Predicting Real-World Penny Auction Durations by Integrating Game Theory and Machine Learning

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Abstract

Game theory and machine learning are two widely used techniques for predicting the outcomes of strategic interactions among humans. However, the game theory-based approach often relies on strong rationality and informational assumptions, while the machine learning-based approach typically requires the testing data to come from the same distribution as the training data. Our work studies how to integrate the two techniques to address these weaknesses. We focus on the interactions among real bidders in penny auctions, and develop a three-stage framework to predict the distributions of auction durations, which indicate the numbers of bids and auctioneer revenues. Specifically, we first leverage a pre-trained neural network to encode the descriptions of products in auctions into embeddings. Second, we apply game theory models to make preliminary predictions of auction durations. In particular, we tackle the challenge of accurately inferring parameters in game theory models. Third, we develop a Multi-Branch Mixture Density Network to learn the mapping from product embeddings and game-theoretic predictions to the distributions of actual auction durations. Experiments on real-world penny auction data demonstrate that our framework outperforms both game theory-based and machine learning-based prediction approaches.

1 Introduction

Predicting human behavior in strategic environments is crucial for designing effective economic mechanisms and policies, such as auctions and tax policies. One conventional prediction approach is reasoning about human strategic behavior through game theory. In classical game theory, humans are assumed to be fully rational and always employ equilibrium strategies. Behavioral game theory considers bounded rational humans, and assumes that they behave according to more sophisticated models, such as quantal response equilibria and cognitive hierarchy (Wright and Leyton-Brown 2010; Goeree, Holt, and Palfrey 2005; Kagel and Roth 2020; Camerer 2011). However, game theory models may not accurately predict human behavior in practice (Hartford, Wright, and Leyton-Brown 2016; Kolumbus and Noti 2019; Bourgin et al. 2019). It is challenging to manually design a behavior model that captures all factors influencing human behavior (e.g., risk preference, myopia, framing effect, and altruism). Moreover, deriving the equilibria given a sophisticated behavior model can be computationally intractable, particularly when the economic mechanisms or policies involve complex rules and humans exhibit heterogeneous rationalities and knowledge.

There has been a growing trend to predict human strategic behavior using machine learning models, especially neural networks (Hartford, Wright, and Leyton-Brown 2016; Zhao et al. 2018; Cai et al. 2018; Kolumbus and Noti 2019; Shen et al. 2020; Ben-Porat et al. 2020). With sufficient data about historical behavior, machine learning models can be trained to make accurate behavior predictions, even when analytically characterizing the human behavior model is challenging. However, the machine learning-based prediction approach suffers from the *domain shift* problem (Noti and Syrgkanis 2021; Andrews et al. 2022). For example, if an auction designer collects sufficient data about bidder behavior under a specific auction rule, using the machine learning model trained on these data may not accurately predict bidder behavior under a different auction rule. The reason is that bidders behave differently when the auction rule changes, causing a shift in behavioral data distribution across different auction rules (i.e., domains).

Our work aims at integrating the above two prediction approaches to leverage the strengths of both machine learning and game theory. First, machine learning directly learns behavior patterns from data, eliminating the need for accurate manually designed behavior models. Second, game theory enhances the robustness of behavior predictions to the domain shift. For example, when the auction rule changes, without any actual behavioral data under the new auction rule, game theory can still provide a characterization of bidder behavior by making rationality assumptions and conducting equilibrium analysis. Although the characterization may not be accurate for real bidders, it is informative in predicting changes in real bidder behavior (e.g., whether bidders bid more aggressively). We will show that incorporating this *domain knowledge* from game theory effectively improves the machine learning-based prediction.

We focus on predicting strategic behavior in penny auctions (i.e., pay-to-bid auctions), where bidders should pay a non-refundable bid fee to place a bid and each new bid increases the item price by a constant increment (Hinnosaar

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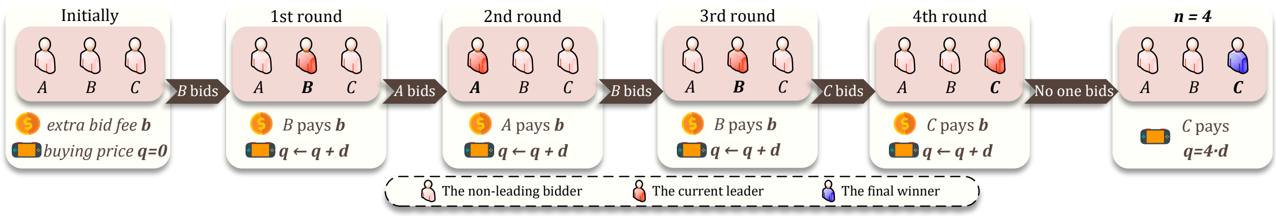

Figure 1: An Example of A Penny Auction.

