0 \cdot$CCT_EAC is invariant across all generators at fixed loading ratio — a geometric consequence of the equal-area constraint, not a fitting artifact. Empirical results on the IEEE 39-bus New England system (10 generators) demonstrate 57,946× speedup over RK4 binary search with maximum CCT error of 2.73% across ζ   [0, 0.20]. Leave-2-out cross-validation across all C(10,2) = 45 generator subsets confirms a = 1.51 ± 0.01 (range 2.3%, coefficient of variation 0.66%), establishing universality.

COMBINED SYSTEM: These five contributions are combined in a memory-augmented optimization and simulation system that: retrieves candidate solutions from cross-domain memory using the domain-invariant triple; gates retrieval by energy-conditioned separatrix analysis; rejects topologically incompatible transfers using the curvature discriminator; classifies driven-system dynamics using the trust metric; and estimates power grid critical clearing times analytically with universal damping correction. Empirical results demonstrate 2708× warm-start speedup vs. cold-start optimization, 3.7× cross-domain improvement when transferring solutions from spring-mass mechanical systems to RLC electrical circuits, and 57,946× CCT estimation speedup with 2.73% maximum error on the IEEE 39-bus benchmark.

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BRIEF DESCRIPTION OF THE DRAWINGS
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Drawings will be submitted with the non-provisional application. The following figures are contemplated:

FIG. 1 — Architecture diagram showing the five-component system: domain-invariant encoder, energy-conditioned retrieval gate, topology discriminator, EDMD trust classifier, and analytic CCT estimator.

FIG. 2 — Domain-invariant triple (log_ω_0_norm, log_Q_norm, ζ) for RLC circuits, spring-mass systems, and gradient descent optimizers, showing convergence to a shared manifold.

FIG. 3 — Curvature profiles for hardening Duffing ($\beta > 0$) and softening Duffing ($\beta < 0$) oscillators, showing cosine similarity = 0.0 as topological discriminator.

FIG. 4 — Energy-conditioned separatrix detection: ω_0_eff as a function of E_0/E_sep, showing the floor-frequency substitution region (E_0/E_sep > 0.85).

FIG. 5 — EDMD trust metric vs. drive amplitude F for the forced Duffing oscillator, showing the periodic-to-chaotic transition.

FIG. 6 — Warm-start speedup plot: cold-start vs. retrieved-start optimization convergence at 750 Hz target, demonstrating the 2708× speedup.

FIG. 7 — Cross-domain transfer: spring-mass system solutions retrieved and transferred to RLC circuits at matched (ω_0, Q) pairs.

FIG. 8 — Poincaré section with orbit-range normalization for period-1, period-2, and chaotic attractors.

FIG. 9 — CCT error vs. damping ratio ζ for IEEE 39-bus (10 generators). Red: raw EAC (D=0 formula). Blue: corrected EAC (a = 1.51). Shaded bands show min/max spread across generators. Dashed horizontal: 5% accuracy gate. Validated envelope: $\zeta \le 0.20$.

FIG. 10 — Leave-2-out cross-validation of scalar a across C(10,2) = 45 generator subsets. a range = 2.3% confirms universality.

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DETAILED DESCRIPTION OF THE INVENTION

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The following description sets forth specific embodiments of the invention. Alternative embodiments and variations will be apparent to those skilled in the art of dynamical systems, signal processing, and computational optimization.

SECTION 1: DOMAIN-INVARIANT DYNAMICAL TRIPLE

1.1 Overview

Physical systems governed by second-order linear ordinary differential equations of the form:

$m\cdot\ \ + b\cdot\ \ + k\cdot x = 0$ (mechanical)
$L\cdot\ddot{q} + R\cdot\dot{q} + (1/C)\cdot q = 0$ (electrical)

share the same normalized form when parameterized by natural frequency $\omega_0 = \sqrt{(k/m)} = 1/\sqrt{(LC)}$ and quality factor $Q = \omega_0\cdot m/b = \omega_0\cdot L/R$.

Gradient descent with momentum on a quadratic objective $f(x) = \kappa x^2/2$ obeys:

$x_{t+1} = x_t - lr\cdot \nabla f(x_t) + \beta\cdot(x_t - x_{t-1})$

which is equivalent to a damped oscillator with $\omega_0 = \sqrt{(lr\cdot\kappa)}$ [rad/step] and $Q = \omega_0 / (1 - \beta)$ (where β is the momentum coefficient, β [0, 1)).

1.2 The Domain-Invariant Triple

The invention defines a three-dimensional domain-invariant representation:

$v = (\log\_\omega_0\_norm, \log\_Q\_norm, \zeta)$

where:
$\log\_\omega_0\_norm = \log(\omega_0 / \omega_0\_ref(domain))$
$\log\_Q\_norm = \log(Q / 1)$ [Q dimensionless, reference Q=1]
$\zeta = 1 / (2Q)$ [damping ratio]

and $\omega_0\_ref(domain)$ is a domain-specific reference frequency:
- Physical systems (RLC, spring-mass): $\omega_0\_ref = 2\pi \times 1000$ rad/s (1 kHz)
- Gradient descent: $\omega_0\_ref = 1$ rad/step

1.3 Retrieval Using the Domain-Invariant Triple

A memory system stores evaluated (parameter, objective_value) pairs indexed by their domain-invariant triple. Given a query system in any domain:

(a) Compute (ω_0_query, Q_query) from domain-specific parameters.
(b) Compute $v\_query = (\log(\omega_0\_query/\omega_0\_ref), \log(Q\_query), 1/(2Q\_query))$.
(c) Retrieve k nearest neighbors in triple-space by Euclidean distance.
(d) Infer domain-native parameters from retrieved ($\omega_0\_retrieved$, $Q\_retrieved$) using the inverse mapping for the target domain.
(e) Initialize optimization from retrieved parameter values.

This enables warm-starting an RLC circuit optimization from a previously solved spring-mass problem at the same (ω_0, Q) operating point.

1.4 Empirical Results

The method achieves:
- 2708× reduction in objective function evaluations for a 750 Hz target frequency retrieval when a nearby prior solution exists in memory.
- 3.7× improvement in cross-domain transfer: solutions from spring-mass memory warm-start RLC optimization with convergence improvement factor

of 3.7× relative to cold-start RLC optimization.

SECTION 2: ENERGY-CONDITIONED SEPARATRIX RETRIEVAL

2.1 Nonlinear Oscillator Background

A Duffing oscillator with softening potential obeys:

$$+ \delta \; + \alpha x + \beta x^3 = 0 \quad \text{(unforced, } \alpha > 0,\ \beta < 0\text{)}$$

The potential energy $V(x) = \alpha x^2/2 + \beta x^4/4$ has a local minimum at $x=0$ and saddle points at $x = \pm\sqrt{\alpha/|\beta|}$ when $\beta < 0$. The separatrix energy is:

$$E_{sep} = \alpha^2 / (4|\beta|)$$

For total mechanical energy $E_0 = \; ^2/2 + V(x)$:
- $E_0 < E_{sep}$: bounded oscillation in the potential well
- $E_0 > E_{sep}$: unbounded trajectory (escape from well)

2.2 Energy-Based Regime Detection

The invention computes E_0 from initial conditions and compares to E_sep:

near_separatrix = (E_0 / E_sep > η_near) [η_near = 0.85 empirically]

The detection is performed in energy space, not position space, because:
(a) Topology is an energy-level concept (connected sublevel sets).
(b) Position-based detection fails under coordinate transformations.
(c) Energy is invariant under Hamiltonian coordinate changes.

2.3 Retrieval Conditioning

When near_separatrix = True:
(a) EDMD cannot reliably fit a Koopman operator because the trajectory spends time near the saddle point where period -> ∞.
(b) The effective natural frequency ω_0_eff computed from EDMD eigenvalues approaches zero, but this limit is not observable with finite trajectory length.
(c) The invention substitutes a floor frequency ω_floor and sets near_separatrix=True in the memory entry.
(d) Retrieval queries whose E_0/E_sep > η_near are only matched against entries also flagged near_separatrix=True, preventing incorrect transfer of sub-separatrix solutions to near-separatrix queries.

2.4 Implementation

The retrieval gate is implemented as:

```
def retrieve(query, memory):
v_query = compute_triple(query)
E_ratio = query.E0 / separatrix_energy(query)
```

```
if E_ratio > ETA_NEAR:
candidates = [m for m in memory if m.near_separatrix]
else:
candidates = [m for m in memory if not m.near_separatrix]
return nearest_neighbor(v_query, candidates)
```

SECTION 3: TOPOLOGY-AWARE MANIFOLD DISCRIMINATION

3.1 The Curvature Profile

For a nonlinear oscillator, the effective natural frequency ω_0_eff varies with amplitude A (or equivalently, with energy E_0). The invention characterizes this variation via the curvature profile:

$$\kappa(\text{log\_E}) = d(\log(\omega_0\text{\_eff} / \omega_0\text{\_linear})) / d(\text{log\_E})$$

where ω_0_linear = $\sqrt{\alpha}$ is the small-oscillation linear frequency.

For a hardening Duffing oscillator ($\beta > 0$):
- ω_0_eff increases with A: positive curvature, bounded
- No separatrix exists; trajectory is always bounded
- Curvature profile is approximately flat on log-log scale

For a softening Duffing oscillator ($\beta < 0$) below separatrix:
- ω_0_eff decreases toward zero as E_0 -> E_sep
- Curvature profile exhibits a downward spike near E_sep
- The spike is a topological signature of separatrix existence

3.2 Geometric Signature

The curvature profile κ(log_E) is binned into n_bins equal intervals over the energy range of the trajectory and normalized to unit norm. This produces a fixed-length geometric signature vector s   R^n_bins.

Two systems are compared using cosine similarity:
$\text{sim}(s_1, s_2) = s_1 \cdot s_2 / (||s_1|| \cdot ||s_2||)$

Key result: For hardening (flat profile) vs. softening (spiked profile), cosine similarity ≈ 0.0. This constitutes topological discrimination — the manifold separates qualitatively distinct dynamical regimes.

Degenerate case: If both profiles are flat (both $\beta = 0$ or both far from separatrix), similarity = 1.0 (profiles are identical up to normalization).

3.3 Bifurcation Approach Detection

The invention further implements a three-criterion bifurcation approach detector:

(a) Curvature spike: max(|κ|) in recent energy window exceeds threshold
(b) Frequency floor: ω_0_eff(E) < ω_floor for multiple consecutive points
(c) Unit-circle proximity: the dominant oscillatory EDMD eigenvalue

satisfies |λ_dominant| -> 1.0 (period doubling precursor)

Criterion (c) uses only the oscillatory eigenvalue with smallest |Im(log λ)|, not the full EDMD polynomial eigenvalue set, to avoid false positives from non-physical eigenvalue branches introduced by polynomial basis expansion.

3.4 Cross-Domain Restriction

A cross-domain transfer is permitted only when:
- The source and target systems share the same topological regime
(both sub-separatrix, or both hardening, determined by sign(β) and E_0/E_sep)
- The curvature profile similarity exceeds a threshold (empirically 0.7)
- The energy log-ratio |log_E_query - log_E_stored| is within a window

When these conditions are not met, the retrieval system returns no result and the optimization proceeds cold-start.

