Homography

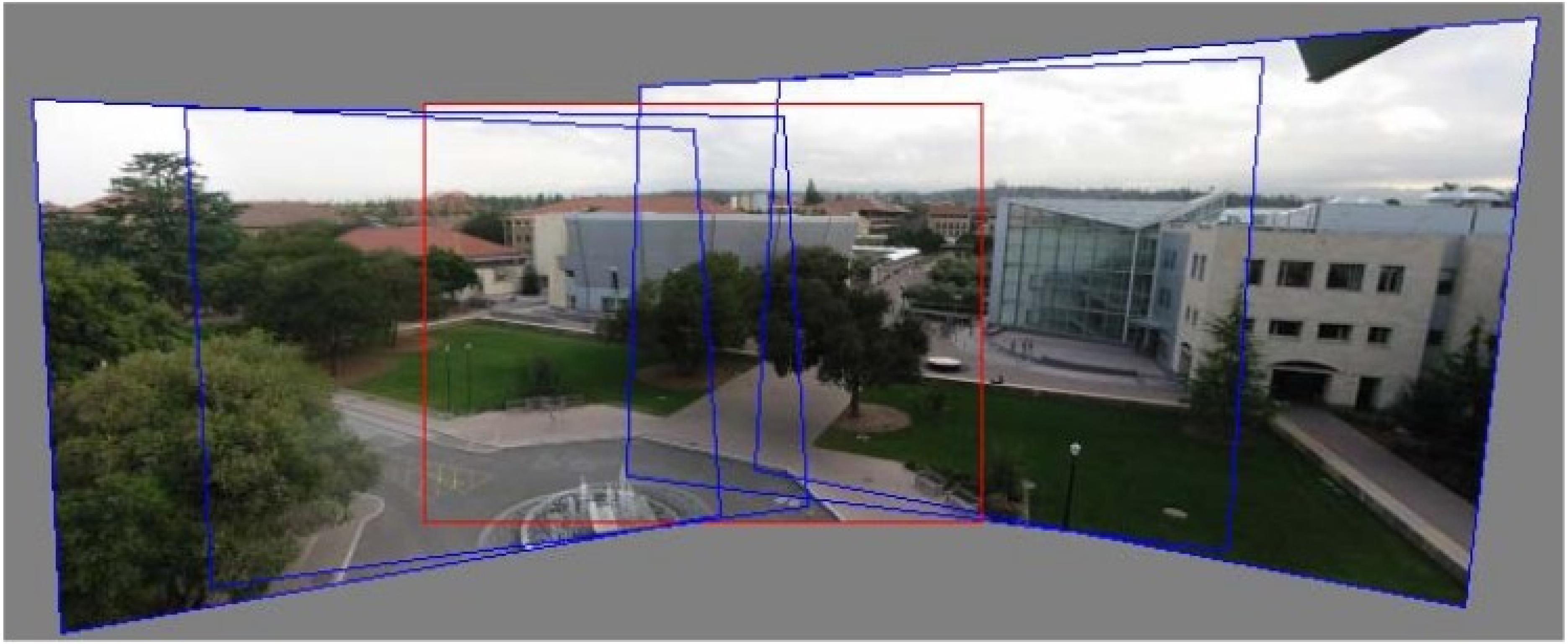
In the field of computer vision, any two images of the **same planar** surface in space are related by a homography.

