Data-driven deep learning neural networks for predicting the cumulative number of COVID-19 infections

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***Abstract*—Infectious disease epidemics are a challenge for medical and public health practitioners. They require prompt treatment, but it is challenging to recognize and define epidemics in real-time. By knowing the short-term prediction of an infectious disease epidemic, the disease's impact can be evaluated by preventive efforts. Real-time mathematical epidemic models such as logistic differential equations and deep learning methods are key preventative tools. Data-driven deep learning enables effective algorithms for identifying parameters in mathematical models. This paper introduces a logistic-informed neural networks algorithm inspired by applying a physics-informed neutral network to a logistic differential equation to learn the constant parameter and the time-dependent function from the existing data involving the accumulative number of individuals reported to be infected by COVID-19 in a given country. The learned parameters from the mathematical models produced from the time-dependent function and the analytical solutions of these mathematical models are used to predict the time that a plateau will be reached and the cumulative number of individuals reported to be infected by COVID-19. Furthermore, the same data were used for the training of recurrent neural networks, namely gated recurrent units and long short-term memory networks. The recurrent neural networks were also used to predict the time that a plateau will be reached and the cumulative number of individuals reported to be infected by COVID-19. The accuracy of these models is demonstrated using error metrics on COVID-19 data for Italy and Sweden.**

***Index Terms*—Deep Learning, Data-driven, COVID-19, Logistic Differential Equation, Time-dependent function, Logistic Informed Neural Network (LINN), Recurrent neural(RNN)**

I. INTRODUCTION

The Chinese city of Wuhan was the site of the first incidence of COVID-19 [1]. However, on a global scale, the disease broke out and spread fast, creating one of human history's most deadly pandemics [2]. COVID-19 is always a threat to human health because it spreads quickly, has terrible effects on health, and changes its genetic makeup. "In addition, the virus maybe facilitated by human-to-human transmission through the air [3]. As a result, on the 31st of January 2020, the World Health Organization (WHO) proclaimed a global health emergency. Despite careful supervision, the disease has spread to over a hundred nations worldwide since its discovery and has become a global pandemic [4]. The success of the measures put in place can only be ascertained by looking into COVID-19's dynamic behavior. Epidemic data affects model accuracy. Due to stochastic variations and unpredictability, certain epidemic curves have many turning points (like peaks and valleys). Surveillance, case definitions, and turning points can indicate a slowing epidemic. It can help with disease management plans, peak-phase waves, and the natural growth delay of susceptible individuals caused by infection. A prediction model is required to prevent the disease's spread. Unfortunately, due to limited public information on emerging epidemics and diseases, creating realistic models during public health emergencies is difficult [5]. Some epidemics can be predicted, while others cannot but are still likely. Predicting turning points and epidemic waves helps create and evaluate response plans [6]. Many researchers from the most severe countries and around the world have created COVID-19 trend-predicting methodologies. In order to better understand and control this pandemic, an infectious disease model, estimation methods, and forecasting tools have been created. Kermack and McKendrick, in 1927 [7], introduced a mathematical model called Susceptible-Infectious-Recovery (SIR) model. This variational model calculates the estimated number of people infected and later recovered from the disease. Under specific circumstances, the logistic differential equation can be derived from the SIR model.

We aim to introduce a deep-learning neural network algorithm called Logistic-Informed Neural Network (LINN). The LINN algorithms are a viable choice to learn the time-varying transmission rate and identify the government's effects on mitigation measures in the data. Some well-known logistical data regarding infectious diseases are added to the LINN loss function. Cubic spline interpolation is used to create enough training data to detect hidden characteristics in the training data. The learned time-varying parameters and the analytical solutions of the mathematical models are used to predict the time that a plateau will be reached and the cumulative number of individuals reported to be infected by COVID-19 in a given country. At the same time, we employed long short-term memory networks that strongly provide a generalization of recurrent neural networks. Furthermore, the accuracy of these models using error metrics are discussed. The paper is organized as follows. Section II gives an overview of the background. Section III presents the methods used in this work. The results are presented in section IV, and the discussion is done in section V.

