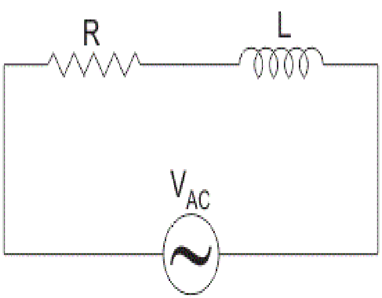
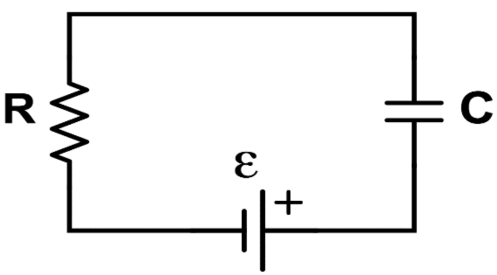
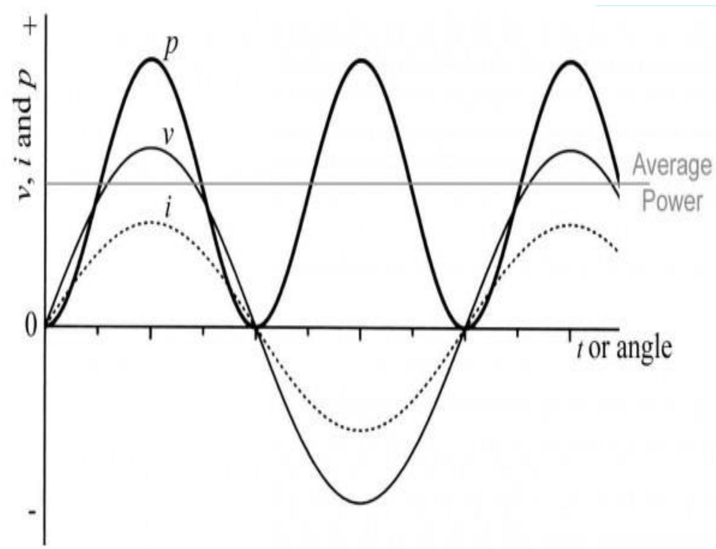
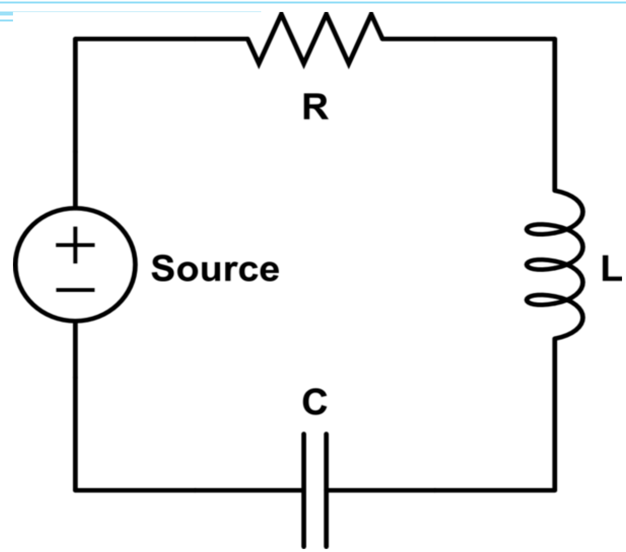
Unit-6
Single Phase AC Series Circuits

Prepared by:
Jigar Sarda
M & V Patel Department of Electrical Engineering
CHARUSAT
jigarsarda.ee@charusat.ac.in









