



Academic Year	Module	Assessment Number	Assessment Type
S20	Introductory Data Structures and Algorithms (DipIT02)	A1	Assignment Submission

[Assignment Submission]

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Module Leader : [Mr. Prakash Gautam]

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Tutorial - 4

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1a-> For any positive integer n , a tree with n vertices has $n-1$ edges.

It is proof by Mathematical induction.

* Base case

A tree with one vertex has 0 edges.

* Inductive Hypothesis

Suppose a tree having ' n ' vertices has ' $n-1$ ' edges.

* Inductive Step

Given from Inductive hypothesis a tree having ' n ' vertices has ' $n-1$ ' edges.

Add a single vertex in any leaf node of tree having n vertices - tree.

$$\text{number of vertices} = n+1$$

$$\text{number of edges} = n$$

b) The # leaves in a non-empty full binary tree is one more than the # internal nodes.

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2) In a perfect binary tree, T : total number of nodes with height h is $n = 2^{h+1} - 1$

total number of nodes =

$$2^0 + 2^1 + 2^2 + \dots + 2^h$$

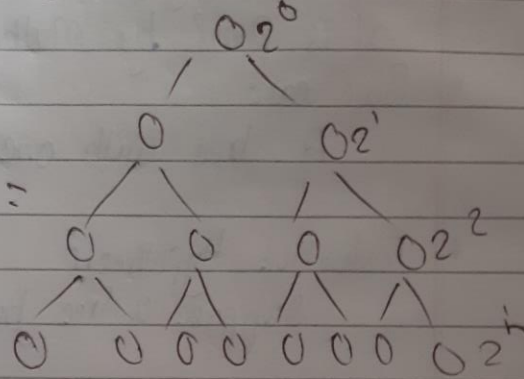
Sum of Geometric series =

$$= \frac{a(r^n - 1)}{r - 1}$$

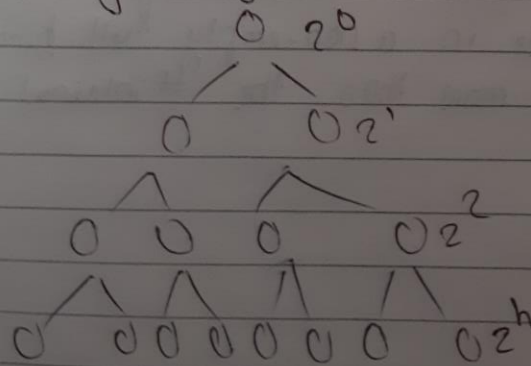
$$\text{Or, } n = \frac{2^0(2^{h+1} - 1)}{2 - 1}$$

$$\text{Or, } n = \frac{2^{h+1} - 1}{2 - 1}$$

$$\therefore n = \frac{2^{h+1} - 1}{1}$$



3) In a complete binary tree, T : the max # nodes with height h is: $n = 2^{h+1} - 1 = 2^{\text{#level}} - 1$



total number of nodes = $2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^h$

$$\text{Or, } n = \frac{2^0(2^{h+1} - 1)}{2 - 1}$$



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$$\text{or } n = 1 \times (2^{h+1} - 1)$$

$$\text{or } n = 2^{h+1} - 1$$

4. In a perfect binary tree T , the height of T having total n nodes is: $h = \log_2(n+1) - 1$
 total number of nodes $(n) = 2^{h+1} - 1$

taking \log on both sides, we get

$$n+1 = 2^{h+1}$$

$$\text{or } \log_2(n+1) = \log_2(2^{h+1})$$

$$\text{or } h+1 = \log_2(n+1)$$

$$\text{or } h = \log_2(n+1) - 1$$

- Q5. In a complete binary tree T , the height of T having total n nodes is: $h = \log_2(n+1) - 1$
 total number of nodes for complete binary

$$n = 2^{h+1} - 1$$

Taking \log with base 2 on both sides

$$\log_2(n+1) = \log_2(2^{h+1})$$

$$\text{or } \log_2(n+1) = h+1$$

$$\text{or } h = \log_2(n+1) - 1$$

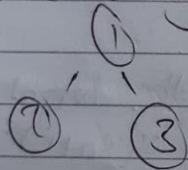
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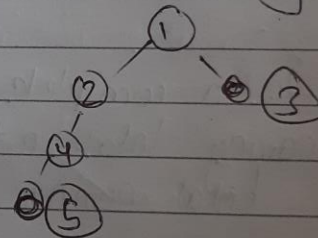
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- a) All proper binary trees are perfect : (F)
 - b) All perfect binary trees are complete & vice-versa. (F)
 - c) All complete binary trees are complete. (T)
 - d) All complete binary trees are balanced. (T)
 - e) All complete binary trees are balanced & vice-versa. (T)
 - f) Proper binary trees can never be perfect. (F)
 - g) Perfect binary trees can always be 2-trees. (T)
 - h) Balanced trees can never be pathological. (F)