II. BACKGROUND

Mathematical models, machine learning, and deep learning have been used to predict how an epidemic will change over time [8], [9]. The Susceptible-Exposed-Infected-Recovered (SEIR) and the Susceptible-Infected-Recovered (SIR) mathematical models are two of the most persuasive and widely used models in epidemiology. For example, the Susceptible-Exposed-Infectious-Removed (SEIR) model is proposed to investigate the impact of preventative measures on epidemic dynamics [10]. The logistic model is frequently used in fitting regression models to time series data because its underlying theory is straightforward and efficient calculation. Fitting and analyzing epidemic prediction has been made using logistic, Bertalanffy, and Gompertz models [11]. Feedforward neural networks (FNN) have been employed to learn approximate solutions to differential equations. Physics-Informed Neural Network (PINN) is a Feedforward neural network (FNN) and an Artificial Neural Network (ANN) design that subjects the created neural network to datasets and controls constraints during training [12]. It is used to learn and identify parameters in an ordinary differential equations (ODE), or Partial differential equation (PDE), which is also good in prediction. For example, Dandekar et al. in [13] applied PINN to the Susceptible, Exposed, Infected, Removed(SEIR model) to scrutinize the spread of COVID-19. Predicting the spread of COVID-19 is a complex and time-consuming endeavor. However, Recurrent Neural Network methods (RNN) like BILSTM, LSTM, VAE, and GRU, which are most frequently used for time-series data, are highperforming machine learning approaches to predict the infected cases and recovered cases of COVID-19. For example, Shahid et al. predict the deaths, infected cases, and recovered cases using LSTM, BILSTM, and GRU models[14].

Fokas et al. predict the time that a plateau will be reached, as well as the cumulative number of individuals reported to be infected with COVID-19 using mathematical models and a deep learning model using an error-minimizing algorithm [8]. Unfortunately, the constant model did not provide an accurate prediction and underestimated the cumulative number of individuals reported to be infected at time t. The major limitation of the constant model is that it could not capture various time-dependent factors in the data. After experimenting with more than 50 different time-dependent forms that could capture various time-dependent factors in the data, they introduced two novel mathematical models called rational and birational models.

A. Logistic Differential Equation

The infectious diseases caused by COVID-19, where the disease is transmitted from one host to another, are examined in this section. Let us assume that we have a constant number of people, N, and that they are separated into the three states, susceptible (S), infected (I), and recovered (R) [7]. Therefore, the SIR model has the following system of ordinary differential equations.

$$\begin{aligned} \frac{dS(t)}{dt} &= -\frac{\alpha(t)S(t)I(t)}{N} + \Gamma(t)R(t) \\ \frac{dI(t)}{dt} &= \frac{\alpha(t)S(t)I(t)}{N} + \Theta(t)I(t) \\ \frac{dR(t)}{dt} &= \Theta(t)I(t) + \Gamma(t)R(t) \end{aligned} \tag{1}$$

The constant $\alpha(t)$ represents the transmission rate at time t, $\Gamma(t)$ represents the rate of loss of immunity at time t and $\Theta(t)$ represents the recovery rate at time t.

Consider the simplest of these models, in which an infected individual remains infectious. The model has the following system of ordinary differential equations.

$$\begin{aligned} \frac{dS(t)}{dt} &= -\frac{\alpha(t)S(t)I(t)}{N} \\ \frac{dI(t)}{dt} &= \frac{\alpha(t)S(t)I(t)}{N} \end{aligned} \tag{2}$$

Since $N(t) = S(t) + I(t)$, Therefore $S(t) = N(t) - I(t)$ which reduces the above model to a simple model called logistic differential equation.

$$\frac{dI(t)}{dt} = \alpha(t)I(t)\left(1 - \frac{I(t)}{N}\right) \tag{3}$$

Therefore, the ordinary differential equation that satisfies the cumulative number of individuals reported to be infected by COVID-19 at time t is

$$\frac{dC(t)}{dt} = \alpha(t)C(t)\left(1 - \frac{C(t)}{N_p}\right) \tag{4}$$

Where, $C(t)$ represents the cumulative number of individuals reported to be infected by COVID-19 at time t, N_p is the constant parameter and $\alpha(t)$ is the time-dependent function. The constant and the function $\alpha(t)$ depend on the fundamental properties of the particular virus and the effect of the various steps taken by the country to limit the spread of the virus [8].

B. Learning Time-Varying Transmission Rate

The fact that $\alpha(t)$ is time-dependent indicates various time-dependent elements, including integrating the effect of public health efforts and the public's response to the actions [15] and the effect of various government policies to prevent the virus from spreading further also depends on t [8]. Early in COVID-19 transmission, the key public health activity was lockdown, followed by early detection of infectives, social distance, contact tracing, masking, etc. Suppose $\alpha(t)$ is replaced by a constant k, then equation becomes.

$$\frac{dC(t)}{dt} = kC(t)\left(1 - \frac{C(t)}{N_p}\right) \tag{5}$$

The analytical solution to equation (5) results to equation (6) called a constant model

$$C(t) = \frac{N_p}{1 + \gamma e^{-kt}} \tag{6}$$

C represent the cumulative number of infected cases at time t, k indicates the growth rate of COVID-19 cases, and N_p represents the total number of infected cases in the final phase of the epidemic. $C(t) = \frac{N_p}{2}$. Where $C(t) = \frac{N_p}{2}$ is called the infection point, meaning that the point where the growth curve changes its concavity. Also, $T = \frac{ln(\gamma)}{k}$ is the estimation of time at which the epidemic reaches its maximum growth rate. Therefore, the slope of equation (6) is symmetrical at the inflection point (peak time) for the value of $C = \frac{N_p}{2}$ defined as $T = \frac{ln(\gamma)}{k}$ where the curve assumes its sigmoid-shaped.

