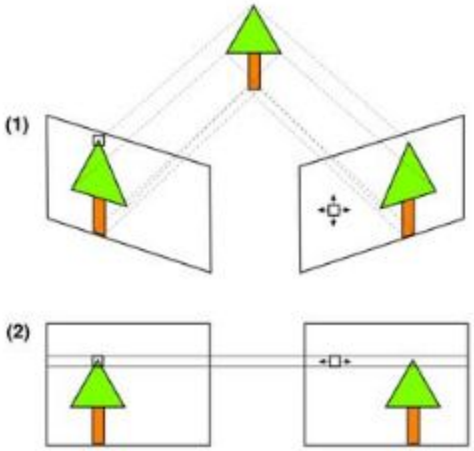
Essential and Fundamental Matrix

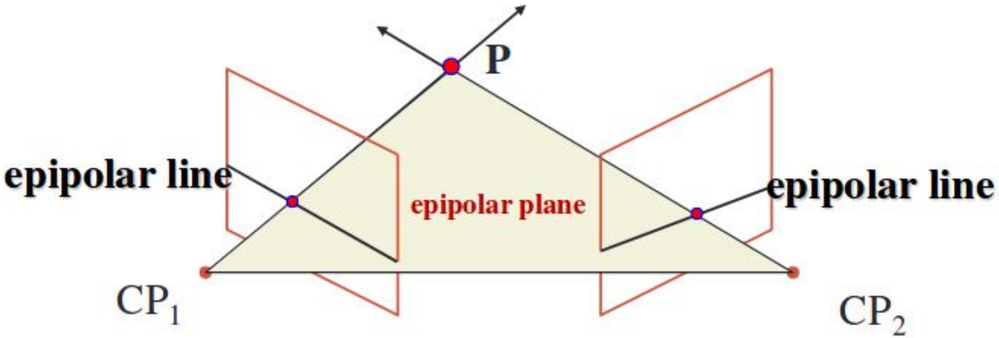
Stereo Correspondence



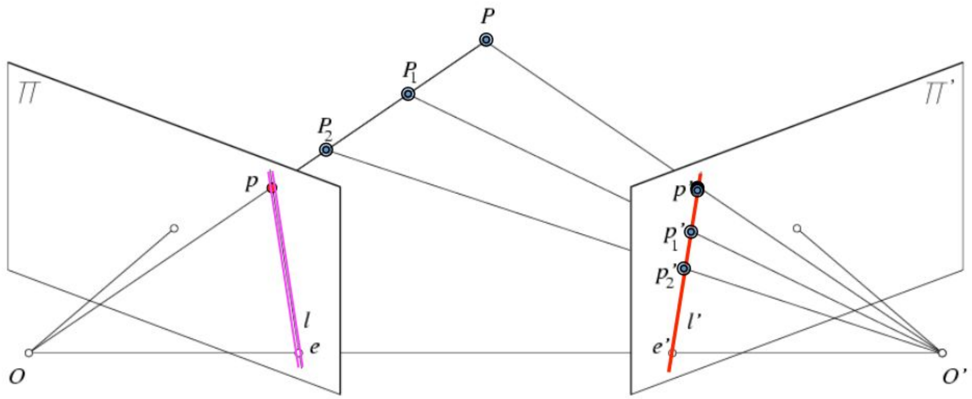
Search problem: Given an element in the left image, we search for the element in the right image.This involves two decisions:

1. Which image element to match and
2. Which similarity measure to adopt

- Determine Pixel Correspondence
    - Pairs of points that correspond to same scene point.