SECTION 4: BIFURCATION-AWARE EDMD TRUST METRIC

4.1 Failure Mode of Spectral Gap for Driven Systems

For driven nonlinear systems (forced oscillator: + δ + αx + $βx^3$ = F·cos(Ωt)), the system settles to a limit cycle attractor in steady state. The EDMD spectral gap — defined as the magnitude difference between the two dominant eigenvalues — collapses to approximately zero because all limit-cycle eigenvalues lie on or near the unit circle. This makes spectral gap an unreliable classifier for distinguishing periodic from chaotic attractors in driven systems.

4.2 Reconstruction-Error Trust Metric

The invention defines trust as a function of EDMD reconstruction error:

trust = max(0, 1 - recon_error / η_max)

where:
- recon_error = ||Φ(X') - K·Φ(X)||_F / ||Φ(X')||_F
(Frobenius-norm relative error of Koopman propagation)
- η_max is calibrated on purely random (white noise) trajectories as the maximum expected reconstruction error when no Koopman structure exists
- trust   [0, 1], where trust = 1 means perfect Koopman structure

Classification:
is_periodic (Abelian) : trust ≥ 0.3
is_chaotic : trust < 0.3

Physical basis: A periodic attractor has a finite-dimensional Koopman invariant subspace that the EDMD polynomial basis can span, resulting in low reconstruction error. A chaotic attractor has an infinite-dimensional or fractal Koopman spectrum that the polynomial basis cannot capture, resulting in high error.

4.3 Period Classification via Poincaré Section

Given classification as periodic, the period multiplicity is determined by a Poincaré section with orbit-range normalization:

(a) Sample state x at $t = n \cdot T_{drive}$ for $n = n_{transient}, ..., n_{total}$ (stroboscopic sampling synchronized to the drive period T_{drive})
(b) Compute orbit_range = max(x_Poincaré) - min(x_Poincaré)
(c) Compute normalized span: span_norm = span / orbit_range
(d) Count clusters: period-k identified by k distinct clusters with inter-cluster gaps > 12% of section span

Orbit-range normalization is required because phase drift (when T_drive/dt is not an integer) introduces a 1-3% smearing of Poincaré points that would otherwise be misclassified as multi-period oscillations.

The invention thereby classifies driven Duffing dynamics as:
period-1, period-2, period-4 (period-doubling cascade), or chaotic,
using only trajectory data and without access to the governing equations.

SECTION 5: ANALYTIC CRITICAL CLEARING TIME WITH UNIVERSAL DAMPING CORRECTION

5.1 Problem Statement

Transient stability assessment of a power grid requires determining the Critical Clearing Time (CCT) — the maximum fault duration for which a generator maintains synchronism after a fault. N-1 contingency screening requires $O(G \times F)$ CCT computations, where G is the number of generators and F is the number of credible faults. The standard reference method — binary search over fault duration with 4th-order Runge-Kutta (RK4) integration — requires approximately:

~13 binary-search iterations × 3,000 settle-steps × 4 RK4 evaluations
≈ 156,000 ODE evaluations per CCT

This is computationally prohibitive for real-time or near-real-time screening.

5.2 System Model

The Single-Machine Infinite Bus (SMIB) classical swing equation is:

$$M \cdot \ddot{\delta} + D \cdot \dot{\delta} = P - P \cdot \sin(\delta)$$

where δ is rotor angle [rad], $\omega = \dot{\delta}$ is speed deviation [rad/s], $M = 2H/\omega s$ is inertia [pu·s^2/rad] (H is inertia constant [s], $\omega s = 2\pi \cdot 60$ rad/s), D is damping [pu·s/rad], P is mechanical power [pu], and P is peak electrical power [pu].

The stable equilibrium is $\delta s = \arcsin(P/P)$. The unstable equilibrium is $\delta u = \pi - \delta s$. The separatrix energy (energy barrier to loss of synchronism) is:

$$E\_sep = 2P \cdot \cos(\delta s) - P \cdot (\pi - 2\delta s)$$

This is structurally identical to the separatrix energy $\alpha^2/(4|\beta|)$ of the softening Duffing oscillator — both define the energy boundary of the stability region in the invariant manifold shared across dynamical domains.

5.3 Equal-Area Criterion (Undamped, Three-Phase Fault)

For a complete three-phase fault ($P \to 0$ during fault) and zero damping ($D=0$), the equal-area criterion gives the critical clearing angle analytically:

$$\cos(\delta c) = P \cdot (\pi - 2\delta s) / P - \cos(\delta s) \quad \text{(EAC-1)}$$

The critical clearing time follows from exact integration of $M \cdot \ddot{\delta} = P$:

$$CCT\_EAC = \sqrt{2M \cdot (\delta c - \delta s) / P} \quad \text{(EAC-2)}$$

This formula requires zero ODE evaluations. It is analytically exact for $D = 0$ and a complete three-phase fault.

5.4 Universal Invariant: $\omega_0 \cdot$ CCT_EAC

The linearized natural frequency at stable equilibrium is:

$$\omega_0 = \sqrt{P \cdot \cos(\delta s) / M}$$

For all generators operating at the same loading ratio $P/P = 2$ ($\delta s = 30°$), the product:

$$\omega_0 \cdot CCT\_EAC = \sqrt{2 \cdot \sqrt{3} \cdot (\delta c - \delta s)} \approx 1.73 \quad \text{(INV-1)}$$

is a universal constant, independent of M, P, P individually. This invariant is a consequence of the geometry of the equal-area constraint — the same geometric structure that defines the invariant manifold of Contribution 1. Equation (INV-1) guarantees that a single scalar correction to CCT_EAC applies universally across all generators at the same loading ratio.

5.5 Damping Correction

For $D > 0$, damping reduces rotor kinetic energy during the fault phase, causing the true CCT (reference) to exceed CCT_EAC. The first-order correction is:

$$CCT\_corrected = CCT\_EAC / (1 - a \cdot \zeta) \quad \text{(CORR-1)}$$

where $\zeta = D/(2M\omega_0)$ is the damping ratio and a is fitted by ordinary least squares (OLS). Defining $e = CCT\_EAC,i - CCT\_ref,i < 0$ and $x = \zeta \cdot CCT\_ref,i$:

$$a = -\Sigma(e \cdot x) / \Sigma(x^2) \quad \text{(OLS-1)}$$

The universality of a follows from (INV-1): since $\omega_0 \cdot$CCT_EAC is constant, the first-order perturbation slope $C = \omega_0 \cdot CCT\_EAC/3 \approx 0.577$ is also constant. The fitted a absorbs higher-order terms and is therefore generator-independent.

5.6 Cross-Validation of Universality

To verify that a is a structural property and not a fitting artifact, leave-2-out cross-validation is performed across all C(G,2) generator subsets. For each split, a is fitted on (G−2) generators and tested on the 2 held-out generators.

On the IEEE 39-bus New England system (G=10, Anderson & Fouad 2003):
- a (mean): 1.5109
- a (std): 0.0100 (CV = 0.66%)
- a (range/mean): 2.3% (stability gate: < 10%)
- Max test error on held-out generators: 2.90%

The near-zero coefficient of variation (0.66%) confirms that a is determined by the geometry of the equal-area constraint (Equation INV-1), not by which machines appear in the training set.

5.7 Invariant Embedding Distance

The embedding distance of a generator in the domain-invariant triple space (log_ω_0_norm, log_Q_norm, ζ) varies by less than 10 ^{8} across all 10 generators at the same ζ — confirming that the triple is generator-independent and that the damping correction operates on a geometrically universal quantity.

5.8 Empirical Results — IEEE 39-Bus Benchmark

Reference data: Anderson & Fouad (2003), Table 2.7, 10-generator New England equivalent of the IEEE 39-bus system. All generators at P = 2P (δs = 30°). Damping sweep: ζ   {0.01, 0.03, 0.05, 0.10, 0.20}.

Undamped (D=0): mean CCT error = 0.8%, mean speedup = 57,946×
Corrected (D>0): max CCT error = 2.73% across all 50 (generator, ζ) pairs
Validated range: ζ   [0, 0.20], Q   [2, ∞) — full inter-area and local mode range

The corrected formula:
CCT_corrected = √(2M·(δc−δs)/P) / (1 − 1.51·ζ)

requires zero ODE evaluations per machine per contingency.

SECTION 6: SYSTEM ARCHITECTURE AND INTEGRATION

5.1 System Components

The invention is implemented as an integrated computational system comprising:

Component A: Domain Encoder
Input: domain-specific parameters (L, R, C for RLC; k, m, b for spring-mass; lr, κ, β for gradient descent; α, β_nl, δ for Duffing oscillator)
Output: domain-invariant triple (log_ω_0_norm, log_Q_norm, ζ)
Method: domain-specific mapping to (ω_0, Q), followed by normalization

Component B: EDMD Koopman Fitter
Input: trajectory data {x(t_0), x(t_1), ..., x(t_N)}
Output: Koopman matrix K, eigenvalues {λ }, eigenvectors {φ },
reconstruction error, spectral gap

Method: Extended Dynamic Mode Decomposition with polynomial observable basis
$G = \Phi(X)\cdot\Phi(X)$, $A = \Phi(X')\cdot\Phi(X)$, $K = G \cdot A$
Preprocessing: Discard near-zero amplitude tail (amplitude < 3% of peak)
before fitting to avoid fitting to numerical noise

Component C: Energy-Conditioned Retrieval Gate
Input: query (ω_0, Q, E_0), memory entries
Output: filtered memory candidates
Method: Compute E_sep from query parameters; classify query energy regime;
filter memory to matching regime before nearest-neighbor retrieval

Component D: Topology Discriminator
Input: stored curvature profile, query curvature profile
Output: compatibility score   [0, 1]
Method: Cosine similarity of geometric signatures; reject if score < 0.7

Component E: Trust Classifier
Input: EDMD reconstruction error
Output: {is_periodic, is_chaotic, period_multiplicity}
Method: Reconstruction-error trust metric + Poincaré section analysis

Component F: Memory Store
Structure: List of entries {v, params, E_regime, curvature_signature,
trust, near_separatrix, objective_value}
Indexed by: domain-invariant triple v for approximate nearest-neighbor search
Operations: insert, retrieve_candidates(v_query, regime_filter)

5.2 Optimization Workflow

Given a new optimization target in domain D with target (ω_0_target, Q_target):

Step 1: Compute v_query from (ω_0_target, Q_target, domain=D)
Step 2: Run energy estimation on representative initial condition
Step 3: Apply retrieval gate (Components C, D) to get candidates
Step 4: If candidates exist:
Infer domain-D native parameters from best candidate
Initialize optimizer at inferred parameters
Else:
Initialize optimizer at domain-D default (cold start)
Step 5: Run local optimization; evaluate objective
Step 6: Fit EDMD on resulting trajectory (Component B)
Step 7: Classify dynamics (Component E)
Step 8: Store result in Memory (Component F) with domain-invariant triple

5.3 Multi-Objective Extension

The domain-invariant triple naturally extends to multi-objective optimization
over (ω_0, Q) simultaneously:

Objective: $J = w\_freq \cdot |\omega_0\_achieved - \omega_0\_target| / \omega_0\_target$
$+ w\_Q \cdot |Q\_achieved - Q\_target| / Q\_target$

Memory entries store the 2D operating point (ω_0, Q) and retrieval is by

Euclidean distance in the 2D (log_ω_0_norm, log_Q_norm) subspace.

Spearman rank correlation of retrieved-solution quality vs. memory distance achieves $\rho \geq 0.9$ for both RLC and spring-mass domains across Q   {0.5, 1, 2, 5}.

SECTION 7: ALTERNATIVE EMBODIMENTS

7.1 Alternative Basis Functions for EDMD
The polynomial observable basis can be replaced with radial basis functions, Fourier modes, or neural network embeddings (Deep EDMD) without departing from the scope of the invention. The trust metric and reconstruction-error formulation apply to any EDMD variant.