The parameters k, γ and N_p generally remain unchanged if we use short series of existing data of the COVID-19, that is using the subset of the existing data, but when considering a long series of existing data where the remaining available data is added the constant model does not make accurate predicting in which the constant model underestimates $C(t)$. This results in a question that was raised in [8]: since the constant model gives a lower bound of $C(t)$, can there be a mathematical model that can give an upper bound of $C(t)$ and capture the remain data that will provide a good fit and more accurate prediction of a long series of existing data? After testing more than 50 distinct forms of $\alpha(t)$, Fokas et al [8] found two innovative formulas called rational and birational that address the question.

Given that $\alpha(t) = \frac{ab}{1+bt}$ which is an algebraic function, the analytical solution becomes

$$C(t) = \frac{N_p}{1 + \gamma(1 + bt)^{-a}} \tag{7}$$

which is called the rational model with four parameters which are N, γ, b and a.

Given that

$$\alpha(t)=\begin{cases}\frac{ab}{1+bt}, & t\le M\\ \frac{a_1b_1}{1+b_1t}\frac{1}{1+(1-(d_1/N_p))(1+b_1t)^{-a_1}}, & t>M\end{cases}$$

the analytical solution becomes

$$C(t)=\begin{cases}\frac{d}{1+\gamma(1+bt)^{-a}}, & t\le M\\ \frac{d}{1+\gamma(1+bM)^{-a}}-\frac{d_1}{1+\gamma_1(1+b_1M)^{-a_1}} & \\ \qquad\qquad +\frac{d_1}{1+\gamma_1(1+b_1t)^{-a_1}}, & t>M\end{cases}$$

which is called the birational model with eight parameters which are a, a_1, b, b_1, d, d_1, γ and γ_1, where the constant parameter M is located close to the value of T. These two models provide an accurate prediction. However, these models fail when predicting the cumulative number of individuals infected with COVID-19 in some countries with partial mitigation measures. The fact that time-dependent function reflects various time-dependent factors, including the cumulative effect of the different measures taken by the government of a given country to prevent the spread of the viral infection. For example, Lockdown was the first public health precaution used early in the COVID-19 outbreak. Other actions like social isolation, contact tracing, masking, and early infection identification. This show that there may be multiple forms of time-dependent function. To address the model's failure to predict the plateau and the cumulative number of individuals infected by COVID-19 in a given country with partial mitigation, we introduce a new model called the time-series model.

III. METHODS

A. Time-Series Model

The time-series model is a neural network base. This model guide the deep learning neural network toward learning the form of the time-dependent function using the LINN algorithm from the COVID-19 data. The suggested LINN algorithm can capture the dynamics of the disease's propagation and the impact of different mitigation measures since the actual number of COVID-19 infections differs significantly from the number of cases that have been reported. Taking $\alpha(t)$ as a subject of the formula in equation (4) we have

$$\alpha(t)=\frac{\frac{dC(t)}{dt}}{C(t)\left(1-\frac{C(t)}{N_p}\right)} \tag{8}$$

As a result, the neural network learns every vital information of $\alpha(t)$ going on in the data.

B. Deep Learning Algorithms

1) Logistic Informed Neural Network (LINN): Feedforward Neural Network architecture makes up the Physics-Informed Neural Network (PINN) used to learn the parameters. In addition, we offer a Logistic Informed Neural Network (LINN), which was motivated by using a PINN on logistic models to overcome the limitations of statistical techniques and was implemented in Tensorflow. Figure 1 is a schematic LINN architecture with one network used for learning the parameters, the solution for $\alpha(t)$, and the models of constant, rational, and birational. Figure 2 is a schematic LINN architecture with two networks in which the first network learned the constant parameter and the cumulative initial value of the COVID-19 infection. The second network learned the form of the time-dependent function ($\alpha(t)$ that helps capture all the various time-dependent factors or elements, including the impact of public health actions and the public response to the actions in preventing the spread of COVID-19.

