

# INTRO TO AI AND ML

## (EE1390)

### MATRIX PROJECT

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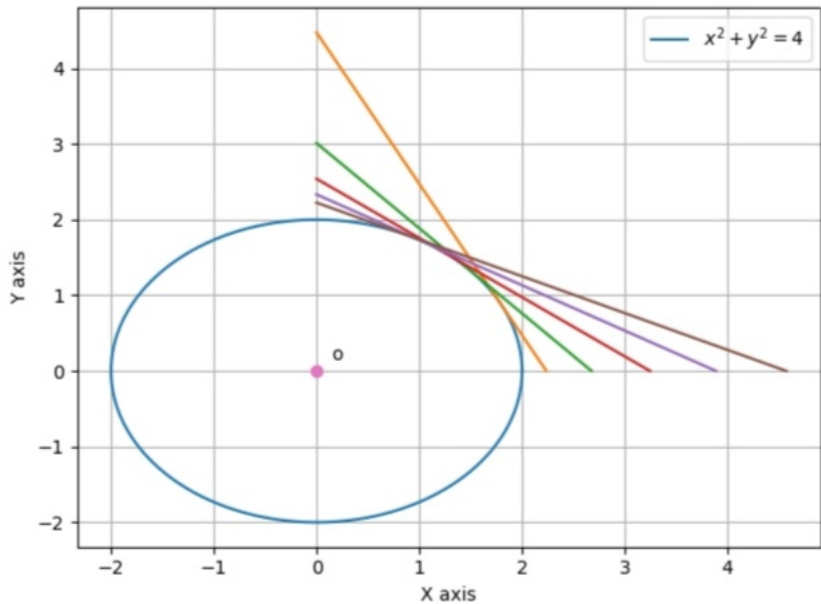
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## Question

Let  $S$  be the circle in the  $xy$ -plane defined by the equation

$$x^2 + y^2 = 4$$

Let  $P$  be a point on the circle  $S$  with both coordinates being positive. Let the tangent to  $S$  at  $P$  intersect the coordinate axes at the points  $M$  and  $N$ . Find the locus of mid-point of the line segment  $MN$



## Solution

General equation of a quadratic curve is given by

$$Ax_1^2 + Bx_1x_2 + Cx_2^2 + Dx_1 + Ex_2 + F = 0$$

This can be expressed as

$$x^T V x + 2u^T + F = 0$$

where

$$x = \begin{bmatrix} x \\ y \end{bmatrix} \quad V = \begin{bmatrix} A & B/2 \\ B/2 & C \end{bmatrix} \quad u^T = [D \quad E]$$

For a circle with centre at origin

$$V = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$u^T = [0 \quad 0]$$

Therefore equation of the circle  $S$  is  $xx^T - 4 = 0$

The tangent to any point  $P$  on the curve is given by

$$\begin{bmatrix} P^T & 1 \end{bmatrix} \begin{bmatrix} V & u \\ u^T & F \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} = 0$$

This can be expressed as

$$\begin{bmatrix} P^T V + u^T \end{bmatrix} x + P^T u + F = 0$$

Substitute the values of  $V$  and  $u$

Equation of tangent  $MN$  is  $P^T x - 4 = 0$

Parametric form of the circle is

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\text{radius } r = 2$$

Taking  $P = \begin{bmatrix} 2\cos\theta \\ 2\sin\theta \end{bmatrix}$

$$0 < \theta < \pi/2$$

$$P^T = [2\cos\theta \quad 2\sin\theta]$$

Therefore

$$[2\cos\theta \quad 2\sin\theta] \begin{bmatrix} x \\ y \end{bmatrix} = 4$$

Dividing by 4 on both sides ,we get

$$[\cos\theta/2 \quad \sin\theta/2] \begin{bmatrix} x \\ y \end{bmatrix} = 1$$



Intercept form of a straight line :

$$\begin{bmatrix} \frac{1}{a} & \frac{1}{b} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 1$$

$x$  intercept =  $a$

$y$  intercept =  $b$

Comparing the the equations ,we get

$$a = \frac{2}{\cos\theta} ; b = \frac{2}{\sin\theta}$$

Midpoint of line segment MN =  $\begin{bmatrix} \frac{1}{\cos\theta} \\ \frac{1}{\sin\theta} \end{bmatrix}$

$$x = \frac{1}{\cos\theta} \qquad y = \frac{1}{\sin\theta}$$

$$(\cos^2\theta) + (\sin^2\theta) = 1$$

Therefore, the locus is

$$x^2 + y^2 = x^2 y^2$$

