Basic Computational Topology
Implementation Assignment
Problem 5

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1 Introduction

The problem is to implement an incremental algorithm to find the Betti numbers of any given simplicial complexes (upto 3 dimension)

2 READme

To run the implementation project, download the zip file and extract it.
Make sure you have python3, sympy and numpy modules installed in your system by using `pip3 install` command.
Run the file `Question5_implementation.py` file, and input the name of any of the `gts format` files containing the data about various 3d shapes, provided in the folder.
Wait for the output.

3 Algorithm

Consider a 2-simplex.
When we attach k-dimensional simplex σ to simplicial complex K.

$\gamma = \delta'_k \sigma$
$K' = K \cup \sigma$

Case 1: γ also bounds in K : $\delta'_k \sigma \in Imd_k$

$\beta_i(K') = \beta_i(K)$, if $i \neq K$ else,

$\beta_i(K') = \beta_i(K) + 1$, if $i = K$.

$Case\ 2$: γ does not bound in K : $\delta'_k \notin Imd_k$

$\beta_i(K') = \beta_i(K)$, if $i \neq K - 1$ else,

$\beta_i(K') = \beta_i(K) - 1$, if $i = K - 1$.

Using this incremental method, Betti numbers of a 2-simplex can be found.

4 Screenshots of Outputs

- Betti numbers for cube

```
Adding Edge:  8 - 7
Betti-0:  1
Betti-1:  11
Betti-2:  0

Adding Face:  9 - 13 - 1
Betti-0:  1
Betti-1:  10
Betti-2:  0

Adding Face:  6 - 1 - 2
Betti-0:  1
Betti-1:  9
Betti-2:  0

Adding Face:  4 - 2 - 3
Betti-0:  1
Betti-1:  8
Betti-2:  0

Adding Face:  14 - 3 - 11
Betti-0:  1
Betti-1:  7
Betti-2:  0

Adding Face:  4 - 15 - 5
Betti-0:  1
Betti-1:  6
Betti-2:  0

Adding Face:  6 - 5 - 7
Betti-0:  1
Betti-1:  5
Betti-2:  0

Adding Face:  7 - 18 - 8
Betti-0:  1
Betti-1:  4
Betti-2:  0

Adding Face:  9 - 8 - 10
Betti-0:  1
Betti-1:  3
Betti-2:  0

Adding Face:  10 - 16 - 12
Betti-0:  1
Betti-1:  2
Betti-2:  0
```

```
Adding Face:  13 - 12 - 11
Betti-0:  1
Betti-1:  1
Betti-2:  0

Adding Face:  15 - 14 - 17
Betti-0:  1
Betti-1:  0
Betti-2:  0

Adding Face:  18 - 17 - 16
Betti-0:  1
Betti-1:  0
Betti-2:  1

Final Result...
Betti-0:  1
Betti-1:  0
Betti-2:  1
```