7 →

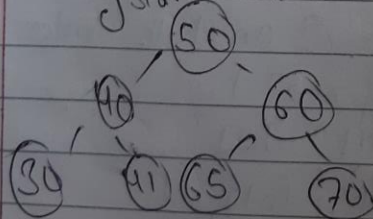
a) Strict binary tree



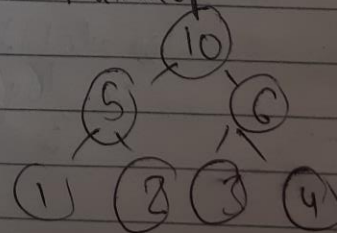
b) Unbalanced binary tree



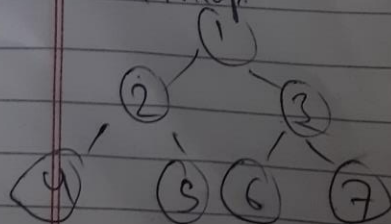
c) Binary search tree



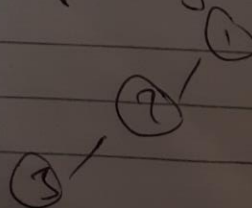
d) Max heap



e) min-heap



f) Pathological binary tree

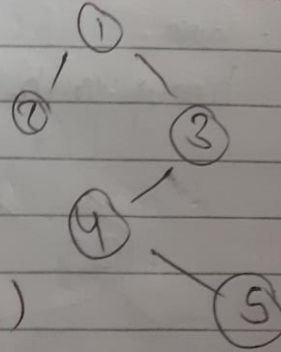


8 a

Inorder - (Left, Root, Right)
= 21453

Postorder - (Left, Right, Root)
= 25431

Preorder - (Root, Left, Right)
= 12345

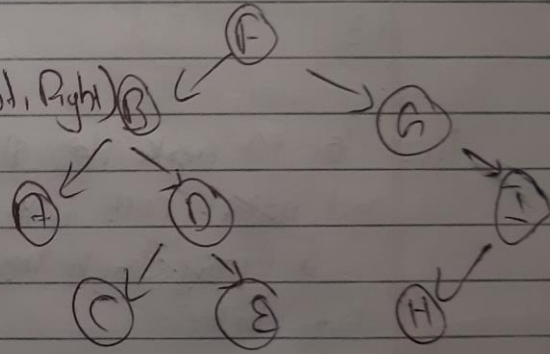


b)

Inorder - P B C D E F G H I (Left, Root, Right)

Preorder - F B A D C E G I H
(Root, Left, Right)

Postorder - A C E D B H I G F
(Left, Right, Root)

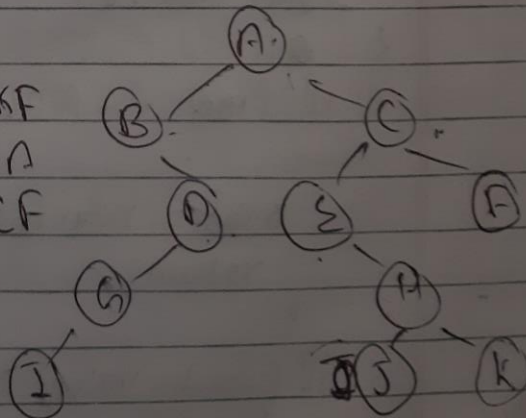


c)

Pre-order - A B D G J C E H K F

Post order - J G D B J K H E F C A

Inorder - B J G D A E J H K C F

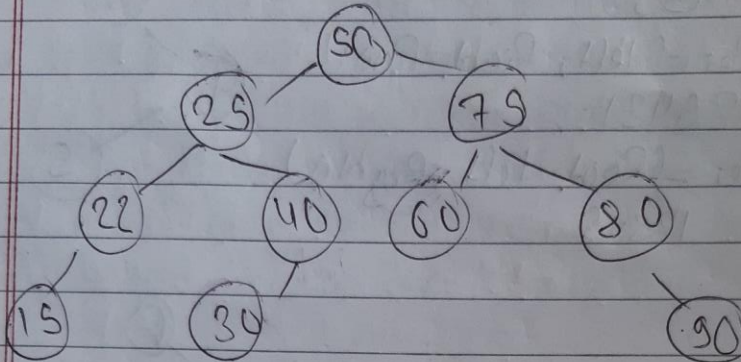


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9. → - Given.