7.2 Alternative Distance Metrics
The domain-invariant triple can be indexed using any metric supporting approximate nearest-neighbor search: cosine distance, Mahalanobis distance with learned covariance, or graph-based indices (HNSW, FAISS). The energy-conditioning gate applies regardless of the chosen metric.

7.3 Non-Duffing Nonlinear Oscillators
The energy-conditioned separatrix detection applies to any potential V(x) with a local maximum. The separatrix energy E_sep generalizes to the saddle-point potential energy. For van der Pol oscillators, limit-cycle energy replaces E_sep as the conditioning quantity.

7.4 Discrete-Time Systems
The domain-invariant triple applies to discrete-time dynamical systems by interpreting ω_0 as frequency in [rad/step] rather than [rad/s]. The trust metric applies without modification to EDMD fitted to discrete-time trajectories.

7.5 Higher-Dimensional Systems
The curvature profile generalizes to multi-dimensional systems by tracking the dominant Koopman eigenvalue frequency as a function of energy in the dominant mode direction. The geometric signature remains a 1D profile parameterized by energy level.

7.6 Neural Network Koopman Approximation
The EDMD Koopman matrix K can be approximated by a neural network trained to minimize reconstruction error. The trust metric — reconstruction error relative to random-noise baseline — applies directly to the neural approximation.

SECTION 8: DOMAIN APPLICABILITY — THE UNIVERSAL GEOMETRIC PATTERN

8.1 The Unifying Mathematical Structure

All domains covered by this invention share the same normalized governing equation. Any physical, computational, or biological system that can be characterized — exactly or approximately — by a second-order ordinary differential equation:

$M\cdot\ \ + D\cdot\ \ + K\cdot x = F(t)$ (GEN-1)

or its discrete-time equivalent, is characterizable by the domain-invariant triple (ω_0, Q, ζ) where $\omega_0 = \sqrt{(K/M)}$, $Q = M\omega_0/D$, $\zeta = 1/(2Q)$.

The invention's methods — domain-invariant indexing, energy-conditioned retrieval, topology discrimination, EDMD trust classification, and universal correction scalar fitting — apply to any system of this form, regardless of the physical interpretation of the variables x, M, D, K.

The core IP claim: the geometric structure of the invariant manifold (log_ω_0_norm, log_Q_norm, ζ) is universal. Any domain-specific quantity whose characteristic time T_char satisfies the universality condition:

$\omega_0 \cdot$ T_char = f(loading_ratio_only) (UNI-1)

admits a loading-class-specific scalar correction a such that:

T_corrected = T_analytic / $(1 - a\cdot\zeta)$ (UNI-2)

where a is invariant across all instances within the same loading class. This is the geometric fingerprint of the equal-area / separatrix structure in the invariant manifold.

8.2 Validated Domains

The following domains have been explicitly validated in this invention:

Domain 1 — RLC Electrical Circuits
Parameters: L (inductance), R (resistance), C (capacitance)
$\omega_0 = 1/\sqrt{(LC)}$, $Q = \omega_0 L/R$
Application: cross-domain warm-start retrieval (2708× speedup validated)

Domain 2 — Spring-Mass-Damper Mechanical Systems
Parameters: k (stiffness), m (mass), b (damping coefficient)
$\omega_0 = \sqrt{(k/m)}$, $Q = m\omega_0/b$
Application: cross-domain source for RLC retrieval (3.7× improvement)

Domain 3 — Gradient Descent with Momentum
Parameters: lr (learning rate), κ (curvature), β (momentum coefficient)
$\omega_0 = \sqrt{(lr\cdot\kappa)}$ [rad/step], $Q = \omega_0/(1-\beta)$
Application: optimizer regime indexing; stability gate ($\omega_0^2 < 2(1+\beta)\cdot 0.95$)

Domain 4 — Duffing Oscillator (Hardening and Softening)
Parameters: α (linear stiffness), β (cubic coefficient), δ (damping)
ω_0_linear = $\sqrt{\alpha}$, $Q = \sqrt{\alpha}/\delta$; E_sep = $\alpha^2/(4|\beta|)$ for $\beta < 0$
Application: topology discrimination, separatrix gate, bifurcation detection

Domain 5 — Power Grid SMIB (Transient Stability)
Parameters: M (inertia), D (damping), P (mechanical), P (electrical)
$\omega_0 = \sqrt{(P\ \cos(\delta s)/M)}$, $Q = M\omega_0/D$; E_sep = $2P\ \cos(\delta s) - P\ (\pi - 2\delta s)$

Application: CCT estimation — 57,946× speedup, 2.73% max error validated

8.3 Claimed Domains by Analogy

The following domains are claimed as embodiments of the invention by direct mathematical analogy to GEN-1. For each, the domain-invariant triple (ω_0, Q) is identified and the applicable inventive contribution is stated.

Domain 6 — Superconducting Quantum Circuits (Transmon Qubits)
Governing equation: the Josephson junction circuit is governed by:
$C\cdot\ddot{\varphi} + (1/R)\cdot\dot{\varphi} + Ic\cdot\sin(\varphi) = I\_ext$
where φ is the superconducting phase. This is exactly the SMIB swing equation under the substitution $\delta \to \varphi$, $P \to I\_ext$, $P \to Ic$. The cosine potential creates a separatrix at the junction critical current.
The Kerr nonlinearity of a transmon qubit is the $\beta < 0$ Duffing term.
$\omega_0 = \sqrt{(Ic/C\cdot\hbar/2e)}$ (plasma frequency), $Q = \omega_0\cdot C\cdot R$.
Applicable contributions: Contributions 2, 3, 5 (energy-conditioned gate, topology discrimination, universal characteristic-time correction).
The escape time from the superconducting state (switching time) is the quantum analog of CCT and admits the universal correction (UNI-2).

Domain 7 — Phase-Locked Loops (PLLs)
A second-order charge-pump PLL with loop filter has:
$\ddot{\varphi} + (Kd\cdot Kv\cdot\tau_2/\tau_1)\cdot\dot{\varphi} + (Kd\cdot Kv/\tau_1)\cdot\varphi = \omega\_ref$
where Kd is detector gain, Kv is VCO gain, τ_1, τ_2 are filter constants.
$\omega_0 = \sqrt{(Kd\cdot Kv/\tau_1)}$, $Q = \omega_0\cdot\tau_1/(1 + Kd\cdot Kv\cdot\tau_2)$.
Lock acquisition time T_lock is the PLL analog of CCT. When $\omega_0\cdot T\_lock$ is loop-bandwidth-determined (fixed ω_0/ω_ref ratio), the universal correction (UNI-2) applies for damping ζ in the PLL's operating range.
Applicable contributions: Contributions 1, 5.

Domain 8 — Structural / Civil Engineering (Seismic Response)
A single-degree-of-freedom building model under seismic excitation obeys:
m· + c· + k·x = −m· _ground
This is GEN-1 with F = −m· _ground. $\omega_0 = \sqrt{(k/m)}$ (natural frequency), $Q = m\omega_0/c$. The collapse capacity ratio (CCR) — maximum ground motion intensity before structural collapse — is structurally identical to CCT.
At fixed ductility ratio (loading ratio equivalent), $\omega_0\cdot T\_collapse$ is approximately constant across structures, admitting the universal correction.
Applicable contributions: Contributions 1, 5.

Domain 9 — Nuclear Reactor Point Kinetics (Prompt Criticality)
The one-group point kinetics equation with a single delayed neutron group:
$(1/v)\cdot d\varphi/dt = (k-1)\cdot\varphi/L + \lambda\cdot C$
$dC/dt = \beta\cdot k\cdot\varphi/L - \lambda\cdot C$
near steady state linearizes to a second-order system with:
$\omega_0^2 \approx \lambda\cdot(k-1)/L$ (for k near 1), $Q \approx \omega_0\cdot L/\beta\cdot v$
The scram time (time to shut down on a reactivity transient) is the nuclear analog of CCT. Applicable contributions: Contributions 1, 5.

Domain 10 — Acoustic Resonators (Loudspeakers, Microphones, MEMS)
An electrodynamic loudspeaker moving mass obeys:
M · + R · + (1/C)·x = Bl·i

where M , R , C are mechanical mass, resistance, compliance; Bl is the force factor; i is coil current.
$\omega_0 = 1/\sqrt{(M \cdot C)}$ (resonance frequency), $Q = M \cdot \omega_0/R$.
MEMS gyroscopes and accelerometers are spring-mass systems at microscale with identical mathematical form ($Q = 10^3$–10^6 typical).
Applicable contributions: Contributions 1, 2, 3.

Domain 11 — Laser Cavities and Optical Resonators
A laser cavity with gain saturation near threshold obeys a rate equation system linearizable to GEN-1 around steady-state photon density:
$\omega_0^2 \approx g_0 \cdot (1/\tau \cdot \tau)$, $Q \approx \omega_0 \cdot \tau$
where g_0 is small-signal gain, τ photon lifetime, τ carrier lifetime.
The relaxation oscillation frequency is ω_0; damping ratio $\zeta \approx 1/(2Q)$.
Turn-on delay (time to reach threshold photon density from injection) is the laser analog of CCT. Applicable contributions: Contributions 1, 5.

Domain 12 — RF / Antenna Resonators and Tank Circuits
Any parallel or series RLC tank circuit in RF systems:
$\omega_0 = 1/\sqrt{(LC)}$, $Q = R\cdot\sqrt{(C/L)}$ (parallel), $Q = (1/R)\cdot\sqrt{(L/C)}$ (series)
Transmitter frequency stabilization, impedance matching networks, and crystal oscillators ($Q = 10^4$–10^6) are embodiments.
Applicable contributions: Contributions 1, 3, 4.

Domain 13 — Thermodynamic / Thermal Systems
A two-body thermal system (e.g., building HVAC, heat exchanger):
$C_{th}\cdot dT/dt = (T_{env} - T)/R_{th} + P_{heater}$
Second-order thermal systems (two-zone models) obey GEN-1 with:
$\omega_0 = 1/\sqrt{(C_1\cdot C_2\cdot R_1\cdot R_2)}$, $Q = \omega_0\cdot\sqrt{(C_1\cdot C_2)}/(1/R_1 + 1/R_2)$
The thermal settling time (time to reach setpoint from disturbance) is the thermal analog of T_char. Applicable contributions: Contributions 1, 5.

Domain 14 — Biological Neural Oscillators
The Hodgkin-Huxley model linearized around resting potential is:
$Cm\cdot dV/dt \approx -gL\cdot V - gNa\cdot m_\infty(V)\cdot V - gK\cdot n_\infty(V)\cdot V + I_{ext}$
which at the subthreshold regime reduces to GEN-1 with effective:
$\omega_0 \approx \sqrt{(gNa\cdot gK/Cm^2)}$, $Q \approx Cm\cdot\omega_0/gL$
Firing threshold crossing time (interspike interval) is the neural analog of CCT. Applicable contributions: Contributions 1, 2, 5.

Domain 15 — Economic and Business Cycle Oscillators
The Samuelson-Hicks multiplier-accelerator model and its extensions obey:
$\ddot{Y} + (s - a)\cdot \ + s\cdot Y = G$
where Y is output, s is the marginal propensity to save, a is the accelerator coefficient. $\omega_0 = \sqrt{(s)}$, $Q = s/(s-a)$ for $a < s$.
The recession depth and recovery time are business-cycle analogs of CCT.
Applicable contributions: Contributions 1, 5 (universality of recovery time correction when $\omega_0\cdot T_{recovery}$ is savings-rate-determined).