2016; Platt, Price, and Tappen 2013). Since penny auctions have rich configuration details (e.g., bid fees and bid increments) and bidder behavior changes with these configurations, penny auctions provide an ideal setting for evaluating behavior prediction approaches. Predicting when a particular bidder places a bid in a penny auction is meaningless and intractable. Instead, our focus lies in predicting the distribution of the *auction duration*, which indicates the time when all bidders decide not to bid. The auction duration reveals the overall number of valid bids, and determines the item's final price and the total payment from bidders to the auctioneer.

We propose a three-stage auction duration prediction framework. Our key idea is first making predictions using different game theory models and then developing a Multi-Branch Mixture Density Network, which learns the mapping from this domain knowledge and other available auction information to the actual distribution of the auction duration. Our contributions are two-fold:

- On the application side, we develop a framework to predict the auction duration under different auction configurations and compare it with other methods. Experiments on both synthetic and real data show that our framework achieves accurate predictions, even under a large domain shift. Our framework can help auctioneers better estimate their revenues under different auction configurations and choose the optimal configuration.
- On the methodology side, we propose a novel strategic behavior prediction approach that combines the strengths of game theory and machine learning. We tackle the associated technical challenges, such as inferring parameters in game theory models. The idea of our prediction approach is general and can be extended to predict human behavior in other strategic environments.

2 Related Work

Behavior Prediction in Penny Auctions Prior studies have analyzed human behavior in penny auctions using either laboratory data (Caldara 2012) or real-world data (Zheng, Goh, and Huang 2011; Wang and Xu 2016). Built upon a common fundamental game theory model, some studies investigated the impacts of different factors (e.g., risk preference, sunk cost, and information asymmetry) on human behavior (Platt, Price, and Tappen 2013; Augenblick 2016; Byers, Mitzenmacher, and Zervas 2010; Hinnosaar 2016). Furthermore, a few studies applied behavioral game theory to model bidders' bounded rationality and cognitive biases. For example, Prospect Theory was used to capture the probability misperception and loss aversion of bidders (Gnutzmann 2014; Brünner et al. 2019). They provide a good understanding of human behavior in penny auctions but still lack predictive accuracy due to the challenge of manually designing an accurate behavior model. In fact, the models in most of these studies can be expressed through a generic form, as will be discussed in Section 4.3.

Integration of Theoretical Behavior Models and Machine Learning To better predict human behavior, a few studies combined theoretical models with machine learning (Radford and Joseph 2020; Plonsky et al. 2019; Hofman et al. 2021). Most references focused on predicting human choices among monetary gambles (Plonsky et al. 2017; Noti et al. 2016), where each gamble encompasses several possible outcomes (Erev et al. 2015; Plonsky et al. 2018). For example, (Bourgin et al. 2019) utilized theoretical models developed by cognitive psychologists to generate synthetic data, alleviating the scarcity of real behavioral data available for training neural networks. In these studies, human payoffs are not affected by the decisions of others, which is different from penny auctions. Moreover, unlike these studies, we propose to learn the relation between predictions of theoretical models and actual human behavior.

3 Problem Formulation

3.1 Penny Auctions

Consider a penny auction lasting for multiple rounds indexed by $t \in \mathbb{N}^+$. To begin with, all bidders are allowed to place a bid. The first bid will make the auction enter its first round. Whoever places the first bid will become the leading bidder in this round. After the first round, any non-leading bidder in the t-th round can place a bid, triggering the start of the $(t+1)$-th round and resulting in the bidder becoming the new leading bidder.

In the penny auction, every bid will raise the buying price q by a constant increment $d \geq 0$. Every time a bidder bids and becomes the leading bidder, she has to pay a small and non-refundable bid fee $b > 0$ to the auctioneer.

If no further bids occur after t rounds, the auction will conclude and we record the auction's duration n as t. The leading bidder in the t-th round will win the auction and pay q for the product. We illustrate an example of $n = 4$ in Figure 1, where three bidders bid for a game controller. The auctioneer charges b for each bid, and $q = 4 \cdot d$ for selling the controller to the winner (bidder C).

Penny auctions can vary in terms of bid fees b and bid increments d. Moreover, they may differ in the products to