- Betti numbers for cube with top and bottom as holes.

```
vaibhav--vt@DESKTOP-6LPO68S:/mnt/c/users/DL_va/OneDrive/Desktop/padhai 4/topo/project$ python3 Question5_implementation.py
Enter the name of the file to be read with extension:cube2.gts
The Number Of Vertices are: 8
The Number Of Edges are: 16
The Number Of Faces are: 8
Initial Betti numbers after adding all 0-simplices...
Betti-0:  8
Betti-1:  0
Betti-2:  0

Adding Edge:  2 - 1
Betti-0:  7
Betti-1:  0
Betti-2:  0

Adding Edge:  6 - 1
Betti-0:  6
Betti-1:  0
Betti-2:  0

Adding Edge:  2 - 6
Betti-0:  6
Betti-1:  1
Betti-2:  0

Adding Edge:  2 - 7
Betti-0:  5
Betti-1:  1
Betti-2:  0

Adding Edge:  3 - 2
Betti-0:  4
Betti-1:  1
Betti-2:  0

Adding Edge:  3 - 7
Betti-0:  4
Betti-1:  2
Betti-2:  0

Adding Edge:  3 - 8
Betti-0:  3
Betti-1:  2
Betti-2:  0

Adding Edge:  4 - 3
Betti-0:  2
Betti-1:  2
Betti-2:  0
```

```
Adding Edge:  4 - 8
Betti-0:  2
Betti-1:  3
Betti-2:  0

Adding Edge:  5 - 1
Betti-0:  1
Betti-1:  3
Betti-2:  0

Adding Edge:  4 - 5
Betti-0:  1
Betti-1:  4
Betti-2:  0

Adding Edge:  1 - 4
Betti-0:  1
Betti-1:  5
Betti-2:  0

Adding Edge:  6 - 5
Betti-0:  1
Betti-1:  6
Betti-2:  0

Adding Edge:  7 - 6
Betti-0:  1
Betti-1:  7
Betti-2:  0

Adding Edge:  5 - 8
Betti-0:  1
Betti-1:  8
Betti-2:  0

Adding Edge:  8 - 7
Betti-0:  1
Betti-1:  9
Betti-2:  0

Adding Face:  1 - 3 - 2
Betti-0:  1
Betti-1:  8
Betti-2:  0

Adding Face:  2 - 10 - 13
Betti-0:  1
Betti-1:  7
Betti-2:  0
```

```
Adding Face:  10 - 11 - 12
Betti-0:  1
Betti-1:  6
Betti-2:  0

Adding Face:  11 - 15 - 9
Betti-0:  1
Betti-1:  5
Betti-2:  0

Adding Face:  7 - 8 - 9
Betti-0:  1
Betti-1:  4
Betti-2:  0

Adding Face:  7 - 6 - 16
Betti-0:  1
Betti-1:  3
Betti-2:  0

Adding Face:  6 - 4 - 5
Betti-0:  1
Betti-1:  2
Betti-2:  0

Adding Face:  3 - 4 - 14
Betti-0:  1
Betti-1:  1
Betti-2:  0

Final Result...
Betti-0:  1
Betti-1:  1
Betti-2:  0
```

- Betti numbers for cone

```
vaibhav--vt@DESKTOP-6LPO68S:/mnt/c/users/DL_va/OneDrive/Desktop/padhai 4/topo/project$ python3 Question5_implementation.py
Enter the name of the file to be read with extension:cone.gts
The Number Of Vertices are: 22
The Number Of Edges are: 60
The Number Of Faces are: 40
Initial Betti numbers after adding all 0-simplices...
Betti-0:  22
Betti-1:  0
Betti-2:  0

Adding Edge:  1 - 20
Betti-0:  21
Betti-1:  0
Betti-2:  0

Adding Edge:  21 - 1
Betti-0:  20
Betti-1:  0
Betti-2:  0

Adding Edge:  1 - 22
Betti-0:  19
Betti-1:  0
Betti-2:  0

Adding Edge:  2 - 1
Betti-0:  18
Betti-1:  0
Betti-2:  0

Adding Edge:  21 - 2
Betti-0:  18
Betti-1:  1
Betti-2:  0

Adding Edge:  2 - 22
Betti-0:  18
Betti-1:  2
Betti-2:  0
```

```
Adding Face:  52 - 51 - 54
Betti-0:  1
Betti-1:  3
Betti-2:  0

Adding Face:  55 - 56 - 53
Betti-0:  1
Betti-1:  2
Betti-2:  0

Adding Face:  55 - 54 - 59
Betti-0:  1
Betti-1:  1
Betti-2:  0

Adding Face:  60 - 57 - 56
Betti-0:  1
Betti-1:  0
Betti-2:  0

Adding Face:  60 - 59 - 58
Betti-0:  1
Betti-1:  0
Betti-2:  1

Final Result...
Betti-0:  1
Betti-1:  0
Betti-2:  1
```

