Mean Shift Segmentation

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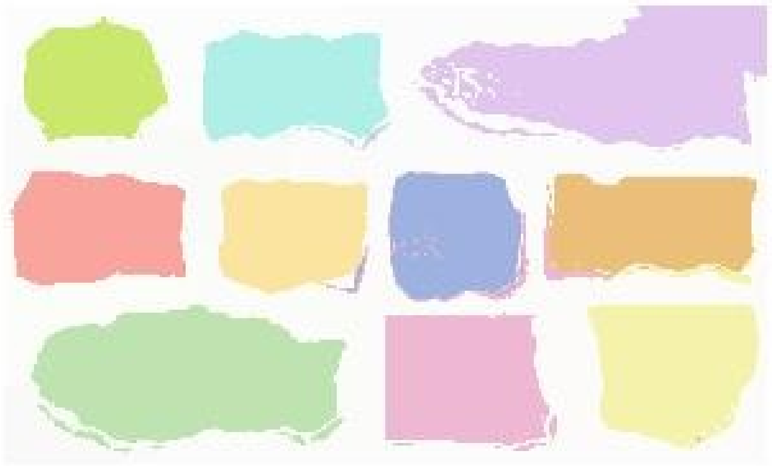
Input

Output



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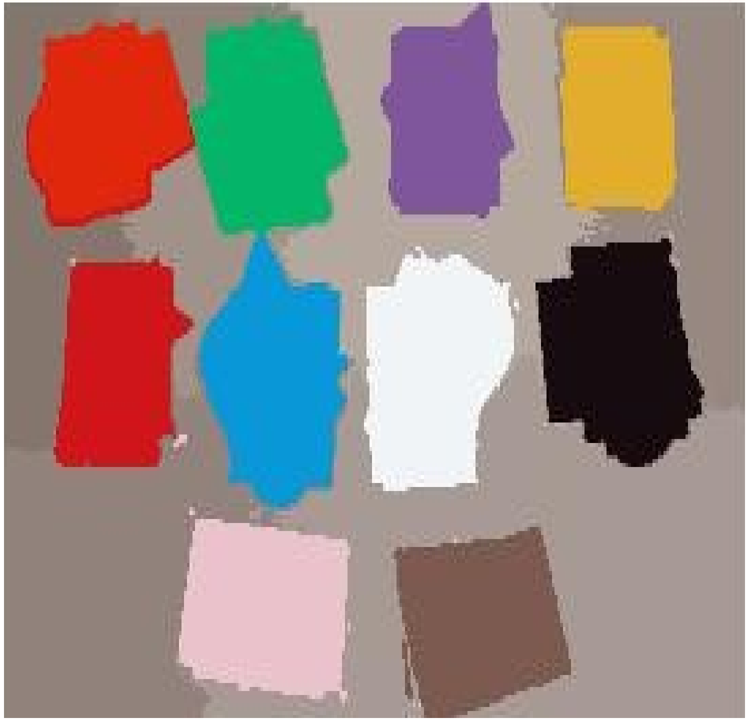
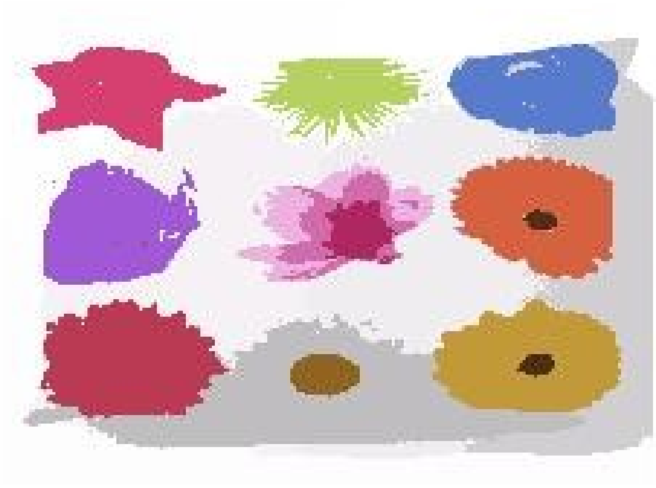