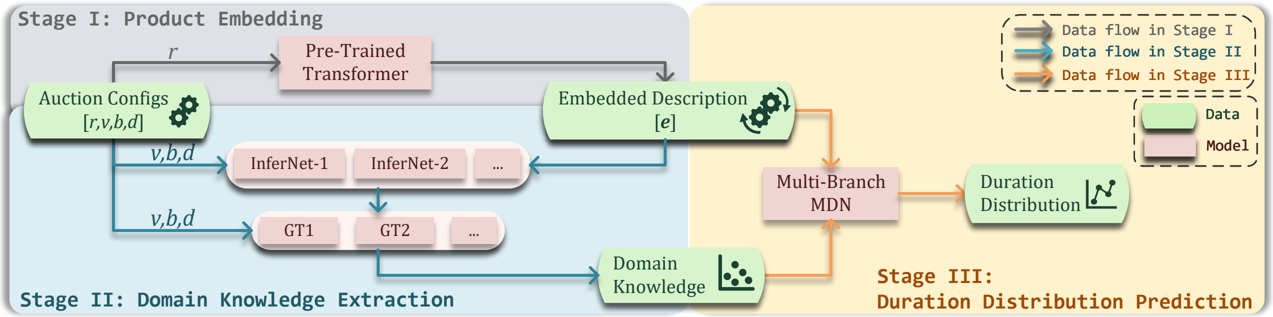

Figure 2: Our Three-Stage Auction Duration Prediction (ADAPT) Framework.

be auctioned off, and we use r to identify a product. Additionally, we use v to denote the *retail price* of a product. It is announced by the auctioneer on the auction website, and can help bidders better assess the value of the product.

Formally, we categorize penny auctions by four features: product r, retail price v, bid fee b, and bid increment d. Thus, by numbering different categories with i, we can define the *auction configuration* as $s_i = \{r_i, v_i, b_i, d_i\}$.

3.2 Auction Duration Prediction Problem

Penny auctions with the same configuration can be held multiple times, and our data record the auction durations observed under each configuration. To capture the randomness in bidder behavior, we aim to predict the *distribution* of auction durations given the configuration information. Formally, if $\mathcal{S}$ denotes the set of all auction configurations that we study, our problem is to predict $p_{i,n}$ (i.e., the probability that the auction ends after n rounds) for each $s_i \in \mathcal{S}$.

4 Our Prediction Framework

4.1 Overview

We propose a three-stage **A**uction **D**ur**a**tion **P**redic**t**ion (ADAPT) framework to predict auction durations. As shown in Figure 2, our framework consists of the following stages.

Stage I (*Product Embedding*): We utilize a pre-trained neural network to encode product descriptions and get their embeddings, which will be used in the next two stages.

Stage II (*Domain Knowledge Extraction*): We leverage game theory models to get preliminary predictions of auction durations, regarded as domain knowledge. A key challenge is inferring the parameters of game theory models, especially some parameters that may depend on the product features. We address it by proposing an **InferNet** to learn the mapping between product features and parameter values.

Stage III (*Duration Distribution Prediction*): We propose a Multi-Branch Mixture Density Network, which takes the outputs of the first two stages as inputs and predicts the distribution of the auction duration.

4.2 Stage I: Product Embedding

For each product r_i, its description is visible to all bidders on the auction website and offers basic information about product specifications. Considering that the product description is in the length of a sentence, we use a pre-trained Sentence Transformer (Reimers and Gurevych 2019) to encode it and get a fixed-length embedding $\boldsymbol{e}_i$. The embedding $\boldsymbol{e}_i$ is a vector representation of the product features and can be easily processed by the neural networks in the next two stages.

4.3 Stage II: Domain Knowledge Extraction

In this subsection, we first discuss the connection between auction durations and the equilibria characterized by game theory models. Second, we present a generic equilibrium condition, and characterize equilibria considering different decision-making theories. Third, we address the challenge of inferring the parameters of the game theory models.

Connection Between Auction Durations and Equilibria Some prior studies conducted game-theoretic analysis of penny auctions and mainly focused on deriving symmetric subgame perfect Nash equilibria (Platt, Price, and Tappen 2013; Hinnosaar 2016). Specifically, we use $m \in \{1, 2, \dots, M\}$ to index game theory models. For game theory model m and auction configuration s_i, a symmetric subgame perfect Nash equilibrium is characterized by the probability that at least one bidder bids after $t-1$ rounds. We denote it by $u_{i,t}^{(m)}$. According to the auction rule, it equals the probability of the penny auction s_i entering its t-th round, conditioning on that $t-1$ rounds have elapsed. Note that we focus on auctions that can be successfully initiated, meaning that the auction durations exceed 0. Therefore, we have $u_{i,1}^{(m)} = 1$. Given $u_{i,t}^{(m)}$, we can compute the distribution of auction duration predicted by model m, and derive the probability $p_{i,n}^{(m)}$ (defined in Section 3.2) as:

$$p_{i,n}^{(m)} = (1 - u_{i,n+1}^{(m)}) \cdot \prod_{t=1}^{n} u_{i,t}^{(m)}. \tag{1}$$