- Betti numbers for sphere

```
vaibhav--vt@DESKTOP-6LPO68S:/mnt/c/users/DL_va/OneDrive/Desktop/padhai 4/topo/project$ python3 Question5_implementation.py
Enter the name of the file to be read with extension:sphere5.gts
The Number Of Vertices are: 252
The Number Of Edges are: 750
The Number Of Faces are: 500
Initial Betti numbers after adding all 0-simplices...
Betti-0: 252
Betti-1: 0
Betti-2: 0

Adding Edge: 39 - 7
Betti-0: 251
Betti-1: 0
Betti-2: 0

Adding Edge: 39 - 1
Betti-0: 250
Betti-1: 0
Betti-2: 0

Adding Edge: 7 - 1
Betti-0: 250
Betti-1: 1
Betti-2: 0

Adding Edge: 42 - 1
Betti-0: 249
Betti-1: 1
Betti-2: 0

Adding Edge: 42 - 2
Betti-0: 248
Betti-1: 1
Betti-2: 0

Adding Edge: 1 - 8
Betti-0: 247
Betti-1: 1
Betti-2: 0

Adding Edge: 1 - 2
Betti-0: 247
Betti-1: 2
Betti-2: 0
```
```
Adding Face: 739 - 740 - 748
Betti-0: 1
Betti-1: 3
Betti-2: 0

Adding Face: 740 - 738 - 742
Betti-0: 1
Betti-1: 2
Betti-2: 0

Adding Face: 742 - 743 - 749
Betti-0: 1
Betti-1: 1
Betti-2: 0

Adding Face: 743 - 741 - 744
Betti-0: 1
Betti-1: 0
Betti-2: 0

Adding Face: 744 - 745 - 750
Betti-0: 1
Betti-1: 0
Betti-2: 1

Final Result...
Betti-0: 1
Betti-1: 0
Betti-2: 1
```