Domain 16 — Railway Vehicle Hunting Oscillations
Lateral hunting of a railway bogie obeys:
$m\cdot\ddot{y} + b\cdot \ + (k\psi/R^2)\cdot y = 0$
where m is bogie mass, b is lateral damping, kψ is yaw stiffness, R is wheel conicity radius. $\omega_0 = \sqrt{(k\psi/(m\cdot R^2))}$, $Q = m\cdot\omega_0/b$.

Critical speed (speed above which hunting becomes unstable) is structurally equivalent to a CCT threshold. Applicable contributions: Contributions 1, 5.

Domain 17 — Transmission Lines (Distributed RLC)
A finite transmission line segment (telegrapher's equations) reduces to a lumped RLC model for wavelength >> segment length:
$L'\cdot C'\cdot \ddot{Y} + R'\cdot C'\cdot \; + Y = Y_{source}$
$\omega_0 = 1/\sqrt{L'C'}$, $Q = \sqrt{L'/C'}/R'$. Signal propagation delay (time for step to traverse line within tolerance) is the transmission analog of CCT.
Applicable contributions: Contributions 1, 5.

Domain 18 — Gyroscope Precession Dynamics
A gyroscope with rotor spin angular momentum H and applied torque τ:
$I\cdot\ddot{\theta} + b\cdot\dot{\theta} + (H^2/I)\cdot\theta = \tau$
$\omega_0 = H/I$ (precession frequency), $Q = \omega_0\cdot I/b$.
Settling time after angular disturbance is the gyroscope analog of CCT.
Applicable contributions: Contributions 1, 2.

8.4 The Universal Correction Principle (Domain-Agnostic Claim)

For any system in any domain satisfying GEN-1 with a characteristic time T_char(params, loading) that obeys universality condition (UNI-1):

STEP 1: Compute T_analytic using the undamped (D=0) analytic formula applicable to that domain.
STEP 2: Compute $\zeta = D/(2M\omega_0)$ from domain parameters.
STEP 3: Apply correction: $T_{corrected} = T_{analytic} / (1 - a\cdot\zeta)$
STEP 4: Fit a once per loading class by OLS across representative instances; verify universality by cross-validation.

This procedure is claimed for all domains listed in Sections 8.2, 8.3, and 9, and for any future domain where universality condition (UNI-1) holds.

8.5 Nonlinear Extension of the Domain-Invariant Triple

For nonlinear systems where K and D are state-dependent, the domain-invariant triple is generalized as follows:

(a) EDMD-extracted triple: ω_{0_eff} and Q_eff are computed from the dominant Koopman eigenvalue pair extracted from trajectory data, as specified in Claim 20. This is valid for any nonlinear system admitting a finite-dimensional Koopman invariant subspace (periodic and quasi-periodic attractors).

(b) Energy-parameterized triple: For systems with a known potential V(x), $\omega_{0\_eff}(E) = \sqrt{V''(x_{eq}(E)) / M_{eff}(E)}$ varies with energy E. The triple is evaluated at the operating energy level and the energy-conditioned gate (Contribution 2) partitions the nonlinear state space into distinct retrieval regimes separated by the separatrix.

(c) Linearized triple at equilibrium: $\omega_0 = \sqrt{K(x_{eq})/M}$, $Q = M\cdot\omega_0/D(x_{eq},0)$ evaluated at the stable equilibrium x_eq provides a first-order index valid for small-to-moderate deviations. The curvature profile (Contribution 3) quantifies the energy range over which this linearization remains accurate.

The invention thereby covers the full spectrum from weakly nonlinear systems (where the linearized triple suffices) to strongly nonlinear systems near bifurcation or separatrix crossing (where the energy-conditioned gate and topology discriminator are essential). All three nonlinear extension modes are implemented and claimed in Claims 1-29.

SECTION 9: EXTENDED DOMAIN APPLICABILITY

The inventive methods of Contributions 1-5 apply to any domain in which a dynamical regime can be characterized by a frequency-like scale ω_0, a dissipation-like ratio Q (or $\zeta = 1/(2Q)$), and a topological energy boundary (separatrix or stability limit). This section enumerates additional domains organized by field. For each domain, the role of ω_0, Q, T_char, and the applicable inventive contribution are identified.

9.A COMPUTATIONAL SCIENCE AND PHYSICS-INFORMED AI

9.A.1 Nonlinear PDE Solving and Physics-Informed Neural Networks (PINNs)
PINNs embed a PDE residual into a neural network loss and minimize over a collocation grid. The loss landscape near a trained solution is governed by the PDE's spectral structure: the dominant eigenvalue of the linearized residual operator defines an effective ω_0, and the ratio of dominant to subdominant eigenvalues defines an effective Q. The invention's domain-invariant triple indexes PINN training regimes; the energy-conditioned gate identifies when residual energy exceeds a stability threshold (separatrix analog in loss space); and the EDMD trust metric classifies whether training has converged to a periodic attractor (limit cycle in gradient flow) or is chaotic (stochastic gradient noise dominated). The universal correction (UNI-2) applies to convergence time estimates: T_convergence = T_analytic_linear / ($1 - a \cdot \zeta$_eff) where ζ_eff is the effective damping ratio of the gradient flow around the solution manifold.

9.A.2 Neural Operators for Parametric PDE Families
Neural operators (DeepONet, Fourier Neural Operator) learn mappings from PDE parameters to solution fields. For a parametric family sharing the same operator topology (same differential order, same boundary condition type, same coupling structure), solutions share a dominant frequency ω_0(parameters) and quality factor Q(parameters). The invention indexes neural operator regimes by the domain-invariant triple of the PDE's linearized spectral profile; retrieves stored operator weights from a nearby regime as warm initialization; and applies the topology discriminator (Contribution 3) to reject initialization from topologically distinct operator classes (e.g., elliptic vs. parabolic PDE).

9.A.3 Surrogate Modeling for CFD and Finite Element Analysis (FEA)
Computational fluid dynamics and structural FEA models have dominant resonant modes (ω_0 = structural natural frequency; Q = modal quality factor). The invention stores prior simulation results indexed by the modal triple and retrieves warm-start basis vectors (POD modes, reduced basis) for a new simulation in the same dynamical regime. The energy-conditioned gate identifies

when the query operating point is near a flutter or buckling bifurcation (separatrix analog in parameter space), preventing incorrect basis transfer. T_char = simulation wall-clock time to convergence; the universal correction applies when $\omega_0 \cdot T\_converge$ is mesh-resolution-determined.

9.A.4 Turbulence Closure Modeling

Turbulent flows exhibit a cascade of eddy structures with characteristic frequencies $\omega_0(k)$ at wavenumber k and effective quality factors $Q(k) = \omega_0(k)/\gamma(k)$ where $\gamma(k)$ is the eddy decay rate. The Koopman EDMD decomposition (Contribution 4) extracts dominant turbulent modes; the trust metric classifies whether the flow is in an organized (low-dimensional Koopman) or chaotic regime. The universal correction applies to large-eddy simulation subgrid closure: the subgrid stress tensor correction scales as $1/(1 - a\cdot\zeta\_subgrid)$ where $\zeta\_subgrid$ is the damping ratio of the subgrid model.

9.A.5 Inverse Parameter Identification

Inverse problems (parameter estimation from observations) reduce to minimization of a residual over parameter space. The gradient flow of the residual near the true parameter is a damped oscillator with $\omega_0 = \sqrt{(\lambda\_min(nabla^2 J))}$ and Q inversely proportional to the condition number. The invention retrieves warm-start parameter estimates from memory entries at nearby (ω_0, Q) operating points, accelerating convergence. The topology discriminator rejects retrieval when the query parameter space has a different separatrix structure (multimodal posteriors).

9.A.6 Chaotic System Prediction

Directly covered by Contribution 4 (EDMD trust metric) and Contribution 3 (curvature profile bifurcation approach). Extended application: for any observable time series from an unknown system, the invention classifies the generating system as periodic, quasi-periodic, or chaotic using Koopman reconstruction error, without access to the governing equations. Short-term prediction is achievable in the Abelian (low-chaos) regime; long-term prediction fails gracefully in the chaotic regime (trust < threshold) with explicit uncertainty quantification.

9.A.7 Multi-Scale Physical Simulation Acceleration

Multi-scale systems (molecular dynamics + continuum, micro + macro-structural) operate at multiple ω_0 scales. The invention stores Koopman representations at each scale indexed by the domain-invariant triple and retrieves compatible fine-scale solutions given a coarse-scale regime query. The energy-conditioned gate identifies when scales are coupled (near-separatrix in scale coupling energy), preventing inappropriate single-scale retrieval. T_char is the fine-scale integration time per coarse timestep; universal correction applies when $\omega_0\_fine \cdot T\_fine$ is ratio-of-scales determined.

9.A.8 Quantum System Approximation

Extended from Claim 11. Beyond Josephson junctions, applies to:

(a) Quantum harmonic oscillator arrays (ion traps, optical lattices): ω_0 from trap frequency, Q from motional coherence time, T_char = thermalization time.

(b) Quantum annealing: the transverse field Ising model near the phase transition is a softening Duffing analog ($\beta < 0$, separatrix = gap closing point). The annealing time $T\_anneal \sim 1/gap^2$ is the CCT analog; universal correction applies when the gap closing rate is loading-class-determined.

(c) Variational quantum eigensolvers (VQE): gradient flow on the variational manifold has ω_0 from the Hessian spectrum and Q from measurement noise; the invention warm-starts VQE from memory of nearby Hamiltonians.

9.A.9 Materials Discovery and Lattice Property Prediction
Crystal lattice vibrations (phonons) are characterized by a phonon dispersion relation $\omega(k)$ and phonon lifetimes $\tau(k) = Q(k)/\omega(k)$. The domain-invariant triple at each wavevector k indexes phonon regimes. The invention retrieves Koopman representations of lattice dynamics from memory of structurally similar materials (same space group symmetry, similar force constants -> similar ω_0/Q topology), enabling rapid phonon band structure prediction without full DFT calculation. The energy-conditioned gate identifies phonon instabilities (imaginary frequency modes = $\beta < 0$ Duffing, separatrix = phase transition energy).

9.A.10 Semiconductor Device Modeling
Carrier transport in semiconductor devices is governed by drift-diffusion equations that in the small-signal regime reduce to GEN-1:
$Cg \cdot \ddot{V} + (1/Rds) \cdot \dot{V} + (1/Ls) \cdot V = I\_gate$
where Cg is gate capacitance, Rds drain-source resistance, Ls parasitic inductance. $\omega_0 = 1/\sqrt{(Cg \cdot Ls)}$, $Q = Rds/\sqrt{(Ls/Cg)}$. The invention retrieves device operating point warm-starts from memory indexed by (ω_0, Q); topology discriminator identifies punch-through or avalanche breakdown bifurcations (separatrix in carrier density space).

9.A.11 Electromagnetic Field Approximation
Maxwell's wave equation in a resonant cavity:
$\varepsilon_0 \cdot \ddot{E} + \sigma \cdot \dot{E} + (1/\mu_0) \cdot \text{nabla}^2 E = J\_source$
reduces to GEN-1 with $\omega_0 = c/\sqrt{(\mu\varepsilon)} \cdot (m\pi/L)$ for mode (m) in a cavity of length L, and $Q = \omega_0 \cdot \varepsilon_0/\sigma$. The invention retrieves cavity mode Koopman representations from memory indexed by (ω_0, Q) for rapid field approximation across geometries.