A Generic Equilibrium Condition We show a generic condition that $u_{i,t}^{(m)}$ satisfies in the equilibrium. For $t \geq 2$, when a non-leading bidder bids after the $(t-2)$-th round, it is possible that she becomes the leader in the $(t-1)$-th round and other bidders continue to bid, leading the auction to enter the t-th round. Each non-leading bidder should decide between bidding or not after the $(t-2)$-th round, considering the abovementioned possibility. In the symmetric subgame perfect equilibrium with $u_{i,t}^{(m)} > 0$, each non-leading bidder is indifferent between the two choices, i.e., the utilities are equal (Brünner et al. 2019):

$$\begin{aligned}
&\pi_i^{(m)}(1-u_{i,t}^{(m)})\cdot f_i^{(m)}(v_i-(t-1)\cdot d_i-b_i-h_i^{(m)}(t-2))\\
+&\pi_i^{(m)}(u_{i,t}^{(m)})\cdot f_i^{(m)}(-b_i-h_i^{(m)}(t-2))\\
=&f_i^{(m)}(-h_i^{(m)}(t-2)). \qquad (2)
\end{aligned}$$

The left side of (2) computes the bidder's perceived expected utility, considering whether her bid after the $(t-2)$-th round will conclude the auction in the $(t-1)$-th round. The right side of (2) computes the bidder's utility when she does not bid, and the utility only depends on the *sunk cost* of placing bids in the preceding $t-2$ rounds.

There are three functions in (2) that can take different specific forms in different game theory model m:

- $f_i^{(m)}(\cdot)$ is the utility function of bidders and reflects bidders' valuation of the outcomes resulting from different actions.
- $\pi_i^{(m)}(\cdot)$ is the probability weighting function, which captures how bidders perceive probabilities differently from the actual probabilities of uncertain outcomes (Kahneman and Tversky 1979).
- $h_i^{(m)}(\cdot)$ denotes the sunk cost incurred by the accumulated historical bid fees (Augenblick 2016).

Next, we introduce two representative game theory models (*GT1* and *GT2*), where we choose specific forms of the above three functions based on Expected Utility Theory (EUT) and Prospect Theory (PT), respectively.

GT1 (EUT-Based Model) Expected Utility Theory (EUT) assumes that bidders are fully rational and can correctly perceive the probabilities of uncertain outcomes, so $\pi_i^{(1)}(u)=u$ in *GT1* (indexed by $m=1$). If the bidder utility function is linear and the bidders disregard the sunk cost (Platt, Price, and Tappen 2013), we have $f_i^{(1)}(v)=v$ and $h_i^{(1)}(t)=0$.

Substituting $f_i^{(1)}(\cdot)$, $h_i^{(1)}(\cdot)$, and $\pi_i^{(1)}(\cdot)$ into (2) yields a simplified equilibrium condition:

$$(1-u_{i,t}^{(1)})\cdot(v_i-(t-1)\cdot d_i-b_i)+u_{i,t}^{(1)}\cdot(-b_i)=0. \qquad (3)$$

Solving the equation gives $u_{i,t}^{(1)}$, which can be used to derive $p_{i,n}^{(1)}$ based on (1).

GT2 (PT-Based Model) Compared with EUT, Prospect Theory (PT) considers more sophisticated bidder behavior, and assumes that bidders evaluate gains and losses asymmetrically (Kahneman and Tversky 1979). *GT2* (indexed by $m=2$) considers the following utility function (Brünner et al. 2019): $f_i^{(2)}\left(v;\alpha_i^{(2)},\lambda_i^{(2)}\right)=\frac{1-\exp\left(-\alpha_i^{(2)}v\right)}{\alpha_i^{(2)}}$ for $v\geq 0$, and $f_i^{(2)}\left(v;\alpha_i^{(2)},\lambda_i^{(2)}\right)=\frac{-\lambda_i^{(2)}\left(1-\exp\left(\alpha_i^{(2)}v\right)\right)}{\alpha_i^{(2)}}$ for $v<0$. Here, $\alpha_i^{(2)}$ and $\lambda_i^{(2)}$ are parameters denoting bidders' preference of risk and attitude toward loss, respectively. *GT2* also assumes that bidders have inaccurate perception of uncertainty and consider the sunk cost. Due to the space limit, we give concrete forms of $\pi_i^{(2)}(\cdot)$ and $h_i^{(2)}(\cdot)$ in the supplementary material. Similar to (3), we can substitute these functions to (2), and solve it to get $u_{i,t}^{(2)}$.

Challenge of Parameter Inference Let $\boldsymbol{\theta}_i^{(m)}$ denote the parameters of model m. For example, $\boldsymbol{\theta}_i^{(2)}$ includes $\alpha_i^{(2)}$, $\lambda_i^{(2)}$, and the parameters in the probability weighting function and suck cost function in *GT2*. A prerequisite for computing $u_{i,t}^{(m)}$ and $p_{i,t}^{(m)}$ is inferring $\boldsymbol{\theta}_i^{(m)}$. Specifically, we can infer $\boldsymbol{\theta}_i^{(m)}$ based on the actual auction durations under auction configuration s_i in the training data. We can use algorithms, such as Simulated Annealing (Bertsimas and Tsitsiklis 1993), to search for the $\boldsymbol{\theta}_i^{(m)}$ that maximizes the likelihood given the data.