Definition

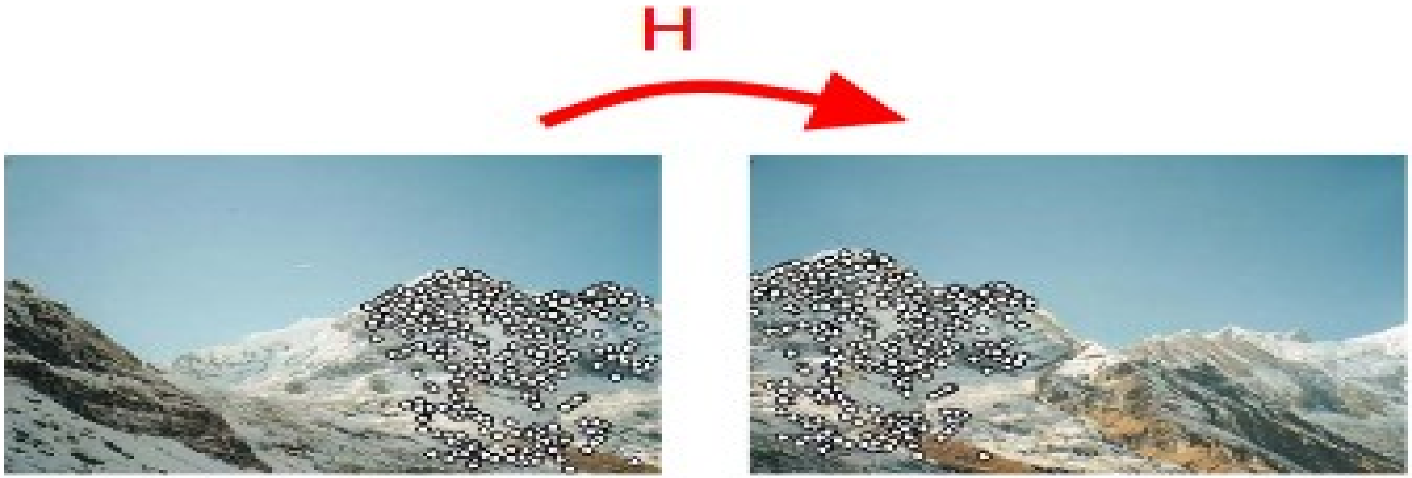
- A homography is a non-singular, projective mapping $H: P^n \rightarrow P^n$. It is represented by a square $(n + 1)$ - dim matrix with $(n + 1)^2 - 1$ Degree Of Freedom
- A mapping from $P^2 \rightarrow P^2$ is a projectivity if and only if there exists a non-singular 3×3 matrix H such that for any point in P^2 represented by vector x it is true that its mapped point equals Hx.

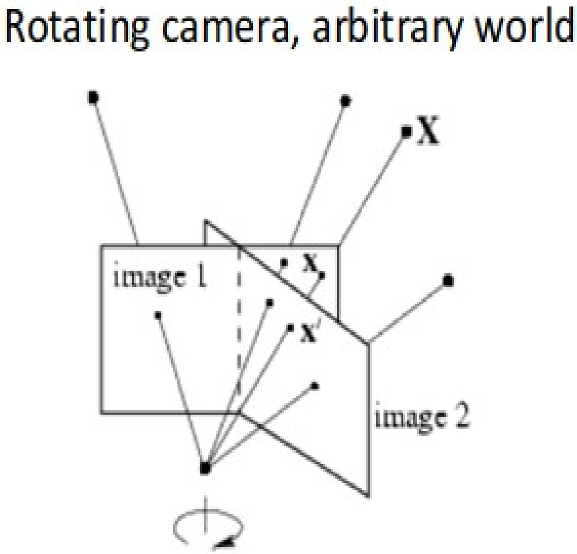
$$x' = Hx$$

Application of Homography - Mosaic



Mosaic





Homography - aligns the image2 relative to that image1

virtual wide-angle camera

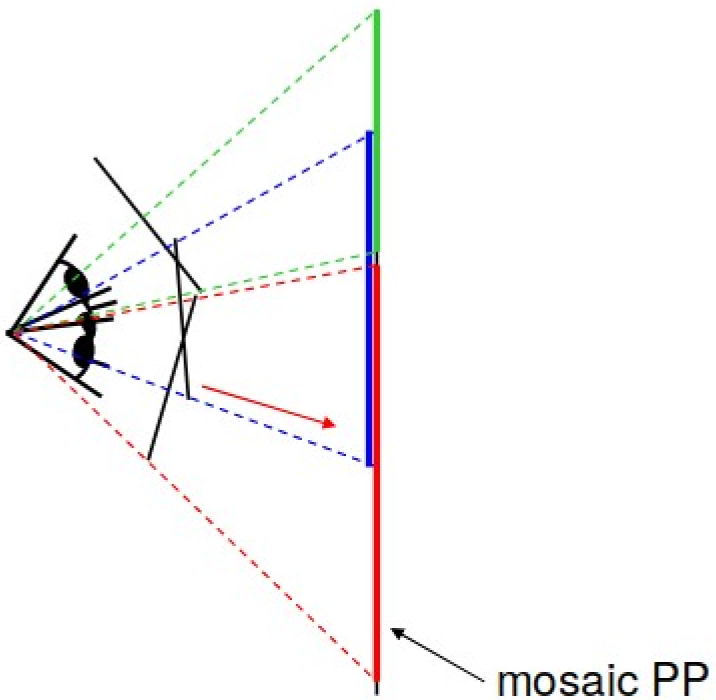
How to Align the images

Is translation sufficient?

How to do it?

