Deep Learning:

HWO:-

A. $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $C = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

1). $2A - B$.

$$2A - B = 2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

2) $||A||$

$$= \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$$

3) Unit vector in direction of A:

$$U_A = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$$

4) Direction cosines of A $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $|A| = \sqrt{14}$

$$\cos\alpha = \frac{1}{\sqrt{14}}$$

$$\cos\beta = \frac{2}{\sqrt{14}}$$

$$\cos\gamma = \frac{3}{\sqrt{14}}$$

5. A·B and B·A.

$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

$A \cdot B = 1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6$

$= 4 + 10 + 18 = 32$

$B \cdot A = 4 \cdot 1 + 5 \cdot 2 + 6 \cdot 3$

$= 4 + 10 + 18 = 32$

6. Angle between A and B.

$A \cdot B = 1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6 = 4 + 10 + 18 = 32.$

$|A| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

$|B| = \sqrt{4^2 + 5^2 + 6^2} = \sqrt{16 + 25 + 36} = \sqrt{77}$

$\cos \alpha = \frac{A \cdot B}{|A| \cdot |B|} = \frac{32}{\sqrt{14 \times 77}}$

$= \frac{32}{32.83}$

$\alpha = \cos^{-1}(0.97).$

7. vector perpendicular to A

let the vector be $D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

Perpendicular $= 1d_1 + 2d_2 + 3d_3 = 0$

Let $d_1 = 1$, $d_2 = 1$, $d_3 = -1$.

$\cdot \quad 1(1) + 2(1) + 3(-1) = 0.$

$\therefore$ D is perpendicular to A.

8. $A \times B$ and $B \times A$

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} i & j & k \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = i(12-15) - j(6-12) + k(5-8)$$
$$= -3i + 6j - 3k.$$

$$B \times A = \begin{bmatrix} i & j & k \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix} = i(15-12) - j(12-6) + k(8-5)$$
$$= 3i - 6j + 3k.$$

9. vector perpendicular to A and B

let the vector be $D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

$$d_1 + 2d_2 + 3d_3 = 0.$$
$$4d_1 + 5d_2 + 6d_3 = 0.$$
$$\therefore d_1 + 2d_2 + 3d_3 = 0 = 4d_1 + 5d_2 + 6d_3$$
$$4d_1 - d_1 + 5d_2 - 2d_2 + 6d_3 - 3d_3 = 0.$$
$$3d_1 + 3d_2 + 3d_3 = 0.$$

let $D = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

10. Linear Dependency: Solving A, B, C by Row elimination the linear dependency is

$$3A - B = C.$$

11. A^TB and B^TA.

$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $A^T \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $B^T = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$

$A^TB = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = 1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6$

$= 4 + 10 + 18 = 32.$

$B^TA = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 4 \cdot 1 + 5 \cdot 2 + 6 \cdot 3$

$= 32.$

B. $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}$

1. $2A - B$ $\quad 2\begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2-1 & 4-2 & 6-1 \\ 8-2 & -4-1 & 6+4 \\ 0-3 & 10+2 & -2-1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 12 & -3 \end{bmatrix}$

2. AB

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 2+2-6 & 1-8+3 \\ 4-4+9 & 8-2-6 & 4+8+3 \\ 0+10-3 & 0+5+2 & 0-20-1 \end{bmatrix} = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

BA

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1+8+0 & 2-4+5 & 3+6-1 \\ 2+4+0 & 4-2-20 & 6+3+4 \\ 3-8+0 & 6+4+5 & 9-6-1 \end{bmatrix} = \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$$

3. $(AB)^T$

$$AB = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix} \quad (AB)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

$B^T A^T$

$$B^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1+4+9 & 4-4+9 & 0+10-3 \\ 2+2-6 & 8-2-6 & 0+5+2 \\ 1-8+3 & 4+8+3 & -20-1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

4. $|A|$.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$$

$$|A| = 1(2-15) - 2(-4-0) + 3(20+0)$$
$$= 1(-13) - 2(-4) + 3(20)$$
$$= -13 + 8 + 60 = 68 - 13 = 55.$$

$|C|$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}$$

$$|C| = 1(15-6) - 2(12+6) + 3(4+5)$$
$$= 9 - 2(18) + 3(9)$$
$$= 9 - 36 + 27 = 0.$$

6. A^{-1}

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A$$

$$\operatorname{adj} A = \begin{bmatrix} 2-15 & -(-4) & 20 \\ -(-2-15) & -1 & 5 \\ 6+6 & -(3-12) & -2-8 \end{bmatrix} = \begin{bmatrix} 13 & 4 & 20 \\ 17 & -1 & 5 \\ 12 & 9 & -10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{55} \begin{bmatrix} 13 & 4 & 20 \\ 17 & -1 & 5 \\ 12 & 9 & -10 \end{bmatrix}$$

$$C^{-1} = \frac{1}{|C|} \operatorname{adj} C$$

$$\operatorname{adj} C = \begin{bmatrix} 15-6 & -(12+6) & 4+5 \\ -(6-3) & (3+3) & 1+2 \\ 12-15 & -(6-12) & 5-8 \end{bmatrix} = \begin{bmatrix} 9 & -18 & 9 \\ -3 & 6 & 3 \\ -3 & 6 & -3 \end{bmatrix}$$

