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Q1A. Start with an empty stack of integers. You will attempt to do a sequence of pushes and pops so that the sequence of pops will be a specified permutation of 1, 2, 3 ,4 , 5, 6. You will be able to do exactly 6 push operations and 6 pop operations. The first push pushes 1 onto the stack; the next pushes 2; and so forth. The sixth push pushes 6 onto the stack. For this exercise, we will let S denote a push operation and X a pop operation. Example: The sequence SSSSSSXXXXXX outputs 654321.

A. Describe a sequence of pushes and pops that would produce output 325641 (or explain why it is not possible)

Stack Simulation Rules:

- Elements are pushed in increasing order: 1 2 3 4 5 6
- At any point, you can only pop the top of the stack.
- You must perform 6 pushes and 6 pops exactly.
- The output sequence must be the result of the pops.

a. Output sequence: 325641

We'll simulate this with step tracking.

Target output: 3 2 5 6 4 1

Let's simulate step-by-step:

Step	Stack	Action	Next to output	Comment
1	1	S	3	Push 1
2	1 2	S	3	Push 2
3	1 2 3	S	3	Push 3
4	1 2	X	2	Pop 3
5	1	X	5	Pop 2
6	1 4	S	5	Push 4
7	1 4 5	S	5	Push 5
8	1 4	X	6	Pop 5
9	1 4 6	S	6	Push 6
10	1 4	X	4	Pop 6
11	1	X	1	Pop 4
12		X	-	Pop 1

Output matches: 3 2 5 6 4 1

Valid sequence

Answer:

SSSXXSSXSXX produces 325641

B. Output sequence: 154623

Target output: 1 5 4 6 2 3

Let's simulate step-by-step:

Step	Stack	Action	Next to output	Comment
1	1	S	1	Push 1
2		X	5	Pop 1
3	2	S	5	Push 2
4	2 3	S	5	Push 3
5	2 3 4	S	5	Push 4
6	2 3 4 5	S	5	Push 5
7	2 3 4	X	4	Pop 5
8	2 3	X	6	Pop 4
9	2 3 6	S	6	Push 6
10	2 3	X	2	Pop 6
11	2	X	3	Pop 3 ❌ but expected 2

Problem: Stack top is 3, but next expected is 2. That violates the stack's LIFO rule.

You cannot pop 2 after 3 unless 3 is popped first, which would break the required sequence.

Answer

Not possible, the sequence 154623 cannot be produced by any valid sequence of stack operations.

Q1 B. Suppose we store n keys in a hash table of size m = n^2 using a hash function h randomly chosen from a Universal class H of hash functions. Assume that X is a random variable that counts the number of collisions. Show that the Expected number of Collisions is < 1/2.

We are given:

- n keys
- A hash table of size m= n^2
- A universal hash function hh, where for any two different keys $X_i \neq X_j$, the chance of collision is at most 1/m
- Let X be the number of collisions (i.e., how many pairs of keys get the same hash value)

Step-by-step:

1. There are (n 2)=n(n−1)/2 possible key pairs.

2. For each pair, the chance of a collision is at most 1/m, since we are using a universal hash function.

3. So, the expected number of collisions is:

$$E[X] \leq (n\ 2) \times 1/m$$
$$= (n(n-1)/2) \times (1/n^2)$$
$$= (n - 1) / 2n$$

4. Since (n - 1) / 2n < 1 / 2 for all n >= 1, we conclude:

$$E[X] < 1/2$$

So, when using a universal hash function and a table of size n^2, the expected number of collisions is less than 0.5.