9.A.12 Circuit Nonlinear Component Modeling
Nonlinear circuit elements (varactor diodes, Josephson junctions, memristors) introduce $\beta \neq 0$ terms into GEN-1, creating Duffing-type dynamics. The topology discriminator (Contribution 3) classifies hardening ($\beta > 0$, e.g., saturable inductor) vs. softening ($\beta < 0$, e.g., tunnel diode) nonlinearities. The energy-conditioned gate identifies when the operating point approaches the nonlinear region ($E_0/E\_sep > 0.85$), preventing incorrect linear-regime retrieval.

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9.B ENERGY, POWER, AND INDUSTRIAL SYSTEMS
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9.B.1 Smart Grid Optimization
Extension of Contribution 5. Multi-machine power systems with center-of-inertia corrections: each generator's SMIB representation is indexed by its domain-invariant triple; the invention identifies inter-area oscillation modes by clustering generators with similar (ω_0, Q) in triple-space. Universal correction parameter a is fitted per inter-area mode class (loading ratio class). T_char = inter-area oscillation damping time (time for a contingency disturbance to damp to ±5% of steady state).

9.B.2 Renewable Energy Output Forecasting
Wind turbine mechanical output obeys GEN-1 as a torsional oscillator:

$$J\cdot\ddot{\Omega} + b\cdot\dot{\Omega} + k\cdot\Omega = \tau_{wind}$$

where J is rotor inertia, b is damping, k is drivetrain stiffness, τ_{wind} is wind torque. $\omega_0 = \sqrt{k/J}$, $Q = J\omega_0/b$. Solar irradiance has dominant harmonic at $\omega_0 = 2\pi/24h$ (diurnal) with seasonal amplitude modulation. The invention indexes forecast regimes by (ω_0, Q, ζ) and retrieves stored production profiles; topology discriminator identifies intermittency regime shifts (β-change events).

9.B.3 Battery Degradation Modeling

The Randles circuit model of a battery cell is:

$$L\cdot\dot{I} + (R_s + R_{ct})\cdot I + V_c = V_{OCV} - V_{terminal}$$

where R_{ct}, C_{dl} (charge-transfer resistance, double-layer capacitance) form an RC circuit with $\omega_0 = 1/\sqrt{L\cdot C_{dl}}$, $Q = \omega_0\cdot C_{dl}\cdot R_{ct}$. Degradation shifts (ω_0, Q) over charge cycles; the invention tracks this shift in triple-space as a degradation trajectory. The energy-conditioned gate identifies when the cell approaches lithium plating or thermal runaway (separatrix in electrochemical energy space). T_{char} = time to reach end-of-life capacity threshold.

9.B.4 Predictive Maintenance for Industrial Systems

Rotating machinery (bearings, gears, motors) exhibits resonances at ω_0 (bearing race frequency, gear mesh frequency) with Q determined by lubrication and structural damping. Incipient faults shift ω_0 and reduce Q. The invention indexes healthy-state Koopman representations in triple-space; fault detection is triggered when the live (ω_0, Q) deviates beyond a retrieval threshold from the nearest healthy memory entry. The topology discriminator identifies qualitative fault type changes (crack initiation = new $\beta < 0$ term in GEN-1).

9.B.5 Digital Twin Industrial Modeling

A digital twin mirrors a physical asset's dynamical state. The invention provides the indexing infrastructure: each physical operating regime is a memory entry $(\omega_0, Q, \text{trust}, \text{curvature\_profile})$; the twin retrieves the matching entry and uses stored Koopman eigenfunctions as a reduced-order model. Regime transitions (startup, shutdown, fault) are flagged by the topology discriminator and trust metric. Universal correction: T_{char} = time to synchronize twin state to physical state after a disturbance.

9.B.6 Oil and Gas Reservoir Modeling

Pressure transient analysis in porous media (diffusivity equation) admits a second-order telegraph approximation at high frequencies:

$$(\rho\cdot c/\kappa)\cdot\ddot{p} + (1/\kappa)\cdot \quad - \text{nabla}^2 p = q_{source}$$

with $\omega_0 = \sqrt{\kappa\cdot c}/L$ (reservoir diffusion frequency), Q inversely proportional to viscous dissipation. The invention indexes pressure transient regimes by (ω_0, Q); retrieves stored decline curve models from nearby reservoir types; applies universal correction to well productivity index (CCT analog = time to pseudosteady state).

9.C ENGINEERING, AEROSPACE, AND TRANSPORTATION

9.C.1 Structural Health Monitoring

Extension of Claim 13. Structural modes (beams, plates, shells, bridges) shift ω_0 downward and Q decreases as damage accumulates (stiffness reduction, damping increase). The invention provides a baseline Koopman memory indexed by healthy modal triples; continuous monitoring detects deviation from baseline in triple-space. The curvature profile detects onset of nonlinear behavior (crack breathing = β-type nonlinearity). T_char = remaining useful life; universal correction applies when $\omega_0 \cdot T_{RUL}$ is fatigue-loading-class-determined.

9.C.2 Autonomous Robotics Control

Robot joint dynamics (series elastic actuator, flexible link):

$$I \cdot \ddot{\theta} + b \cdot \dot{\theta} + k \cdot \theta = \tau_{motor}$$

obey GEN-1 with $\omega_0 = \sqrt{k/I}$, $Q = I\omega_0/b$. The invention indexes joint control regimes and retrieves warm-start PID/impedance controller gains from memory at nearby (ω_0, Q) operating points (load-dependent ω_0). Multi-joint systems are indexed by their dominant mode triple; the topology discriminator prevents coupling of modes in different stability regimes.

9.C.3 Multi-Agent Robotic Coordination

N coupled robots with elastic interconnections obey a network of GEN-1 equations:

M ·   + D ·   + Σ K ·(x − x) = F

The network's collective modes have frequencies $\omega_{0,k} = \sqrt{\lambda k/M_{eff}}$ where λk is the kth Laplacian eigenvalue. The invention indexes collective modes by ($\omega_{0,k}$, Qk); the topology discriminator identifies synchronization vs. desynchronization regimes (separatrix in coupling energy space). T_char = synchronization time; universal correction applies per coupling-topology class.

9.C.4 Aerospace Trajectory Optimization

Orbital mechanics near circular orbit reduces to the Clohessy-Wiltshire equations:

  − 2n· − 3n²·x = Fx/m

ÿ + 2n· = Fy/m

The in-plane oscillation frequency ω_0 = n (mean motion), Q determined by atmospheric drag. Transfer trajectory time (CCT analog = time to reach target orbit) admits the universal correction when $\omega_0 \cdot T_{transfer}$ is orbit-ratio-determined. The invention retrieves warm-start maneuver sequences from memory indexed by (n, Q_drag).

9.C.5 Smart City Traffic Modeling and Transportation Networks

Macroscopic traffic flow (Lighthill-Whitham-Richards model) supports kinematic waves with $\omega_0 = \sqrt{\partial F/\partial k \cdot \partial^2 F/\partial k^2}$ (speed of wave propagation × curvature of flow-density curve), Q from traffic damping. Stop-and-go oscillations are a nonlinear Duffing-type phenomenon (traffic flow softening at jam density). The invention detects approach to traffic breakdown (separatrix = capacity drop) using the energy-conditioned gate; T_char = clearance time for incident.

9.C.6 Materials Stress and Fatigue Failure Forecasting

Cyclic fatigue loading at frequency ω_0 with amplitude-dependent damage:

 _fail = C·(ΔK/K_Ic)^m·ω₀

where ΔK is stress intensity range and K_Ic is fracture toughness. This maps to GEN-1 near resonance. The energy-conditioned gate identifies when the stress intensity ratio ΔK/K_Ic exceeds the separatrix threshold. T_char = cycles to failure (CCT analog); universal correction applies per material-class loading ratio.

9.D ENVIRONMENTAL, CLIMATE, AND EARTH SYSTEMS

9.D.1 Climate Simulation Acceleration

General circulation models (GCMs) solve coupled PDE systems for atmosphere, ocean, and land. Dominant modes (planetary waves, ENSO, AMO) have characteristic ω_0 (annual, decadal) and Q (inter-annual persistence). The invention indexes climate mode regimes in triple-space; regime-memory reuse provides warm-start basis vectors for model initialization; topology discriminator identifies abrupt regime transitions (tipping points = separatrix crossing in energy space).

9.D.2 Weather Forecasting

Numerical weather prediction solves the primitive equations (PDE system). Dominant synoptic-scale eddies have $\omega_0 \approx 2\pi/5$ days, Q from baroclinic growth rate. The invention retrieves stored Koopman mode decompositions from memory at matching (ω_0, Q) for rapid forecast initialization. Universal correction: T_char = forecast skill horizon; universal correction applies per atmospheric regime class (blocking, cyclone, anticyclone loading classes).

9.D.3 Flood and Hydraulic Prediction

Saint-Venant equations for open channel flow:

$$\partial A/\partial t + \partial Q/\partial x = 0$$

$$\partial Q/\partial t + \partial(Q^2/A)/\partial x + gA\cdot\partial h/\partial x + gA\cdot(Sf - S_0) = 0$$

linearized around steady flow yield a wave equation with $\omega_0 = c/L$ (celerity/channel length), Q from friction slope Sf. T_char = peak flow arrival time (flood CCT analog); universal correction applies per catchment loading class.

9.D.4 Wildfire Prediction

Reaction-diffusion PDE for fire spread:

$$\partial T/\partial t = D\cdot \text{nabla}^2 T - \lambda\cdot T + S\cdot\exp(-Ea/RT)$$

near the ignition threshold reduces to GEN-1 with ω_0 from the linearized reaction rate (Damköhler frequency), Q from the ratio of diffusion to reaction timescales. T_char = time to fire perimeter escape (CCT analog = energy to exceed containment). The energy-conditioned gate identifies near-critical fire conditions.

9.D.5 Ocean Current Modeling

Geostrophic and quasi-geostrophic ocean dynamics support Rossby waves:

$$\partial q/\partial t + J(\psi, q + \beta y) = F - r\cdot q$$

with $\omega_0 = \beta\cdot k/(k^2 + l^2 + f^2/N^2H^2)$, Q from dissipation rate r. The invention indexes ocean regime Koopman representations; retrieves warm-start basis for mesoscale eddy prediction; topology discriminator identifies eddy merging events (separatrix in potential vorticity space).

9.D.6 Agricultural Yield Prediction

Crop growth dynamics (logistic + seasonal forcing):

$$\ \ = r\cdot x\cdot(1 - x/K) + A\cdot\cos(\omega_0 t)\cdot x$$

where $\omega_0 = 2\pi/365$ days (annual), Q from weather variability damping. The invention indexes crop growth regimes by dominant ω_0 and Q (climate zone class); retrieves stored yield trajectories for similar agroclimatic regimes. Separatrix analog = wilting point or frost threshold in water/temperature space.

9.D.7 Seismic Risk Modeling

Extension of Claim 13. Site response amplification:

$$m\cdot\ \ + c\cdot\ \ + k\cdot x = -m\cdot a\_ground(t)$$

Site resonance at $\omega_0 = \sqrt{k/m}$ amplifies ground motion. $Q = m\omega_0/c$ (site quality factor). The invention retrieves Koopman response representations from memory indexed by site (ω_0, Q); universal correction: T_char = duration of strong shaking at site; a is fitted per soil-class loading ratio.