A challenge is that the training data cannot include all possible auction configurations. Without observations of actual auction durations under configuration s_i, the conventional method cannot infer $\boldsymbol{\theta}_i^{(m)}$.

Inference with InferNet The key idea of our solution is learning the mapping between auction configurations (such as the embedded description $\boldsymbol{e}_i$) and model parameters $\boldsymbol{\theta}_i^{(m)}$ using the training data. Given the learned mapping, even if the training data do not include any observation of the duration for configuration s_i, we can still estimate $\boldsymbol{\theta}_i^{(m)}$ based on the configuration information such as $\boldsymbol{e}_i$.

Neural networks have a strong capability to capture the underlying pattern inherent in the data. For each model m, we propose an **InferNet** (a multi-layer fully connected neural network). The inputs include configuration information (e.g., $\boldsymbol{e}_i$), and the outputs include the inferred parameters, denoted as $\hat{\boldsymbol{\theta}}_i^{(m)}$.

To train the **InferNet** on the training data, we design a loss function. Specifically, with $\hat{\boldsymbol{\theta}}_i^{(m)}$, we can compute $u_{i,t}^{(m)}$ using (2), and then get $p_{i,n}^{(m)}$ based on (1). To evaluate $\hat{\boldsymbol{\theta}}_i^{(m)}$, we can resort to $p_{i,n}^{(m)}$, and measure the consistency between $p_{i,n}^{(m)}$ with respect to different n and the actual auction durations observed in the data. Consequently, we choose the loss function to be the following negative log-likelihood:

$$\mathrm{Loss}_{\mathrm{InferNet}}=-\sum_{i:|\mathcal{N}_i|\neq 0}\frac{1}{|\mathcal{N}_i|}\sum_{n\in\mathcal{N}_i}\log(p_{i,n}^{(m)}), \qquad (4)$$

where $\mathcal{N}_i$ is the set of actual auction durations under configuration s_i observed in the data. A lower negative log-likelihood implies a more precise mapping between configuration information and model parameters. After training **InferNet** to minimize the negative log-likelihood, we can use it to predict model parameters for the configurations that do not appear in the training data.

4.4 Stage III: Duration Distribution Prediction

We propose a Multi-Branch Mixture Density Network (MB-MDN) to predict the duration distribution. We first discuss

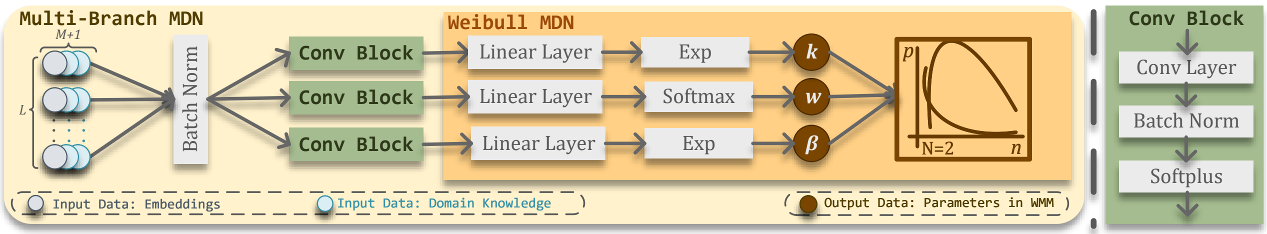

Figure 3: Structure of Our Multi-Branch Mixture Density Network.

the network inputs and MDN, and then introduce MB-MDN.

Network Inputs Given the configuration s_i, the input of our duration prediction model is a stack of $M + 1$ vectors. The first vector is the embedded product description $\boldsymbol{e}_i$. Each of the remaining M vectors corresponds to the domain knowledge extracted using each game theory model, i.e., it consists of $p_{i,1}^{(m)}$, $p_{i,2}^{(m)}$, etc. To facilitate the batch training of our prediction model, we extend these vectors to the same length L, where L is a sufficiently large integer. We apply the padding technique (adding zeros to the end of short vectors) and mask the padded zeros during model training.

Mixture Density Network The Mixture Density Network (MDN) can effectively learn the mapping from features to the probability distribution of random events. Specifically, it combines a conventional neural network with a mixture density model (Bishop 1994). MDN outputs parameters of the mixture density model. For example, taking the stacked vectors under configuration s_i as the input, a Weibull MDN can output $\boldsymbol{w}_i$, $\boldsymbol{k}_i$, and $\boldsymbol{\beta}_i$ as the parameters of a Weibull mixture model. Given $\boldsymbol{w}_i$, $\boldsymbol{k}_i$, and $\boldsymbol{\beta}_i$, the predicted probability density function of n is as follows:

