Practice Examples

Example 1: Find Eigen values and corresponding Eigen vectors of $\begin{bmatrix}-1 & -2\\ -4 & -3\end{bmatrix}$.

Example 2: Find Eigenvalues and corresponding Eigenvector of $\begin{bmatrix}5 & 4\\ 1 & 2\end{bmatrix}$.

Example 3: Find eigenvalues and corresponding eigenvectors of $\begin{bmatrix}1/4 & -1/4\\ -1/12 & 5/12\end{bmatrix}$.

(Note: This is Inverse of matrix $A = \begin{bmatrix}5 & 3\\ 1 & 3\end{bmatrix}$)

Example 4: Find eigenvalues and corresponding eigenvectors of $\begin{bmatrix}28 & 24\\ 8 & 12\end{bmatrix}$.

(Note: This is square of matrix $A = \begin{bmatrix}5 & 3\\ 1 & 3\end{bmatrix}$)

Example 5: If $A = \begin{bmatrix}3 & -1 & 1\\ -1 & 5 & -1\\ 1 & -1 & 3\end{bmatrix}$, find eigenvalues for the following matrices:

a) A b) A^T C) A^{-1} D) $4A^{-1}$ E) A^2.

Example 6: Find Eigen values and corresponding Eigen vectors of $\begin{bmatrix}2 & -2 & 3\\ 1 & 1 & 1\\ 1 & 3 & -1\end{bmatrix}$.

Example 7: Find Eigen values and corresponding Eigen vectors of $\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$.

Example 8: Find eigenvalues and corresponding eigenvectors of $\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$.

Example 9: Find eigenvalues and corresponding eigenvectors of $\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$.

(non-repeated eigenvalues with symmetric matrix)

Example 10: Find eigenvalues and corresponding eigenvectors of $\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$.

Example 11: Find eigenvalues and eigenvectors of

1) $\begin{bmatrix}-5 & 2\\ 2 & -2\end{bmatrix}$ 2) $\begin{bmatrix}8 & -4\\ 2 & 2\end{bmatrix}$ 3) $\begin{bmatrix}0 & 0 & 0\\ 0 & 2 & 5\\ 0 & 0 & -1\end{bmatrix}$ 4) $\begin{bmatrix}4 & 0 & 0\\ 0 & 0 & 0\\ 1 & 0 & 3\end{bmatrix}$ 5) $\begin{bmatrix}3 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 3\end{bmatrix}$

6) $\begin{bmatrix}0 & 0 & -2\\ 1 & 2 & 1\\ 1 & 0 & 3\end{bmatrix}$ 7) $\begin{bmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{bmatrix}$ 8) $\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{bmatrix}$ 9) $\begin{bmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{bmatrix}$ 10) $\begin{bmatrix}8 & 0 & 3\\ 2 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$

11) $\begin{bmatrix}3 & 4 & 2\\ 0 & 1 & 2\\ 0 & 0 & 0\end{bmatrix}$ 12) $\begin{bmatrix}0 & 0 & 2\\ 0 & 2 & 0\\ 2 & 0 & 0\end{bmatrix}$

Example 12: Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}1 & 4\\ 5 & -1\end{bmatrix}$ and find its inverse.

Example 13: If $A=\begin{bmatrix}2 & 4\\ 1 & 1\end{bmatrix}$, find A^3 and A^{-3} using cayley- Hamilton theorem.

Example 14 : If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$then find A^2, A^3, A^{-1}, A^{-2} using Cayley-Hamilton theorem.

Example 15: Verify Cayley-Hamilton theorem for the following matrices

1) $\begin{bmatrix}1 & 0\\ 4 & 5\end{bmatrix}$ 2) $\begin{bmatrix}5 & 4\\ 1 & 2\end{bmatrix}$

Example 16: Using Cayley-Hamilton theorem, find the inverse of

1) $\begin{bmatrix}2 & 3\\ 3 & 5\end{bmatrix}$ 2) $\begin{bmatrix}5 & 3\\ 3 & 2\end{bmatrix}$.

Example 17: Find the characteristic equation of the matrix $A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$ and hence compute A^{-1}. Also express $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$ as a quadratic polynomial.

Example 18: Verify Cayley Hamilton theorem for the matrix $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ and hence find A^{-1}.

Example 19: If $A=\begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix}$, prove that $A^5-3A^4+A^3-7A^2+5A+I=61I-43A$.

Example 20: Find LU decomposition of following matrices (if exist):

1) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ 3) $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ 4) $\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

5) $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ 6) $\begin{bmatrix} 8 & 0 & 3 \\ 2 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ 7) $\begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ 8) $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$ 9) $\begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

Example 21: Find the LU decomposition of following matrices:

1) $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

2) $B = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

3) $E = \begin{bmatrix} 3 & 1 & 4 \\ 1 & 2 & 6 \\ 4 & 6 & 5 \end{bmatrix}$

4) $F = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ -4 & -6 & -3 \end{bmatrix}$

5) $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

6) $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

Example 22: Find the LU decomposition of following metrices:

1) $C = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

2) $D = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ -4 & -6 & -3 \end{bmatrix}$.