We employed three hidden layers with 64 neurons each in all of the simulations discussed in this study, the learning rate was 0.001, and the training loss was minimized in 25,000 iterations. The hyperbolic tangent function serves as the activation function for the hidden layers. The output of LINN is the learned solution of the models denoted by $C(t_n;\delta;\theta), n=1,...,X$. Where δ represents the neural network weights and biases, and θ represents the model's solution parameters. X is the number of the training set. The same network also produces the time-varying transmission rate denoted by $\alpha(t_n;\delta;\theta), n=1,...,X$. Cubic spline interpolation is used to create enough training data to detect hidden characteristics in the training data. We notice that training data for all compartments in the models are unavailable; however, the model's residual is included in the MSE loss function, which allows LINN to capture the interactions between the compartments. The loss function of LINN and the fundamental framework, which is FNN, are shown in Figures 1 and 2. In figure 1, the LINN's loss function is divided into two components called RE_{loss} and ME_{loss}. Subtracting the right side from the left side of the logistic model equation (4) and the analytical solution produces the residuals that are represented by the RE_{loss}. The mean squared error between the outputs of the neural network and the data is denoted as ME_{loss}. Figure 2, the mean square error (MSE) of this neural network's loss function includes the known logistic dynamics, while the time-series transmission rate detects various time-dependent factors in the COVID-19 infected data.

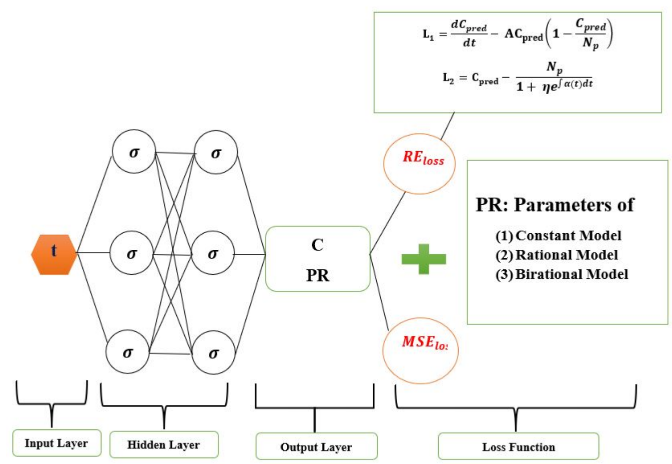


Fig. 1. schematic diagram of the Logistic Informed Neural Network with non-linear time-varying transmission rate.

$$MSE=\frac{1}{X}\sum_{n=1}^{X}||C(t_n)-C_{pred}(t_n)||_2^2+\frac{1}{X}\sum_{i=1}^{2}\sum_{n=1}^{X}||L_i(t_n)||_2^2$$

where the residual L_i, $i=1,2$ is as follows:

$$L_1=\frac{dC(t)}{dt}-AC(t)\left(1-\frac{C(t)}{N_p}\right)$$

$$L_2=C(t)-\frac{N_p}{1+\gamma e^{-\int Adt}}$$

Where A represents the time-varying transmission rate.

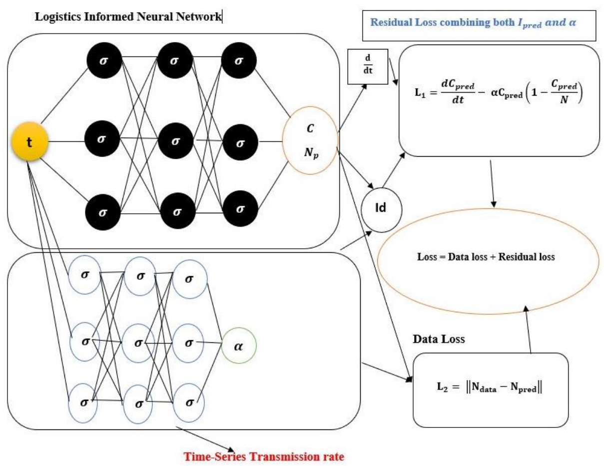


Fig. 2. Schematic diagram of the Logistic Informed Neural Network with non-linear time-series transmission rate.

2) Recurrent Neural Network (RNN): The recurrent neural network is a neural network used for learning sequential data because of the existence of time [16]. In this work, we use RNN namely Long Short Term Memory (LSTM) and Gated Recurrent Unit (GRU) and the implementations were done in PyTorch. We employed three hidden layers with 16 neurons each for the RNN, the learning rate was 0.01, the batch size is 32 for GRU and 45 for LSTM and the training loss was minimized with 1500 epochs.