9.D.8 Carbon Cycle Modeling

The coupled atmosphere-ocean-land carbon cycle is governed by a system of ODEs with dominant relaxation frequencies ω_0 (decadal = ocean uptake, century = deep ocean saturation) and effective Q (carbon sink efficiency). The invention indexes carbon cycle regimes; topology discriminator identifies abrupt transitions (permafrost thaw, Amazon dieback = separatrix crossings).

9.E BIOLOGICAL, MEDICAL, AND NEURAL SYSTEMS

9.E.1 ECG and EEG Anomaly Detection

Extension of Claim 17. Cardiac rhythm: the sinoatrial pacemaker is a nonlinear oscillator with $\omega_0 = 2\pi \times$ (heart rate/60) and Q from autonomic regulation. Arrhythmia = bifurcation (ω_0 shift + Q collapse). EEG oscillations: alpha (8–12 Hz), beta (13–30 Hz), gamma (>30 Hz) bands each have domain-invariant triples indexed by frequency and coherence (Q analog). The invention detects anomalies as deviations from healthy-state triple in memory and applies the trust metric to classify seizure (chaotic) vs. sleep (periodic) dynamics.

9.E.2 ICU Risk Prediction

Heart rate variability (HRV) is characterized by ω_0 (LF: 0.04–0.15 Hz; HF: 0.15–0.4 Hz) and Q (reciprocal of band relative power). Sepsis, cardiovascular failure, and neurological deterioration manifest as Q collapse in one or more bands. The invention monitors ICU patient HRV triples over time; retrieves nearest historical patient trajectories from memory for risk stratification.

9.E.3 Epidemiological Forecasting

SIR model:

$= -\beta \cdot S \cdot I$

$\dot{I} = \beta \cdot S \cdot I - \gamma \cdot I$

$= \gamma \cdot I$

linearized around endemic equilibrium gives oscillations with:

$\omega_0 = \sqrt{\beta\gamma \cdot S^* - \gamma^2}$, $Q = \omega_0/(2\gamma)$

Epidemic waves are periodic for $\omega_0 > 0$ (above herd immunity threshold). The energy-conditioned gate identifies when effective reproduction number Reff ≈ 1 (near-separatrix = near herd immunity). T_char = epidemic peak time; universal correction applies per disease-class R_0 loading ratio.

9.E.4 Wearable Health Data Integration

Physiological signals from wearables (accelerometer, PPG, skin conductance) contain oscillatory components at ω_0 (respiration ≈ 0.25 Hz, cardiac ≈ 1 Hz, circadian ≈ 11.6 μHz) with Q from inter-individual variability. The invention provides real-time regime monitoring: the live (ω_0, Q) triple is compared to memory of the individual's baseline healthy states; deviations trigger alerts.

9.E.5 Gene Regulatory Network Inference

The Goodwin oscillator (negative feedback gene regulation):

$= \alpha/(1 + z\ ) - \beta \cdot x$

$= k_1 \cdot x - k_2 \cdot y$

$\dot{z} = k_3 \cdot y - k_4 \cdot z$
supports limit-cycle oscillations with ω_0 from feedback delay and Q from degradation rates. The topology discriminator classifies GRN dynamics as periodic (circadian, cell cycle) or chaotic. The invention retrieves Koopman representations of known regulatory networks from memory to warm-start inference on unknown networks with similar oscillatory signatures.

9.E.6 Population Dynamics and Ecological Modeling
Lotka-Volterra predator-prey:
$= (\alpha - \beta y) \cdot x$, $= (\delta x - \gamma) \cdot y$
linearized around coexistence equilibrium: $\omega_0 = \sqrt{(\alpha\gamma)}$, Q from environmental damping (real ecosystems have weak dissipation). The curvature profile detects approach to extinction bifurcation (separatrix in population state space). Extended: competitive exclusion, mutualism, trophic cascade networks are all GEN-1 analogs at the dominant mode level.

9.E.7 Brain-Computer Interface and Neural Decoding
Extension of Claims 17, 9.E.1. Motor cortex local field potentials have dominant oscillatory modes (beta suppression at $\omega_0 \approx 20$ Hz during movement). The invention fits EDMD Koopman operators to neural trajectories in electrode space; the trust metric classifies decoded intention states (periodic = clear intent; chaotic = ambiguous). The domain-invariant triple of neural oscillations is retrieved from a library of movement-associated Koopman modes to decode intended motor commands without retraining per session.

9.E.8 Molecular Docking and Drug-Receptor Interaction
Ligand binding to a receptor is governed by a potential energy surface V(r) with a binding well separated from the unbound state by a saddle-point energy E_barrier (= E_sep analog). The softening Duffing model ($\beta < 0$) approximates the binding well near the saddle. The energy-conditioned gate (Contribution 2) identifies near-barrier configurations (E_0/E_barrier > 0.85); the topology discriminator classifies binding pocket type (hardening = tight binding; softening = loose/non-specific binding). T_char = binding dwell time; universal correction applies per receptor-class loading ratio.

9.F ECONOMIC, FINANCIAL, AND SOCIAL SYSTEMS

9.F.1 Financial Time-Series Forecasting
The Ornstein-Uhlenbeck (OU) process for mean-reverting asset prices:
$dX = -\kappa \cdot (X - \mu) \cdot dt + \sigma \cdot dW$
is the stochastic analog of GEN-1 with $\omega_0 = \sqrt{\kappa}$ (mean-reversion rate), $Q = \kappa/\sigma^2$ (signal-to-noise ratio). The invention indexes financial regimes (bull, bear, volatile) by (ω_0, Q) in triple-space; retrieves stored regime statistics for risk modeling; topology discriminator identifies regime change events. T_char = time to return to mean after shock; universal correction applies per asset-class mean-reversion loading ratio.

9.F.2 Macroeconomic and Business Cycle Forecasting
Extension of Claim 15 (Samuelson-Hicks). Real Business Cycle (RBC) and DSGE models produce oscillatory impulse responses with ω_0 (cycle frequency $\approx 2\pi/8$yr) and Q from monetary policy damping. The invention indexes macroeconomic regime

Koopman representations; retrieves recession/expansion dynamics from historical memory. Universal correction: T_{char} = recovery time from recession trough; a is fitted per monetary-regime loading class.

9.F.3 Energy Demand Forecasting

Electricity demand follows GEN-1 forced oscillator dynamics (diurnal $\omega_0 = 2\pi/24h$, weekly, seasonal harmonics) with Q from thermal inertia of buildings. The invention retrieves demand profile templates from memory indexed by (ω_0, Q, season_class); topology discriminator identifies demand regime shifts (heat wave, holiday = loading class change). Universal correction: T_{char} = ramp-up time for generation dispatch; a fitted per demand-class.

9.F.4 Commodity and Energy Market Prediction

Commodity price cycles (Cobweb model, storage dynamics):

$$\ddot{P}_t + (1/\tau_d + 1/\tau_s)\cdot\ {}_t + (1/(\tau_d\cdot\tau_s))\cdot P_t = \text{const}$$

$\omega_0 = 1/\sqrt{\tau_d\cdot\tau_s}$, $Q = \sqrt{\tau_s/\tau_d}$. The invention indexes commodity cycle regimes; retrieves historical supercycle analogs from memory for forward price estimation.

9.F.5 Epidemiological and Social Contagion Modeling

Social influence propagation, misinformation diffusion, and information cascades follow SIR-analog dynamics (same mathematical form as 9.E.3) with:

$$\omega_0 = \sqrt{\beta\cdot\gamma\cdot S^* - \gamma^2},\quad Q = \omega_0/(2\gamma)$$

where β is transmission rate and γ is recovery/forgetting rate. Viral phenomena correspond to near-critical (near-separatrix) dynamics. The energy-conditioned gate identifies when contagion is near the spreading threshold ($R_{eff} \approx 1$).

9.F.6 Systemic Risk and Financial Network Modeling

Interbank contagion dynamics are governed by coupled GEN-1 equations (bank balance sheet dynamics with interbank exposures as spring terms). Dominant collective modes have ω_0 from network Laplacian eigenvalues (as in 9.C.3). Systemic crisis = synchronization of failure modes (separatrix crossing in network energy space). The topology discriminator identifies systemic risk regime shifts before they occur.

9.G ARTIFICIAL INTELLIGENCE AND AUTONOMOUS SYSTEMS

9.G.1 Cross-Domain Transfer Learning Systems

Contribution 1 is itself a cross-domain transfer learning system in the dynamical regime sense. Extended application: any machine learning system that maps input distributions to outputs, where the input distribution's dominant variation mode has characteristic frequency ω_0 (e.g., dominant PCA eigenvalue), quality factor Q (ratio of dominant to subdominant eigenvalue), and damping ratio ζ (distribution flatness), can be indexed by the domain-invariant triple. Knowledge (model weights, optimizer states, architecture configurations) from a previously solved task at (ω_0, Q) is retrieved and transferred to a new task at a nearby triple.

9.G.2 Meta-Learning and Few-Shot Optimization

Memory-augmented meta-learning (MAML, Prototypical Networks) stores prior task solutions indexed by task difficulty metrics. The invention replaces naive task-similarity metrics with the domain-invariant triple (ω_0 = dominant gradient curvature, Q = inverse condition number, ζ = learning-rate-to-curvature ratio) for more physically grounded similarity. Regime-memory retrieval provides a warm-start initialization that is topologically compatible with the query task.

9.G.3 Autonomous Robotics and Multi-Agent Coordination
Extension of 9.C.2, 9.C.3. Reinforcement learning agents controlling physical systems (robot arm, legged robot, drone) operate in environments governed by GEN-1 dynamics. The invention provides the agent's world model with a structured regime memory indexed by (ω_0, Q) of the environment's mechanical modes. Transfer between robot platforms (sim-to-real, robot-to-robot) uses the topology discriminator to verify regime compatibility before weight transfer.

9.G.4 Swarm Intelligence Systems
N-agent swarms with local interaction rules (Reynolds flocking, Vicsek model) exhibit collective oscillatory modes. The linearized mean-field dynamics obey GEN-1 with ω_0 from flock cohesion stiffness and Q from alignment damping. The invention indexes swarm behavioral regimes (ordered, disordered, near-transition) by (ω_0, Q); the energy-conditioned gate identifies near-phase-transition swarm states; the trust metric classifies emergent swarm dynamics as periodic (ordered flight) or chaotic (dispersal).

9.G.5 Autonomous Scientific Experiment Design
Scientific experiments explore parameter spaces of physical or biological systems. The invention provides the experimental design agent with a structured memory of previously explored dynamical regimes. New experiment parameters are chosen by navigating the triple-space to unexplored (ω_0, Q) regions (Thompson sampling on the invariant manifold). The energy-conditioned gate prevents experimental designs that would drive the system beyond the separatrix (destructive operating conditions).

9.G.6 Federated Learning Across Institutions
In federated learning, distributed clients share model updates without sharing raw data. The invention provides a regime-aware aggregation mechanism: clients with the same (ω_0, Q) dynamical regime (same data distribution type) are identified and their updates are aggregated using the domain-invariant triple as a compatibility gate, preventing topologically incompatible gradient mixing.

9.G.7 Autonomous Defense, Surveillance, and Security Systems
Radar and sonar signal processing: target returns have ω_0 (Doppler frequency) and Q (coherence time). The invention indexes target dynamic regimes and retrieves stored Koopman representations for rapid track initiation. Anomaly detection: the trust metric classifies sensor time series as structured (periodic attractor, known target type) or anomalous (chaotic, unknown contact). Trajectory prediction: the universal correction applies to intercept time estimation when $\omega_0 \cdot T_{intercept}$ is geometry-class-determined.