binary search tree: 50, 25, 75, 22, 40, 60, 80, 90, 15, 30



10. → To create algorithm and insert node with key k in a Binary Search tree, it is given below:

→ Insert (root, k)

{

if (root == NULL)

{

root = new node (k)

return;

}

if (root->value > k)

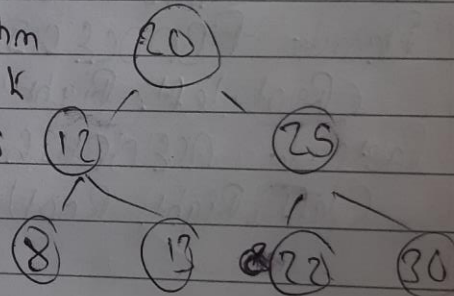
{

if (root->left == NULL)

{ root->left = new node (k);

else { insert (root->left, (k));

}



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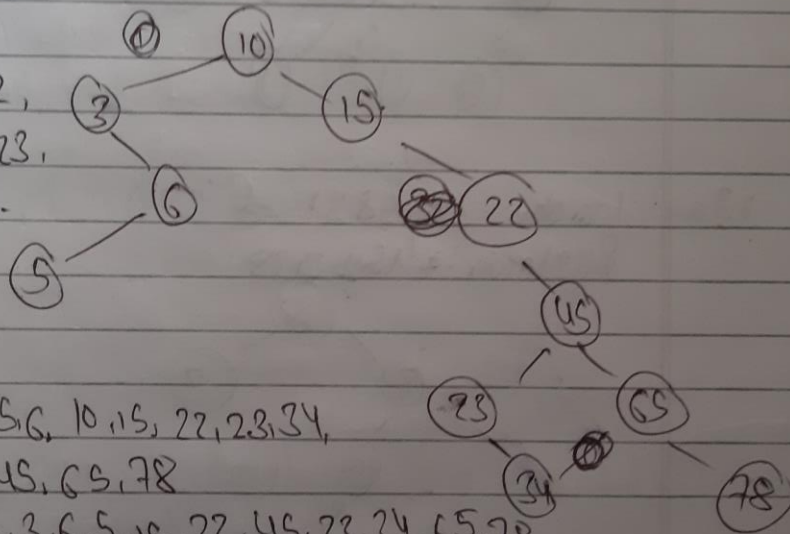
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```

if (root->value < k)
{
    if (root->right == NULL)
    {
        root->right = new node(k)
    }
    else insert (root->right, (k))
}
    
