OPTIMAL SYNCHRONIZATION OF INVERTED PENDULUM

by

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BONAFIDE CERTIFICATE

Certified that this project report entitled "**OPTIMAL SYNCHRONIZATION OF INVERTED PENDULUM"** is a bonafide work of **VISHWATH KUMAR B S -18BEC1289, YOGEESHWAR S – 18BEC1343** who carried out the Project work under my supervision and guidance for **ECE2010 – CONTROL SYSTEMS.**

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ABSTRACT

Inverted pendulum is a system in which the center of the mass is above the pivot point, where the mass can freely rotate. The inverted pendulum, a highly nonlinear unstable system, is used as a benchmark for implementing the control methods. This research mainly focusses on balancing an inverted pendulum with reaction wheel. The study objective is to control the system such that the cart reaches a desired position and the inverted pendulum stabilizes in the upright position. In this paper, the modelling and simulation for optimal control design of nonlinear inverted pendulum-cart dynamic system using PID controller and LQR have been presented. The finding in this research is that torque is generated by the acceleration of the reaction wheel. Higher acceleration gives a high torque. Others findings is the PID parameter; Proportional gain increases the response rate; Integral gain is used to eliminate steady state error; Derivative gain is used to lessen the overshoot. The simulation results justify the comparative advantage of LQR over PID control method.

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VISHWATH KUMAR B S **YOGEESHWAR S**

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INTRODUCTION

1.1 OBJECTIVES

To design various controllers for stabilizing the unstable inverted pendulum and compare the results between them.

1. Proportional Controller (P)
2. Proportional-Integral Controller (PI)
3. Proportional-Derivative Controller (PD)
4. Proportional-Integral-Derivative Controller (PID)
5. Linear Quadratic Regulator Controller (LQR)

1.2 BENEFITS

Our project proves that an unstable chaotic inverted pendulum can be stabilised by Proportional-Integral-Derivative (PID) controller and Linear Quadratic Regulator (LQR) controller.

1.3 FEATURES

- Practical parameters of unstable inverted pendulum have been taken and simulated in two different platforms (MATLAB & SIMULINK) to stabilize it with the time domain specifications of the response time graph.

CHAPTER 2

INVERTED PENDULUM CONTROLLER – DESIGN

2.1.1 FLOWCHART

The work flow of the stabilization of Inverted Pendulum using PID/LQR controller is given in **Figure 1**.

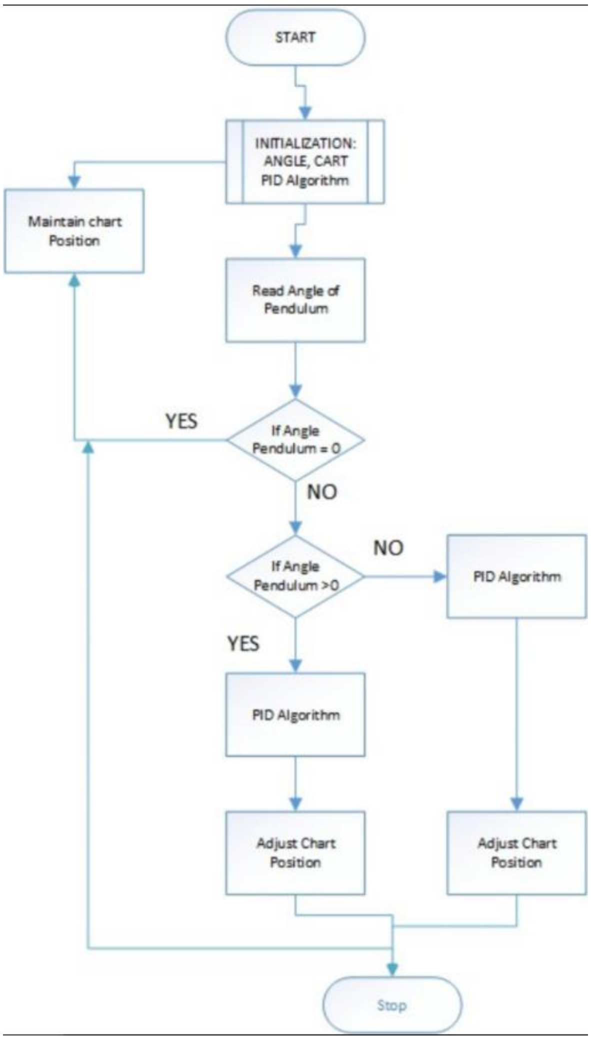

Figure 1. FLOWCHART OF STABILIZATION OF INVERTED PENDULUM

2.1.2 BLOCK DIAGRAM

The detailed working of the PID controller to stabilize Inverted Pendulum is given in **Figure 2**:

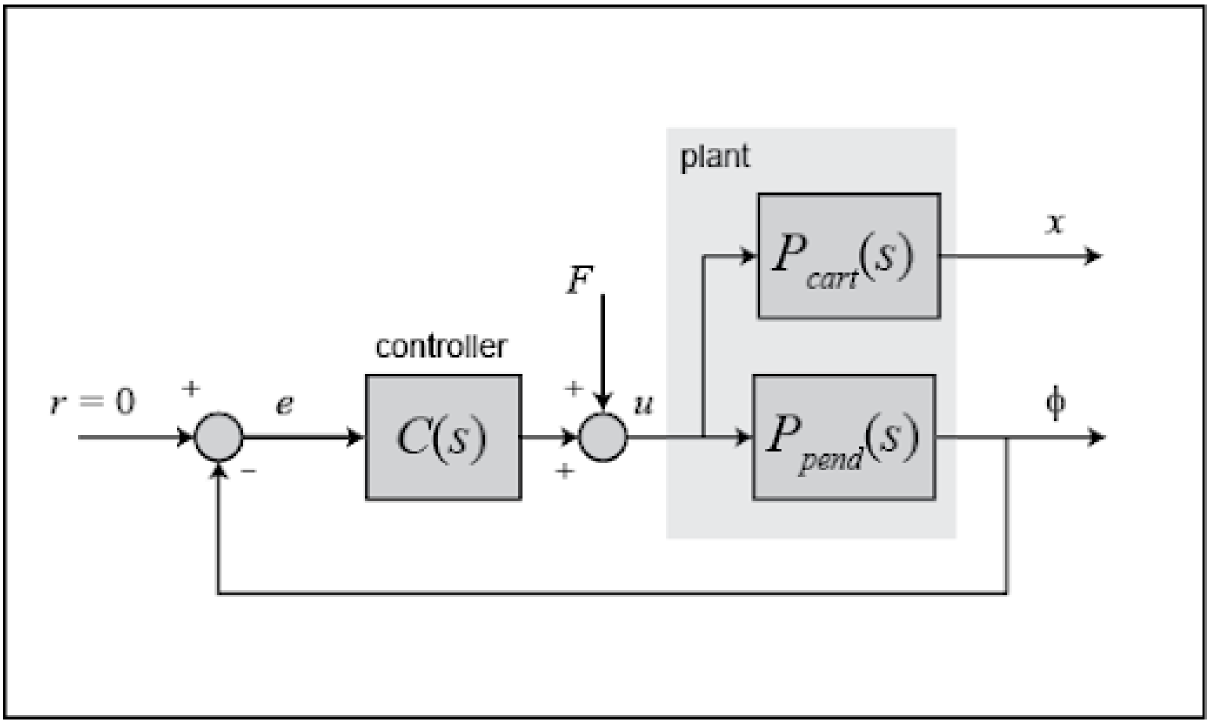

Figure 2. Block diagram for PID controller for Pendulum-Cart system.