$$\hat{g}_i(n; \boldsymbol{w}_i, \boldsymbol{k}_i, \boldsymbol{\beta}_i) = \sum_{j=1}^{J} w_{i,j} \cdot \phi_{i,j}(n; k_{i,j}, \beta_{i,j}), \tag{5}$$

where J is a hyper-parameter indicating the number of mixture components, $w_{i,j}$ is the mixing coefficient satisfying $w_{i,j} > 0$, and $\sum_{j=1}^{J} w_{i,j} = 1$, $\phi_{i,j}(\cdot)$ is a Weibull probability density function parameterized by the shape parameter $k_{i,j} > 0$ and the scale parameter $\beta_{i,j} > 0$. By combining different components $\phi_{i,j}(\cdot)$ with the weight $w_{i,j}$, $\hat{g}_i(\cdot)$ gains the capability to represent complex distributions.

Multi-Branch Mixture Density Network The conventional MDN typically uses a fully connected neural network to learn the mapping from input features to the parameters of the mixture model. We propose a novel MB-MDN, which uses multiple branches to predict different mixture model parameters separately. Each branch focuses on capturing the relationship between input features and a specific type of model parameters, leading to a more fine-grained prediction for each type of parameters. Leveraging the multi-branch structure (Ye et al. 2020; Leroux, Li, and Simoens 2022), our network effectively predicts heterogeneous model parameters while requiring fewer trainable neural network weights.

As shown in Figure 3, the input of MB-MDN is a stack of L-dimensional vectors. We use a batch normalization layer to normalize the input data over each mini-batch. Each branch is dedicated to predicting one type of parameters of the Weibull mixture model. For each branch, we utilize a convolutional block to extract features from the input data. Within the convolutional blocks, we use convolutional kernels with customized sizes and strides. We choose the softplus function ($\mathrm{Softplus}(x) = \log(1 + e^x)$) as the activation function, which always has a non-zero gradient. For the branch corresponding to $\boldsymbol{w}$, we include a softmax layer to ensure that $\sum_{j=1}^{J} w_j = 1$ and $w_j > 0$. For the branches associated with $\boldsymbol{k}$ and $\boldsymbol{\beta}$, we apply an exponential function to the output, which guarantees that $k_j > 0$ and $\beta_j > 0$.

Loss Function The loss function used to train MB-MDN measures the consistency between the predicted auction duration and the actual duration. Note that the output of MB-MDN is a continuous probability distribution and the actual duration can only be an integer. Let $\hat{p}_{i,n}$ denote the prediction of $p_{i,n}$ given by our framework. We can compute it as $\hat{p}_{i,n} = \int_{n-0.5}^{n+0.5} \hat{g}_i(\tilde{n})\, d\tilde{n}$. Then we can define the loss function as the following negative log-likelihood:

$$\mathrm{Loss}_{\mathrm{MB-MDN}} = -\sum_{i:|\mathcal{N}_i| \neq 0} \frac{1}{|\mathcal{N}_i|} \sum_{n \in \mathcal{N}_i} \log(\hat{p}_{i,n}), \tag{6}$$

where $\mathcal{N}_i$ is the set of actual auction durations under configuration s_i.

5 Experiments

5.1 Dataset Description

We evaluate our duration prediction framework on a synthetic dataset and a real dataset. We use game theory models to generate the synthetic dataset. Given an auction configuration s_i, we first select a game theory model m from the M models according to a predefined mapping from auction configurations to the choices of game theory models. Next, we compute the corresponding $p_{i,n}^{(m)}$ for different n. Then, we perturb each $p_{i,n}^{(m)}$ using a Gaussian noise, subject to the constraint that the sum of probabilities remains one. We randomly generate samples of auction durations according to the perturbed distribution. We leave more details of generating the synthetic data to our supplementary material.

Our real data are collected from online penny auction websites (Byers, Mitzenmacher, and Zervas 2010; Augenblick 2016). Our dataset includes auction configurations consisting of the product information, retail price, bid fee,

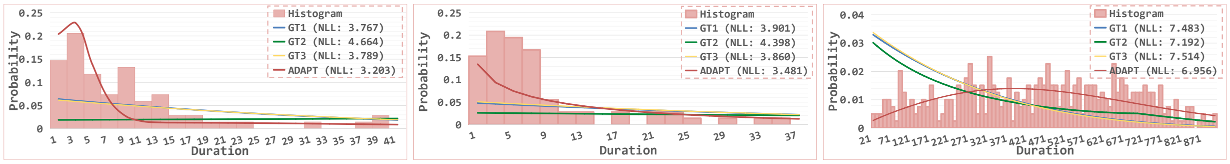

Figure 4: Examples of Predicted Distributions and Normalized Histogram of Durations in Real Testing Data.

and bid increment. Under each configuration, multiple auctions may be held. The dataset records the duration of each of these auctions. It contains $115,831$ records covering $1,276$ auction configurations.