```

11.→ Given,

10, 3, 15, 22,
6, 45, 65, 23,
78, 34, 5.



~~10, 3, 15~~

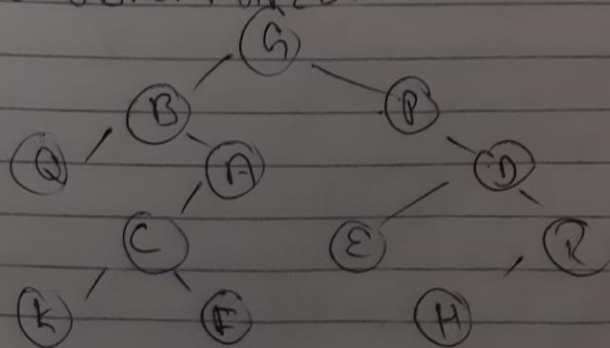
In order = 3, 5, 6, 10, 15, 22, 23, 34,
45, 65, 78

Preorder = 10, 3, 6, 5, 15, 22, 45, 23, 34, 65, 78

Post order = 5, 6, 3, 34, 23, 78, 65, 45, 22, 15, 10

12.→ Preorder: G B A C H F P D E R

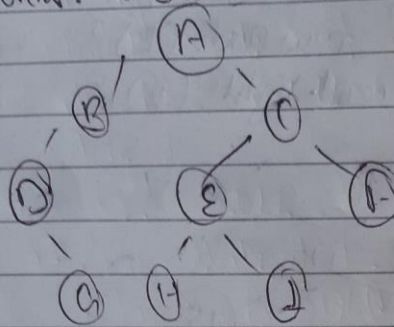
Inorder: G B K C F A G R E D H R



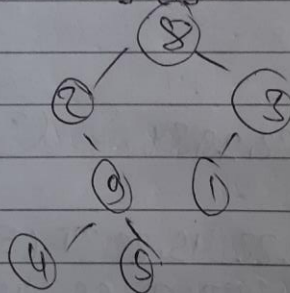
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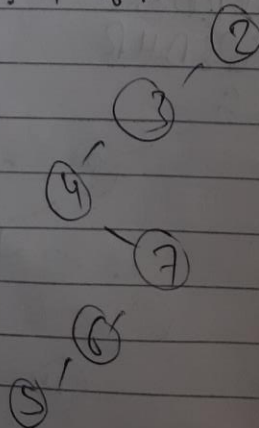
b) Inorder: D G B A H E I C F
Postorder: G D B H T E F C A



13a) Inorder: 2 4 9 5 8 3 1
Postorder: 4 5 9 2 3 8



b) Inorder: 4, 5, 6, 7, 3, 2
Preorder: 2, 3, 4, 7, 6, 5

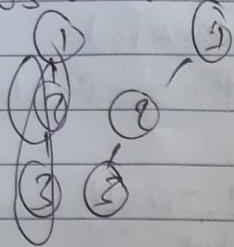


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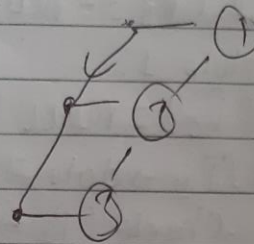
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③ Preorder: 123

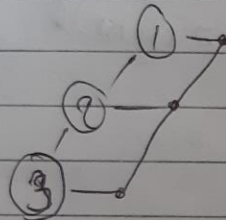
Postorder: 321



Check (using flag) (Pre order)



Check (Post order)



321 ✓

14. → ~~Preorder~~

16. → An algorithm to search a data in BST.

Search (w.root, k) {

if (w == NULL)

{

return null;

if (w.get data() == k)

{ return w;

if (w.get data() > k)

{

Search (w.left, k)

if (w.get data() < k)

Search (w.right, k)

}

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17. → Given

Two binary trees, ...

int check (node * p₁, node * p₂)

{ if (p₁ == NULL && p₂ == NULL)

return 1;

if (p₁ != NULL && p₂ != NULL)

if (p₁ → data == p₂ → data &&

check (p₁ → left, p₂ → left) &&

check (p₁ → right, p₂ → right))

{

return 1;

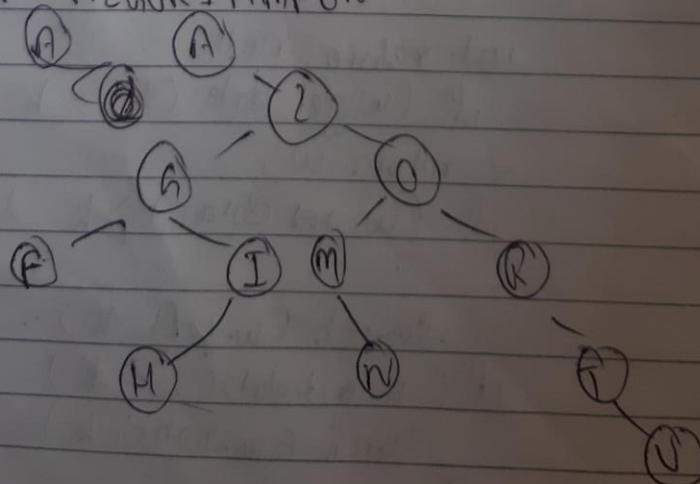
}

}

return 0;

}

14. → Inorder : A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
Preorder : A L G O R I T H M F U N



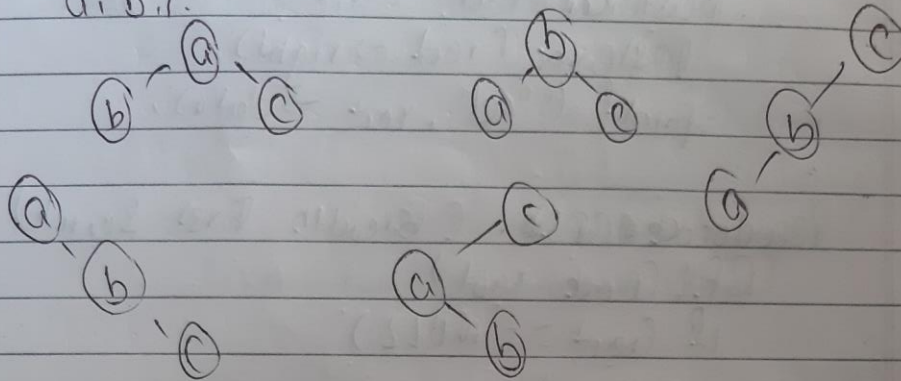
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15)

Given.

a, b, c.



19)

Pseudocode DFS (Depth First Search)

void inorder (struct node *root)

{

if (root == NULL)

{ return;

}

inorder (root -> left);

printf ("%d", root->data);

inorder (root -> right);

}

void preorder (struct node *root)

{

if (root == NULL)

{ return;

printf ("%d", root->data);

preorder (root -> left);

preorder (root -> right);

}

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```
void postorder (struct node * root)
```

```
    postorder (root -> left);
```

```
    postorder (root -> right);
```

```
    printf ("%d", root -> data);
```

```
Pseudocode BFS (Breadth First Search)
```

```
BFS (Node root)
```

```
    if (root == NULL)
```

```
        return 1
```

```
    Queue q
```

```
    q.enqueue (root)
```

```
    while (!q.is.empty)
```

```
        Node x = q.dequeue()
```

```
        Print x
```

```
        if (x.left != null)
```

```
            q.enqueue (x.left)
```

```
        if (x.right != null)
```

```
            q.enqueue (x.right)
```