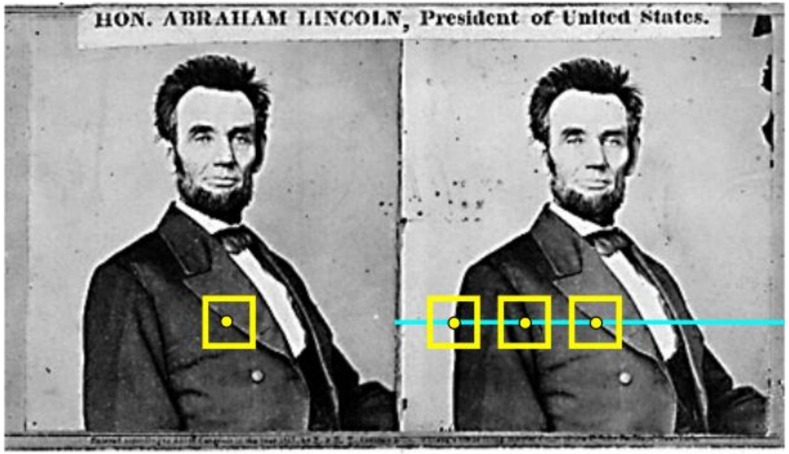


Epipolar Property



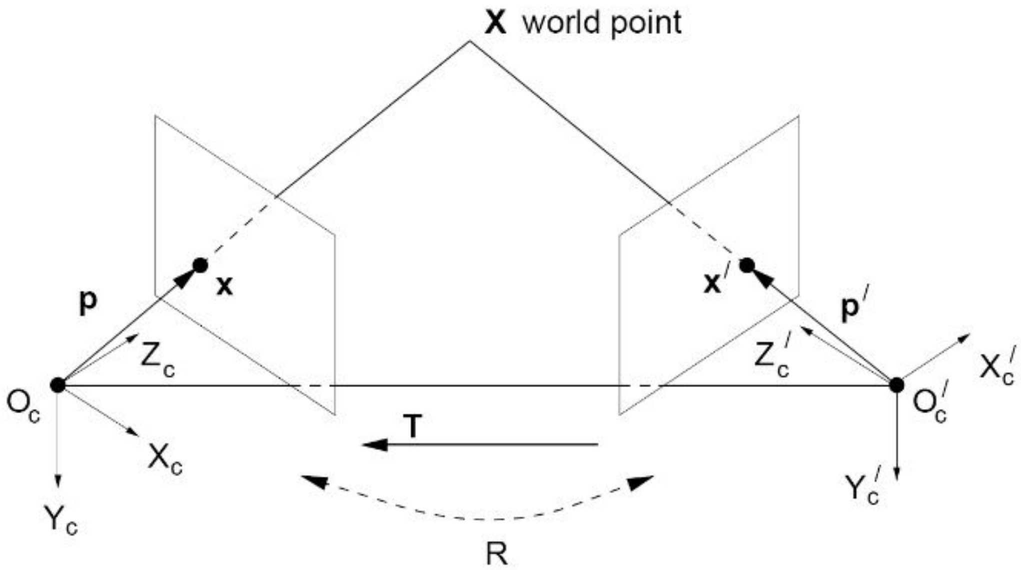
- Potential matches for *p* have to lie on the corresponding epipolar line *l'*.
- Potential matches for *p'* have to lie on the corresponding epipolar line *l*.

Why is the epipolar property useful?



- The epipolar property reduces the correspondence problem to a 1D search along the epipolar line.

Stereo Geometry, with calibrated cameras



$$X' = RX + T$$

Cross product with T on both sides..

$T \times X' = T \times (RX + T)$

$T \times X' = T \times RX + T \times T$

$T \times X' = T \times RX$ (since $T \times T = 0$)

Dot product with X' on both sides..

$X' \cdot (T \times X') = X' \cdot (T \times RX)$

$(X' \cdot (T \times X') = 0)$

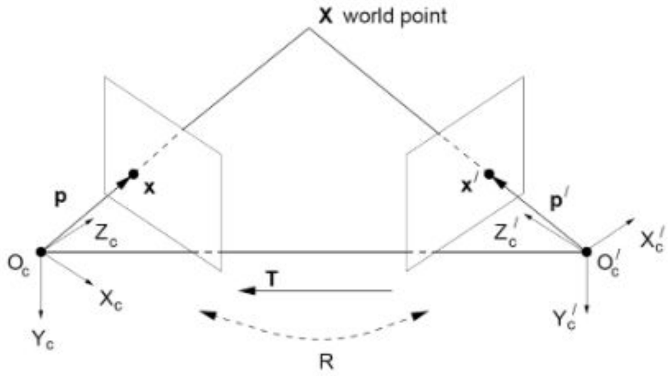
Essential Matrix

$$\boxed{\mathbf{X}'\cdot(\mathbf{T}\times\mathbf{R}\mathbf{X})=0}$$

$$\mathbf{X}'\cdot([\mathrm{T}_x]\mathbf{R}\mathbf{X})=0$$

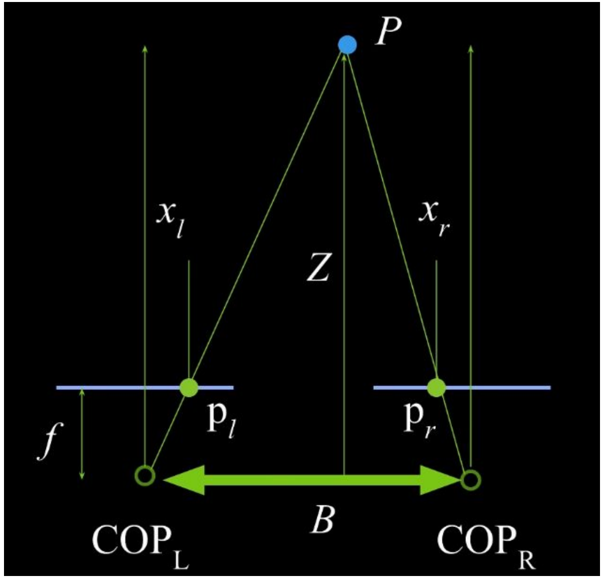
Let $\mathbf{E}=[\mathrm{T}_x]\mathbf{R}$

$$\mathbf{X}'^T\mathbf{E}\mathbf{X}=0$$



- **E** is called the **essential matrix** and it relates corresponding image points between both cameras, given the rotation and translation.