Mean Shift Segmentation

Input

Output

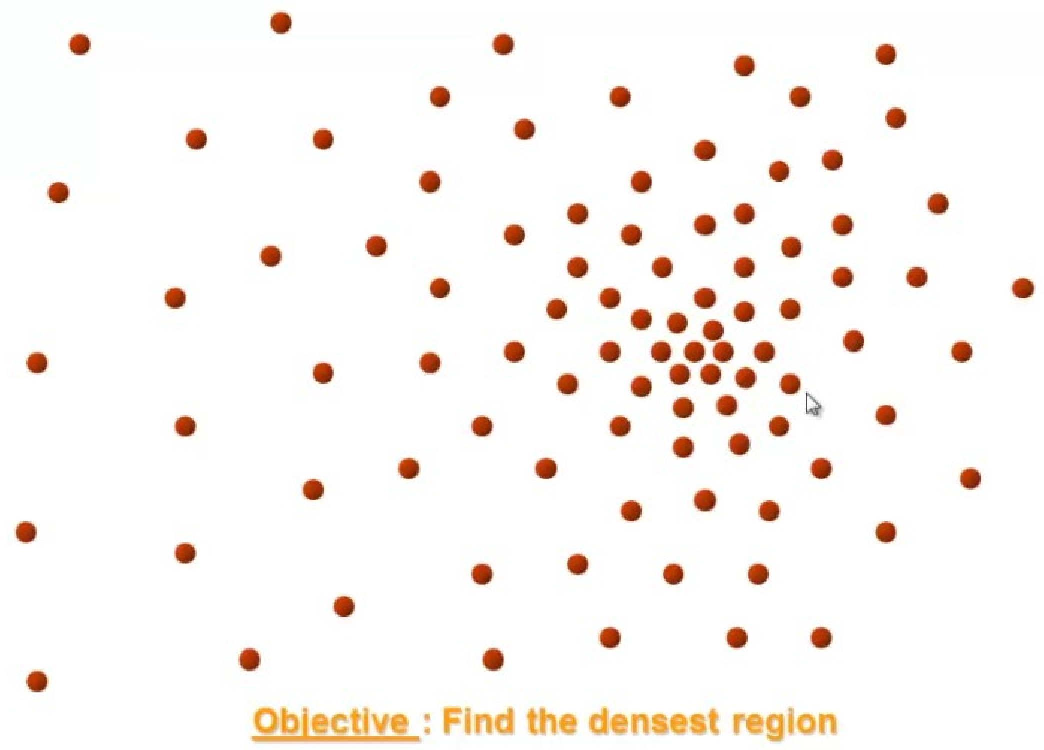


Mean Shift Segmentation

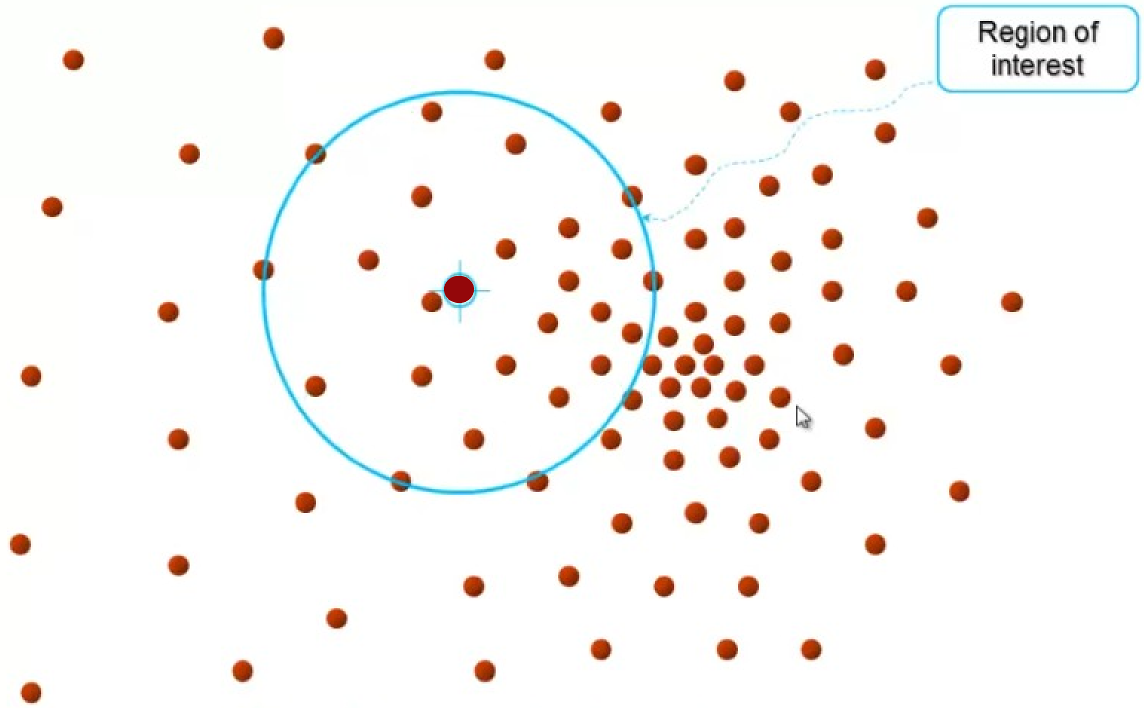
Input

Output

Mean Shift: Given a distribution of points and a point (x), mean shift is a procedure for finding the densest region corresponding to that x.



