# Thermal Study of Lithium Battery

## PH 213 Fall 2020


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#### Proposal

I would like to understand more complex circuitry and analyze the relationship between electricity and thermal dynamics. Specifically I would like to adapt an arduino powered charging circuit to do a study on the temperature changes of a battery. I will be modeling the temperature and increase in temperature of a battery as it relates to current, voltage, and time. I aim to use the experimental data collected from the arduino to further my models. I believe this project offers much opportunity for me to bring to bear a host of skills and tools. I will expand my circuitry knowledge from 212 by designing and wiring the circuit. There will be plenty of utilization of junction and loop laws to ensure it works as intended.The modeling of heat transfer and temperature rise will require a combination of both ohm's law and thermodynamics. I would not be surprised to see a differential equation or two pop up in the process. There is additional opportunity to explore waves and periodicity in the manipulation of the charging current, though this will be reserved as a secondary study.

The most simple form of my problem? I push a fixed current through a battery and calculate the heat generated with Joules equation then model the temperature rise with thermodynamics. In other words: I'll be calculating how many degrees per second the battery will rise at I current and V voltage.

I doubt the temperature change will be linear, who's to say how it will behave at very high temperatures or at very low temperatures. Perhaps the curve is quadratic, exponential, logarithmic, or even bounded. Modeling the temperature as a changing element will provide better insight. Beyond that, I will want to run iterative models to describe the relationship between temperature change and current change. What happens if we back off on current, will the heating slow down? I haven't even begun to wonder how many iterations will be needed to describe the cooling that happens due to the air around the battery.

I have already touched on this a bit. I think a first improvement is to compare model data with experimental data to see if there are any striking differences. A solid second step would be to iterate over different currents as previously mentioned. A big third step would be to include the heat transfer from battery to ambient air, perhaps Newton's cooling laws and differential calc can be employed ( I would be head over heels if this came to pass!). Long term thoughts: Modeling what happens at different extremes, prettier graphs of course, what happens between different batteries, measuring capacity of battery, using data for charging algorithms, and the list continues ad infinitum.

If even moderately successful I hope to return to this project in terms to come.

#### The Study (in process)

The purpose of this study will be to explore the relationship current has on battery temperatures during charging. To begin the study I will need to model the most basic approach as well as set up a circut to obtain experimental data

#### Variables

Our first task will be to mathematically describe the relationship in its most simple terms. Below is a table to keep easier track of variables and their units.

| Variable | Representing | Unit | Reference value / initial value |
|---|---|---|---|
| $H$ | energy | Joules | $I^2R$ |
| $I$ | current | amperes | 1,3.4 |
| $R$ | resistance | ohms | 0.013 |
| $t$ | time | seconds | 3600 |
| $m$ | mass | kilograms | 0.044 |
| $T_i$ | temp intial | degree Celsius | 20 |
| $T_f$ | temp final | degree Celsius | To Solve |
| $c$ | specific heat capacity | $\frac{J}{Kg*C^{\circ}}$ | 902 |

| Variable | Representing | Unit | Reference value / initial value |
|---|---|---|---|
| $T_a$ | Ambient Temp | degree Celsius | 20 |
| $k$ | Cooling Constant | unitless | ? |

We will ignore any cooling done for the moment.

Using Joule's equation

$$P = I^2 R$$

We know Energy is Power over Time therefore

$$H = I^2 Rt$$

Combining with our heat transfer equation

$$I^2 Rt = mc(T_f - T_i)$$

$$I^2 Rt = mcT_f - mcT_i$$

$$I^2 R = \frac{mcT_f - mcT_i}{t}$$

We will want to seperate out Final Temp for our programs sake, With some algebra we get:

$$T_f = T_i + \frac{I^2 Rt}{mc}$$

Now we will be iterating over 1 second intervals so theoretically we will pull out the t (now I am not entirely sure here, I wonder if I ought to be taking a derivative with respect to time but then again time only exists in once place so It would likely go to zero. More will be revealed) So anyway that gives us the following

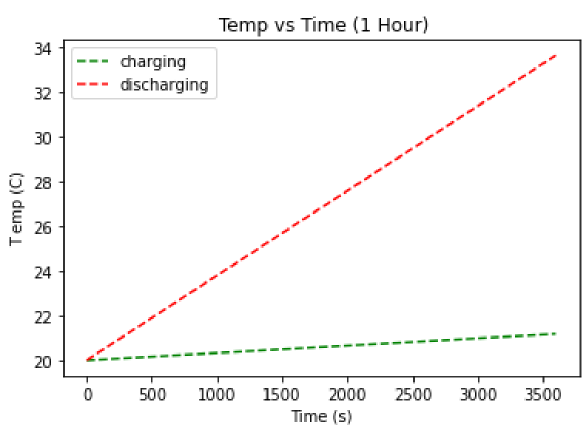
$$T_f = T_i + \frac{I^2 R}{mc}$$

The above equation describes the increase in temperature taking place due to circuit resistance in one second assuming a constant current mass and resistance. As lithium charging chemical process is endothermic and reportedly rather weak I imagine this will serve reasonably well for initial estimates without stepping into Newtons cooling differentials.

