



Academic Year	Module	Assessment Number	Assessment Type
S20	Introductory Data Structures and Algorithms (DipIT02)	A1	Assignment Submission

## [Assignment Submission]

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Tutorial - 627

11. 7, 2, 1, 6, 8, 5, 3, 9, 21, 4

Soln:

Here, Quick Sort Code.

```

Quick-Sort(A, start, End)
if (start < End) {
    Pindex = Partition(A, start, End)
    Quick-Sort(A, start, Pindex-1)
    Quick-Sort(A, Pindex+1, End)
}
}

```

```

Partition(A, start, End)
Pivot = A[End]
Pindex = start;
for (i = start to End-1)
    if (A[i] < Pivot)
        Swap(A[i], A[Pindex]);
        Pindex = Pindex + 1;
Swap(A[Pindex], A[End]);
return Pindex;
}
∴ P-index = 0

```

Q(1)

7	2	1	6	8	5	3	9	21	4
0	1	2	3	4	5	6	7	8	9

7	2	1	6	8	5	3	9	21	4
0	1	2	3	4	5	6	7	8	9

P-index = 1

Now, Swap (A[i], A[P-index])

2	7	1	6	8	5	3	9	21	4
0	1	2	3	4	5	6	7	8	9

P-index = 2

2	1	7	6	8	5	3	9	2	4
0	1	2	3	4	5	6	7	8	9

2	1	7	6	8	5	3	9	2	4
0	1	2	3	4	5	6	7	8	9

2	1	3	6	8	5	7	9	2	4
0	1	2	3	4	5	6	7	8	9

P-index = 3

Again swap( $A[P-index]$ ,  $A[End]$ )

2	1	3	4	8	5	7	9	2	6
0	1	2	3	4	5	6	7	8	9

Now the correct position of Direct element is

2	1	3
0	1	2(P)

8	5	7	9	2	1	6
4	5	6	7	8	9(P)	

2	1
0	1(P)

8	5	7	9	2	1	6
4	5	6	7	8	9(P)	

1	2
	1(P)

8	5	7	9	2	1	6
4	5	6	7	8	9(P)	

5	6	7	9	2	1	8
4	5	6	7	8	9	

5
4

7	8	2	1	8
6	7	8	9	

7
6

9	21
8	9

97
8

1	2	3	4	5	6	7	8	9	21
0	1	2	3	4	5	6	7	8	9

→ Here is pseudocode for merge-sort.

merge-sort(A)

n = length(A);

mid = n/2;

right = Array of size (n - mid)

if (n < 2)

print("is sorted");

else

return;

for (i = 0 to mid - 1

left[i] = A[i]

for i = mid to n - 1

right[i - mid] = A[i]

