Assignment 2
Gödel's incompleteness theorems

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Gödel's incompleteness theorems are two theorems of mathematical logic that are concerned with the limits of provability in formal axiomatic theories.The first incompleteness theorem states that no consistent system of axioms whose theorems can be listed by an effective procedure (i.e., an algorithm) is capable of proving all truths about the arithmetic of natural numbers. For any such consistent formal system, there will always be statements about natural numbers that are true, but that are unprovable within the system. The second incompleteness theorem, an extension of the first, shows that the system cannot demonstrate its own consistency.

Employing a diagonal argument, Gödel's incompleteness theorems were the first of several closely related theorems on the limitations of formal systems.
The incompleteness theorems apply to formal systems that are of sufficient complexity to express the basic arithmetic of the natural numbers and which are consistent, and effectively axiomatized, these concepts being detailed below.
The incompleteness theorems are about formal provability within these systems, rather than about "Provability" in an informal sense.
The incompleteness theorems show that systems which contain a sufficient amount of arithmetic cannot possess all three of these properties.
The incompleteness theorems apply only to formal systems which are able to prove a sufficient collection of facts about the natural numbers.

1 First incompleteness theorem

The first incompleteness theorem states that no consistent system of axioms whose theorems can be listed by an algorithm is capable of proving all truths about the arithmetic of natural numbers.The unprovable statement GF referred to by the theorem is often referred to as "the Gödel sentence"for the system F. The Gödel sentence is designed to refer, indirectly, to itself. The sentence states that, when a particular sequence of steps is used to construct another sentence, that constructed sentence will not be provable in F.However, the sequence of steps is such that the constructed sentence turns out to be GF itself. In this way, the Gödel sentence GF indirectly states its own unprovability within F.

The stronger version of the incompleteness theorem that only assumes consistency, rather than omega-consistency, is now commonly known as Gödel's incompleteness theorem and as the Gödel-Rosser theorem.

2 Second Incompleteness Theorem

Gödel's second incompleteness theorem states no consistent axiomatic system which includes Peano arithmetic can prove its own consistency. Stated more colloquially, any formal system that is interesting enough to formulate its own consistency can prove its own consistency if it is inconsistent.

The proof of the second incompleteness theorem is obtained by formalizing the proof of the first incompleteness theorem within the system F itself. The incompleteness theorems are among a relatively small number of nontrivial theorems that have been transformed into formalized theorems that can be completely verified by proof assistant software.The main difficulty in proving the second incompleteness theorem is to show that various facts about provability used in the proof of the first incompleteness theorem can be formalized within the system using a formal predicate for provability.