The detailed working of the LQR controller to stabilize Inverted Pendulum is given in **Figure 3**:

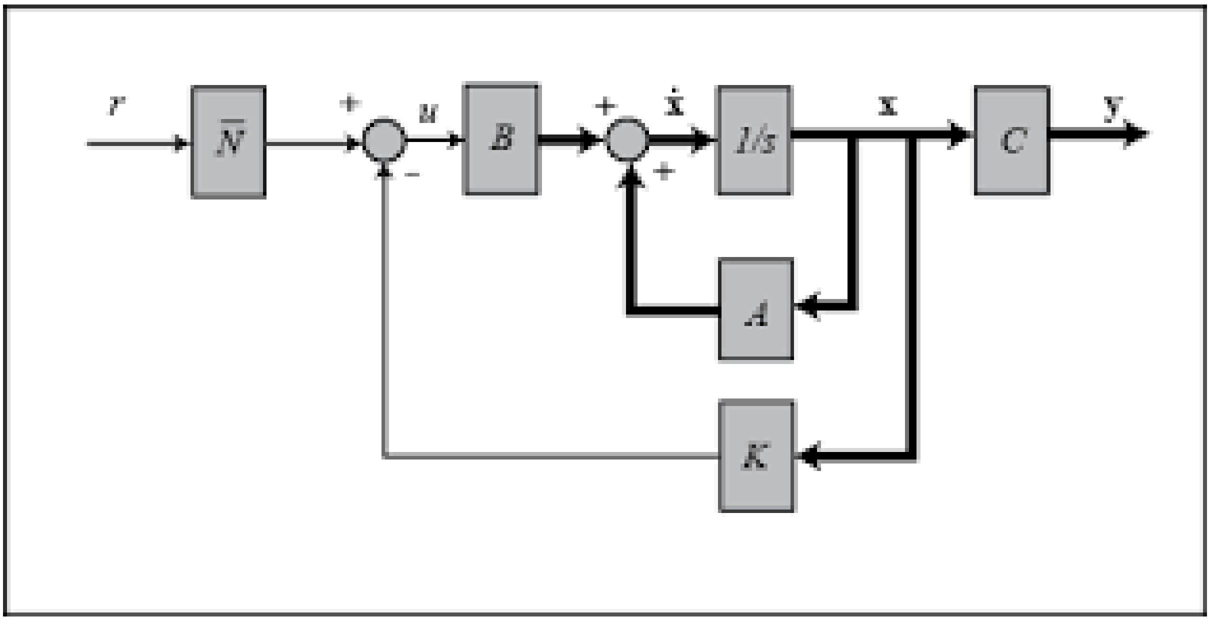

Figure 3. Block diagram for LQR controller for Pendulum-Cart system.

2.2 CONTROL METHODS:

Various control methods used are :

- Proportional Controller (P)
- Proportional-Integral Controller (PI)
- Proportional-Derivative Controller (PD)
- Proportional-Integral-Derivative Controller (PID)
- Linear Quadratic Regulator Controller (LQR)

2.2.1 PROPORTIONAL CONTROLLER (P)

Proportional control is a form of feedback control. It is the simplest form of continuous control that can be used in a closed-looped system. [2] P-only control minimizes the fluctuation in the process variable, but it does not always bring the system to the desired set point. It provides a faster response than most other controllers, initially allowing the P-only controller to respond a few seconds faster. However, as the system becomes more complex (i.e. more complex algorithm) the response time difference could accumulate, allowing the P-controller to possibly respond even a few minutes faster. Although the P-only controller does offer the advantage of faster response time, it produces deviation from the set point. This deviation is known as the offset, and it is usually not desired in a process. [3] The existence of an offset implies that the system could not be maintained at the desired set point at steady state. It is analogous to the systematic error in a calibration curve, where there is always a set, constant error that prevents the line from crossing the origin. The offset can be minimized by combining P-only control as in eq.[1] with another form of control, such as I- or D- control.

$$c(t) = K_c e(t) + b \qquad [1]$$

2.2.1 PROPORTIONAL-INTEGRAL CONTROLLER (PI)

PI-control lacks the D-control of the PID system [4]. PI control is a form of feedback control. It provides a faster response time than I-only control due to the addition of the proportional action. PI control stops the system from fluctuating, and it is also able to return the system to its set point. Although the response time for PI-control is faster than I-only control, it is still up to 50% slower than P-only control. Therefore, in order to increase response time, PI control as in eq.[2] is often combined with D-only control.

$$c(t) = K_c \left(e(t) + \frac{1}{Ti} \int e(t)dt \right) + C \qquad [2]$$

2.2.2 PROPORTIONAL-DERIVATIVE CONTROLLER (PD)

PD-control, which lacks the I-control of the PID system. PD-control is combination of feedforward and feedback control, because it operates on both the current process conditions and predicted process conditions [5]. In PD-control, the control output is a linear combination of the error signal and its derivative. PD-control as in eq.[3] contains the proportional control's damping of the fluctuation and the derivative control's prediction of process error.

$$c(t) = K_c \left(e(t) + T_d \frac{de}{dt} \right) + C \qquad [3]$$

2.2.3 PROPORTIONAL-INTEGRAL-DERIVATIVE CONTROLLER (PID)

Proportional-integral-derivative control is a combination of all three types of control methods. PID-control is most commonly used because it combines the advantages of each type of control. This includes a quicker response time because of the P-only control, along with the decreased/zero offset from the combined derivative and integral controllers [6]. This offset was removed by additionally using the I-control. The addition of D-control greatly increases the controller's response when used in combination because it predicts disturbances to the system by measuring the change in error.

On the contrary, as mentioned previously, when used individually, it has a slower response time compared to the quicker P-only control. However, although the PID controller seems to be the most adequate controller, it is also the most expensive controller. Therefore, it is not used unless the process requires the accuracy and stability provided by the PID controller as in eq[4].

$$c(t) = K_c \left(e(t) + \frac{1}{T_i} \int e(t)dt + T_d \frac{de}{dt} \right) + C \qquad [4]$$

2.2.4 LINEAR QUADRATIC REGULATOR CONTROLLER (LQR)

Linear quadratic regulator (LQR) [7] is one of the optimal control techniques, which takes into account the states of the dynamical system and control input to make the optimal control decisions. This is simple as well as robust.