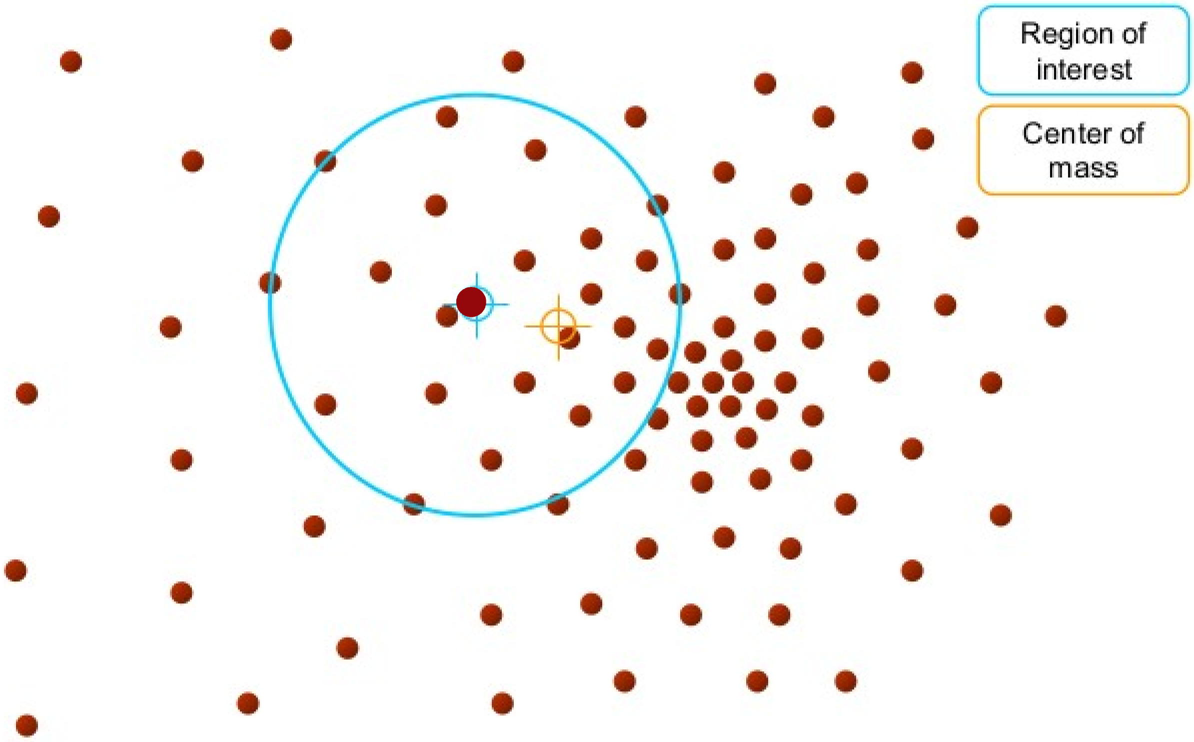
Method 1
Step 1



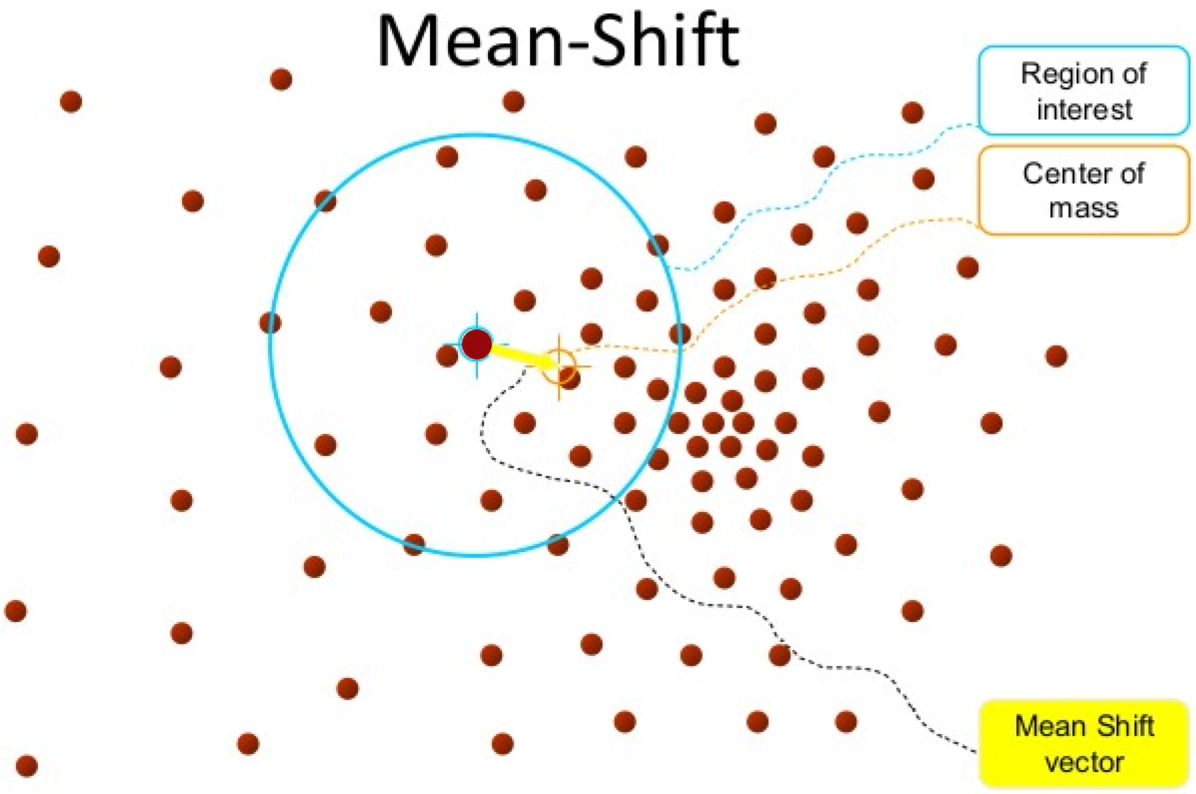
Objective : Find the densest region

