OBJECTIVE TYPE QUESTIONS – VOL I

Choose the correct or most suitable answer :

(1) The rank of the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 4 & -4 & 8 \end{bmatrix}$ is

(1) 1 (2) 2 (3) 3 (4) 4

Ans : R_2 and R_3 are proportional to R_1

$\therefore$ The matrix reduces to a matrix with a single non-zero row.

$\therefore$ Rank is 1

(2) The rank of the diagonal matrix $\begin{bmatrix} -1 & & & & \\ & 2 & & & \\ & & 0 & & \\ & & & -4 & \\ & & & & 0 \end{bmatrix}$

(1) 0 (2) 2 (3) 3 (4) 5

Ans : For the diagonal matrix except leading diagonal, all the other elements are zeroes.

$\therefore$ The matrix has only three non-zero rows.

$\therefore$ Rank is 3.

(3) If $A = [2 \;\; 0 \;\; 1]$, then rank of AA^T is

(1) 1 (2) 2 (3) 3 (4) 0

Ans : The order of A is 1×3 and A^T is 3×1

$\therefore$ Order of AA^T is 1×1

$\therefore$ Rank of AA^T is 1

(4) If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then the rank of AA^T is

(1) 3 (2) 0 (3) 1 (4) 2

Ans : $AA^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

Since R_2 and R_3 are proportional to R_1, it reduces to a matrix with a single non-zero row.

$\therefore$ Rank is 1

(5) If the rank of the matrix $\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{bmatrix}$ is 2, then λ is

(1) 1 (2) 2 (3) 3 (4) any real number

Ans : 'The rank of 3×3 matrix is 2' means the determinant value is 0

i.e., $\lambda^3 - 1 = 0 \Rightarrow \lambda = 1$

(6) If A is a scalar matrix with scalar $k \neq 0$, of order 3, then A^{-1} is

(1) $\frac{1}{k^2}\mathrm{I}$ (2) $\frac{1}{k^3}\mathrm{I}$ (3) $\frac{1}{k}\mathrm{I}$ (4) kI

Ans : $A^{-1} = \frac{\text{adj } A}{|A|}$ and adj $(kI) = k^{n-1}I$ where k is a constant.

re A is a scalar matrix with scalar k of order 3. i.e., $A = kI$

$\therefore adj\ (A) = k^2 I\ ;\ |A| = k^3$

$A^{-1} = \frac{1}{k} I$

(7) If the matrix $\begin{bmatrix} -1 & 3 & 2 \\ 1 & k & -3 \\ 1 & 4 & 5 \end{bmatrix}$ has an inverse then the values of k

(1) k is any real number (2) $k = -4$ (3) $k \neq -4$ (4) $k \neq 4$

Ans : 'It has inverse' means the determinant value is $\neq 0$

i.e. $-7k - 28 \neq 0 \Rightarrow k \neq -4$

(8) If $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, then (adj A) A =

(1) $\begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (3) $\begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix}$ (4) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

Ans : $(adj\ A)A = |A|\ I$

$|A| = 5$

$|A|\ I = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

(9) If A is a square matrix of order n then $|\text{adj } A|$ is

(1) $|A|^2$ (2) $|A|^n$ (3) $|A|^{n-1}$ (4) $|A|$

Ans : $A(adj\ A) = |A|\ I$

Take the determinant on both sides

$|A|\ \ |adj\ A| = |A|^n\ \ |I|$

$|adj\ A| = |A|^{n-1}$

(10) The inverse of the matrix $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ is

(1) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ (3) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (4) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Ans : Take one option. Now find the product with the given matrix. If it is equal to I then choose that option. Otherwise delete it and continue. Here the option is (3).

(11) If A is a matrix of order 3, then det (kA)

(1) k^3det (A) (2) k^2 det(A) (3) k det (A) (4) det (A)

Ans : To multiply a matrix by a scalar, we multiply each element with that scalar. But in the case of determinant, multiplying a determinant by a scalar means multiplying one row elements or one column elements.

$\therefore$ det $(kA) = k^n|A|$

$\therefore$ If A is of order 3 then det $(kA) = k^3|A|$

(12) If I is the unit matrix of order n, where $k \neq 0$ is a constant, then adj(kI) =

(1) k^n (adj I) (2) k (adj I) (3) k^2 (adj (I)) (4) k^{n-1} (adj I)

Ans : It is a formula, $adj(kI) = k^{n-1}(adj\ I)$

(13) If A and B are any two matrices such that $AB = O$ and A is non-singular, then

(1) $B = O$ (2) B is singular (3) B is non-singular (4) $B = A$

Ans : Since A is non-singular, A^{-1} exists

$AB = 0 \Rightarrow A^{-1}(AB) = A^{-1}O \Rightarrow IB = O \Rightarrow B = O$

(14) If $A = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$, then A^{12} is

