Practice Examples

1 Evaluate
$$\int_{0}^{1} \int_{0}^{2} (4-x-y) dx dy$$
. (Ans: 5)

2 Evaluate
$$\int_{0}^{1} \int_{0}^{x} e^{y/x} dA$$
 (Ans: $\frac{e-1}{2}$)

3 Change the order of integration in
$$\int_{0}^{a} \int_{y}^{a} \frac{x}{x^2 + y^2} dy dx$$
 and hence evaluate it. (Ans: $\frac{a\pi}{4}$)

4 Evaluate $\iint_R r^3 dr d\theta$; where R is the region bounded by the curves $r = 2\sin\theta$ and $r = 4\sin\theta$.

5 Evaluate
$$\int_{0}^{2} x^{4} (8-x^{3})^{-\frac{1}{3}} dx$$
. (Ans: $\frac{16}{3}\beta\left(\frac{5}{3},\frac{2}{3}\right)$)

6 Evaluate
$$\int_{0}^{\frac{\pi}{2}} \sqrt{\cot x} \, dx.$$

Find
$$\int_0^\infty 2^{-3x^2} dx$$
.

8 Change the order of integration in
$$\int_0^1 \int_0^{\sqrt{1-x^2}} x \, dy dx$$
 and hence evaluate it. (Ans:\frac{1}{3})

9 Find
$$\int_0^3 \int_0^{\sqrt{9-x^2}} x^2 + y^2 dA$$
 by changing in to polar coordinate system. (Ans: $\frac{81\pi}{8}$)

Find the area of a region enclosed by the cardioid
$$r = 1 - \cos \theta$$
.

Evaluate
$$\iiint dv$$
, where V is the region bounded by $1 \le x \le 2$, $2 \le y \le 4$, $2 \le z \le 5$. (Ans: 6)

12 Evaluate
$$\int_{1}^{2} \int_{2}^{3} \int_{1}^{0} xyz \, dy dz dx$$
. (Ans: $\frac{15}{8}$)