9.G.8 Speech and Audio Signal Analysis
Vocal tract resonances (formants) are characterized by ω_0 (formant frequency) and Q (formant bandwidth). The invention indexes phonetic regimes by formant triples; retrieves acoustic templates from memory for speech recognition, speaker identification, and language processing. The topology discriminator classifies voiced (periodic, high Q) vs. unvoiced (aperiodic, low Q) segments.

Laryngeal pathology (dysphonia) manifests as Q reduction and ω_0 instability, detectable by the trust metric.

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CLAIMS
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Note: These are informal claims for provisional filing purposes. Formal independent and dependent claims will be drafted for the non-provisional application within 12 months of this provisional filing date.

CLAIM 1 (System — Domain-Invariant Retrieval):
A computer-implemented system for cross-domain solution retrieval in dynamical system optimization, comprising:
(a) a domain encoder that maps domain-specific physical or computational parameters to a domain-invariant triple comprising normalized natural frequency, normalized quality factor, and damping ratio, wherein said natural frequency and quality factor are computed either (i) analytically from linearization of the governing equation about an equilibrium, or (ii) from the dominant Koopman eigenvalue extracted by Extended Dynamic Mode Decomposition (EDMD) of observed trajectory data, thereby covering both linear and nonlinear dynamical systems;
(b) a memory store indexed by said domain-invariant triple;
(c) a retrieval module that locates memory entries nearest to a query triple in triple-space and infers domain-native initialization parameters from the retrieved entry; and
(d) an optimizer initialized at said inferred parameters.

CLAIM 2 (System — Energy-Conditioned Gate):
The system of Claim 1, further comprising an energy-conditioned retrieval gate that:
(a) computes separatrix energy E_sep from query system parameters;
(b) computes total energy E_0 from query initial conditions;
(c) when E_0/E_sep exceeds a near-separatrix threshold, restricts retrieval to memory entries flagged as near-separatrix; and
(d) when E_0/E_sep is below said threshold, excludes near-separatrix entries from retrieval.

CLAIM 3 (Method — Topology Discrimination):
A computer-implemented method for detecting topological incompatibility between a stored dynamical system solution and a query dynamical system, comprising:
(a) computing an effective natural frequency ω_0_eff as a function of energy level for both stored and query systems using EDMD eigenvalue extraction;
(b) computing a curvature profile κ(log_E) = d(log(ω_0_eff/ω_0_linear))/d(log_E) for each;
(c) computing a fixed-length geometric signature by binning and normalizing said curvature profile;
(d) computing cosine similarity between the two geometric signatures; and
(e) rejecting cross-domain invariant transfer when said cosine similarity falls below a topological compatibility threshold.

CLAIM 4 (Method — Bifurcation-Aware Trust Metric):
A computer-implemented method for classifying the dynamics of a driven nonlinear system as periodic or chaotic without access to the governing equations, comprising:
(a) collecting steady-state trajectory data after discarding a transient fraction of the simulation;
(b) fitting an Extended Dynamic Mode Decomposition Koopman matrix K to said steady-state data;
(c) computing reconstruction error as the relative Frobenius norm of $K \cdot \Phi(X) - \Phi(X')$;
(d) computing trust = max(0, 1 - reconstruction_error / η_max) where η_max is calibrated on random trajectory data;
(e) classifying the system as periodic (Abelian) when trust ≥ threshold and as chaotic when trust < threshold; and
(f) classifying period multiplicity via Poincaré stroboscopic sampling with orbit-range normalization.

CLAIM 5 (System — Integrated Architecture):
A system that integrates the domain-invariant triple of Claim 1, the energy-conditioned gate of Claim 2, the topology discriminator of Claim 3, and the trust metric of Claim 4 into a memory-augmented dynamical system optimizer capable of cross-domain solution transfer with topology-aware rejection and chaos detection.

CLAIM 6 (Method — Poincaré Orbit-Range Normalization):
The method of Claim 4, wherein classifying period multiplicity comprises:
(a) sampling system state at integer multiples of the drive period T_drive;
(b) computing orbit_range as the range of said stroboscopic samples;
(c) normalizing cluster span by orbit_range to compensate for phase drift when T_drive/dt is non-integer; and
(d) counting period-k clusters as k distinct groupings with inter-cluster gaps exceeding 12% of normalized section span.

CLAIM 7 (Method — Analytic CCT with Universal Damping Correction):
A computer-implemented method for estimating Critical Clearing Time (CCT) of a synchronous generator in a power grid transient stability assessment, comprising:
(a) receiving generator parameters M (inertia), P (mechanical power), P (peak electrical power), and D (damping coefficient);
(b) computing stable equilibrium angle δs = arcsin(P /P);
(c) computing critical clearing angle δc from:
cos(δc) = P ·(π − 2δs)/P − cos(δs)
(d) computing undamped CCT analytically as:
CCT_EAC = √(2M·(δc − δs)/P)
requiring zero numerical ODE evaluations;
(e) computing damping ratio ζ = D/(2Mω_0) where ω_0 = √(P ·cos(δs)/M);
(f) applying a universal scalar correction:
CCT_corrected = CCT_EAC / (1 − a·ζ)
where a is a scalar fitted once by ordinary least squares across a set of generators at the same loading ratio and is invariant across generators because the product ω_0·CCT_EAC is a loading-ratio-determined constant; and
(g) returning CCT_corrected as the estimated critical clearing time.

CLAIM 8 (Method — OLS Fitting of Universal Correction Scalar):

The method of Claim 7, wherein the scalar a is determined by:
(a) collecting (CCT_EAC,i, CCT_ref,i, ζ) for a plurality of generators i at a set of damping ratios ζ , where CCT_ref,i is obtained by RK4 binary-search reference simulation;
(b) defining e = CCT_EAC,i − CCT_ref,i and x = ζ · CCT_ref,i; and
(c) computing a = −Σ(e ·x) / Σ(x ²) by ordinary least squares.

CLAIM 9 (Method — Universality Verification by Cross-Validation):
The method of Claim 8, further comprising verifying universality of a by:
(a) performing leave-k-out cross-validation across all C(G,k) generator subsets;
(b) for each subset, fitting a on (G−k) generators and evaluating corrected error on the k held-out generators; and
(c) confirming universality when (max(a) − min(a))/mean(a) < a stability threshold across all subsets, indicating a is determined by loading-ratio geometry rather than by the specific generators in the training set.

CLAIM 10 (Broad — Universal Geometric Pattern, Domain-Agnostic, Linear and Nonlinear):
A computer-implemented method for cross-domain invariant indexing and characteristic-time correction applicable to any physical, computational, or biological dynamical system whose dynamics are governed by, or well-approximated by, an equation of the form:
M· + D(x,)· + K(x)·x = F(t)
where D(x,) and K(x) may be nonlinear functions of state, or its discrete-time equivalent, comprising:
(a) characterizing a system by its natural frequency $\omega_0 = \sqrt{(K/M)}$ and quality factor $Q = M\omega_0/D$ to form a domain-invariant triple ($\log\_\omega_0\_norm$, log_Q_norm, ζ) where ζ = 1/(2Q) and ω_0_norm is normalized to a domain-specific reference;
(b) storing the domain-invariant triple and associated system behavior data in a memory indexed by said triple;
(c) retrieving stored behavior data for a query system by nearest-neighbor search in triple-space, enabling cross-domain transfer from any stored domain to any query domain sharing the same triple coordinates; and
(d) when a domain-specific analytic formula T_analytic exists for a characteristic time T_char under the assumption D=0, applying the universal first-order correction:
T_corrected = T_analytic / (1 − a·ζ)
where a is a scalar fitted once per loading class from reference data by the method of Claims 7–9.

CLAIM 11 (Embodiment — Superconducting Quantum Circuits):
The method of Claim 10, wherein the dynamical system is a Josephson junction superconducting circuit, the governing equation is:
$C\cdot\ddot{\varphi} + (1/R)\cdot\dot{\varphi} + Ic\cdot\sin(\varphi) = I\_bias$
where φ is superconducting phase, C is junction capacitance, Ic is critical current, and the characteristic time is the switching time from the zero-voltage to finite-voltage state, and wherein the loading ratio is I_bias/Ic and the separatrix energy is the Josephson energy EJ = (ħ/2e)·Ic.

CLAIM 12 (Embodiment — Phase-Locked Loops):
The method of Claim 10, wherein the dynamical system is a second-order charge-pump phase-locked loop with a passive loop filter, the natural frequency is $\omega_0 = \sqrt{(Kd\cdot Kv/\tau_1)}$, the quality factor is $Q = \omega_0\cdot\tau_1/(1 + Kd\cdot Kv\cdot\tau_2)$, and the characteristic time is the lock acquisition time T_lock, and wherein the

loading ratio is the ratio of input frequency step to loop bandwidth.

CLAIM 13 (Embodiment — Structural Seismic Response):
The method of Claim 10, wherein the dynamical system is a single-degree-of-freedom structural model under seismic excitation, the natural frequency is $\omega_0 = \sqrt{(k/m)}$, the quality factor is $Q = m\omega_0/c$, and the characteristic time is the collapse capacity time or maximum displacement time under a normalized ground motion, and wherein the loading ratio is the spectral demand-to-capacity ratio.

CLAIM 14 (Embodiment — Nuclear Reactor Scram):
The method of Claim 10, wherein the dynamical system is a nuclear reactor described by one-group point kinetics equations, the linearized natural frequency is $\omega_0 \approx \sqrt{(\lambda\cdot(k-1)/L)}$, and the characteristic time is the scram time (time to reach safe shutdown flux level following a reactivity transient), and wherein the loading ratio is the reactivity insertion normalized to prompt critical.

CLAIM 15 (Embodiment — Acoustic and MEMS Resonators):
The method of Claim 10, wherein the dynamical system is an acoustic resonator, electrodynamic transducer, or MEMS sensor characterized by mechanical mass M , mechanical resistance R , and mechanical compliance C , with natural frequency $\omega_0 = 1/\sqrt{(M \cdot C)}$ and quality factor $Q = M \cdot \omega_0/R$.

CLAIM 16 (Embodiment — Laser Relaxation Oscillations):
The method of Claim 10, wherein the dynamical system is a semiconductor or solid-state laser near threshold described by coupled photon and carrier rate equations, linearized to GEN-1 with natural frequency $\omega_0 = \sqrt{(g_0/(\tau \cdot \tau))}$ and quality factor $Q = \omega_0 \cdot \tau$, and the characteristic time is the turn-on delay.

CLAIM 17 (Embodiment — Biological Neural Subthreshold Dynamics):
The method of Claim 10, wherein the dynamical system is a biological neuron operating in the subthreshold (non-spiking) regime, modeled as a leaky integrate-and-fire or linearized Hodgkin-Huxley system with membrane capacitance Cm and total conductance g_total, with $\omega_0 = \sqrt{(g_{Na} \cdot g_K/Cm^2)}$ and $\zeta = g_{total}/(2Cm\cdot\omega_0)$, and the characteristic time is the interspike interval or first-spike latency.

CLAIM 18 (Embodiment — Thermal Systems):
The method of Claim 10, wherein the dynamical system is a two-zone thermal system (building envelope, heat exchanger, electronic package) with thermal capacitances C_1, C_2 and thermal resistances R_1, R_2, characterized by $\omega_0 = 1/\sqrt{(C_1\cdot C_2\cdot R_1\cdot R_2)}$, and the characteristic time is the thermal settling time to a prescribed temperature tolerance following a disturbance.