Step 2

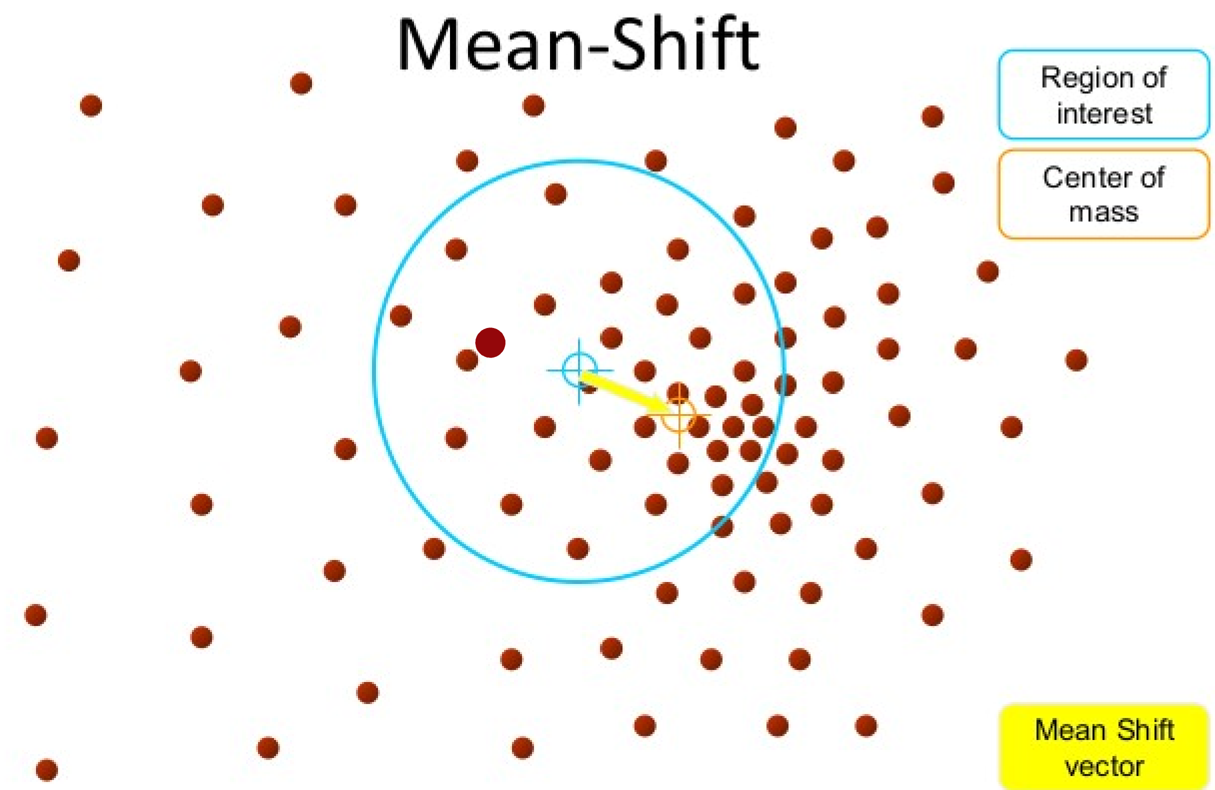
Find mean of the points inside the circle, call the mean as m_1