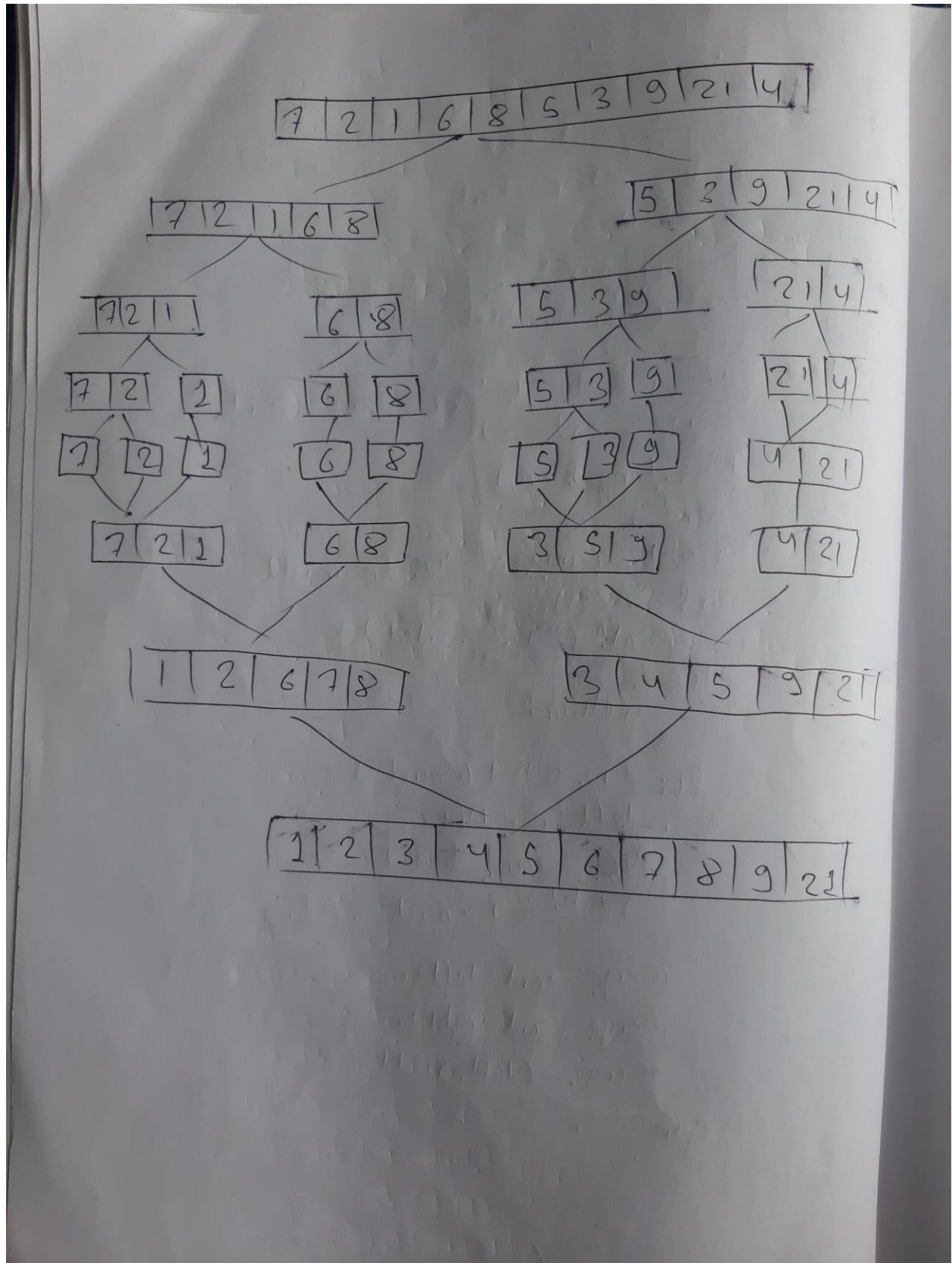
merge-sort(left);

merge-sort(right);

merge(left, right);

Again





2)

2.1.

Bubble sort

```
for (i = 0; i < n-1; i++) {
    for (j = 0; j < n-1-i; j++) {
        if (A[j] > A[j+1]) {
            temp = A[j];
            A[j] = A[j+1];
            A[j+1] = temp;
        }
    }
}
```

→ The bubble sort algorithm is a stable algorithm because the relative order of any two equal elements is preserved in the sorted array.

2.2.

Selection sort

```
for (i = 0; i < n-1; i++) {
    int min = i;
    for (j = i+1; j < n; j++) {
        if (A[j] < A[min]) {
            min = j;
        }
    }
    if (min != i) {
        swap(A[i], A[min]);
    }
}
```

→ The Selection sort is not a stable because it swaps non-adjacent elements. The time complexity is  $O(n^2)$  which is worst case.

### 7.3. Insertion sort

```
for (i = 1; i < n; i++)
```

```
temp = a[i];
```

```
j = i - 1;
```

```
while (j >= 0 && a[j] > temp) {
```

```
    a[j+1] = a[j];
```

```
    j--;
```

```
    a[j+1] = temp;
```

When the array is given in descending order and we are going to sort that array in ascending order that is worst case which is  $O(n^2)$  because loop is also to go to  $i$  too and while loop also go to from  $j$  to temp.  $O(n^2)$ .

### 7.4. merge sort

```
merge(n)
```

```
{
```

```
    n = length(A);
```

```
    mid = n / 2;
```

```
    left = mid
```

```
    right = n - mid;
```

```
    for (i = 0 to mid-1)
```

```
        left[i] = A[i]
```

```
    for (i = mid to n-1)
```

```
        right[i - mid] = A[i]
```

```
    merge-sort(left);
```

```
    merge-sort(right);
```

```
    merge(left, right, A)
```



```

merge (A, L, R, X)
nL = L;
nR = R;
while (nL < nR)
{
    if (L[nL] <= R[nR]);
        A[k] = L[nL]++;
    else
        A[k] = R[nR]++;
    k++;
}
while (i < nL)
    A[k] = L[i]++; k++;
while (j < nR)
    A[k] = R[j]++; k++;

```

→ The merge sort is not in place sorting algorithm as it requires  $O(n)$  extra space. It is stable as it does not change the order of two or more duplicate values.