CLAIM 20 (Broad — Any Observable Oscillatory System):
The method of Claim 10, wherein the domain-invariant triple (ω_0, Q, ζ) is extracted from observed time-series data rather than from explicit governing equations, by:
(a) applying EDMD to the time-series to extract Koopman eigenvalues $\{\lambda\}$;
(b) identifying the dominant oscillatory eigenvalue pair as $\omega_0 = |\mathrm{Im}(\log \lambda)|/\Delta t$;
(c) estimating Q from the eigenvalue decay rate as $Q = |\mathrm{Im}(\log \lambda)|/(2\cdot|\mathrm{Re}(\log \lambda)|)$;
(d) computing $\zeta = 1/(2Q)$;
and wherein the method is applicable to any physical, computational, biological, financial, social, or engineered system that produces time-series data exhibiting

oscillatory structure, including but not limited to all systems described in Sections 8 and 9 of this specification.

CLAIM 21 (Embodiment Group — Computational Science and Physics-Informed AI): The method of Claim 10, wherein the dynamical system is selected from the group consisting of: a physics-informed neural network optimization trajectory on a PDE residual loss landscape; a neural operator applied to a parametric PDE family; a surrogate model for computational fluid dynamics or finite element analysis; a turbulence closure model extracting Koopman modes from turbulent flow data; an inverse parameter identification gradient flow; a chaotic system prediction system applying the EDMD trust metric of Contribution 4; a multi-scale physical simulation regime-memory warm-start system; a variational quantum eigensolver gradient flow; a phonon dispersion calculator for crystal lattice materials; a semiconductor small-signal equivalent circuit; and an electromagnetic cavity resonator mode approximation system.

CLAIM 22 (Embodiment Group — Energy, Power, and Industrial Systems): The method of Claim 10, wherein the dynamical system is selected from the group consisting of: a multi-machine power grid with inter-area oscillation mode indexing; a wind turbine torsional oscillator or solar irradiance harmonic model; a battery Randles circuit model indexed by electrochemical (ω_0, Q); a rotating machinery bearing or gear resonance monitoring system for predictive maintenance; a digital twin regime-memory system for industrial asset monitoring; and an oil and gas reservoir pressure transient analysis system.

CLAIM 23 (Embodiment Group — Civil, Structural, Aerospace, and Transportation): The method of Claim 10, wherein the dynamical system is selected from the group consisting of: a structural health monitoring system tracking modal frequency and quality factor deviation; a robot joint spring-mass dynamics system with load-dependent (ω_0, Q) retrieval; a multi-agent robot coordination system using network Laplacian modal triples; an orbital mechanics transfer trajectory optimization system with mean-motion ω_0; a smart city traffic flow oscillation system with kinematic wave ω_0; and a materials fatigue life prediction system with cyclic loading frequency ω_0.

CLAIM 24 (Embodiment Group — Environmental, Climate, and Earth Systems): The method of Claim 10, wherein the dynamical system is selected from the group consisting of: a general circulation model (GCM) regime warm-start system indexed by planetary wave (ω_0, Q); a numerical weather prediction system with synoptic eddy Koopman retrieval; a Saint-Venant hydraulic flood prediction system with celerity-based ω_0; a wildfire reaction-diffusion PDE with Damköhler frequency ω_0; an ocean quasi-geostrophic eddy system with Rossby wave ω_0; a crop growth phenological oscillator with annual ω_0; and a seismic site response amplification system with structural resonance ω_0.

CLAIM 25 (Embodiment Group — Biological, Medical, and Neural Systems): The method of Claim 10, wherein the dynamical system is selected from the group consisting of: a cardiac rhythm anomaly detection system treating the sinoatrial pacemaker as a nonlinear oscillator; an ICU patient physiological oscillation monitoring system using heart rate variability (ω_0, Q); an SIR-model epidemic forecasting system with endemic equilibrium oscillation ω_0; a wearable health sensor regime monitoring system with physiological band (ω_0, Q); a gene regulatory Goodwin oscillator inference system; a predator-prey or trophic cascade ecological system with Lotka-Volterra mode ω_0; a brain-computer interface neural decoding system using Koopman motor cortex modes; and a molecular docking binding well classification system using separatrix energy.

CLAIM 26 (Embodiment Group — Economic, Financial, and Social Systems):
The method of Claim 10, wherein the dynamical system is selected from the group consisting of: an Ornstein-Uhlenbeck mean-reverting financial asset price model; a DSGE macroeconomic model with business cycle oscillation ω_0; an electricity demand forecasting system with diurnal and seasonal harmonic oscillations; a commodity price Cobweb cycle model with supply-demand delay ω_0; a social influence or misinformation propagation SIR-analog system with spreading ω_0; and a financial interbank network contagion system with Laplacian collective mode.

CLAIM 27 (Embodiment Group — AI, Autonomous Systems, and Signal Processing):
The method of Claim 10, wherein the dynamical system is selected from the group consisting of: a cross-domain machine learning transfer system indexed by gradient curvature triple; a meta-learning warm-start system using domain-invariant task similarity; a federated learning gradient aggregation system with regime-compatibility gating; a swarm intelligence collective oscillation system with Vicsek-model (ω_0, Q); a radar or sonar Doppler target tracking system with Koopman mode retrieval; an autonomous experiment design system navigating the invariant manifold; and a speech formant analysis system treating vocal tract resonances as GEN-1 oscillators.

CLAIM 28 (Broad — Universal Characteristic-Time Correction for Any Oscillatory Domain):
A computer-implemented method for accelerating estimation of any characteristic time T_char in any oscillatory dynamical domain, wherein T_char is defined as the time for the system to cross a stability, convergence, or state-transition threshold, comprising:
(a) identifying the system's natural frequency ω_0 and damping ratio ζ from domain-specific parameters or from EDMD of observed time-series data;
(b) computing T_analytic using an analytic formula valid in the zero-damping ($\zeta = 0$) limit for said system;
(c) applying the universal correction: T_corrected = T_analytic / ($1 - a\cdot\zeta$), where a is a loading-class-specific scalar satisfying $\omega_0\cdot$T_analytic = f(loading_class), verified by leave-k-out cross-validation over system instances within the same loading class; and
(d) returning T_corrected as the estimated characteristic time, requiring no numerical integration of the governing equations,
wherein said method is applicable to all domains enumerated in Sections 8 and 9 and any domain satisfying the universality condition $\omega_0\cdot$T_analytic = f(loading_class).

CLAIM 19 (Broad — Universal Correction Scalar Principle):
A method for computing a universal first-order correction scalar a applicable to any parameterized family of dynamical systems satisfying:
$\omega_0 \cdot$ T_analytic = f(loading_class)
where T_analytic is an analytic characteristic time computed under zero-damping assumptions and f depends only on the system loading class (not on individual system parameters within the class), comprising:
(a) collecting (T_analytic,i, T_ref,i, ζ) across a plurality of instances i within the same loading class, where T_ref,i is a reference value computed by numerical integration or direct measurement;
(b) fitting a = −Σ(e ·x)/Σ(x ²) where e = T_analytic,i − T_ref,i and

x = ζ ·T_ref,i; and
(c) applying T_corrected = T_analytic / (1 − a·ζ) to any new instance in the same loading class without additional numerical integration,
wherein the universality of a within a loading class is guaranteed when the condition ω_0·T_analytic = f(loading_class) holds, and wherein a is applicable to all domains enumerated in Section 8 and any future domain satisfying said condition.

CLAIM 29 (Nonlinear Systems — Explicit Coverage):
A computer-implemented method for invariant indexing and regime-aware retrieval in nonlinear dynamical systems, comprising:
(a) receiving trajectory data from a nonlinear system governed by an equation of the form M· + D(x,)· + K(x)·x = F(t), where D and K are nonlinear functions of state;
(b) extracting an effective natural frequency ω_0_eff and effective quality factor Q_eff from said trajectory data using one or more of:
(i) Extended Dynamic Mode Decomposition (EDMD) on observed state trajectories, identifying the dominant oscillatory Koopman eigenvalue pair;
(ii) energy-parameterized linearization, computing ω_0_eff(E) = sqrt(K(x_eq(E))/M_eff(E)) as a function of the system's total energy E; or
(iii) linearization at the stable equilibrium x_eq of the nonlinear system;
(c) forming the domain-invariant triple (log(ω_0_eff/ω_0_ref), log(Q_eff), ζ_eff) where ζ_eff = 1/(2Q_eff);
(d) applying the energy-conditioned separatrix gate of Contribution 2 to classify the operating regime as sub-separatrix, near-separatrix, or beyond-separatrix based on the ratio E_0/E_sep, where E_sep is the separatrix energy of the nonlinear potential;
(e) applying the topology discriminator of Contribution 3 to classify the nonlinear regime type (hardening, softening, or linear) and reject cross-regime retrieval when curvature profile cosine similarity is below threshold; and
(f) retrieving from memory the nearest stored solution in the same topological regime and using it to warm-start optimization or simulation of the nonlinear system,
wherein said method is applicable to any nonlinear oscillatory system including but not limited to: Duffing oscillators (hardening $\beta>0$ and softening $\beta<0$), van der Pol oscillators, Lorenz and other chaotic systems, Josephson junction circuits, SMIB power grid swing equation with sin(δ) nonlinearity, molecular dynamics with anharmonic potentials, traffic flow with nonlinear flux-density relations, biological oscillators with sigmoidal coupling, and any system admitting a Koopman invariant subspace of finite or approximate dimension.

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ABSTRACT
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A system and method for topology-aware invariant indexing, cross-domain dynamical memory, and analytic power grid stability estimation exploits dynamical equivalence across physical domains. A domain-invariant triple (normalized natural frequency, normalized quality factor, damping ratio) indexes memory entries across physically disparate domains (electrical circuits, mechanical systems, gradient-descent optimizers, power grid generators) that obey equivalent governing dynamics. An energy-conditioned separatrix gate

restricts retrieval to entries in the same topological energy regime, preventing incorrect transfer across the separatrix boundary of softening nonlinear oscillators. A curvature-profile discriminator computes cosine similarity between geometric signatures of Koopman effective-frequency-vs-energy profiles, achieving zero similarity between topologically distinct (hardening vs. softening) regimes. A bifurcation-aware trust metric classifies driven nonlinear system dynamics as periodic or chaotic using EDMD reconstruction error relative to a random-trajectory baseline, overcoming the failure of spectral gap in limit-cycle systems. An analytic Critical Clearing Time (CCT) method applies the Equal-Area Criterion with a single universal damping correction scalar — fitted once by ordinary least squares and invariant across generators because the product ω_0·CCT_EAC is loading-ratio-determined — to estimate power grid transient stability without ODE integration. Together these components achieve 2708× warm-start speedup over cold-start optimization, 3.7× cross-domain transfer improvement for RLC circuit optimization initialized from spring-mass mechanical system solutions, and 57,946× CCT estimation speedup with 2.73% maximum error across the full realistic damping range (ζ [0, 0.20]) validated on the IEEE 39-bus New England system (10 generators).

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INVENTOR DECLARATION (to be completed at filing)

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I hereby declare that:
(a) I am the sole inventor of the subject matter described in this application.
(b) I have reviewed and understand the contents of this application.
(c) I acknowledge my duty to disclose to the USPTO all information known to me to be material to patentability.

Inventor: Nikolas Yoo
Date: _______________________
Signature: _______________________
Address: _______________________

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END OF PROVISIONAL PATENT SPECIFICATION

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