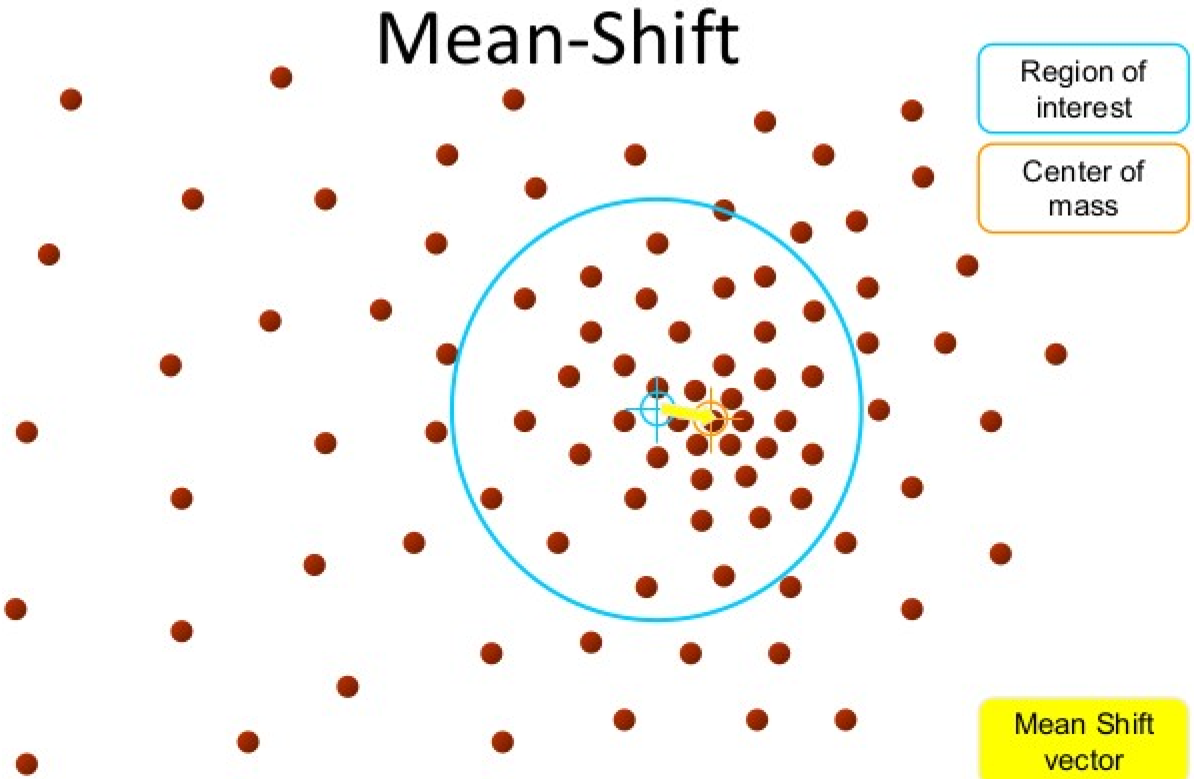
Step 3 Consider a new circle centered at m_1