The internal resistance of a battery lessens as its temperature rises. For now we will ignore this. Resistance then is constant, our current will be prechosen, mass and specific heat capactiy will be constants as well. This gives us the only changing variables to be time and our temperature. Using an iterative solution we can loop over each second, solving for final temp and using our return from our previous iteration as our initial conditions. While it will create a quadratic order of growth we can nest the operation inside of another iteration to walk over the pre chosen currents thus returning us several data arrays that can be modeled.

In [3]:

```python
# Purpose: To model Thermal Cooling and heating of a lithium ion battery during charging.
# written for a single sony 18650 cell

# --------------- Here we will list variables-----------------------
import numpy as np
import matplotlib.pyplot as plt
from decimal import *

getcontext().prec = 8

def solve_temp (current,time):
    ''' Inputs: current : decimal, time: interger
        outputs:  returns numpy array of time and T_f '''

    # initializing variables
    r = Decimal(.013)  # Resistance in Ohms
    m = Decimal(.044)  # Mass of battery in kg
    t_i = Decimal(20)  # initial Temperature of Battery (room temp celsius)
    t_f = Decimal(0)  # Final temperature of Battery (will be solving for this )
    c = Decimal(902)  # Specific heat capacity in Joules per kg celsius
    time_array = np.array([])
    temp_array = np.array([])
    for t in range(1,time):
        step_1 = current ** 2
        step_2 = step_1 * r
        step_3 = step_2 / m
        step_4 = step_3 / c
        t_f = t_i + step_4
        t_i = t_f
        time_array = np.append(time_array, [t])
        temp_array = np.append(temp_array, [t_f])
        t+= 1

    return time_array, temp_array


time = 3600  # time (our iterable) in seconds
current =  [Decimal(1), Decimal(3.4)]  # Current in Amps recommend to be 1A charging and 3.4 Amps discharge
charging = solve_temp(current[0], time)
discharge = solve_temp(current[1],time)

plt.figure()

plt .subplot().set(xlabel = 'Time (s)', ylabel = 'T emp (C)', title = ' Temp vs Time (1 Hour)', )
plt.plot(charging[0], charging[1] , "g--", label = "charging")
#plt.subplot().set(xlabel = 'Time (s)', ylabel = 'Temperature (C)', title = ' Discharging',)
plt.plot(discharge[0],discharge[1],"r--",label = "discharging")
plt.legend()
plt.show()
```




Our next step is to add Newtons cooling equation to this model.

$$\frac{dT}{dt} = -k(T_o - T_a)$$

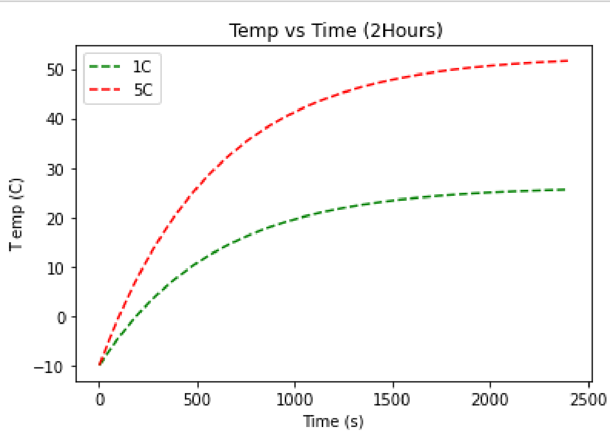
This being a differential equation, now if we think for a moment. We are iterating over time, and so we can shrink time by increasing our iterations. So feasibly we could multiply over the dt. We can do this in our loop but not on a piece of paper because we are constantly updating the temperature of our battery with each increment of time. Something a human couldn't do. What we will insert into the code will look something like this:

$$T_f = T_i - k(T_i - T_a)$$

As of this moment I am unsure what k will be however it seems feasible I can track it down. Now to plug this into our code.

In [31]:

```
# Purpose: To model Thermal Cooling and heating of a lithium ion battery during charging.
# written for a single sony 18650 cell

# --------------- Here we will list variables-----------------------
import numpy as np
import matplotlib.pyplot as plt
from decimal import *

getcontext().prec = 8

def solve_temp (current,time):
    ''' Inputs: current : decimal, time: interger
        outputs:  returns numpy array of time and T_f '''

    # initializing variables
    r = Decimal(.013)  # Resistance in Ohms
    m = Decimal(.044)  # Mass of battery in kg
    t_i = Decimal(-10) #initial Temperature of Battery (room temp celsius)
    t_f = Decimal(0)  # Final temperature of Battery (will be solving for this )
    t_a = Decimal(25) # Ambient temperature
    c = Decimal(902)  # Specific heat capacity in Joules per kg celsius
    k = Decimal(.0017)     # Cooling Constant (very roughly estimated from other papers)
    time_array = np.array([])
    temp_array = np.array([])
    t_change = 0

    for t in range(1,time):
        t_heating = t_i + (((current ** 2) * r)/m)/c  # Heating through internal resistance
        t_cooling =(k*(t_heating-t_a))  #newtons cooling
        t_f = t_heating - t_cooling
        t_i = t_f
        time_array = np.append(time_array, [t])
        temp_array = np.append(temp_array, [t_f])
        t+= 1

    return time_array, temp_array


time = 2400  # time (our iterable) in seconds
current =  [Decimal(2.5), Decimal(12)]  # Current in Amps recommend to be 1A charging and 3.4 Amps discharge
charging = solve_temp(current[0], time)
discharge = solve_temp(current[1],time)

plt.figure()

plt .subplot().set(xlabel = 'Time (s)', ylabel = 'T emp (C)', title = ' Temp vs Time (2Hours)', )
plt.plot(charging[0], charging[1] , "g--", label = "1C")
#plt.subplot().set(xlabel = 'Time (s)', ylabel = 'Temperature (C)', title = ' Discharging',)
plt.plot(discharge[0],discharge[1],"r--",label = "5C")
plt.legend()
plt.show()
```