Content

R-L Series Circuit

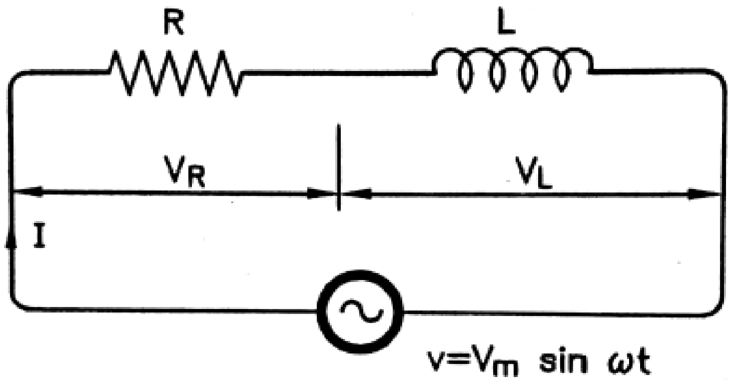
A pure resistance R and pure inductive coil of inductance L are connected in series.

V = Supply voltage

V_R = Potential difference across $R = IR$

V_L = Potential difference across $L = IX_L$

f = Supply frequency (Hz)

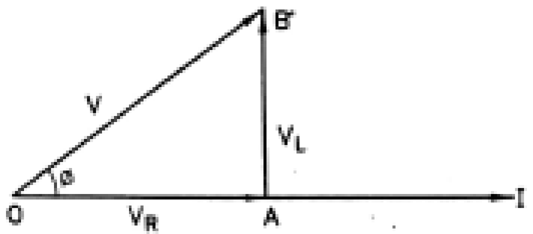


In series connection current will flow through same for each of elements.

So, We take current as a refrence phasor.

Phasor diagram

From diagram



$$OB^2 = OA^2 + AB^2$$

$$V = \sqrt{(V_R)^2 + (V_L)^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I\sqrt{R^2 + X_L{}^2}$$

$$\therefore \quad I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

Where $\sqrt{R^2 + X_L^2}$ = impedance of the circuit = Z

$$\therefore \quad I = \frac{V}{Z}$$

$\boxed{\textbf{Phase Angle} = \phi}$

$$\tan\phi = \frac{VL}{VR} = \frac{IX_L}{IR} = \frac{XL}{R}$$

Equation For Current

$$i = I_m \sin(\omega t - \phi)$$

Current I lags behind the applied voltage V by an angle ϕ

Instantaneous Power :

$$= V \times i$$

$$= V_m \sin \omega t \times I_m \sin (\omega t - \phi)$$

$$= V_m I_m \sin \omega t \cdot \sin (\omega t - \phi)$$

$$= \frac{V_m I_m}{2} 2 \sin \omega t \cdot \sin (\omega t - \phi)$$

$$= \frac{V_m I_m}{2} \left[\cos \phi - \cos (2\omega t - \phi)\right]$$

$$= \frac{1}{2} V_m I_m \cos \phi - \frac{1}{2} V_m I_m \cos (2\omega t - \phi)$$

- The average power for $\frac{1}{2} V_m I_m \cos\phi$ remains constant because it is independent of frequency.

The average power for $\frac{1}{2} V_m I_m \cos(2\omega t - \phi)$ is zero.

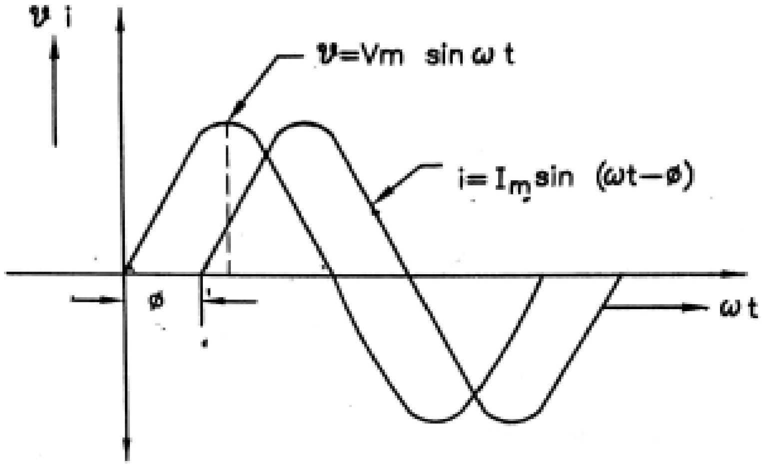
Average power

$$\text{Average power} = \frac{1}{2} V_m I_m \cos\phi$$

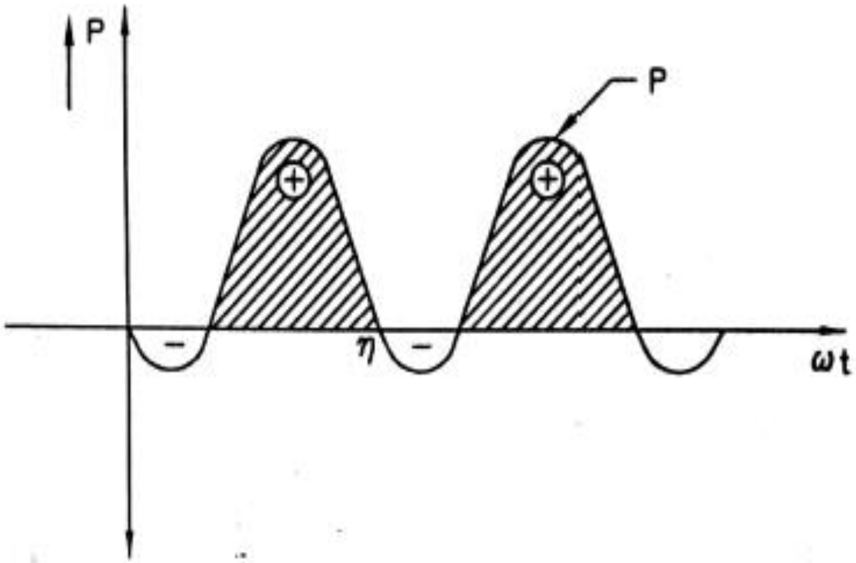
$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos\phi$$

$$= VI \cos\phi$$

- Waveform of v and i:-



- Power Waveform:-



Power factor :

$$\cos\phi = \frac{VR}{V}$$

$$= \frac{IR}{IZ}$$

$$= \frac{R}{Z}$$

Active, Reactive and Apparent Power

1. Apparent Power:-

- The product of r.m.s values of voltage and current in a.c. circuit is called apparent power.
- It is denoted by 'S'
- Apparent power = VI
- It is measured in volt amp (VA)

2. Active Power:-

- The power which is actually consumed in an a.c. circuit is called active power.

OR

- It is the power which is actually dissipated in the circuit resistance.

- It is denoted by ‘P’
- Active Power= Voltage*active component of current

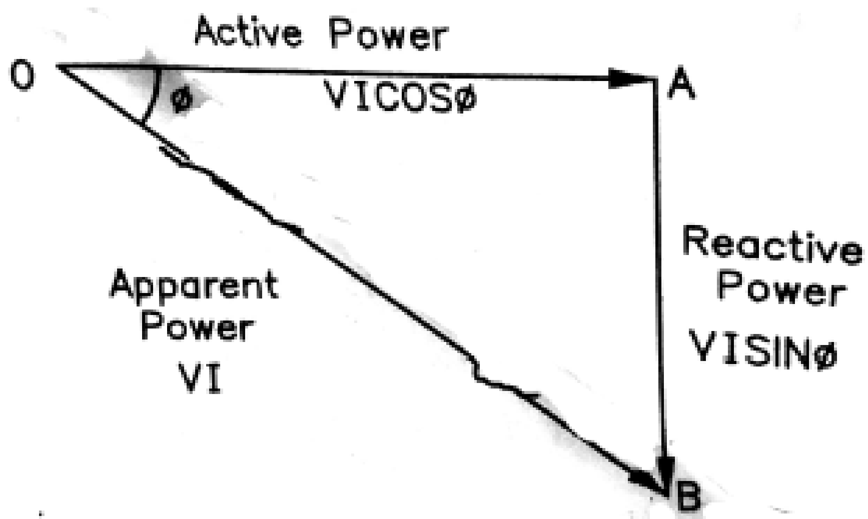
  = V*Icosø
- It is measured in Watt (W).