(1) $\begin{bmatrix} 0 & 0 \\ 0 & 60 \end{bmatrix}$ (2) $\begin{bmatrix} 0 & 0 \\ 0 & 5^{12} \end{bmatrix}$ (3) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Ans : Clearly $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 5^2 \end{bmatrix}$, $A^3 = \begin{bmatrix} 0 & 0 \\ 0 & 5^3 \end{bmatrix}$, ... $A^{12} = \begin{bmatrix} 0 & 0 \\ 0 & 5^{12} \end{bmatrix}$

(15) Inverse of $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ is

(1) $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$ (2) $\begin{bmatrix} -2 & 5 \\ 1 & -3 \end{bmatrix}$ (3) $\begin{bmatrix} 3 & -1 \\ -5 & -3 \end{bmatrix}$ (4) $\begin{bmatrix} -3 & 5 \\ 1 & -2 \end{bmatrix}$

Ans : Cofactor matrix is $\begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$, adj is $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$, det value is 1

$\therefore$ inverse is $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

(16) In a system of 3 linear non-homogeneous equation with three unknowns, if $\Delta = 0$ and $\Delta_x = 0$, $\Delta_y \neq 0$ and $\Delta_z = 0$ then the system has

(1) unique solution (2) two solutions

(3) infinitely many solutions (4) no solutions

Ans : If $\Delta = 0$ and any one of Δ_x, Δ_y, Δ_z is non-zero then the system is inconsistent and hence no solutions.

(17) The system of equations $ax + y + z = 0$; $x + by + z = 0$; $x + y + cz = 0$ has a non-trivial solution then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} =$

~~(1)~~ 1 (2) 2 (3) -1 (4) 0

Ans : Since it has a non-trivial solution, $\Delta = 0$

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 1-a & 0 & c-1 \end{vmatrix} = 0$$

$\Rightarrow a(1-b)(1-c) + (1-a)(1-c) + (1-a)(1-b) = 0$

$\div$ by $(1-a)(1-b)(1-c)$

$$\Rightarrow \frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$$

i.e. $\left(-1 + \frac{1}{1-a}\right) + \frac{1}{1-b} + \frac{1}{1-c} = 0$

The required value is 1

(18) If $ae^x + be^y = c$; $pe^x + qe^y = d$ and $\Delta_1 = \begin{vmatrix} a & b \\ p & q \end{vmatrix}$; $\Delta_2 = \begin{vmatrix} c & b \\ d & q \end{vmatrix}$,

$\Delta_3 = \begin{vmatrix} a & c \\ p & d \end{vmatrix}$ then the value of (x, y) is

(1) $\left(\frac{\Delta_2}{\Delta_1}, \frac{\Delta_3}{\Delta_1}\right)$ ~~(2)~~ $\left(\log\frac{\Delta_2}{\Delta_1}, \log\frac{\Delta_3}{\Delta_1}\right)$ (3) $\left(\log\frac{\Delta_1}{\Delta_3}, \log\frac{\Delta_1}{\Delta_2}\right)$ (4) $\left(\log\frac{\Delta_1}{\Delta_2}, \log\frac{\Delta_1}{\Delta_3}\right)$

Ans : Solve for e^x and e^y

$$e^x = \frac{\begin{vmatrix} c & b \\ d & q \end{vmatrix}}{\begin{vmatrix} a & b \\ p & q \end{vmatrix}} = \frac{\Delta_2}{\Delta_1} \Rightarrow x = \log\left(\frac{\Delta_2}{\Delta_1}\right)$$

Similarly $e^y = \frac{\Delta_3}{\Delta_1} \Rightarrow y = \log\left(\frac{\Delta_3}{\Delta_1}\right)$ $\Rightarrow$ option (2)

(19) If the equations

$$-2x + y + z = l$$
$$x - 2y + z = m$$
$$x + y - 2z = n$$

such that $l + m + n = 0$, then the system has

(1) a non-zero unique solution

(2) trivial solution

~~(3)~~ Infinitely many solution

(4) No Solution

Ans : $[A, B] = \left|\begin{array}{cccc} -2 & 1 & 1 & l \\ 1 & -2 & 1 & m \\ 1 & 1 & -2 & n \end{array}\right| \sim \left|\begin{array}{cccc} -2 & 1 & 1 & l \\ 1 & -2 & 1 & m \\ 0 & 0 & 0 & 0 \end{array}\right| R_3 \to R_3 + R_3 + R_1$

$\Rightarrow \rho(A) = \rho(A, B) = 2$

$\Rightarrow$ has infinitely many solutions.

(20) If $\vec{a}$ is a non-zero vector and m is a non-zero scalar then $m\vec{a}$ is a unit vector if

(1) $m = \pm 1$ (2) $a = |m|$ (3) $a = \dfrac{1}{|m|}$ (4) $a = 1$

Ans : $\left|m\vec{a}\right| = 1 \Rightarrow m\left|\vec{a}\right| = 1 \Rightarrow \left|\vec{a}\right| = \dfrac{1}{m}$