After linearization of nonlinear system equations about the upright (unstable) equilibrium position having initial conditions as $X_0 = [0, 0, 0, 0]^T$, the linear state-space equation is obtained as in eq[5].

$$\dot{X} = AX + Bu \qquad [5]$$

where, $X = [\theta, \dot{\theta}, x, \dot{x}]^T$.

The state feedback control $u = -KX$ leads to

$$\dot{X} = (A - BK)X \qquad [6]$$

where, K is derived from minimization of the cost function given by eq.[7]

$$J = \int (X^T QX + u^T Ru)dt \qquad [7]$$

where, Q and R are positive semi-definite and positive definite symmetric constant matrices respectively. The LQR gain vector K is given by eq[8]

$$K = R^{-1}B^T P \qquad [8]$$

where, P is a positive definite symmetric constant matrix obtained from the solution of matrix algebraic riccatti equation eq[9] (ARE)

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \qquad [9]$$

In the optimal control of nonlinear inverted pendulum dynamical system using PID controller & LQR approach, all the instantaneous states of the nonlinear system, pendulum angle θ, angular velocity $\dot{\theta}$, cart position x, and cart velocity $\dot{x}$ have been considered available for measurement which are directly fed to the LQR. The LQR is designed using the linear state-space model of the system[8].

The design procedure for finding the feedback gain K for LQR can be formulated to 3 simple steps:

1. Select the design parameter matrices Q and R.
2. Find P by solving the ARE.
3. Find the state feedback matrix K using :[9].

The LQR guarantees pole placement and stability to the closed loop system as long as two LQR theorems [10] hold:

LQR theorem 1:

Let the system (A, B) be reachable. Let R be positive definite and Q be positive definite. Then the closed loop system (A-BK) is asymptotically stable.

LQR theorem 2:

Let the system (A, B) be stabilizable. Let R be positive definite, Q be positive semi definite, and be observable. Then the closed loop system (A-BK) is asymptotically stable. The Simulink model for state feedback controller.

The SVFB gain K is found using lqr command in Matlab and this gain is given in the Simulink model to obtain the output.

2.3 SOFTWARE SPECIFICATIONS

2.3.1 MATLAB: VERSION: R2020a

MATLAB is a proprietary multi-paradigm programming language and numerical computing environment developed by MathWorks. MATLAB allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages.
TOOLBOX: Control systems toolbox

2.3.2 SIMULINK: VERSION: R2020a - Simulink 10.1

Simulink is a MATLAB-based graphical programming environment for modelling, simulating and analysing multidomain dynamical systems. Its primary interface is a graphical block diagramming tool and a customizable set of block libraries. It offers tight integration with the rest of the MATLAB environment and can either drive MATLAB or be scripted from it. Simulink is widely used in automatic control and digital signal processing for multidomain simulation and model-based design.
COMPILERS: MinGW-w64 C/C++ Compiler

CHAPTER 3
SYSTEM IMPLEMENTATION AND ANALYSIS

This section describes system implementation and results with inferences.

3.1 MATHEMATICAL MODELLING:

A. Inverted Pendulum System Equations

The free body diagram of an inverted pendulum [11] mounted on a motor driven cart is shown in Figure. 4.

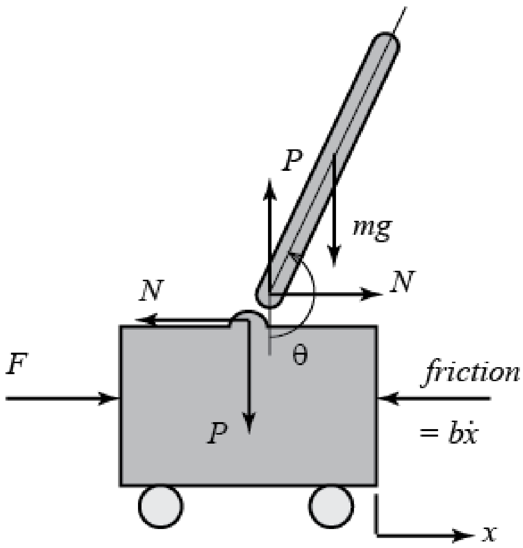

Figure 4. Free-body diagrams of the inverted pendulum system.

Summing the forces in the free-body diagram of the cart in the horizontal direction, we get eq,[10]:

$$M\ddot{x} + b\dot{x} + N = F \qquad [10]$$

Summing the forces in the vertical direction for the cart would produce no useful information. Summing the forces in the free-body diagram of the pendulum in the horizontal direction, we get the following expression eq.[11] for the reaction force N.

$$N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \qquad [11]$$

Substituting this equation into the first equation, we get one of the two governing equations eq.[12] for this system.

$$(M+m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \qquad [12]$$

For the second equation of motion for this system, sum the forces perpendicular to the pendulum. Solving the system along this axis greatly simplifies the mathematics as in eq.[13].

$$P\sin\theta + N\cos\theta - mg\sin\theta = ml\ddot{\theta} + m\ddot{x}\cos\theta \qquad [13]$$

To get rid of the P and N terms in the equation above, sum the moments about the centroid of the pendulum as in eq[14].

$$-Pl\sin\theta - Nl\cos\theta = I\ddot{\theta} \qquad [14]$$

Combining these last two expressions, we get the second governing equation eq.[15].

$$(I+ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta \qquad [15]$$

Since the analysis and control design techniques we will be employing in this example apply only to linear systems, this set of equations needs to be linearized. Linearizing the equations about the vertically upward equilibrium position, $\theta = \pi$, and will assume that the system stays within a small neighbourhood of this equilibrium. This assumption should be reasonably valid since under control we desire that the pendulum not deviate more than 20 degrees from the vertically upward position. Let ϕ represent the deviation of the pendulum's position from equilibrium, that is, $\theta = \pi + \phi$. Again, presuming a small deviation (ϕ) from equilibrium, we can use the following small angle approximations of the nonlinear functions in our system equations eq.[16],[17],[18]:

$$\cos\theta = \cos(\pi+\phi) \approx -1 \qquad [16]$$

$$\sin\theta = \sin(\pi+\phi) \approx -\phi \qquad [17]$$

$$\dot{\theta}^2 = \dot{\phi}^2 \approx 0 \qquad [18]$$

After substituting the above approximations into our nonlinear governing equations, we arrive at the two linearized equations of motion. Note u has been substituted for the input F.

$$(I+ml^2)\ddot{\phi} - mgl\phi = ml\ddot{x} \qquad [19]$$

$$(M+m)\ddot{x}+b\dot{x}-ml\ddot{\phi}=u \qquad [20]$$

B. Transfer Function of Inverted Pendulum

To obtain the transfer functions of the linearized system equations [12], we must first take the Laplace transform of the system equations assuming zero initial conditions.