2.5) Quick sort

Quick sort (A, lb, ub)

if (lb < ub)

{

int loc = partition (A, lb, ub)

Quick sort (A, lb, loc - 1);

Quick sort (A, loc + 1, ub)

Partition (A, lb, ub)

Pivot = a[lb]

start = lb;

end = ub

while (lb < ub)

while (a[start] <= pivot)

start ++

while (a[end] > pivot)

end --

if (start < end)

swap (a[start], a[end])

swap (a[lb], a[end])

return end;

-> The Quick sort is in place sorting algorithm as it takes extra space but only for recursive function calls. No. Quick sort is not stable as we do swapping of elements according to pivot's position without considering original position.

2.6) Shell sort

Shell-Sort(A, size)

for (gap = gap/2; gap >= 1; gap/2)

for (j = gap; j < size; j++)

for (i = j - gap; i >= 0; i - gap)

if (A[i+gap] > A[i])

break;

else swap (A[i+gap], A[i])

→ The Shell sort is in-place sorting algorithm as it takes  $O(1)$  space for exchanging elements. It is not stable as it does not examine elements lying between gap.

2.7) Heap sort

Heapify(A, n, i)

largest = i;

l = 2 \* i + 1

r = 2 \* i + 2

if (l <= n and A[l] > A[largest])

largest = l;

if (r <= n and A[r] > A[largest])

largest = r;

if (largest != i)

swap (A[i], A[largest])

heapify (A, n, largest)

→ The Heap sort is in-place as it only requires extra space for swapping elements. It is unstable.

3). Soln

Let denote the function  $T(n)$   
now,

Partition  $\propto n \rightarrow$  linear time complexity  $\pm a \pm b$ ;

For two recursive calls of Quick Sort  $T(n/2) + T(n/2)$   
Then,

$$T(n) = 2T(n/2) + a \pm b \quad \text{--- (i)}$$

$$T(n/2) = 2T(n/4) + (n/2) \quad \text{--- (ii)}$$

$$T(n/4) = 2T(n/8) + (n/4) \quad \text{--- (iii)}$$

From (iii) and (ii)

$$T(n/2) = 2[2T(n/8) + (n/4)] + (n/2) \quad \text{--- iv}$$

$$\begin{aligned} T(n) &= 2[2[2T(n/8) + (n/4)] + (n/2)] + (n) \\ &= 8T(n/8) + 3n \\ &= 2^3 T(n/2^3) + 3n \end{aligned}$$

Again,

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + k(n - (n/2^k))$$

Best case

$$\frac{n}{2^k} = 1, \quad 2^k = n$$

Taking  $\log_2$  on both side

$$\log 2^k = \log_2 n$$

$$\therefore k = \log_2 n \quad \text{--- (v)}$$

Now, from (v) and (vi)

$$T(n) = 2 \log_2 n + T(1) + n \log_2 n$$

$$T(n) = n + n \log_2 n$$

$$\therefore \text{Time complexity} = O(n \log n)$$



worst case

for partition for  $a$  and  $b$

for two recursive call  $T(n-1)$  and  $T(n-1)$

$$\therefore T(n) = T(n-1) + T(n-1) + O(1)$$

$$\therefore T(n) = T(n-1) + O(n)$$

4  $\rightarrow$  Per. Inorder traversal produces sorted list  
key values because it prints left and root  
and right element.

5  $\rightarrow$  The Quick sort is more popular sorting  
algorithm than merge-sort because ~~it~~ it is  
not in-place so it requires additional  
memory space to store the auxiliary  
array.

6  $\rightarrow$  Purpose of while (values[start]  $\leq$  pivot)  
& start++ is to increase the index  
 $\rightarrow$  start until we get element smaller than  
pivot. So, at the end we could swap  
element at last. while (values[end]  $>$  pivot)  
& end-- is to decrease the index and  
which starts from the end of array  
when element is greater than pivot.

7  $\rightarrow$  The array ~~be~~ would be balanced if  
values from 10-22 by providing values  
greater than 28 & less than 3. we would get  
worst case time complexity  $O(n^2)$ .