5.2 Experiment Settings

Our duration prediction framework uses **InferNet** to infer the parameters of game theory models for out-of-sample configurations, which do not appear in training data. We compare **InferNet** with two inference methods:

- **SA-Avg** (Booij, Van Praag, and Van De Kuilen 2010): For each configuration that appears in the training data, this method applies Simulated Annealing to infer the parameters of game theory models. Then, it computes the **av**era**g**es of the inferred parameters across these in-sample configurations, which serve as the model parameters for all out-of-sample configurations.
- **SA-Unified** (Brünner et al. 2019): For all configurations that appear in the training data, this method applies Simulated Annealing to infer a **unified** set of model parameters, and uses them for all out-of-sample configurations.

To evaluate these inference methods, we compare game theory models whose parameters are inferred using different inference methods and test their prediction performance under out-of-sample configurations. Specifically, we focus on *GT2*, and compute the negative log-likelihood (NLL) achieved by *GT2* with parameters inferred by each inference method. The NLL measures the consistency between the predicted duration distribution and the actual auction durations (Lakshminarayanan, Pritzel, and Blundell 2017). This metric has been widely used to evaluate the performance of prediction methods in strategic environments (Stahl and Wilson 1995; Hartford, Wright, and Leyton-Brown 2016; Wright and Leyton-Brown 2019).

We compare our **ADAPT** framework (which integrates the two aforementioned game theory models with machine learning) with the following methods in terms of auction duration prediction:

- **GT1**: It is a representative **g**ame **t**heory model that applies Expected Utility Theory to analyze bidder behavior and predict durations (as introduced in Section 4.3).
- **GT2**: It is a representative **g**ame **t**heory model established on Prospect Theory (as introduced in Section 4.3). We infer the model parameters by **InferNet**.
- **GT1+MDN**: It first applies **GT1** to predict the auction distribution, which is then fed into the Multi-Branch Mixture Density Network to further refine the prediction. The MB-MDN is used to learn the mapping from the prediction of **GT1** to the actual duration distribution.
- **GT2+MDN**, **GT1+GT2+MDN**, **GT1+EMB+MDN**, **GT2+EMB+MDN**, **EMB+MDN**: These methods are similar to **GT1+MDN**, but differ in terms of the inputs to MB-MDN, which can include the prediction of **GT1**, the prediction of **GT2**, and the product **emb**edding.

	SA-Avg	SA-Unified	**InferNet**
Synthetic Data	7.89 ± 0.23	6.73 ± 0.74	$\mathbf{6.00 \pm 0.09}$
Real Data	7.48 ± 0.16	7.50 ± 0.41	$\mathbf{7.00 \pm 0.18}$

Table 1: NLLs of Game Theory Models with Parameters Inferred by Different Methods.

In addition to **GT1** and **GT2**, prior studies have investigated another game theory model (referred to as **GT3**) (Platt, Price, and Tappen 2013). It is essentially a generalization of **GT1** and a special case of **GT2**. In **GT3**, $\pi_i^{(m)}(\cdot)$ and $h_i^{(m)}(\cdot)$ are the same as those of **GT1**, and $f_i^{(m)}(\cdot)$ is the same as that of **GT2** to capture the risk preference of bidders. Due to its similarities with **GT1** and **GT2**, we will only show the performance of **GT3**, and omit the performance of methods like **GT1+GT3+MDN** and **GT3+EMB+MDN** to reduce the number of similar methods in each figure.

Implementation Details We conduct experiments using five random seeds for both synthetic and real data. Under each seed, we randomly select 70% of data for training, 20% for validation, and 10% for testing. We apply the Adam optimizer to train neural networks. Our supplementary material provides more details about network training, our codes, and datasets (which will be made publicly available).

5.3 Experiment Results

Model Parameter Inference Table 1 presents the comparison between different inference methods on synthetic testing data and real testing data. The game theory model (**GT2**) with parameters inferred by **InferNet** achieves the best prediction performance (i.e., the lowest NLL) under out-of-sample auction configurations. This implies that **InferNet** can effectively learn the inherent connection between auction configurations (e.g., product information) and parameters characterizing bidder behavior patterns.

Auction Duration Prediction In Figure 4, we show three examples comparing the predicted distributions with the normalized histogram of auction durations in real testing data.

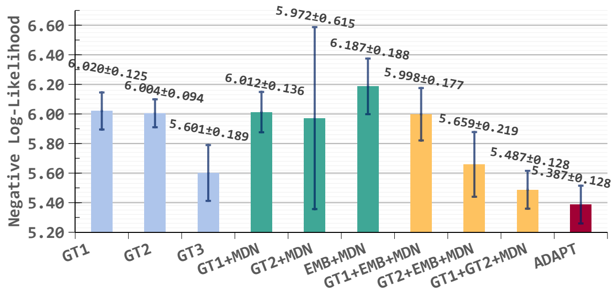

Figure 5: NLLs of Methods on Synthetic Testing Data.