$$(I+ml^2)\Phi(s)s^2-mgl\Phi(s)=mlX(s)s^2 \qquad [21]$$

$$(M+m)X(s)s^2+bX(s)s-ml\Phi(s)s^2=U(s) \qquad [22]$$

Transfer function represents the relationship between a single input and a single output at a time. To find the first transfer function for the output $\Phi(s)$ and an input of U(s) we need to eliminate X(s) from the above equations. Solve the first equation for X(s) eq.[23].

$$X(s)=\left[\frac{I+ml^2}{ml}-\frac{g}{s^2}\right]\Phi(s) \qquad [23]$$

Substituting the above into the second equation.

$$(M+m)\left[\frac{I+ml^2}{ml}-\frac{g}{s^2}\right]\Phi(s)s^2+b\left[\frac{I+ml^2}{ml}-\frac{g}{s^2}\right]\Phi(s)s-ml\Phi(s)s^2=U(s) \qquad [24]$$

Rearranging, the transfer function

$$\frac{\Phi(s)}{U(s)}=\frac{\frac{ml}{q}s^2}{s^4+\frac{b(I+ml^2)}{q}s^3-\frac{(M+m)mgl}{q}s^2-\frac{bmgl}{q}s} \qquad [25]$$

where,

$$q=[(M+m)(I+ml^2)-(ml)^2] \qquad [26]$$

From the transfer function above, it can be seen that there is both a pole and a zero at the origin. These can be cancelled and the transfer function becomes the following eq.[27].

$$P_{pend}(s)=\frac{\Phi(s)}{U(s)}=\frac{\frac{ml}{q}s}{s^3+\frac{b(I+ml^2)}{q}s^2-\frac{(M+m)mgl}{q}s-\frac{bmgl}{q}} \qquad [\frac{rad}{N}] \qquad [27]$$

Second, the transfer function with the cart position X(s) as the output can be derived in a similar manner to arrive at the following eq.[27].

$$P_{cart}(s) = \frac{X(s)}{U(s)} = \frac{\frac{(I+ml^2)s^2 - gml}{q}}{s^4 + \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s} \qquad [\frac{m}{N}] \quad [28]$$

C. State Space Modelling of Inverted Pendulum

The linearized equations of motion from above can also be represented in state-space form [13] if they are rearranged into a series of first order differential equations. Since the equations are linear, they can then be put into the standard matrix form as shown in eq.[29]

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{I+ml^2}{I(M+m)+Mml^2} \\ 0 \\ \frac{ml}{I(M+m)+Mml^2} \end{bmatrix} u \quad [29]$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u \quad [30]$$

The C matrix has 2 rows because both the cart's position and the pendulum's position are part of the output. Specifically, the cart's position is the first element of the output Y and the pendulum's deviation from its equilibrium position is the second element of Y.

3.1 SIMULATION RESULTS:

The MATLAB models for the simulation of modelling, analysis, and control of nonlinear inverted pendulum-cart dynamical system have been developed. The typical parameters of inverted pendulum-cart system setup are selected as [16,20]: mass of the cart (M): 5 kg, mass of the pendulum (m): 1 kg, length of the pendulum (l): 2 m, length of the cart track (L): ± 1 m, friction coefficient of the cart & pole rotation is assumed negligible. After linearization the system state space matrices are as below;

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1.96 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 5.88 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0.2 \\ 0 \\ 0.1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad D = [0]$$

A. *PID Tuning and Parameters:*

K_i and K_d values to zero. Increase the K_p until the output of the loop oscillates, then the K_p should be set to approximately half of that value for a "quarter amplitude decay" type response. Then increase K_i until any offset is corrected in sufficient time for the process. However, too much K_i will cause instability. Finally, increase K_d, if required, until the loop is acceptably quick to reach its reference after a load disturbance. However, too much K_d will cause excessive response and overshoot. A fast PID loop tuning usually overshoots slightly to reach the setpoint more quickly; however, some systems cannot accept overshoot, in which case an overdamped closed-loop system is required, which will require a K_p setting significantly less than half that of the K_p setting that was causing oscillation. The effect of changing various parameters are shown in Table 1.

Parameter	Rise time	Overshoot	Settling time	Steady-state error	Stability
K_p	Decrease	Increase	Small change	Decrease	Degrade
K_i	Decrease	Increase	Increase	Eliminate	Degrade
K_d	Minor change	Decrease	Decrease	No effect in theory	Improve if K_d small

Table 1. Effects of increasing a PID parameter independently.

Tuned Parameters:

1. Proportional Controller - $K_p = 45$
2. Proportional Integral Controller - $K_p = 100$, $K_i = 5$
3. Proportional Derivative Controller - $K_p = 100$, $K_d = 2$
4. Proportional Integral Derivative Controller – $K_p = 350$, $K_i = 300$, $K_d = 50$

B. LQR Tuning and Parameters:

After linearization the system matrices used to design LQR are computed as below;

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 500 & 0 \\ 0 & 0 & 0 & 250 \end{bmatrix} \qquad R = 1$$

we obtain LQR gain vector as following:

$$K = \begin{bmatrix} -1.000 & -4.551 & 155.380 & 66.735 \end{bmatrix}$$

C. MATLAB Simulation:

i. Proportional Controller:

Transfer function of the proportional controller of the pend-cart system:

```
TP =

                 1141 s
  -------------------------------------
  127.7 s^3 + 2.551 s^2 + 2633 s + 24.85

Continuous-time transfer function.
```

The time domain specification graph of the P controller system is shown in Figure 5.

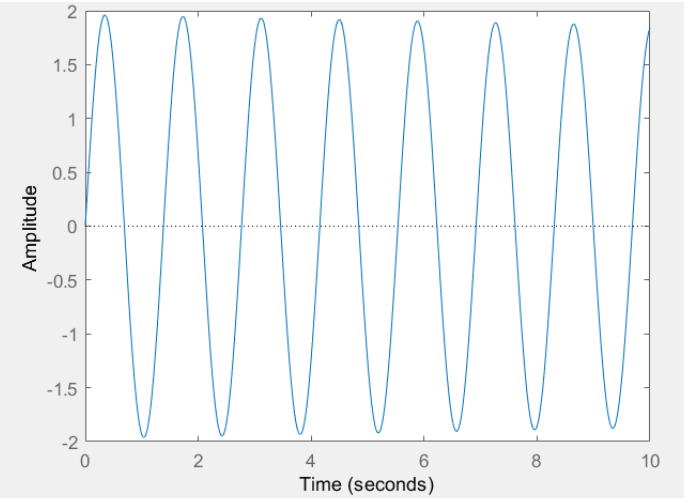

Figure 5. Response of Pendulum position to an impulse disturbance under P controller.

From Figure 5, we can see the response time graph is approximately marginally stable. At infinity the response tends to 0 but in practical scenario, the controller should stabilize the system within 10sec inorder to make the system practically feasible.

ii. *Proportional-Integral Controller:*

Transfer function of the PI controller of the pend-cart system:

```
TPI =
 
              2536 s^2 + 126.8 s
  ------------------------------------------
  127.7 s^4 + 2.551 s^3 + 4027 s^2 + 151.7 s
 
Continuous-time transfer function.
```

The time domain specification graph of the PI controller system is shown in Figure 6.