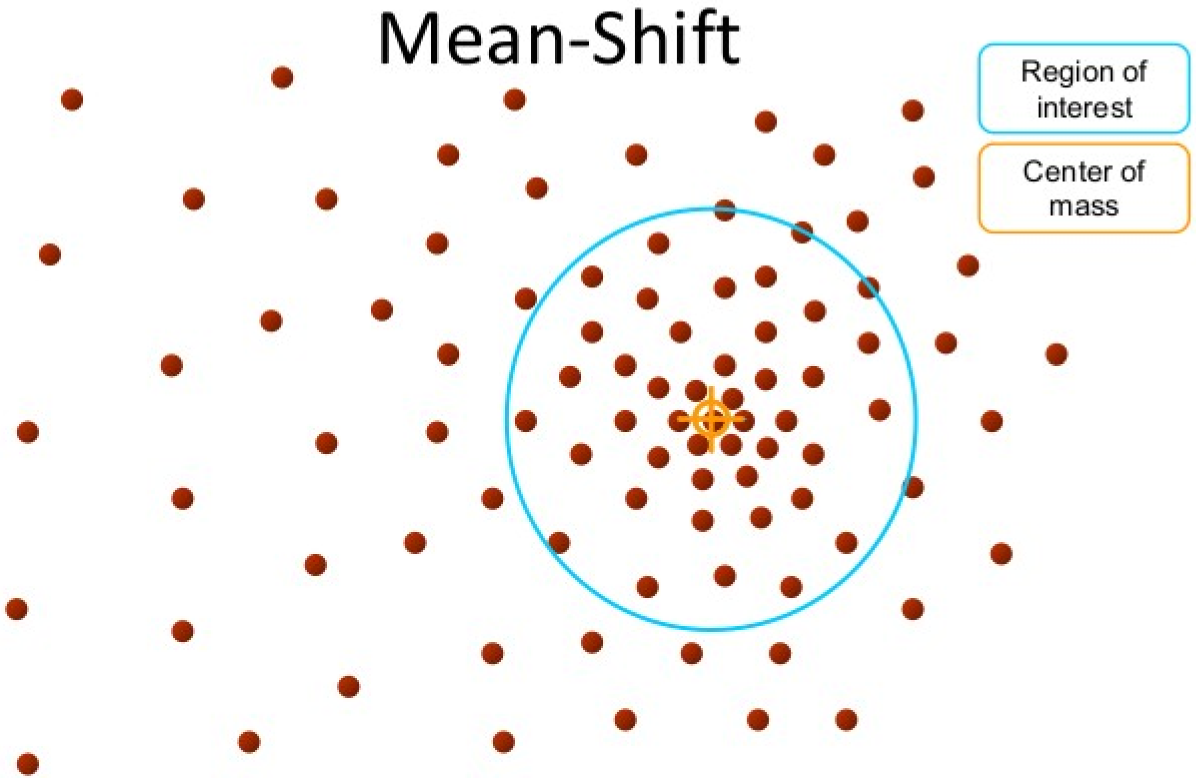
Repeat step 2 and 3 until converge



Repeat step 2 and 3 until converge



Repeat step 2 and 3 until converge



Method 1: Mean shift procedure

Input: Set of points and a point x
Output: Centre of local denest region corresponding to x

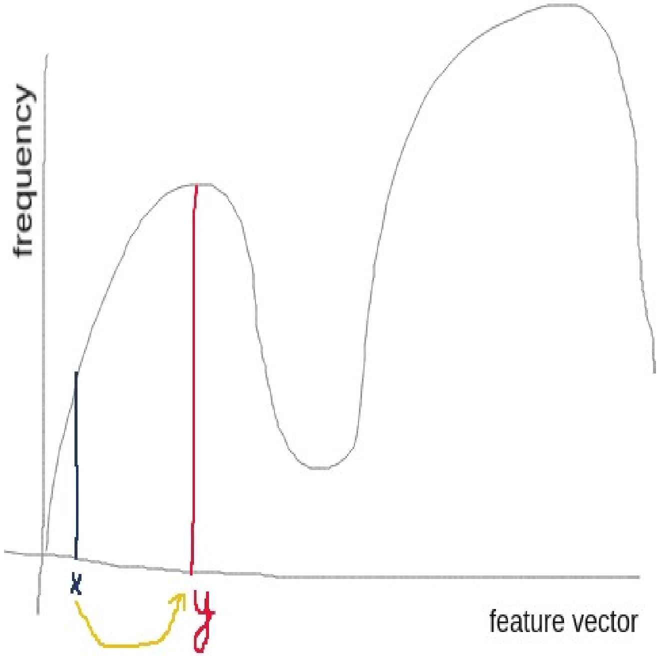
1. Consider a circle centered at x with a fixed redius r.
2. Determine a centroid (mean) of the data inside the circle.
3. Consider new circular region centered at mean computed at step 2 with the radius r.
4. Repeat step 2 and 3 until convergence (the magnitude of the motion vector is ϵ).

