PERMUTATION

Permutation of a set of distinct objects is an ordered arrangement of these objects.

Example:

Let $S = \{A, B, C\}$

permutation of S :

1. A B C
2. A C B
3. B A C
4. B C A
5. C A B
6. C B A

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(3,3) = \frac{3!}{(3-3)!} = \frac{3!}{0!} = 3! = 6$$

r-permutation :

An ordered ~~arrangement~~ arrangement of r-elements of a set is called r-permutation

~~Example:~~

~~Let $S = \{A, B, C, D\}$~~

Example :

a) In how many ways can we select two students from a group of four students to stand in line for a picture ?

b) In how many ways can we arrange all four students in a line for a picture.

Sol: $S = \{1, 2, 3, 4\}$

a) r-permutation = 2-permutation (r=2, selection of 2 students)

$$P(4,2) = \frac{n!}{(n-r)!} = \frac{4!}{(4-2)!} = \frac{4!}{2!} = \frac{4\times3\times2!}{2!} = 12$$

$$\boxed{= n(n-1)(n-2)\cdots(n-r+1) = 4*3}$$

Standing Pos.

12
③ 13
14

21
③ 23
24

31
③ 32
34

41
③ 42
43

Total = 12

b) $$P(4,4) = \frac{n!}{(n-r)!} = \frac{4!}{(4-4)!} = \frac{4!}{0!} = 4\times3\times2\times1 = 24$$

$n = 4$

4 3 2 1 $= 4*3*2*1$

ways for student 1 · ways for student 2 · ways for student 3 · ways for student 4

$$\boxed{= \underline{1}\ \underline{2}\ \underline{3}\ \underline{4}\ \underline{5} \ldots \underline{n} = n(n-1)(n-2)\cdots 1}$$

Permutation with Repetition

Counting permutations when repetition of elements is allowed can easily be done using production rule.

Example:

How many strings of length 4 can be formed from English alphabet

Sol:

$S = \{A, B, C, D, \ldots, Z\}$

4-permutation : (here $n=26$)

As repetition is allowed

$\underline{26}\ \underline{26}\ \underline{26}\ \underline{26} = 26^4$

r-permutation
Here $n=26$
$r=4$
$\underbrace{\underline{n}\ \underline{n}\ \underline{n}\ \underline{n}}_{r} = n^r$

Note:

In case of all permutations, we have $r=n$, ($n=26$)

$\underline{n}\ \underline{n}\ \underline{n}\ \underline{n} \ldots \underline{n}$

$\underline{26}\ \underline{26}\ \underline{26} \ldots \underline{26} = 26^{26}$

$S = \{A, B, C, D\}$
Repetition Allowed:
1. ABCD
≡
24. DCBA
25. AAAA
AAAB
≡
256: ——
$4^4 = 256$

Permutation with Indistinguishable (Identical Elements):

Some elements may be indistinguishable in counting problems. When this is the case, care must be taken to avoid counting things more than once.

Example:

How many different strings can be made by reordering the letters of the word "SUCCESS"?

"DAD"
1. DAD
2. ADD
3. DDA
Half Permutation

Sol:- In SUCCESS, there are 3 S's, 2 C's, 1 U and 1 E.

Here $n=7$ $\quad n_1 = 3$ (3 S's), $n_2 = 2$ (2 C's) $n_3 = 1$ (1U), $n_4 = 1$ (1E)

$$= \frac{n!}{n_1!\, n_2!\, n_3!\, n_4!} = \frac{7!}{3!\,2!\,1!\,1!} = \frac{7 \times 6 \times 5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{3!}\ \cancel{2!}} = 420$$

It can also be calculated as:

$$= C(7,3) * C(4,2) * C(2,1) * C(1,1) = \frac{7!}{3!\cancel{(7-3)!}} * \frac{\cancel{4!}}{2!\,\cancel{2!}} * \frac{\cancel{2!}}{1!\,1!} * \frac{1!}{1!\,0!} = \frac{7!}{3!\,2!}$$

(Same as before)

COMBINATION

Unordered selection of objects.

EXAMPLE:

How many different Committees of three Students Canbe formed from a group of four Students?

Sol:

We need only to find the number of ~~Subje~~ Subsets with three elements from the Set Containing the four Students.

$S = \{1, 2, 3, 4\}$

Subsets of length 3 are :

$\{1, 2, 3\}$
$\{1, 2, 4\}$
$\{1, 3, 4\}$
$\{2, 3, 4\}$

Four Committies each having 3 Students Canbe formed from a group of four Students.

Using formula :

3-Combination : $C(\overset{n}{4}, \overset{r}{3}) = \dfrac{n!}{r!\,(n-r)!} = \dfrac{4!}{3!\,(4-3)!} = \dfrac{\cancel{4}\times 3!}{3!\ 1!} = 4$

* Similarly, Committies of 2 Students (Subsets)

$C(4,2) = \dfrac{4!}{2!\,2!} = \dfrac{\overset{2}{\cancel{4}}\times 3\times\cancel{2}\times 1}{\cancel{2!}\,\cancel{2!}} = 6$

Committees
$\{1,2\}$
$\{1,3\}$
$\{1,4\}$
$\{2,3\}$
$\{2,4\}$
$\{3,4\}$
6

* Committtes of 4 Students from the Set Containg four students:

$C(4,4) = \dfrac{\cancel{4!}}{\cancel{4!}\ 1!} = 1$ (only one Committe $\{1,2,3,4\}$)

COMBINATION WITH REPETITION

Example :

Suppose that a Cookie Shop has four different kinds of cookies. How many different ways can six cookies be choosen? Assume that only the type of cookie, and not the individual cookies or the order in which they are choosen matters.

Sol:-

Theorem :

There are $C(n+r-1, r) = C(n+r-1, n-1)$ r-combinations from a set with n-elements when repetition of element is allowed.

four different kinds of cookies $= n = 4$

Selection : Choose six cookies $= r = 6$

~~C(n+r-1) = C(47~~

$$C(n+r-1, r) = C(4+6-1, 6) = C(9,6)$$

$$= \frac{9!}{6!\,3!} = \frac{\cancel{9}^{3} \times \cancel{8}^{4} \times 7 \times \cancel{6!}}{\cancel{6!} \cdot \cancel{3} \times \cancel{2}} = 84$$