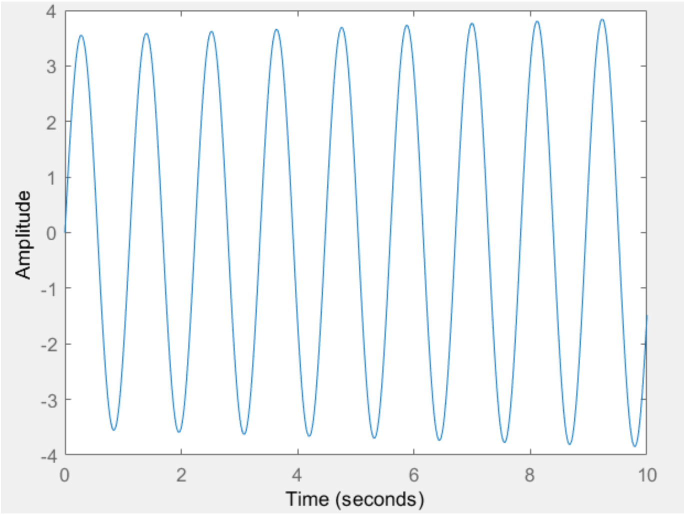

Figure 6. Response of pendulum position to an impulse disturbance under PI controller.

From Figure 6, we can see the response time graph is marginally stable. At infinity the response does not tend to 0 therefore system practically not feasible.

iii. Proportional-Derivative Controller:

Transfer function of the PD controller of the pend-cart system:

```
TPD =

            50.72 s^2 + 2536 s
  --------------------------------------
  127.7 s^3 + 53.27 s^2 + 4027 s + 24.85

Continuous-time transfer function.
```

The time domain specification graph of the PD controller system is shown in Figure 7.

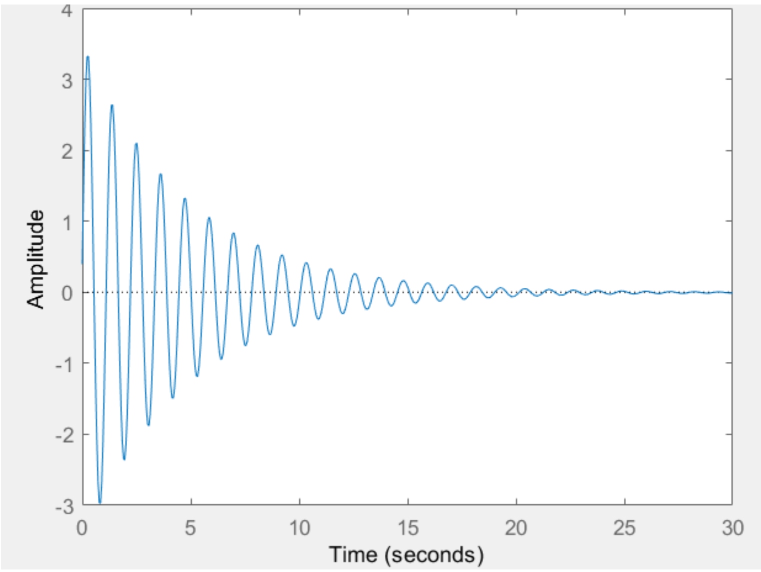

Figure 7. Response of pendulum position to an impulse disturbance under PD controller.

From Figure 7, we can see the response time graph is stable as at around 30sec the response tends to 0 but in practical scenario, the controller should stabilize the system within 10sec inorder to make the system practically feasible.

iv. Proportional-Integral-Derivative Controller:

Transfer function of the PID controller of the pend-cart system:

```
T =

          1268 s^3 + 8876 s^2 + 7608 s
  --------------------------------------------
  127.7 s^4 + 1271 s^3 + 1.037e04 s^2 + 7633 s

Continuous-time transfer function.
```

The time domain specification graph of the PID controller system is shown in Figure 8.

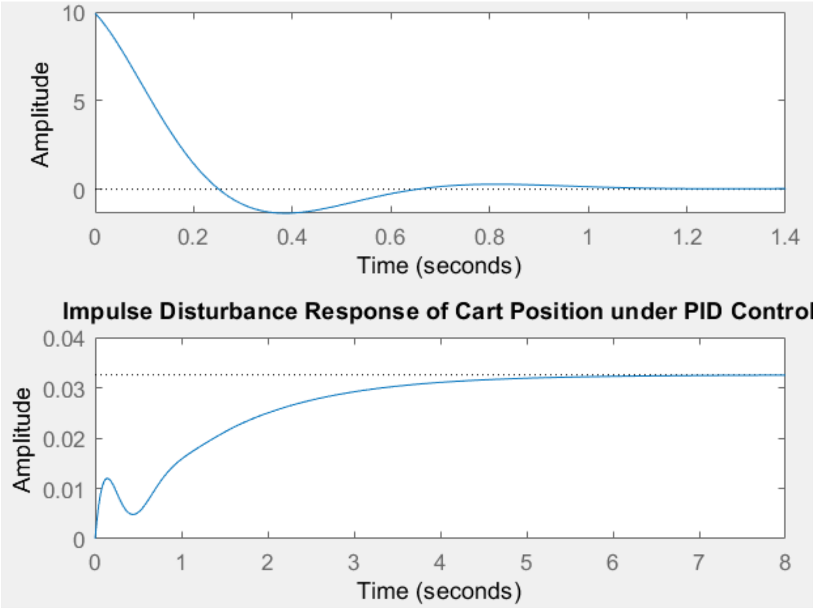

Figure 8. Response of pendulum position and cart position to an impulse disturbance under PID controller.

From Figure 8, we can see the response time graph is stable as at around 5sec the response tends to 0 therefore the controller stabilizes the system to make the system practically feasible.

v. *Linear-Quadratic-Regulator Controller:*

The time domain specification graph of the LQR controller system is shown in Figure 9.

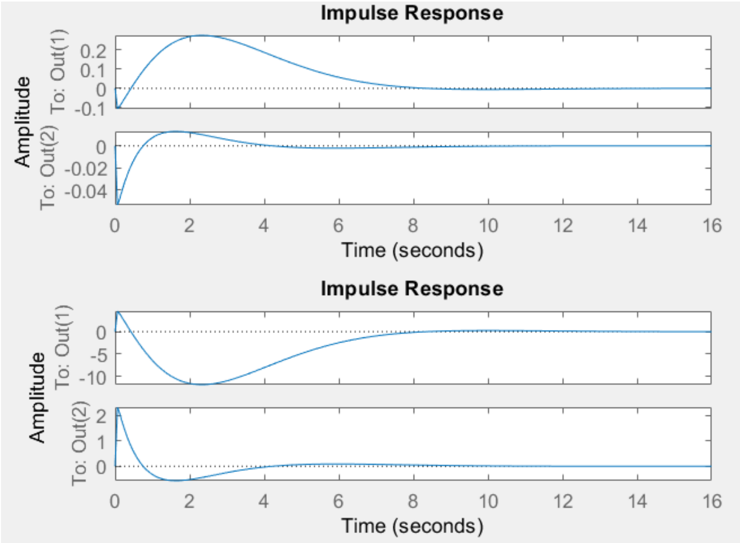

Figure 9. Response of pendulum position and cart position to an impulse disturbance of cart and pendulum respectively under LQR controller.