C^{-1} = not defined

$$B^{-1} = \frac{1}{|B|} \operatorname{adj} B$$

$$|B| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{vmatrix} = 1(1-8) - 2(2+12) + 1(-4-3)$$

$$-7 - 2(14) + 1(-7)$$

$$-7 - 28 - 7 = -42$$

$$\text{adj } B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-8 & -(2+12) & -4-3 \\ 2+2 & -(1-3) & -2-6 \\ & -(-4-2) & 1-4 \end{bmatrix}$$

$$-8-1$$

$$= \begin{bmatrix} -7 & -14 & -7 \\ 4 & 2 & -8 \\ -9 & 6 & -3 \end{bmatrix}$$

$$B^{-1} = \frac{1}{-42} \begin{bmatrix} -7 & -14 & -7 \\ 4 & 2 & -8 \\ -9 & 6 & -3 \end{bmatrix}$$

5. Orthogonal Set:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$$

$a_1 \cdot a_2 = 1 \cdot 4 + 2(-2) + 3(3)$

$= 4 - 4 + 9 \neq 0$.

$\therefore$ not orthogonal.

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$b_1 \cdot b_2 = 1 \cdot 2 + 2 \cdot 1 + 1 \cdot (-4) = 0$

$b_2 \cdot b_3 = 2 \cdot 3 + 1 \cdot (-2) + (-4) \cdot 1 = 0$

$b_1 \cdot b_3 = 1 \cdot 3 + 2 \cdot (-2) + 1 \cdot (1) = 0$.

Orthogonal.

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix} = c_1 \cdot c_2 = 1 \cdot (4) + 2(5) + 3(6) \neq 0$$

= Not orthogonal

C. $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ $\quad B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$

1. Eigenvalues of A:

$$A - \lambda I = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix}$$

$$= (1-\lambda)(2-\lambda) - 6$$

$$= 2 - 2\lambda - \lambda + \lambda^2 - 6 = \lambda^2 - 3\lambda - 4 = 0$$

$$(\lambda - 4)(\lambda + 1)$$

$$\lambda = 4, -1$$

-4

$-4 \quad 1$

Eigenvector: $\lambda = 4$.

$$\begin{bmatrix} 1-4 & 2 \\ 3 & 2-4 \end{bmatrix} = \underbrace{\begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix}}_{C}$$

$Cx = 0$

$$\begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-3x_1 + 2x_2 = 0$$

$$3x_1 - 2x_2 = 0$$

Vector is $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$A+I$:

$$\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}.$$

By row generation $\rightarrow \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ $R_2 \Rightarrow R_2 - R_1$.

$x_1 + x_2 = 0 \Rightarrow x_1 = -x_2$ $\therefore$ vector $= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

2). $V^{-1}AV$.

$V = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$ $V^{-1} = \frac{1}{|V|} \text{adj } V$.

$$V^{-1} = \frac{1}{-2-3} \begin{bmatrix} -1 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$$

$$= \frac{1}{-5} \begin{bmatrix} -1 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2+2 & 3+(-2) \\ 6+2 & 9-2 \end{bmatrix}$$

$$= \frac{1}{-5} \begin{bmatrix} -1 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 8 & 7 \end{bmatrix} = \frac{-1}{5} \begin{bmatrix} -4 \overset{-24}{+16} & -1 \overset{-}{+} 21 \\ -4+16 & -1+14 \end{bmatrix}$$

$$= \frac{-1}{5} \begin{bmatrix} -28 & -22 \\ 12 & 13 \end{bmatrix}$$

3. Dot matrix between Eigenvectors of A.

$$\begin{bmatrix} 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \end{bmatrix} = 2\cdot(1) + 3\cdot(-1)$$
$$2 - 3 = -1.$$

4. Dot product b/w eigenvectors of B.

$$B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$$

$$B - \lambda I = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 2-\lambda & -2 \\ -2 & 5-\lambda \end{bmatrix}$$

$$(2-\lambda)(5-\lambda) - 4 \Rightarrow 10 - 5\lambda - 2\lambda - 4 + \lambda^2$$
$$\Rightarrow \lambda^2 - 7\lambda + 6 = 0.$$
$$(\lambda - 6)(\lambda - 1) = 0.$$

6
6 1

Eigenvalues $\Rightarrow +6, +1.$

Eigenvectors $\Rightarrow \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Dot product = $2\cdot1 + 1\cdot2 = 4.$

5) Property of Eigenvector of B.

One of eigenvector when multiplied by B yields the ~~vector~~ Eigenvector.

D. $f(x) = x^2 + 3$ $g(x,y) = x^2 + y^2$

1) $f'(x) = 2x$

$f''(x) = 2$

2) Partial derivative:

$$\frac{\partial g}{\partial x} = 2x \qquad \frac{\partial g}{\partial y} = 2y$$

3) Gradient vector $\nabla g(x,y)$.

$$\begin{bmatrix} 2x \\ 2y \end{bmatrix}$$