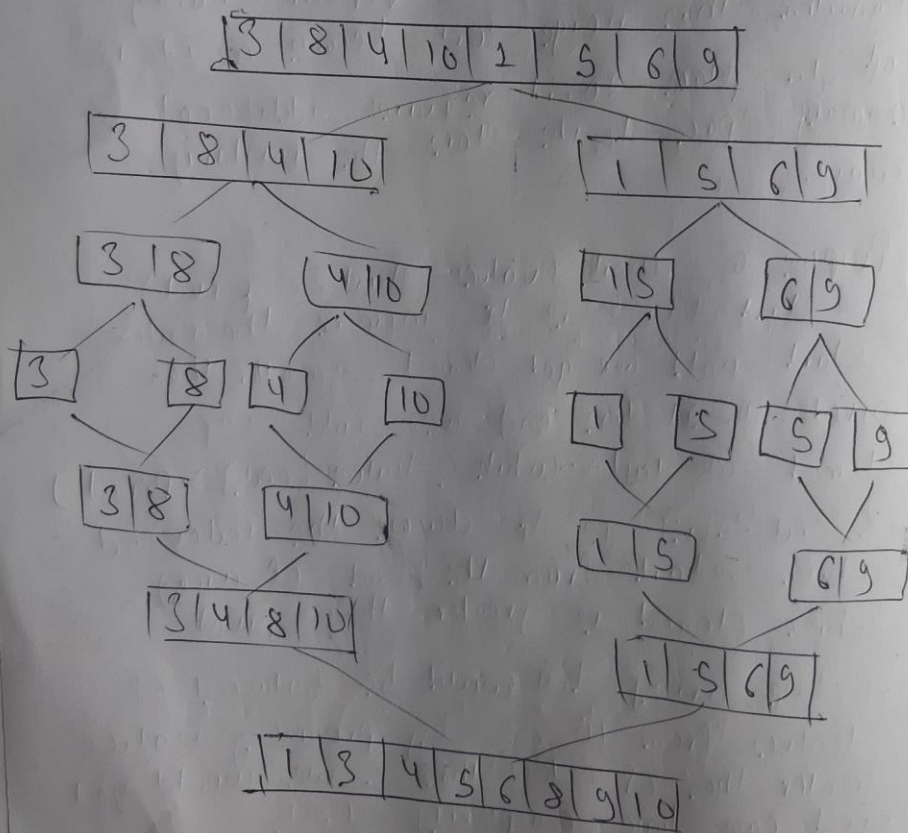
Time complexity for Quick Sort.

$$\text{Best case} = O(n \log n)$$

$$\text{worst case} = O(n^2)$$

$$\therefore \text{Average case} = O(n \log n)$$

8.) The Quick sort is the more popular sorting algorithm as it is in place because it takes extra space for recursive calls where merge sort takes  $O(n)$  extra space during process. To create Quick sort the recursive function will divide the array into left side first merged then after dividing toward left it is complete then dividing on right side is performed and merge for.



9). Soln

Bubble Sort

Best Case =  $O(n)$

Worst Case =  $O(n^2)$

Average Case =  $O(n^2)$

Insertion Sort

Best Case =  $O(n)$

Worst Case =  $O(n^2)$

Average Case =  $O(n^2)$

10). Soln

Merge Method (A, L, R)

$A[2] = L$ ,  $nR = R$ ,  $i = j = k = 0$ ;

while ( $i \leq nL$  &  $j \leq nR$ )

if ( $L[i] < R[j]$ )

$A[k] = L[i]$ ;

$i++$ ;

}

else

$A[k] = R[j]$ ;

$j++$ ;

}

$k++$ ;

}

while ( $i \leq nL$ )

$A[k] = L[i]$ ;

$i++$ ;

$j++$ ;

}

while ( $j \leq nR$ )

$A[k] = R[j]$

$j++$ ;

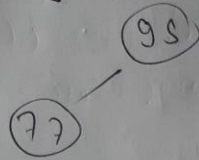
$k++$ ;

}

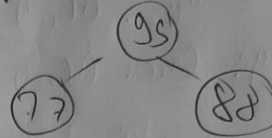


11) Sol: Max Heap.