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20)

Soln

Build maxheap (Array n)

2

int i:
for (i = $\lfloor \frac{n-1}{2} \rfloor$ to 0) Time Complexity = $O(n \log n)$

{ max-heapify (Array, n, i) }

2 max-heapify (Array, n, largest(i))

int largest = i;

int l = 2 * i + 1;

int r = 2 * i + 2;

if (l ≤ n and A[l] > A[largest])
{ largest = l }

if (r ≤ n and A[r] > A[largest])
{ largest = r }

if (largest != i)

{ Swap (A[i] and A[largest]) }

max-heapify (Array, n, largest)

}

Time Complexity = $O(n \log n)$

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2128 Build & Pseudocode to build mini-heap
and to illustrate Min-heapify procedure.
build-minheap (Array, n)

\leftarrow int i ;

 for ($i = \lfloor \frac{n-1}{2} \rfloor$ to 0)

\leftarrow min-heapify (Array, n, i)

Min-heapify (Array, n, i)

 int

 int largest = i;

 int left = $2 * i + 1$;

 int Right = $2 * i + 2$;

 if (left \leq n and $A[\text{left}] < A[\text{largest}]$)

 largest = left;

 if (Right \leq n and $A[\text{Right}] < A[\text{largest}]$)

 largest = Right;

 if (largest \neq i)

\leftarrow

 Swap ($A[i]$ and $A[\text{largest}]$)

 min-heapify (Array, n, largest)

 }

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22) a) getMini (Array) {
 return Array[0];
 }

b) ExtractMin (Array, n) {
 Array[0] = Array[n-1];
 n = n-1;
 MinHeapify (Array, n, 0);
 }

c) Insert (Array, key) {
 ~~Heapify = 0~~
 HeapSize = HeapSize + 1;
 int i = HeapSize;
 A[i] = key;
 while (i > 1 and A[i] < A[i/2])
 {
 Swap (A[i] and A[i/2])
 i = i/2;
 }
 }

d. decreaseKey (Array, key) {
 int i = HeapSize;
 A[i] = key;
 while (i > 1 and A[i] < A[i/2])
 {
 Swap (A[i] and A[i/2])
 i = i/2;
 }
 }

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23.) a) getMax() {
 return Array[a];
}

P

b) extractMax(Array, n) {
 Array[a] = Array[n-2]
 n = n-1;
 maxHeapify(Array, n, a);
}

c) insert(Array, key) {
 HeapSize = HeapSize + 1;
 int i = HeapSize;
 A[i] = key;
 while (i > 1; and A[i/2] < A[i])
 {
 Swap(A[i] and A[i/2])
 i = i/2
 }
}

d. decreaseKey(Array, key) {
 int i = HeapSize;
 A[i] = key;
 while (i > 1; and A[i/2] < A[i])
 {
 Swap(A[i] and A[i/2])
 i = i/2
 }
}

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24. Solkin

maximum(node)

{

if (node == NULL)

{ return -1

else {

Lmax = Max (node.left)

Rmax = Max (node.right)

if (node->value > Lmax and node->value > Rmax)

{ return node->value

else if (Lmax > Rmax)

{ return ~~node->value~~ Lmax

else { return Rmax

}

}

25.