3. Reactive Power:-

- Power drawn by the circuit due to reactive component of current is called reactive power.
- It is denoted by ‘Q’
- Reactive power=Voltage*Reactive component of current

  = V*Isinø
- It is measured in (*)

- A purely inductor and pure capacitor do not consume any power, means that during +ve half cycle they received power and returned to the source in –ve half cycle. This power which flows back is called reactive power.



where OA = Active power= V*Icosø

OB = reactive power= V*Isinø

OC = Apparent power= VI

Examples:

1. A coil takes 2.5A, when connected across 200V,50Hz main. The power consumed by the coil is found to be 400W. Calculate resistance and inductance of coil.

Ans:- Given : $V = 200\ V$ Find :

$I = 2.5\ A$ $R = ?$

$f = 50\ Hz$ $L = ?$

$P = 400\ w$

Power $P = I^2 R = (2.5)^2 \times R$

$\therefore \quad 400 = (2.5)^2 \times R$

$$\therefore \quad R = \frac{400}{(2.5)^2} = \boxed{64\ \Omega}$$

$$\text{Impedance of coil } Z = \frac{V}{I} = \frac{200}{2.5} = 80\ \Omega$$

But $\quad Z = \sqrt{R^2 + X_L^2}$

$\therefore \quad Z^2 = R^2 + X_L{}^2$

$\therefore \quad X_L = \sqrt{R^2 - Z^2}$

$= \sqrt{80^2 - 64^2}$

$= 48\ \Omega$

But, $X_L = 2\pi f L$

$\therefore \quad 48 = 2\pi \times 50 \times L$

$$\therefore \quad L = \frac{48}{100\pi}$$

$$\boxed{= 0.153\ H}$$

2. A power dissipation of 1.2kW occurs when a sinusoidal voltage of 100volts r.m.s. is applied to an R-L series circuit and a current flow is given by i=28.3*sin(314 t - ø). determine: (i) Impedance, resistance and inductance (ii) Power factor of the circuit and (iii) Voltage across R and L. Draw vector diagram showing current and all voltages.

Ans:-

$P = 1.2\ \text{kW} = 1200\ \text{W}.$ $\quad V = 100\ \text{V}$

$\omega = 314\ \text{rad/sec}$ $\quad I = \dfrac{28.3}{\sqrt{2}} = 20\ \text{A}$

$$P = I^2 R$$

$$R = \frac{P}{I^2} = \frac{1200}{(20)^2}$$

$$\boxed{R = 3\ \Omega}$$

$$Z = \frac{V}{I} = \frac{100}{20}$$

$$\boxed{= 5\ \Omega}$$

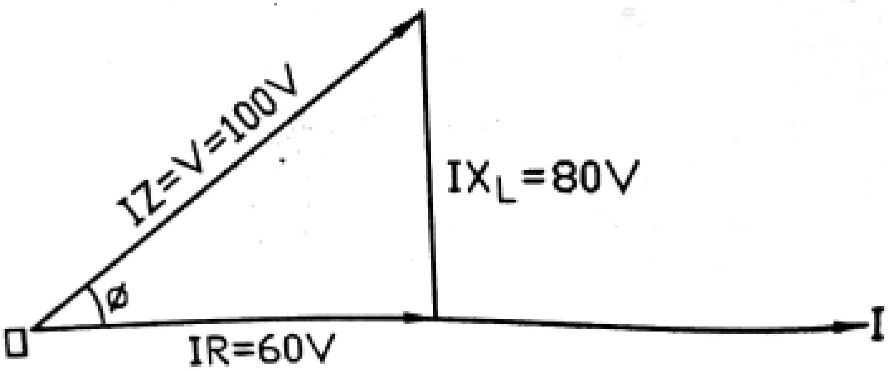
$$X_L = \sqrt{Z^2 - R^2} = \sqrt{5^2 - 3^2}$$