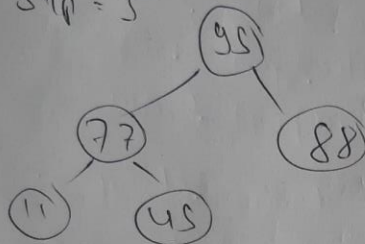
Step = 1



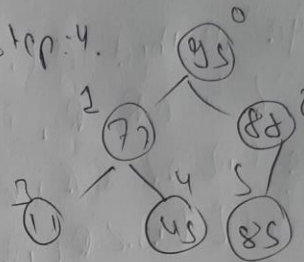
Step = 2



Step = 3

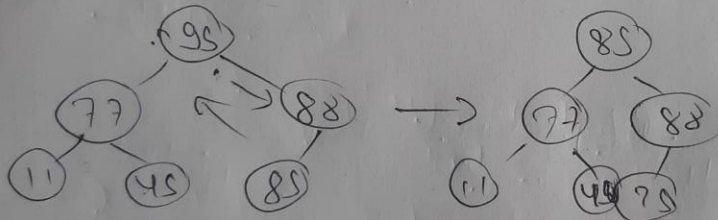


Step = 4



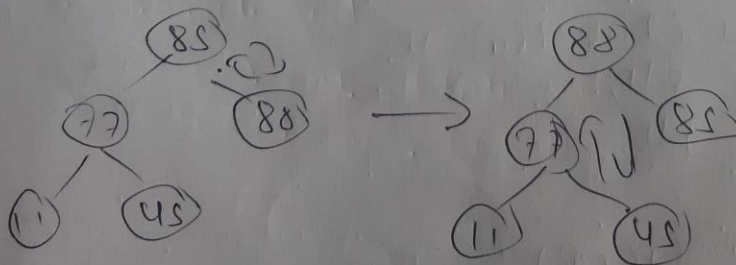
95	77	88	11	45	85
----	----	----	----	----	----

Step = 5



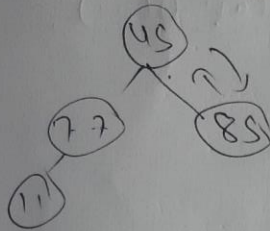
85	77	88	11	45	95
0	1	2	3	4	5

Step = 6

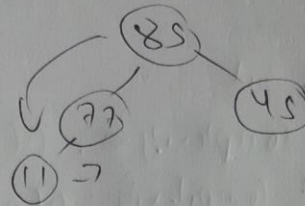


88	77	85	11	45	95
----	----	----	----	----	----

Step: 7

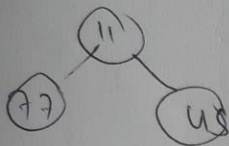


→



85	77	45	11	88	95
----	----	----	----	----	----

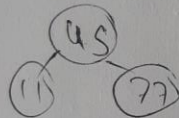
Step: 8



→



77	11	45	85	88	95
----	----	----	----	----	----



45	11	77	85	88	95
----	----	----	----	----	----

Step: 9



11	45	77	85	88	95
0	1	2	3	4	5



11	45	77	85	88	95
0	1	2	3	4	5

∴ This is called Sorted Array

12  $\rightarrow$  Merge sort doesn't have worst-case  $O(n^2)$ .

13  $\Rightarrow$   
a) Insertion Sort

Best complexity  $= O(n)$

Worst complexity  $= O(n^2)$

b) Bubble Sort

Best complexity  $= O(n)$

Worst complexity  $= O(n^2)$

c) Sequential Search

Best complexity  $= O(1)$

Worst complexity  $= O(n)$

d) Merge Sort

Best complexity  $= O(n \log n)$

Worst complexity  $= O(n \log n)$

e) Selection Sort

Best complexity  $= O(n^2)$

Worst complexity  $= O(n^2)$

f) Quick Sort

Best case  $= O(n \log n)$

Worst case  $= O(n^2)$



14)  $\rightarrow$  Insertion Sort should be faster than Quick Sort if an array which already sorted.

15)  $\rightarrow$  Sorting is known as stable if the input contains same keys and under going algorithm's order of input ~~is~~ which does not change.

Stable - Bubble Sort

Unstable - Quick Sort

16)  $\rightarrow$  Quick Sort is a well-known sorting algorithm developed by C.A.R. Hoare and on average, it makes  $O(n \log n)$  comparisons to sort  $n$  items.