The time domain specification graph of the LQR controller for a manual disturbance system is shown in Figure 9.

Reference Position of the Cart: y = [0; 0; pi; 0]
Disturbance/ Initial Condition: y0 = [2; 0; pi-0.8; 0]

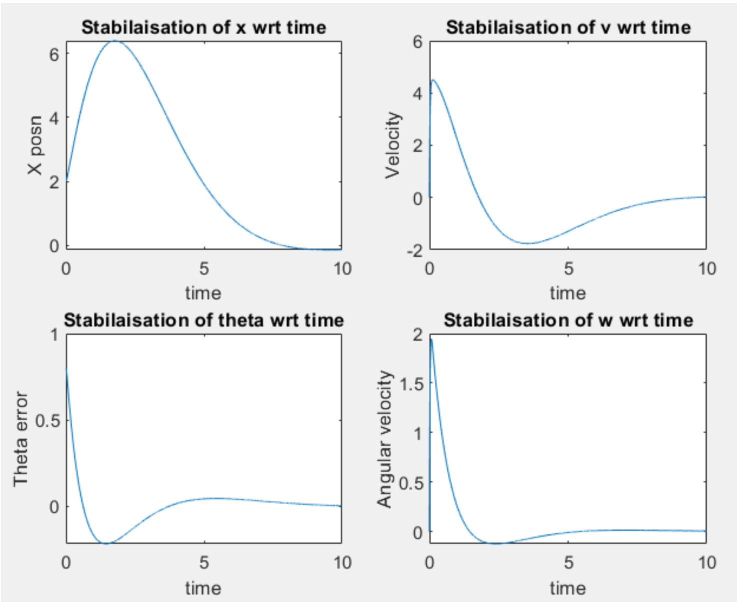

Figure 10. Response of various state variables of the Pend-Cart system to a manual disturbance of cart and pendulum respectively under LQR controller.

From Figure 9, Figure 10, we can see the response time graph is stable as at around 4sec the response tends to 0 therefore the controller stabilizes the system to make the system practically feasible. We could also see that LQR is the best controller as the response time is very low compared to other controllers.

vi. Observations on Free Oscillations:

Without damping, the response of various state space variables of the inverted pendulum is shown in Figure 11.

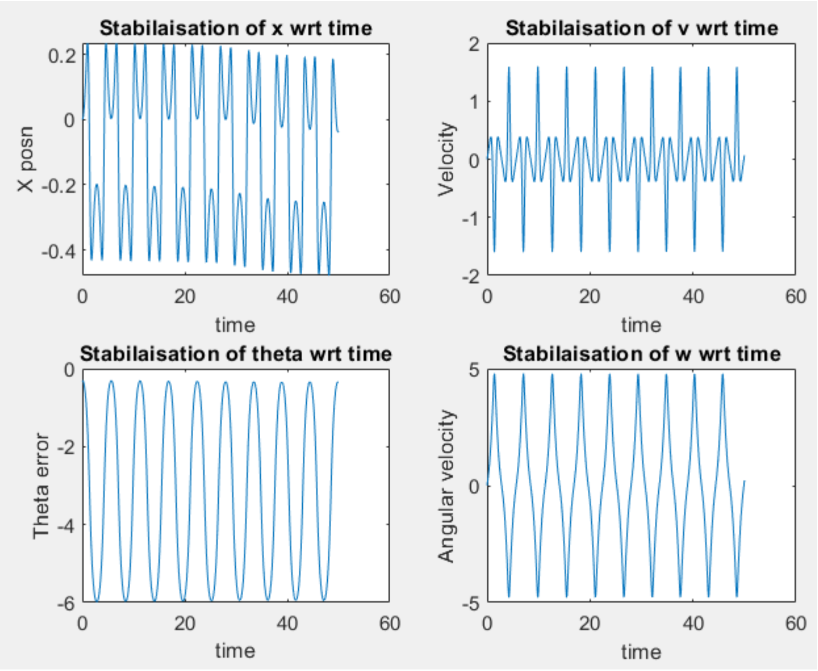

Figure 11. Response of various state variables of the Pend-Cart system under free oscillation without damping.

With the damping constant of b = 0.2, the response of various state space variables of the inverted pendulum is shown in Figure 12.

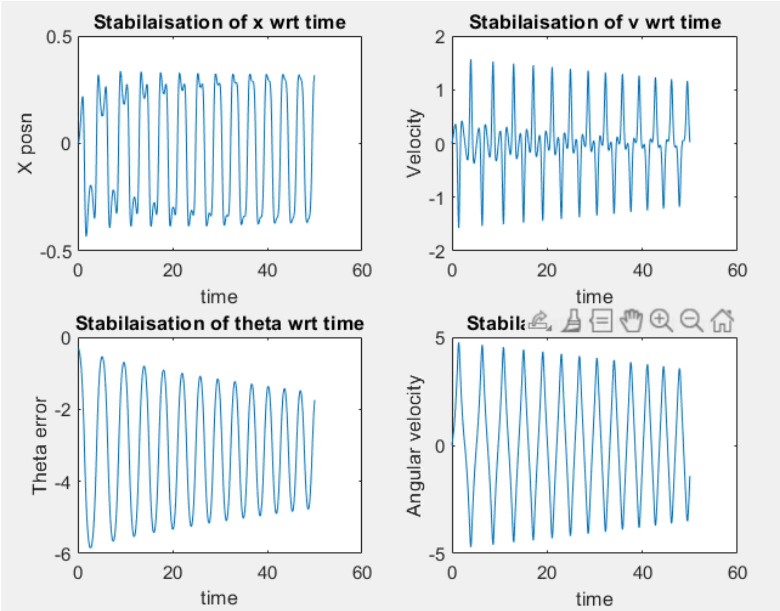

Figure 12. Response of various state variables of the Pend-Cart system under free oscillation with damping.

3.2 SIMULINK IMPLEMENTATION

Simulink implementation of the Pend-Cart system for LQR controller is shown in Figure 13.