C. *Error Metrics*

Error metrics for data-driven simulations are presented. It is examined why and when to employ certain error measures. Root Mean Squared Error (RMSE), Mean Absolute Percentage Error (MAPE), and Explained Variance (EV) are the metrics used to assess the effectiveness of regression-based models. The values of RMSE and MAPE are used to compare many models and choose the best one based on the lowest values. The optimum metric for nonlinear regression is the explained variance in which the model with the highest values between 0 and 1 is established as the best model.

D. *Data and Data Preprocessing*

The time-series data for COVID-19 for Italy and Sweden were obtained from the official site of the European Centre for Disease Prevention and Control [17]. The data comprised the accumulative number of individuals reportedly infected with the COVID-19 epidemic starting from the days after 500 cases were reported to July 10, 2020. As a result, it is observed that the cumulative number of individuals infected with COVID-19 in these countries is different. In addition, the days after 500 cases of COVID-19 were reported are also different, which will make the infection rate of each country different. The data is preprocessed using Min-Max scaling. The Cubic spline interpolation will be used to create enough training data to detect hidden characteristics in the training data. The processed cumulative infected data will be splitted into 80% and 20% training and validation using random splitting and passed into LINN to learn the parameters of the mathematical models. The same processed cumulative infected data splitted into 80% and 20% training and validation ans were passed into LSTM and GRU.

IV. RESULTS

The data simulation results of the mathematical models, recurrent neural networks and prediction of the time that a plateau will be reached as well as the cumulative number of individuals reported to be infected by COVID-19 of Italy and Sweden are presented in this section.

A. *Parameter Identification and Data-Driven Simulations*

The parameters of the mathematical models are learned using the validation data as well as LSTM and GRU. For reliability and accuracy of the learned parameters, the LINN is run ten independent times to quantify the uncertainty in the learned parameters. Table I-IV presents the parameters, plateau days, plateau cases, and error metrics of the mathematical models, LSTM and GRU applied to the fitting accuracy of Italy's and Sweden's cumulative COVID-19 infection data. In addition, the inflection point T on the tables was calculated by fitting all the cumulative infection data, namely data up to July 10, 2020. Figure 3 and 4 presents the learned transmission rate of COVID-19 for the given country using rational, birational, and time-series models.

TABLE I
THE MODEL PARAMETERS, PLATEAU DAYS, PLATEAU CASES AND ERROR METRIC FOR CONSTANT, RATIONAL AND BIRATIONAL MODELS FOR THE CUMULATIVE COVID-19 INFECTIONS IN ITALY.

Parameters	Constant	Parameters	Rational	Param	Birational
N	229856	N	247848	d	224619
k	0.1185	a	3.0335	a	1.3825
γ	80.0903	γ	6497.3755	γ	1500.1837
T	38	b	0.4621	b	1.2052
RMSE	44371	RMSE	43188	d_1	253517
MAPE	0.3158	MAPE	0.3024	a_1	3.0291
EV	0.8993	EV	0.9236	γ_1	1200.6889
days	152	days	166	b_1	0.2571
Cases	229834	Cases	245795	RMSE	43222
				MAPE	0.2863
				EV	0.9243
				days	163
				Cases	243936

TABLE II
MODEL PARAMETERS, PLATEAU DAYS, PLATEAU CASES AND ERROR METRICS OF TIME-SERIES MODEL AS WELL LSTM AND GRU NETWORKS FOR COVID19 EPIDEMICS FOR ITALY.

Parameters	Time-Series	Parameters	LSTM	GRU
N	247940			
Plateau (days)	190	Plateau (days)	184	188
Plateau (Cases)	248653	Plateau (Cases)	239959	254676
RMSE	21449	RMSE	1591	1433
MAPE	0.1267	MAPE	0.05879	0.05971
EV	0.9782	EV	0.9760	0.9890

TABLE III
THE MODEL PARAMETERS, PLATEAU DAYS, PLATEAU CASES AND ERROR METRIC FOR CONSTANT, RATIONAL AND BIRATIONAL MODELS FOR THE CUMULATIVE COVID-19 INFECTIONS IN SWEDEN.

Parameters	Constant	Parameters	Rational	Param	Birational
N	76304	N	193006	d	30666
k	0.0537	a	1.9602	a	8.4132
γ	61.2445	γ	6502.6573	γ	247.9347
T	84	b	0.5518	b	0.0306
RMSE	21596	RMSE	20312	d_1	148351
MAPE	0.5274	MAPE	0.5124	a_1	3.5693
EV	0.6757	EV	0.7121	γ_1	139.2934
days	151	days	192	b_1	0.0239
Cases	74627	Cases	124698	RMSE	20492
				MAPE	0.5188
				EV	0.6987
				days	197
				Cases	117585

TABLE IV
MODEL PARAMETERS, PLATEAU DAYS, PLATEAU CASES AND ERROR METRICS OF TIME-SERIES MODEL AS WELL LSTM FOR COVID19 EPIDEMICS FOR SWEDEN.

Parameters	Time-Series	Parameters	LSTM	GRU
N	77646			
Plateau (days)	172	Plateau (days)	180	184
Plateau (Cases)	77554	Plateau (Cases)	81467	82440
RMSE	10060	RMSE	1770	882
MAPE	0.1899	MAPE	0.02663	0.01328
EV	0.9091	EV	0.9828	0.9912

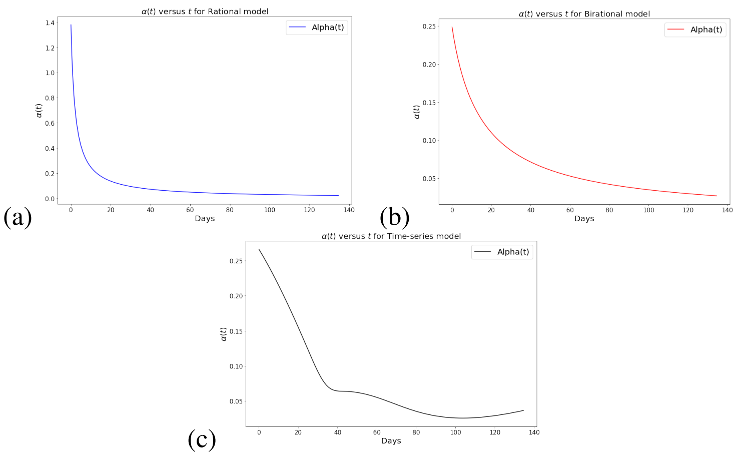


Fig. 3. Simulation results of the rate of transmission ($\alpha(t)$) for the cumulative Omicron infection in Italy using: **(a)** Rational model; **(b)** Birational model; **(c)** Time-series model.

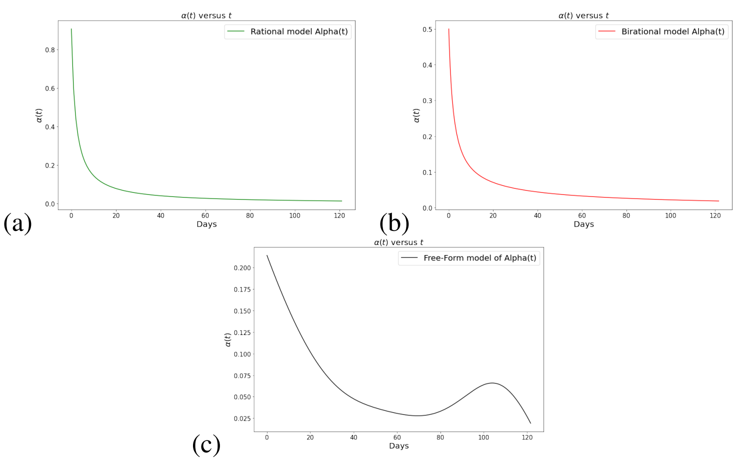


Fig. 4. Simulation results of the rate of transmission ($\alpha(t)$) for the cumulative Omicron infection in Sweden using: **(a)** Rational model; **(b)** Birational model; **(c)** Time-series model.

B. Prediction of the Cumulative Number of COVID-19 Infections.

To predict the time that a plateau will be reached as well as the cumulative number of individuals reported to be infected by COVID-19 in a given country, the learned parameters obtained from these three mathematical models are added to their model's analytical solution were used. In addition, the time-series model's learned parameter, initial values, and time-dependent functions were also passed into a standard ODE solver to predict. Finally, the confidence interval of LSTM and GRU using the Bootstrap algorithm was constructed and was run ten independent times to predict the time that a plateau will be reached and the cumulative number of individuals reported to be infected by COVID-19 in a given country. Figure 5 and 6 present the mathematical models, while Figures 7 present the deep-learning model (GRU) for predicting the time that a plateau will be reached and the cumulative number of individuals reported to be infected by COVID-19 in Italy and Sweden.

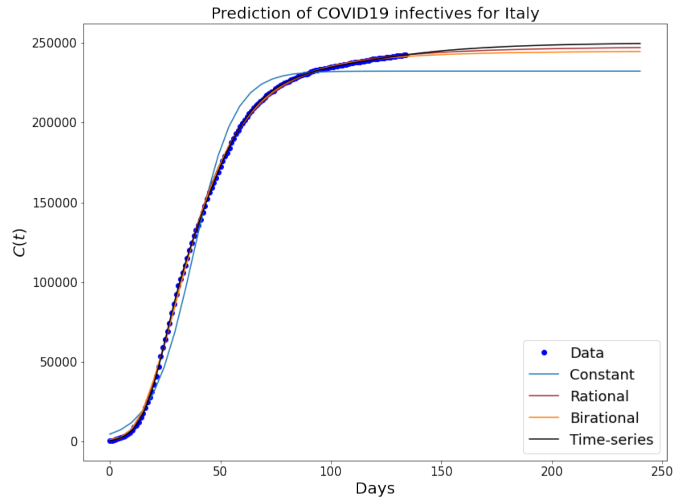


Fig. 5. The mathematical model prediction for the time that a plateau will be reached as well as the cumulative number of individuals reported to be infected by the COVID-19 in Italy

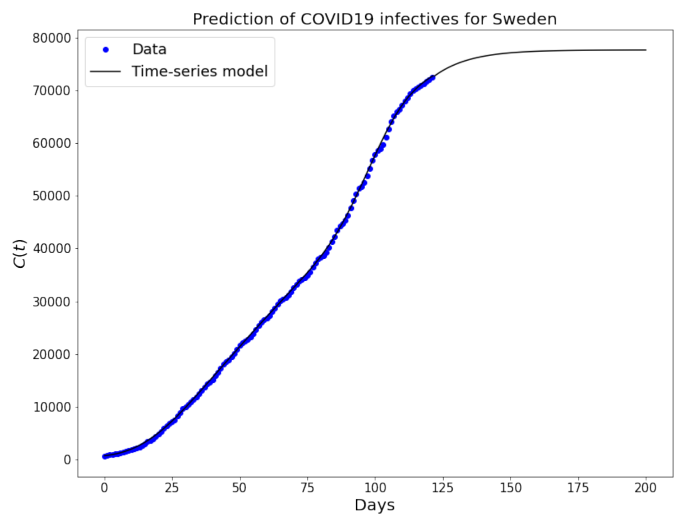
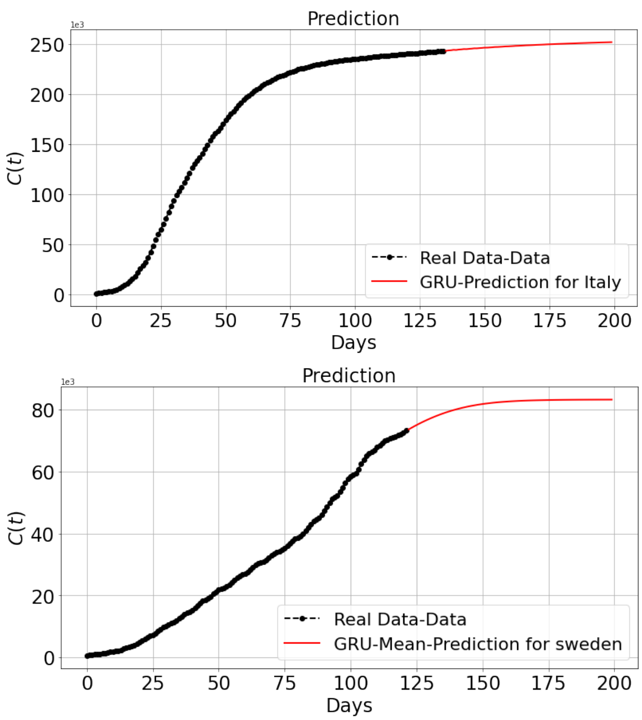


Fig. 6. The mathematical model prediction for the time that a plateau will be reached as well as the cumulative number of individuals reported to be infected by the COVID-19 in Sweden

Table I and II present the mathematical models that yielded the following estimates for the dates that the plateau will be reached, as well as for the number of individuals reported to be infected with COVID-19 when this occurs: the constant model predicted that the COVID-19 outbreak in Italy would plateau on July 27, 2020 (152 days), days after the day 500 cases were reported, with 232368 cumulative cases of people being infected. The rational model indicated a plateau of 245795 cases on August 10, 2020 (166 days), the birational model indicated a plateau of 243936 cases on August 07, 2020 (166 days), the time-series model indicated a plateau of 248653 cases on September 3, 2020 (190 days). Finally, on August 28, 2020 (184 days) and September 5, 2020 (192 days), LSTM and GRU predicted a plateau of 239959 and 254676. However, the cumulative number of individuals reported to be infected by the COVID-19 epidemic in Italy on the last day of acquired data for this study was 243639.