- Take a sequence of images from the **same position**
  - Rotate the camera about its optical center
- **Compute transformation** between second image and first
- Transform the second image to **overlap** with the first
- Blend the two images together to create a mosaic
- If there are more images, repeat

Image Reprojection



The mosaic has a natural interpretation of scene

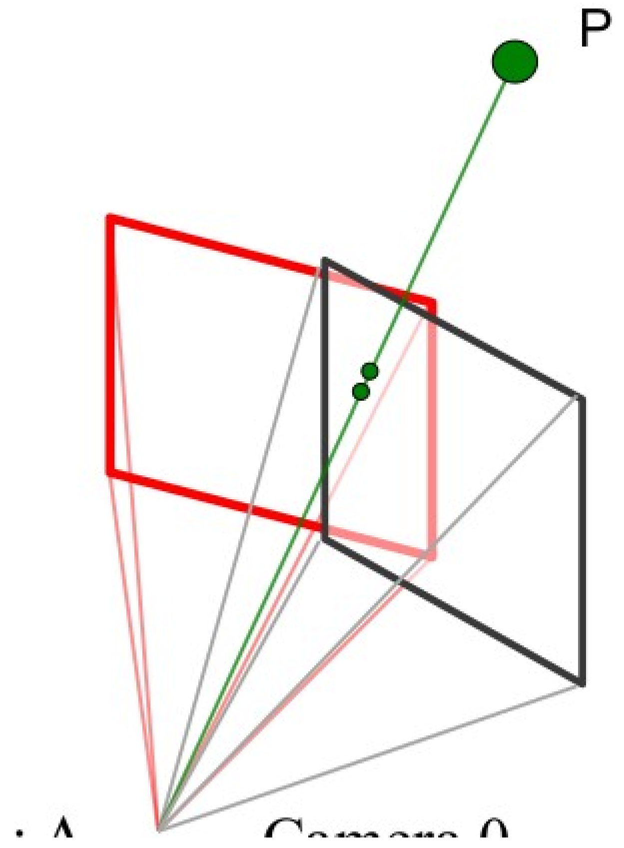
- The images are reprojected onto a common plane
- The mosaic is formed on this plane

Homography conditions

Two images are related by a homography if and only if

- Both images are taken from the **same camera** but from a different angle
  - Camera is rotated about its center of projection without any translation
- Note that the homography relationship is **independent of the scene** structure
  - It does not depend on what the cameras are looking at
  - Relationship holds regardless of what is seen in the images

Homography H?



- If world plane coordinate is P, then
  - $x = AP$ and $x' = A'P$. (ie $A' = AR$)
  - $x' = A'A^{-1}x$. (sub: $P = A^{-1}x$)
  - $x' = Hx$

Computation of Homography (Method 1)

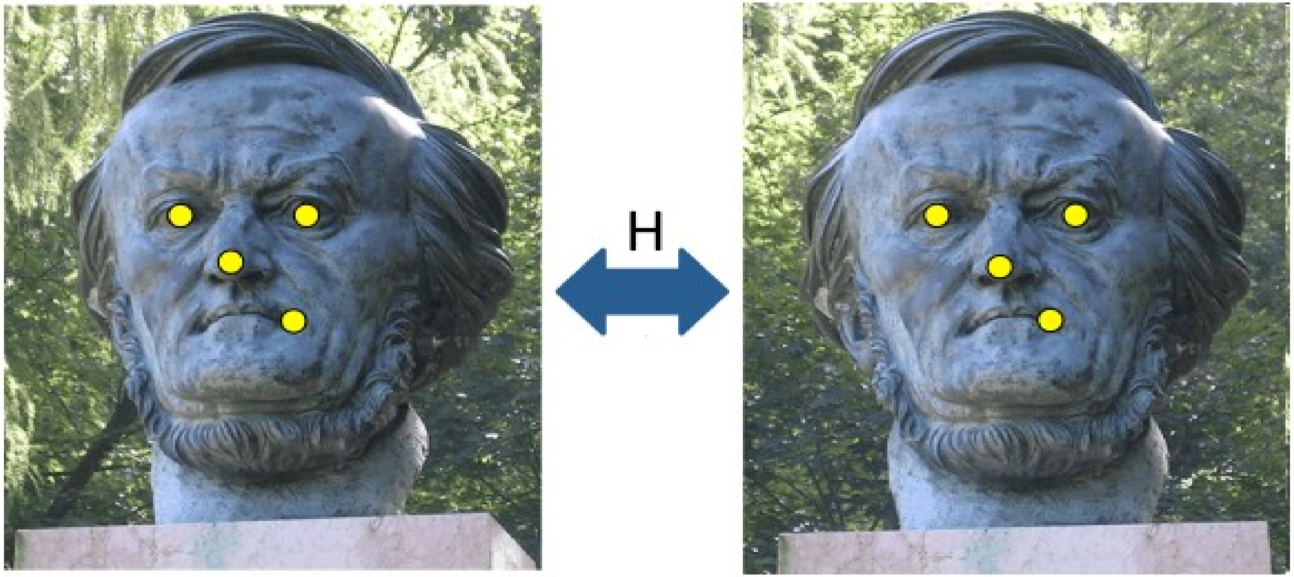
- If there is no translation x' = Rx
- projection equation for x and x'