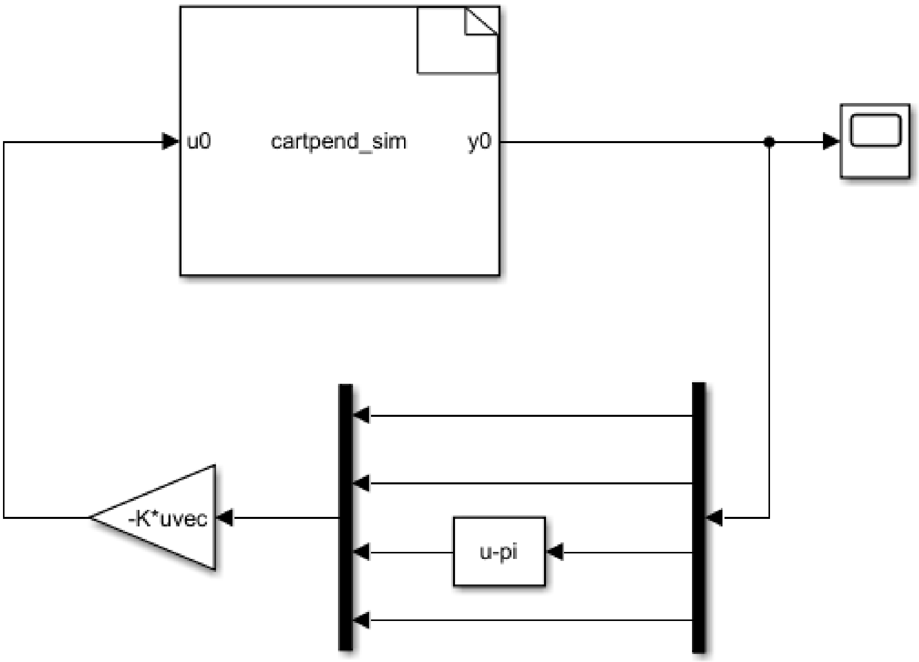

Figure13. Simulink block diagram for the Inverted pendulum system for LQR controller.

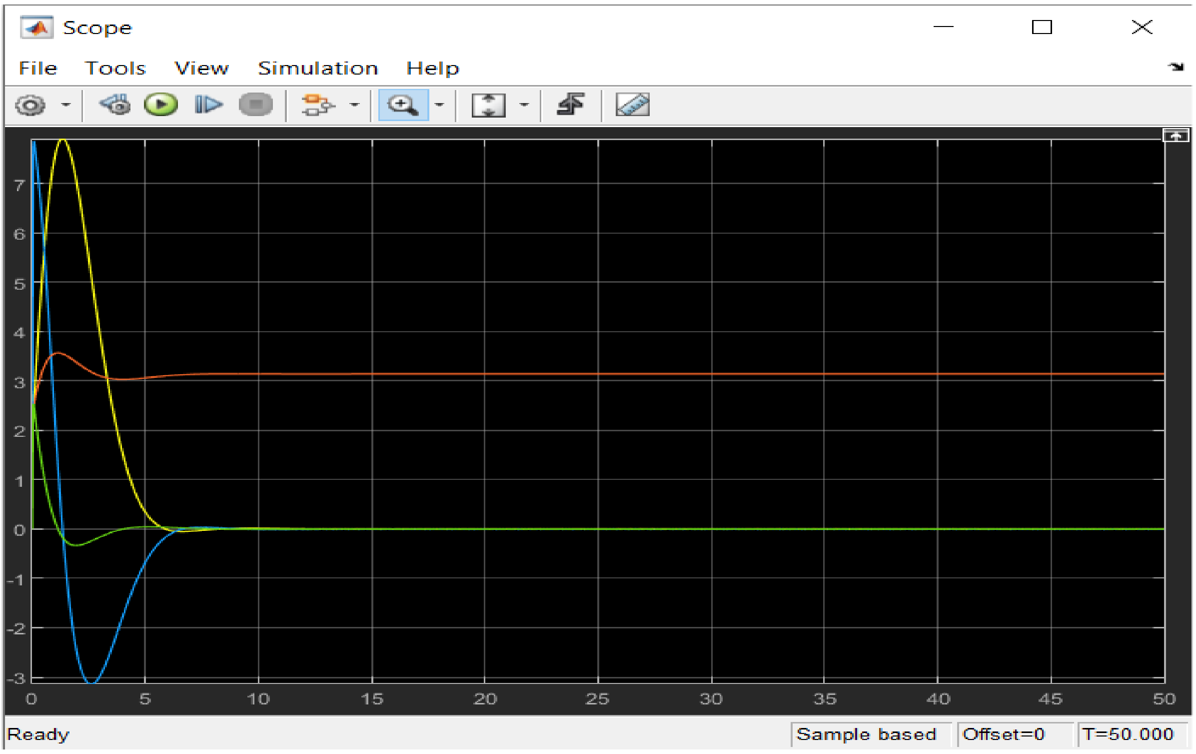

Figure 14. Response of pendulum position and cart position to an impulse disturbance of pendulum respectively under LQR controller in Simulink.

Yellow – Position of the cart
Blue – Velocity of the cart
Red – Angle of the pendulum
Green – Angular Velocity of bob

As we can see from the Figure 9 and Figure 14, the response time graph of Response of pendulum position and cart position to an impulse disturbance of pendulum respectively under LQR controller exactly matches with the output of MATLAB implementation and SIMULINK implementation.

CODE ANALYSIS

- **Number of Lines: 246**

CHAPTER 4

CONCLUSION AND FUTURE WORK

4.1 CONCLUSION

PID control, and LQR, an optimal control technique to make the optimal control decisions, have been implemented to control the nonlinear inverted pendulum-cart system. To compare the results PID control has been implemented. In the optimal control of nonlinear inverted pendulum dynamical system using PID controller & LQR approach, all the instantaneous states of the nonlinear system, are considered available for measurement, which are directly fed to the LQR. The LQR is designed using the linear state-space model of the system. The MATLAB-SIMULINK models have been developed for simulation of the control schemes. The simulation results justify the comparative advantages of optimal control using LQR method. The pendulum stabilizes in upright position justify that the control schemes are effective & robust. The performance of LQR control scheme is better than PID control scheme.

4.2 FUTURE WORK

This project will be further implemented as hardware project and few other control schemes like fuzzy, regenerative control systems will be simulated and comparative results will be produced.