The graph above is a model for a 1C discharge and 5C discharge (2.5 and 12 amps) Initial battery temperature is -10C and ambient temperature is 25C. These numbers were chosen to compare with similar models made in other studies.

The cooling constant will need to be figured out somehow. For the moment it is very much a number I made up that creates data along the lines of what one would expect. After about 2hrs the cooling effect negates the resistance heating and we see a plateau in temperature raise for the battery.

**Comparing with models and data.**

Taking a look at this graph from "Internal Temperature Prediction of Lithium-ion Cell Using Differential Voltage Technique" Which can be found in the resources.

We can see intially the x axis are different, however we can ignore this for now as we see the behavior of both graphs is bounded and we will focus on the end conditions. We see the general shape of the graph is very close, the temperatures are within 10C of the internal temps of this model. We can expect our model to further fit the graph as we have not yet accounted for the decrease in resistance over the rise of temperature.

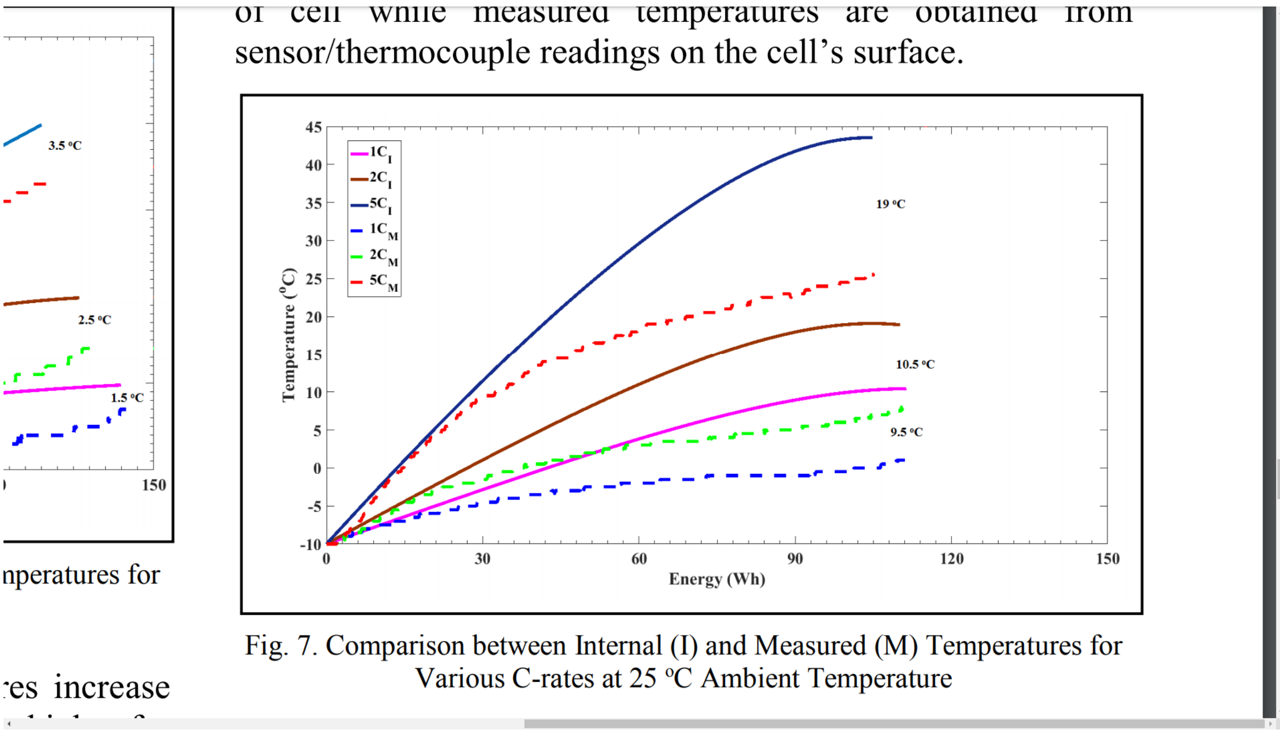



Fig. 7. Comparison between Internal (I) and Measured (M) Temperatures for Various C-rates at 25 °C Ambient Temperature

In [ ]: