Potenzen und Wurzeln

Ioannis Christodoulakis

26. September 2020

1 Übung Potenzen

1. $(-1)^{2n-1} = -1$

2. $-10^2 = -100$

3. $(-10)^{-3} = \frac{1}{(-10)^3} = -\frac{1}{1000}$

4. $((-2)^3)^2 = 64$

5. $(-1)^{-2n} = \frac{1}{(-1)^{2n}} = 1$

6. $-\left(\frac{3}{2}\right)^{-4} = -\left(\frac{2}{3}\right)^{-4} = -\frac{16}{81}$

7. $x^7 \cdot x^{-3} \cdot x^{-2} = x^{7-3-2} = x^2$

8. $\frac{y^3 \cdot x^{-2}}{(x \cdot y)^2} = \frac{y^3}{x^2 \cdot y^2 \cdot x^2} = \frac{y}{x^4}$

9. $\left(\frac{z^{-5}}{z^{-2}}\right)^{-2} = \left(\frac{z^2}{z^5}\right)^{-2} = \left(\frac{z^5}{z^2}\right)^{2} = (z^3)^2 = z^6$

10. $\left(\frac{3}{5}\right)^4 \div \left(\frac{6}{25}\right)^4 = \left(\frac{3}{5} \cdot \frac{25}{6}\right)^4 = \left(\frac{5}{2}\right)^4 = \frac{625}{16}$

11. $a^{n-2} \cdot a^{1-n} = a^{n-2+1-n} = \frac{1}{a}$

12. $(a-b)^3 \cdot (b-a)^{-3} = \frac{(a-b)^3}{(-1 \cdot (a-b))^3} = -1$

13. $b^{2x-1} \cdot b^{2x+1} \div b^{3x-1} = b^{2x-1+2x+1-3x+1} = b^{x+1} = b \cdot b^x$

14. $\left(\frac{a^2}{b^3}\right)^{-2} \cdot \frac{5a^3}{2b^2} \cdot 2ab^{-4} = \left(\frac{b^3}{a^2}\right)^2 \cdot \frac{5a^3}{2b^2} \cdot \frac{2a}{b^4} = 5$

15. $\frac{4n^{-2}m^4}{5c^2x^{-3}} \div \frac{8m^3c^{-1}x}{15n^{-2}c} = \frac{4m^4x^3}{5c^2 \cdot n^2} \cdot \frac{15c \cdot c}{n^2 \cdot 8m^3x} = \frac{3mx^2}{2n^4}$

16. $\frac{u^2 - t^2}{2u^2 + 4ut + 2t^2} = \frac{(u-t) \cdot (u+t)}{2 \cdot (u+t) \cdot (u+t)} = \frac{u-t}{2 \cdot (u+t)}$

17. $(r + r^{-1})^2 - (r - r^{-1})^2 = r^2 + 2 + \frac{1}{r^2} - (r^2 - 2 + \frac{1}{r^2}) = 4$

18. $(2 - p)^3 = 8 - 12p + 6p^2 - p^3$

19. $\frac{a^{15} - a^{10}}{a^5} = \frac{a^5 \cdot (a^{10} - a^5)}{a^5} = a^{10} - a^5$

20. $\frac{\frac{a^2 - b^2}{ab + b^2}}{\frac{(a-b)^2}{ab^2}} = \frac{(a+b) \cdot (a-b)}{b \cdot (a+b)} \cdot \frac{ab \cdot b}{(a-b) \cdot (a-b)} = \frac{ab}{a-b}$

2 Übung Wurzeln

1. $\frac{\sqrt{27}}{\sqrt{3}} = 3$

2. $\sqrt{\frac{4}{49}} = \frac{2}{7}$

3. $\sqrt{\frac{b^8}{25c^2}} = \frac{b^4}{5c}$

4. $\frac{\sqrt{a^3b}}{\sqrt{ab^5}} = \frac{a}{b^2}$

5. $\sqrt{2ac} \cdot \sqrt{\frac{8a}{c}} = 4a$

6. $\sqrt{\frac{9m^3}{5n}} \div \sqrt{\frac{81m}{20n^5}} = \frac{2}{3}m \cdot n^2$

7. $\sqrt[3]{8r^6t^4} = 2r^2 \cdot t^3 \sqrt[3]{t}$

8. $\sqrt{a} \cdot \sqrt[3]{a^2} = a \cdot \sqrt[6]{a}$

9. $\frac{\sqrt{a} \cdot \sqrt[3]{a}}{\sqrt[6]{a}} = \sqrt[3]{a^2}$

10. $\frac{\sqrt{x}}{x^{\frac{1}{3}}} = \sqrt[6]{x}$

11. $\frac{y^{\frac{3}{4}} \cdot \sqrt[6]{y}}{y^{\frac{7}{12}}} = \sqrt[3]{y}$

12. $(75x)^{\frac{1}{2}} \div (3x)^{\frac{1}{2}} = 5$

    Nenner ohne Wurzel:

13. $\frac{5}{\sqrt{3}+1} = \frac{5 \cdot \sqrt{3} - 5}{2}$

14. $\frac{a-b}{\sqrt{a-b}} = \sqrt{a-b}$

15. $\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \dfrac{7+2\cdot\sqrt{10}}{3}$

16. $1000^{-\frac{1}{3}} = \dfrac{1}{10}$

17. $c^{-\frac{2}{3}} \cdot \sqrt[6]{c} \cdot c^{\frac{1}{2}} = 1$

18. $\sqrt[3]{z \cdot \sqrt[4]{\dfrac{1}{z}}} = \sqrt[4]{z}$

19. $(8x^{-9})^{\frac{1}{3}} = \dfrac{2}{x^3}$

20. $\dfrac{\sqrt[3]{x\sqrt{x}} \cdot x^{\frac{1}{6}}}{\sqrt[3]{x}} = \sqrt[3]{x}$

21. $\sqrt[5]{\sqrt[4]{z^{10}}} = \sqrt{z}$