- Betti numbers for tetrahedron

```
vaibhav--vt@DESKTOP-6LPO68S:/mnt/c/users/DL_va/OneDrive/Desktop/padhai 4/topo/project$ python3 Question5_implementation.py
Enter the name of the file to be read with extension:tetrahedron.gts
The Number Of Vertices are: 4
The Number Of Edges are: 6
The Number Of Faces are: 4
Initial Betti numbers after adding all 0-simplices...
Betti-0:  4
Betti-1:  0
Betti-2:  0

Adding Edge:  2 - 1
Betti-0:  3
Betti-1:  0
Betti-2:  0

Adding Edge:  3 - 2
Betti-0:  2
Betti-1:  0
Betti-2:  0

Adding Edge:  2 - 4
Betti-0:  1
Betti-1:  0
Betti-2:  0

Adding Edge:  1 - 3
Betti-0:  1
Betti-1:  1
Betti-2:  0

Adding Edge:  4 - 1
Betti-0:  1
Betti-1:  2
Betti-2:  0

Adding Edge:  3 - 4
Betti-0:  1
Betti-1:  3
Betti-2:  0

Adding Face:  1 - 2 - 4
Betti-0:  1
Betti-1:  2
Betti-2:  0

Adding Face:  5 - 3 - 1
Betti-0:  1
Betti-1:  1
Betti-2:  0

Adding Face:  3 - 6 - 2
Betti-0:  1
Betti-1:  0
Betti-2:  0

Adding Face:  6 - 5 - 4
Betti-0:  1
Betti-1:  0
Betti-2:  1

Final Result...
Betti-0:  1
Betti-1:  0
Betti-2:  1
```
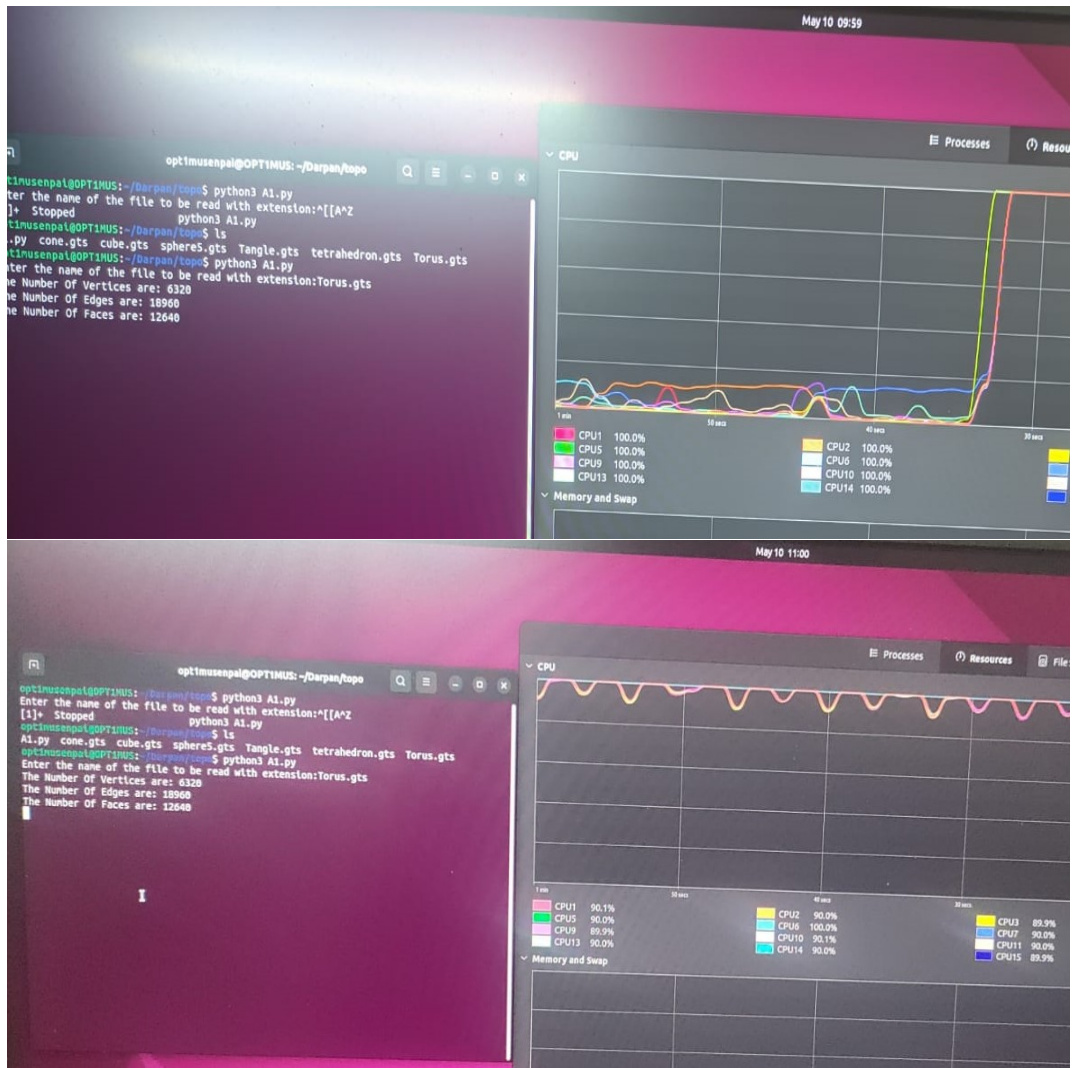
- Betti numbers for torus

Even after one hour of computing, the betti numbers are not calculated
by computer since the data sample is too large.

- Similarly for a tangle, the betti numbers couldn't be calculated since the
  data sample was too large.

References

[1] *Cecil Jose A. Delfinado and Herbert Edelsbrunner. 1993. An incremental algorithm for Betti numbers of simplicial complexes. In Proceedings of the ninth annual symposium on Computational geometry (SCG '93). Association for Computing Machinery, New York, NY, USA, 232–239. https://doi.org/10.1145/160985.161140*

[2] *Course material taught by Prof. Amit Chattopadhyay*