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APPENDIX

SOURCE CODE:

```
clc
clear all
close all

function dx = pendcart(x,m,M,L,g,d,u)
Sx = sin(x(3));
Cx = cos(x(3));
D = m*L*L*(M+m*(1-Cx^2));
dx(1,1) = x(2);
dx(2,1) = (1/D)*(-m^2*L^2*g*Cx*Sx + m*L^2*(m*L*x(4)^2*Sx-
d*x(2))) + m*L*L*(1/D)*u;
dx(3,1) = x(4);
dx(4,1) = (1/D)*((m+M)*m*g*L*Sx - m*L*Cx*(m*L*x(4)^2*Sx -
d*x(2))) - m*L*Cx*(1/D)*u;
end

function drawcartpend(y,m,M,L)
x = y(1);
th = y(3);
% dimensions
W = 1*sqrt(M/5);
H = .5*sqrt(M/5);
wr = .2;
mr = .3*sqrt(m);

% positions
y = wr/2+H/2.2;
w1x = x-.9*W/2;
w1y = 0;
w2x = x+.9*W/2-wr;
w2y = 0;

px = x + L*sin(th);
py = y - L*cos(th);

plot([-10 10],[0 0],'k','LineWidth',2)
hold on
rectangle('Position',[x-W/2,y-
H/2,W,H],'Curvature',.2,'FaceColor',[0.4 0.5 0.9])
rectangle('Position',[w1x,w1y,wr,wr],'Curvature',1,'FaceC
olor',[0 0 0])
```

```
rectangle('Position',[w2x,w2y,wr,wr],'Curvature',1,'FaceC
olor',[0 0 0])

plot([x px],[y py],'k','LineWidth',1.5)

rectangle('Position',[(px-mr/2),(py-
mr/2),mr,mr],'Curvature',1,'FaceColor',[.1 1 0.4])

xlim([-5 5]);
ylim([-2 2.5]);
drawnow
hold off


%system variables
m = 1;
M = 5;
L = 2;
b = 0.1;
I = 0.006;
g = -9.8;
l=1;
d = 0;
s=tf('s');
q = (M+m)*(I+m*l^2)-(m*l)^2;
P_pend = (m*l*s/q)/(s^3 + (b*(I + m*l^2))*s^2/q - ((M +
m)*m*g*l)*s/q - b*m*g*l/q);
P_cart = (((I+m*l^2)/q)*s^2 - (m*g*l/q))/(s^4 + (b*(I +
m*l^2))*s^3/q - ((M + m)*m*g*l)*s^2/q - b*m*g*l*s/q);

c=menu('Inverted pendulum','Free oscillation','Controlled
oscillation');
if(c==1)
    c1=menu('Type of oscillation','Frictionless
oscillation','Damped oscillation');
    if(c1==1)
        d=0;
    else
        d=1;
    end
    tspan = 0:.001:50;
    y0 = [0; 0; pi+0.3; 0];
    [t,y] =
ode45(@(t,y)pendcart(y,m,M,L,g,d,0),tspan,y0);
     %Animating pendcart
     for k=1:100:length(t)
         drawcartpend(y(k,:),m,M,L);
```

```
    end
figure(2);
subplot(2,2,1);
plot(t,y(:,1))
ylabel('X posn');
xlabel('time');
title('Stabilaisation of x wrt time');

subplot(2,2,2);
plot(t,y(:,2))
ylabel('Velocity');
xlabel('time');
title('Stabilaisation of v wrt time');

subplot(2,2,3);
plot(t,pi-y(:,3))
ylabel('Theta error');
xlabel('time');
title('Stabilaisation of theta wrt time');

subplot(2,2,4);
plot(t,y(:,4))
ylabel('Angular velocity');
xlabel('time');
title('Stabilaisation of w wrt time');
else
    ch=menu('Control System:
','P','PI','PD','PID','LQR');

if(ch==1)
    kp=45;
    K=pid(kp);
    %display(P_pend);
    %display(K);
    TP=feedback(K*P_pend,1)
    %display(TP);
    t=0:0.01:10;
    figure(1);
    impulse(TP,t)
    title({'Response of Pendulum Position to an Impulse
Disturbance';'under P Control'});

elseif(ch==2)
    kp=100;
    ki=5;
    K=pid(kp,ki);
    %display(P_pend)
```

```
    %display(K)
    TPI=feedback(K*P_pend,1)
    t=0:0.01:10;
    figure(1);
    impulse(TPI,t)
    title({'Response of Pendulum Position to an Impulse
Disturbance';'under PI Control'});

elseif(ch==3)
    kp=100;
    kd=2;
    K=pid(kp,0,kd);
    TPD=feedback(K*P_pend,1)
    figure(1);
    impulse(TPD)
    title({'Response of Pendulum Position to an Impulse
Disturbance';'under PD Control'});

elseif(ch==4)
    Kp = 350;
    Ki = 300;
    Kd = 50;
    C = pid(Kp,Ki,Kd);
    T = feedback(C*P_pend,1);
    figure(1);
    subplot(211);
    impulse(T)
    title({'Response of Pendulum Position to an Impulse
Disturbance';'under PID Control'});
    P_cart = (((I+m*l^2)/q)*s^2 - (m*g*l/q))/(s^4 + (b*(I
+ m*l^2))*s^3/q - ((M + m)*m*g*l)*s^2/q - b*m*g*l*s/q);
    T2 = feedback(1,P_pend*C)*P_cart;
    t = 0:0.01:6;
    subplot(212)
    impulse(T2);
    title('Impulse Disturbance Response of Cart Position
under PID Control');

elseif(ch==5)
A = [0 1 0 0;
    0 -d/M -m*g/M 0;
    0 0 0 1;
    0 -d/(M*L) -(m+M)*g/(M*L) 0];

B = [0; 1/M; 0; 1/(M*L)];
% eig(A)
```

```
%cost matrix
Q = [1 0 0 0;
    0 1 0 0;
    0 0 500 0;
    0 0 0 250];
R = 0.001;

%checking for stability
rank(ctrb(A,B))

%negative feedback
K = lqr(A,B,Q,R);

% t_f=K*inv((s*eye(4)-A))*B;

%predicting the path of pendcart
tspan = 0:.001:10;
y0 = [2; 0; pi-0.8; 0];
[t,y] = ode45(@(t,y)((A-B*K)*(y-[0; 0; pi;
0])),tspan,y0);

% u=-K*y0;
% [T,J]=ode45(@(T,J)(J*y0.'*Q*y0+u.'*R*u),tspan,0);

%Animating pendcart
for k=1:100:length(t)
    drawcartpend(y(k,:),m,M,L);
end
%Analysing output stability for given disturbance
figure(2);
subplot(2,2,1);
plot(t,y(:,1))
ylabel('X posn');
xlabel('time');
title('Stabilaisation of x wrt time');

subplot(2,2,2);
plot(t,y(:,2))
ylabel('Velocity');
xlabel('time');
title('Stabilaisation of v wrt time');

subplot(2,2,3);
plot(t,pi-y(:,3))
ylabel('Theta error');
xlabel('time');
title('Stabilaisation of theta wrt time');
```

```
subplot(2,2,4);
plot(t,y(:,4))
ylabel('Angular velocity');
xlabel('time');
title('Stabilaisation of w wrt time');

% figure(3);
% step(t_f,t)
%
% figure(4);
% plot(T,J,'g-')
%
% figure(5);
% margin(t_f)

%Analysing output stability for impulse disturbance
figure(3);
C=[1 0 0 0;
   0 0 1 0];
D=0;
sys=ss(A,B,C,D);
subplot(311);
impulse(sys)
n=length(K);
A1=A - B * K;
for i=1:n
    B1(:,i)=B * K(i);
end
C1=C;
D1=D;
sys=ss(A1,B1,C1,D1);
subplot(312);
impulse(sys(:,1))
subplot(313);
impulse(sys(:,3))
end
end
```