Table II and IV present the mathematical models that yielded the following estimates for the dates that the plateau will be reached, as well as for the number of individuals reported to be infected with COVID-19 when this occurs: the constant model predicted that the COVID-19 outbreak in Sweden would plateau on August 8, 2020 (151 days), days after the day 500 cases were reported, with 74627 cumulative cases of people being infected. The rational model indicated a plateau of 124698 cases on September 18, 2020 (192 days), the birational model indicated a plateau of 128788 cases on September 28, 2020 (202 days), the time-series model indicated a plateau of 77554 cases on August 30, 2020 (172 days). Finally, on September 6, 2020 (180 days) and September 10, 2020 (184 days), LSTM and GRU predicted a plateau of 81467 and 82440. However, the cumulative number of individuals reported to be infected by the COVID-19 epidemic in Sweden on the last day of acquired data for this study was 72459.



(a)

(b)

Fig. 7. Deep learning model (GRU) prediction for the time that a plateau will be reached as well as the cumulative number of individuals reported to be infected by the COVID-19 in: **(a)** Italy; **(b)** Sweden

C. *Error Metrics of the Neural Network Training*

The neural network training and validation performance is demonstrated in Figure 8, where the random splits have been used to generate training and validation for the cumulative infected COVID-19 data in Italy. In Figure 8, we present the training and validation MSE at different epochs for the four mathematical models and the deep learning model namely GRU.

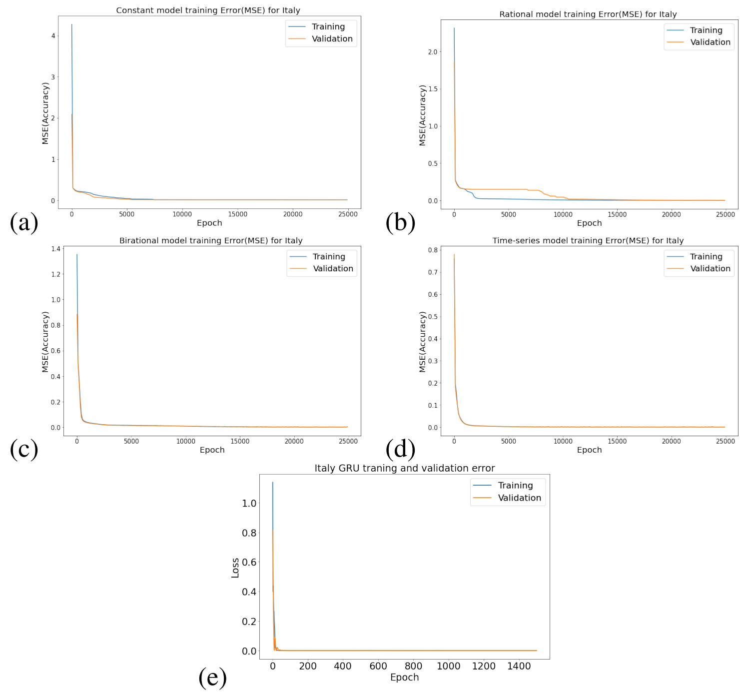


Fig. 8. The training and validation Errors of LINN for Italy COVID-19 infectives using: (a) Constant model; (b) Rational model; (c) Birational model; (d) Time-series model; (e) The training and validation Errors of GRU for Italy COVID-19 infectives

V. Discussion

In this work, the prediction of the time that a plateau will be reached and the cumulative number of individuals reported to be infected by COVID-19 in a given country was made using a deep-learning algorithm called logistic informed neural network on four mathematical models. Furthermore, the logistic-informed neural network was developed to learn the time-varying transmission rate parameters of the mathematical models, and the algorithm can be adapted to most logistical models. It was observed that the time-series model performed better than birational, rational, and constant models based on the error metrics of the models, the prediction of plateau cases, and plateau time, which is shown in table I-IV. We can see that the constant model underestimates the prediction of the plateau of COVID-19 in Figure 5 for Italy because of the long series of existing data on the epidemics of Italy, while the time-series model provides an upper bound, making it preferable to rational and birational models. Figure 6 shows the prediction curve of the COVID-19 plateau in Sweden. The constant, rational, and birational models curves were not obtained because of many infection points in the data caused by partial restrictions instead of strict restrictions and mitigation measures in Sweden. Although the prediction curve was obtained using a time-series model because the model was able to learn the form of the time-dependent function using a neural network. In addition, Figure 7 shows the prediction of the plateau cases and time of COVID-19 using GRU for Italy and Sweden, which provides a powerful generalization of recurrent neural networks. The important part of this work is the presentation of detailed comparisons between the predictions of the time-series model and those obtained via a GRU network. As discussed above, the time-series model yield similar predictions to those of the above networks.

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