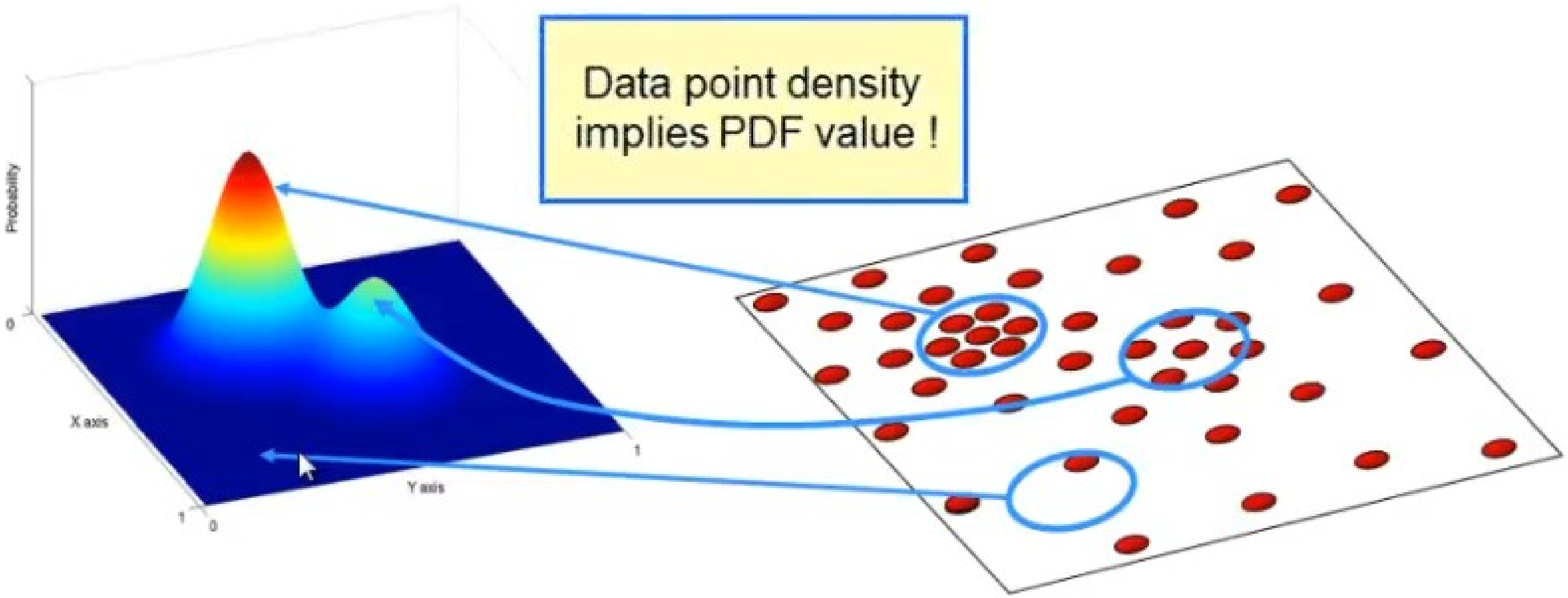
Method 2: Mean Shift

Given: set of n points in the d-dimensional space: {xi } i=1..n

Model: We assume that there is a probability density function (PDF) associated with the set of points, without any assumptions on its parameters.

Goal: for any given point find closest local mode of the density function.





Assumed Underlying PDF

Real Data Samples

Mean Shift Vector Computation

Gaussian Kernal

$$K(x;h) = \frac{1}{(2\pi)^{\frac{d}{2}}(h^d)} e^{\left(\left(\frac{-1}{2}\right)\left(\frac{|x|^2}{h}\right)\right)}$$

Probability density function

$$f(x) = \frac{1}{n}\sum_{i=1}^{n} K(x_i - x; h)$$

$$f(x) = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{(2\pi)^{\frac{d}{2}}(h^d)} e^{\left(\left(-\frac{1}{2}\right)\left(\frac{|x_i - x|^2}{h}\right)\right)} \tag{1}$$

let $k(u) = e^{(-\frac{1}{2}u)}$

Now equation (1) becomes

$$f(x) = \frac{1}{n}\frac{1}{(2\pi)^{\frac{d}{2}}(h^d)}\sum_{i=1}^{n} k\left(\frac{|x_i - x|^2}{h}\right) \tag{2}$$