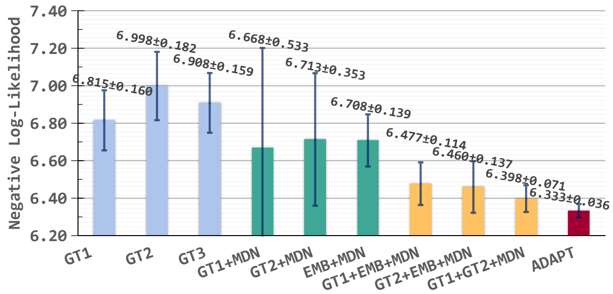

Figure 6: NLLs of Methods on Real Testing Data.

Compared with game theory models **GT1**, **GT2** and **GT3**, the predicted distribution of **ADAPT** exhibits a better fit to the histogram of actual durations, and achieves a lower NLL. This implies that the NLL serves as a reliable indicator of prediction performance.

Figures 5 and 6 show the NLLs achieved by different prediction methods on synthetic and real testing data, respectively. The error bars represent the standard deviations. We categorize the comparison methods into three groups, based on whether they use MB-MDN and the inputs to MB-MDN. We use four colors to differentiate our **ADAPT** and the three groups of comparison methods.

We can observe that our **ADAPT** achieves the lowest average NLL. For example, compared with **GT1**, **ADAPT** reduces the NLL on real data by around 0.5. As shown in Figure 4, a decrease of 0.5 corresponds to a significantly improved alignment with the actual distribution. Moreover, as we feed more information regarding game-theoretic predictions and product embeddings to the MB-MDN, it will achieve a lower NLL. For example, the methods in yellow (e.g., **GT1+GT2+MDN**) generally outperform the methods in green (e.g., **GT1+MDN** and **GT2+MDN**), where some inputs to the MB-MDN are removed. This observation validates our design of **ADAPT**, which feeds both game-theoretic predictions and product embeddings to the MB-MDN. From Figures 5 and 6, we also observe that our **ADAPT** yields a low standard deviation, indicating its robustness to data randomness.

Prediction Under Large Domain Shifts In the above experiments, we randomly split data for training, validation, and testing. Next, we deliberately split data to evaluate different methods under substantial domain shifts. First, we select the data with a bid increment of 0.01 (the lowest positive increment value) as testing data, and use the remaining data for training and validation. Second, we select the data with a bid fee of 0.01 (the lowest positive bid fee) as testing data. In these two cases, the proportions of testing data are 6.3% and 15.8%, respectively. The box plots in Figure 7 show that the distribution of auction durations in the testing data significantly diverges from that in the training and validation data. Note that each box plot in Figure 7 does not display the outliers that notably differ from other data.

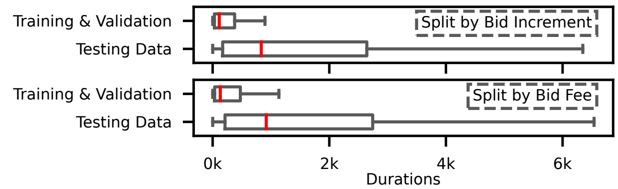

Figure 7: Duration Distributions under Large Domain Shifts.

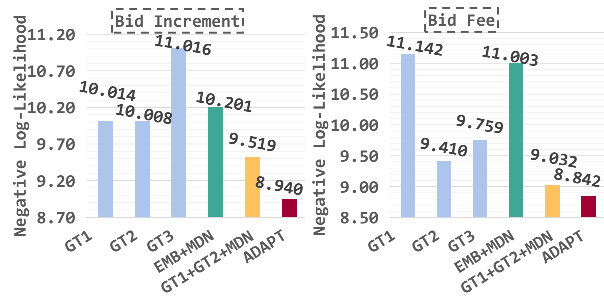

Figure 8: NLLs of Methods Under Large Domain Shifts.

Due to space limit, Figure 8 compares **ADAPT** with the following representative methods: **GT1**, **GT2**, **GT3** (game theory models), **EMB+MDN** (machine learning-based method without using game theory), and **GT1+GT2+MDN** (which is second only to **ADAPT** in previous experiments). We can see that under large domain shifts, the reductions in NLLs under our **ADAPT** over these methods become greater. Furthermore, **EMB+MDN** gives inaccurate predictions, which is consistent with our anticipated performance of machine learning-based methods in the face of large domain shifts.

6 Conclusion

In this paper, we integrated game theory and machine learning to predict penny auction durations. We proposed **Infer-Net** to infer parameters of game theory models for out-of-sample configurations, and developed MB-MDN to learn the mapping from game-theoretic predictions and product embeddings to the actual duration distributions. Our framework outperforms game theory-based and machine learning-based methods on both synthetic and real data, especially under large domain shifts. We plan to extend our prediction framework to other strategic environments in our future work.

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