$$x = K[I \mid 0]\begin{bmatrix} X \\ 1 \end{bmatrix} = KX \qquad\qquad x' = K[R \mid 0]\begin{bmatrix} X \\ 1 \end{bmatrix} = KRX$$

- So $x' = KRK^{-1}x$ where K is intrinsic matrix
- Where KRK^{-1} is a 3by3 matrix H called a homography

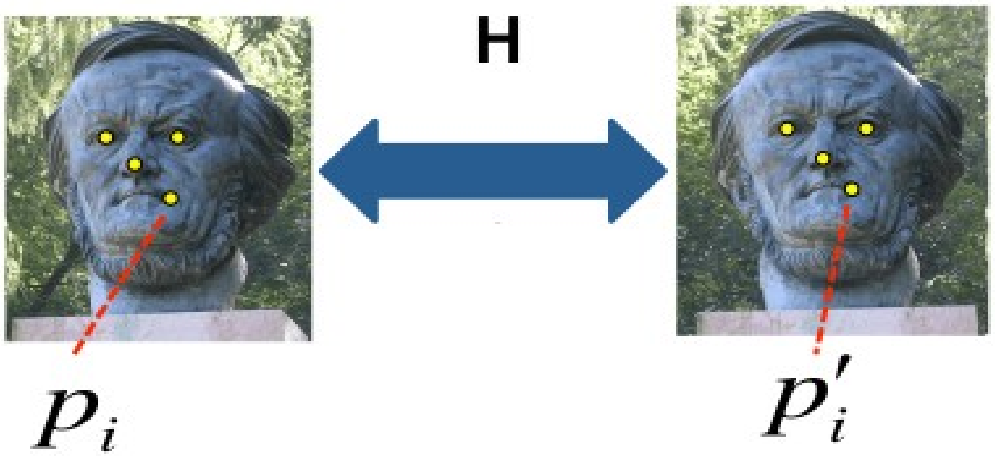
Computing homography (Method 2)

- Estimate the homographic tranformation between two images



Assumption: Given a set of corresponding points.

Homography H?



$$p'_i = H\ p_i$$

Computing Homography

$$\begin{bmatrix} x'' \\ y'' \\ s \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

x’ = x’’ / s
y’ = y’’ / s

$$
\begin{array}{ccccc}
 & \textbf{2N x 8} & \textbf{8 x 1} & & \textbf{2N x 1} \\
\begin{array}{l} \textbf{Point 1} \\ \\ \textbf{Point 2} \\ \\ \textbf{Point 3} \\ \\ \textbf{Point 4} \\ \\ \end{array} &
\begin{bmatrix}
x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x'_1 & -y_1x'_1 \\
0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y'_1 & -y_1y'_1 \\
x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2x'_2 & -y_2x'_2 \\
0 & 0 & 0 & x_2 & y_2 & 1 & -x_2y'_2 & -y_2y'_2 \\
x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3x'_3 & -y_3x'_3 \\
0 & 0 & 0 & x_3 & y_3 & 1 & -x_3y'_3 & -y_3y'_3 \\
x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4x'_4 & -y_4x'_4 \\
0 & 0 & 0 & x_4 & y_4 & 1 & -x_4y'_4 & -y_4y'_4
\end{bmatrix}
&
\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix}
& = &
\begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \\ x'_4 \\ y'_4 \end{bmatrix} \\
\textbf{additional points} & \vdots & & & \vdots
\end{array}
$$

Linear equations

$$\underset{2N\times 8}{A}\ \underset{8\times 1}{h} = \underset{2N\times 1}{b}$$

Solve:

$$\underset{8\times 2N}{A^T}\ \underset{2N\times 8}{A}\ \underset{8\times 1}{h} = \underset{8\times 2N}{A^T}\ \underset{2N\times 1}{b}$$

$$\overbrace{(A^T\ A)}^{8\times 8}\ \overset{8\times 1}{h} = \overbrace{(A^T\ b)}^{8\times 1}$$

$$h = (A^T\ A)^{-1}\ (A^T\ b)$$

Thank You