4 6 23 6 2 7 9

PRIO * R * * 1 * T * 4 * * * QUES * * * U * E

A → 2

low high

Heap
J E

Extracted value

E-1

J E

6 R

J-2

6 R

Q-3

3 4 P

P-4

3 0

Q-5

7 4

R-6

2 1

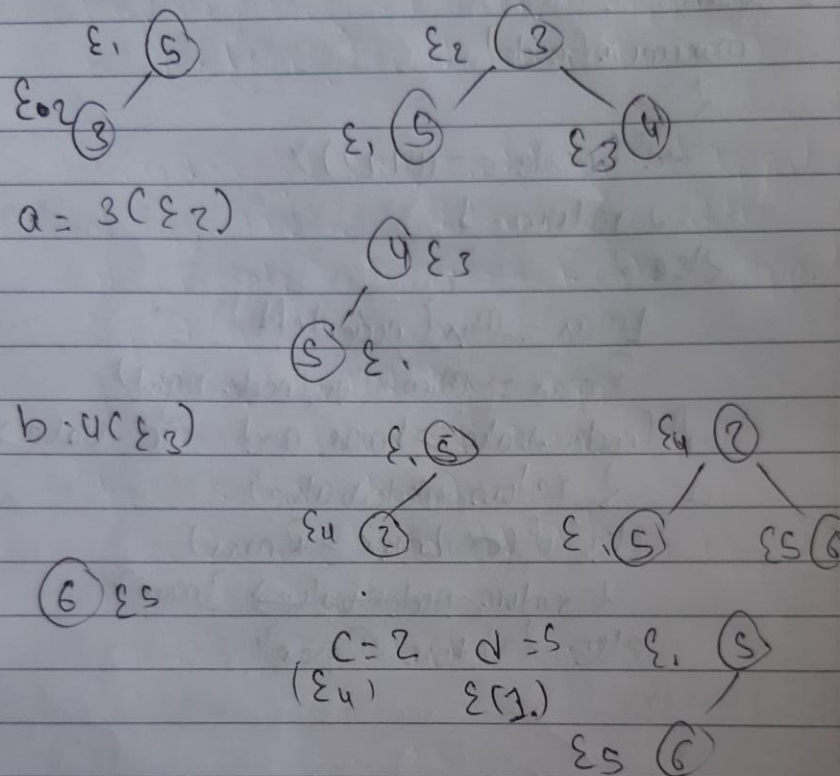
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26.



27.

```

int findMax(Node root)
{
    if (root == NULL)
        return -1;
    }
    else
    {
        int maxLeft = findMax(root->left);
        int maxRight = findMax(root->right);
        int maxMax = maxLeft;
        if (maxRight > maxLeft)
            maxMax = maxRight;
        }
        return maxMax;
    }

```

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28. -> void printRange (int lower, int upper) {
 printRange1 (root, lower, upper) }

```
void printRange1 (Node p, int lower, int upper)
{
    if (p == NULL) { // terminal condition
        return;
    }
    if (p->data >= lower && p->data <= upper)
    {
        print (p->data);
        printRange1 (p->left, lower, upper);
        printRange1 (p->right, lower, upper);
    }
    else if (p->data < lower)
    {
        printRange1 (p->right, lower, upper);
    }
    else
    {
        printRange1 (p->left, lower, upper);
    }
}
```


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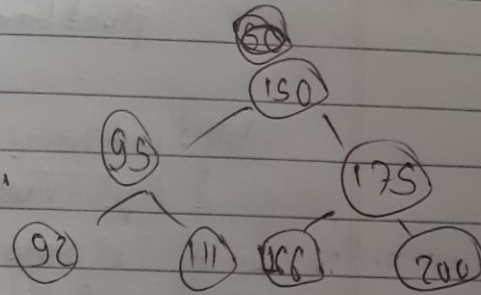
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29- Implement a method that prints all nodes in a
certain level.

Invoking printLevel(2) on the
binary tree above should

Print: ~~92 111 166 92 111~~

Print: 92 111 166 200



```
void printLevel (int level) {
```

```
    printLevel (Node root, int level)
```

```
}
```

```
void print (Node n, int level) {
```

```
    if (n == null) {
```

```
        return;
```

```
    }
```

```
    if (level == 0) { print (n->data); }
```

```
    else
```

```
    {
```

```
        print (n->left, level-1)
```

```
        print (n->right, level-1)
```

```
    }
```

```
}
```