let $c = \frac{1}{n}\frac{1}{(2\pi)^{\frac{d}{2}}(h^d)}$

Now equation (2) becomes

$$f(x) = c\sum_{i=1}^{n} k\left(\frac{|x_i - x|^2}{h}\right)$$

To max f, find x such that $\nabla f(x)|_{y=x} = 0$

$$c\sum_{i=1}^{n} \nabla k\left(\frac{|x_i - y|^2}{h}\right) = c\sum_{i=1}^{n}\left[k'\left(\frac{|x_i - y|^2}{h}\right)\left(\frac{2(x_i - y)}{h}\right)\right]$$

$$= \frac{2c}{h}\sum_{i=1}^{n}\left[k'\left(\frac{|x_i - y|^2}{h}\right)(x_i - y)\right]$$

let g(x) = k'(x)

$$\nabla f(y) = \frac{2c}{h} \sum_{i=1}^{n} \left[g\left(\frac{|x_i - y|^2}{h} \right) (x_i - y) \right]$$

$$= \frac{2c}{h} \left[\sum_{i=1}^{n} x_i g\left(\frac{|x_i - y|^2}{h} \right) - \sum_{i=1}^{n} y g\left(\frac{|x_i - y|^2}{h} \right) \right]$$

$$= \frac{2c}{h} \left[\sum_{i=1}^{n} x_i g\left(\frac{|x_i - y|^2}{h} \right) - y \sum_{i=1}^{n} g\left(\frac{|x_i - y|^2}{h} \right) \right]$$

multiply and divide by $\sum_{i=1}^{n} g\left(\frac{|x_i - y|^2}{h} \right)$

$$= \frac{2c}{h} \left(\frac{\sum_{i=1}^{n} g\left(\frac{|x_i-y|^2}{h} \right)}{\sum_{i=1}^{n} g\left(\frac{|x_i-y|^2}{h} \right)} \right) \left[\sum_{i=1}^{n} x_i g\left(\frac{|x_i - y|^2}{h} \right) - y \sum_{i=1}^{n} g\left(\frac{|x_i - y|^2}{h} \right) \right]$$

$$= \frac{2c}{h} \left(\sum_{i=1}^{n} g\left(\frac{|x_i - y|^2}{h} \right) \right) \left[\frac{\sum_{i=1}^{n} x_i g\left(\frac{|x_i-y|^2}{h} \right)}{\sum_{i=1}^{n} g\left(\frac{|x_i-y|^2}{h} \right)} - \frac{y \sum_{i=1}^{n} g\left(\frac{|x_i-y|^2}{h} \right)}{\sum_{i=1}^{n} g\left(\frac{|x_i-y|^2}{h} \right)} \right]$$

$$= \frac{2c}{h} \left(\sum_{i=1}^{n} g\left(\frac{|x_i - y|^2}{h} \right) \right) \left[\frac{\sum_{i=1}^{n} x_i g\left(\frac{|x_i-y|^2}{h} \right)}{\sum_{i=1}^{n} g\left(\frac{|x_i-y|^2}{h} \right)} - y \right]$$

The third term in the above equation is the mean shift

$$\left[\frac{\sum_{i=1}^{n} x_i g\left(\frac{|x_i - y|^2}{h}\right)}{\sum_{i=1}^{n} g\left(\frac{|x_i - y|^2}{h}\right)} - y\right]$$

since

$g\left(\frac{|x_i - y|^2}{h}\right) < 0 \quad \forall i$

$\sum_{i=1}^{n} g\left(\frac{|x_i - y|^2}{h}\right) < 0$

Therefore,

$$\left[\frac{\sum_{i=1}^{n} x_i g\left(\frac{|x_i - y|^2}{h}\right)}{\sum_{i=1}^{n} g\left(\frac{|x_i - y|^2}{h}\right)} - y\right] = 0$$

where, the first term is the mean obtained from the current iteration and the second term is the mean obtained from previous iteration

$$y = \frac{\sum_{i=1}^{n} x_i g\left(\frac{|x_i - y|^2}{h}\right)}{\sum_{i=1}^{n} g\left(\frac{|x_i - y|^2}{h}\right)}$$

$$y^{j+1} = \frac{\sum_{i=1}^{n} x_i g\left(\frac{|x_i - y^j|^2}{h}\right)}{\sum_{i=1}^{n} g\left(\frac{|x_i - y^j|^2}{h}\right)}$$

Mean shift segmentation

Algorithm:

Input : point x_i

Output : label for x_i

1. For each image pixel x_i, initialize $y_{i,1} = x_i$
2. Iterate the mean shift procedure until convergence, say the convergence value is $y_{i,con.}$
3. Assign the pixel value of $y_{i,con}$ to x_i

Thank you