Essential Matrix example : Parallel Cameras



R = Identity

T = [B,0,0]

$$\mathbf{p}'^{\mathrm{T}}\mathbf{E}\mathbf{p} = 0$$

E = $[T_x]$R = $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & -B & 0 \end{pmatrix}$

$$\begin{bmatrix} x' & y' & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & -B & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} x' & y' & 1 \end{bmatrix} \begin{bmatrix} 0 \\ B \\ -By \end{bmatrix} = 0$$

Given a point (x,y), this is the line where(x’,y’) lies on

$$By' = By \Rightarrow \boxed{\mathbf{y' = y}}$$

Unknown Camera Calibration parameters

- Want to estimate world geometry without requiring calibrated cameras
- Main Idea :
  - Estimate Epipolar geometry from a (redundant) set of point correspondences between two uncalibrated cameras

From before: Projection matrix

$$\begin{bmatrix} wx_{im} \\ wy_{im} \\ w \end{bmatrix} = \mathbf{K}_{int}\mathbf{\Phi}_{ext}\begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\mathbf{p}_{im} = \mathbf{K}_{int}\underbrace{\mathbf{\Phi}_{ext}\mathbf{P}_w}_{\mathbf{p}_c}$$

$$\mathbf{p}_{im} = \mathbf{K}_{int}\mathbf{p}_c$$

Uncalibrated case

For a given camera: $\mathbf{p}_{im} = \mathbf{K}_{int}\mathbf{p}_c$

So, for **two** cameras (left and right):

$$\mathbf{p}_{c,left} = \mathbf{K}^{-1}_{int,left}\mathbf{p}_{im,left}$$

$$\mathbf{p}_{c,right} = \underbrace{\mathbf{K}^{-1}_{int,right}}_{\text{Internal calibration matrices, one per camera}}\mathbf{p}_{im,right}$$

Uncalibrated case

$$\mathbf{p}_{c,right} = \mathbf{K}^{-1}_{int,right}\mathbf{p}_{im,right} \qquad \mathbf{p}_{c,left} = \mathbf{K}^{-1}_{int,left}\mathbf{p}_{im,left}$$

From before, the ***essential*** *matrix* ***E.***

$$\boxed{\mathbf{p}_{c,right}{}^{\mathrm{T}}\mathbf{E}\mathbf{p}_{c,left} = 0}$$

$$\left(\mathbf{K}^{-1}_{int,right}\mathbf{p}_{im,right}\right)^{\mathrm{T}}\mathbf{E}\left(\mathbf{K}^{-1}_{int,left}\mathbf{p}_{im,left}\right) = 0$$

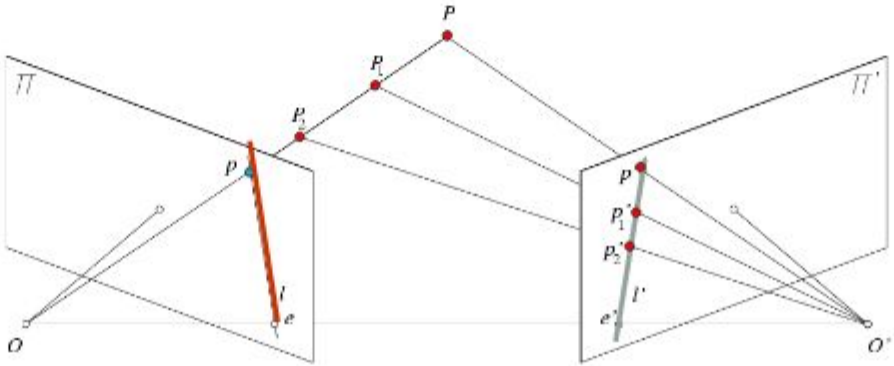
$$\mathbf{p}^{\mathrm{T}}_{im,right}\left(\underbrace{(\mathbf{K}^{-1}_{int,right})^{T}\mathbf{E}\mathbf{K}^{-1}_{int,left}}_{\mathbf{F}}\right)\mathbf{p}_{im,left} = 0$$

"Fundamental matrix" $\mathbf{F}$

$$\boxed{\mathbf{p}^{\mathrm{T}}_{im,right}\mathbf{F}\mathbf{p}_{im,left} = 0} \quad or \quad \mathbf{p}^{T}\mathbf{F}\mathbf{p}' = 0$$

Properties of Fundamental Matrix

$$\mathbf{p}^T \mathbf{F} \mathbf{p}' = 0$$



- $\mathbf{l} = \mathbf{F}\mathbf{p}'$ is the epipolar line associated with $\mathbf{p}'$
- $\mathbf{l}' = \mathbf{F}^T\mathbf{p}$ is the epipolar line associated with $\mathbf{p}$

Computing F from correspondences

Each point correspondence generates one constraint on F

$$\mathbf{p}_{im,right}^{T}\mathbf{F}\mathbf{p}_{im,left} = 0$$

$$\begin{bmatrix} u' & v' & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = 0$$

Collect n of these constraints

$$\begin{bmatrix} u'_1 u_1 & u'_1 v_1 & u'_1 & v'_1 u_1 & v'_1 v_1 & v'_1 & u_1 & v_1 & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = \mathbf{0}$$

Solve for f , vector of parameters.