$$\boxed{= 4\ \Omega}$$

$$\text{Power factor} = \frac{R}{Z}$$

$$= \frac{3}{5}$$

$$\boxed{= 0.6 \text{ (lag)}}$$



$$\text{Voltage across } R = I \times R$$

$$= 20 \times 3$$

$$\boxed{= 60 \text{ V}}$$

$$\text{Voltage across } L = I \times X_L$$

$$= 20 \times 4$$

$$\boxed{= 80 \text{ V}}$$

3. A choke coil having a resistance of 10Ω and a inductance of 63.7mH is connected in series with a resistance of 5 Ω. The circuit is connected across 230V, 50Hz supply. Calculate (i) current (ii) voltage across the coil (iii) power factor (iv) voltage across 5Ω resistor (v) power. Draw complete phasor diagram.

Ans:-

$r = 10\ \Omega \qquad L = 63.7\text{ mH}, \qquad R = 5\ \Omega$

$V = 230\text{ V}. \qquad f = 50\text{ Hz}$

$$Z = \sqrt{(R+r)^2 + X_L^{\,2}}$$

$$= \sqrt{(10+5)^2 + (20)^2} = 25\ \Omega$$

$$X_L = \omega L$$

$$= 314 \times 63.7 \times 10^{-3}$$

$$= 20\ \Omega$$

$$Z_{coil} = \sqrt{r^2 + X_L^{\,2}}$$

$$= \sqrt{(10)^2 + (20)^2}$$

$$= 22.36\ \Omega$$

(i) $I = \dfrac{V}{Z}$

$= \dfrac{230}{25}$

$\boxed{= 9.20\ \text{A}}$

(ii) Voltage across $V_{coil} = I\, Z_{coil}$

$= 9.20 \times 22.36$

$\boxed{= 205.72\ \text{V}}$

(iii) Power factor $= \dfrac{R + r}{Z}$

$= \dfrac{10+5}{25}$

$\boxed{= 0.6\ \text{(lag)}}$

(iv) Voltage across 5 Ω resistor $= I \times R$

$= 9.20 \times 5$

$\boxed{= 46\ \text{V}}$

(v) Power $= I^2 (R + r)$

$= (9.2)^2 \times (10 + 5)$

$\boxed{= 1269.6 \text{ Watts}}$

OR

$P = V I \cos \phi$

$= 230 \times 9.2 \times 0.6$

$\boxed{= 1269.6 \text{ Watts}}$

R-C Series Circuit

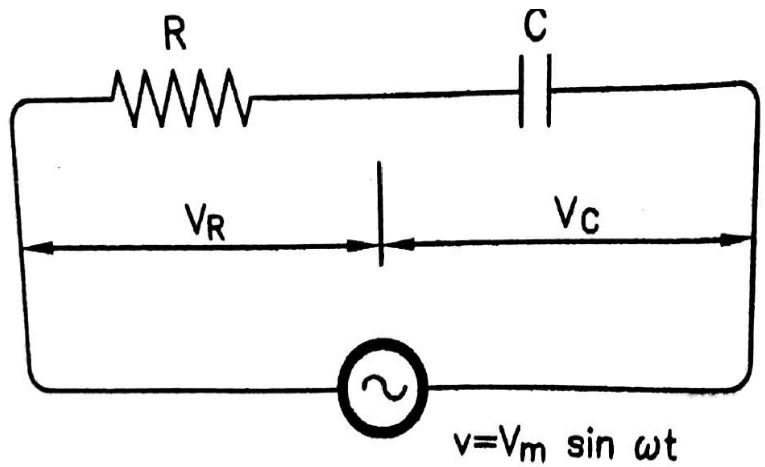
Circuit contain resistance and capacitance in series. With single phase a.c. supply.

V = Supply voltage

V_R = Potential difference across $R = IR$

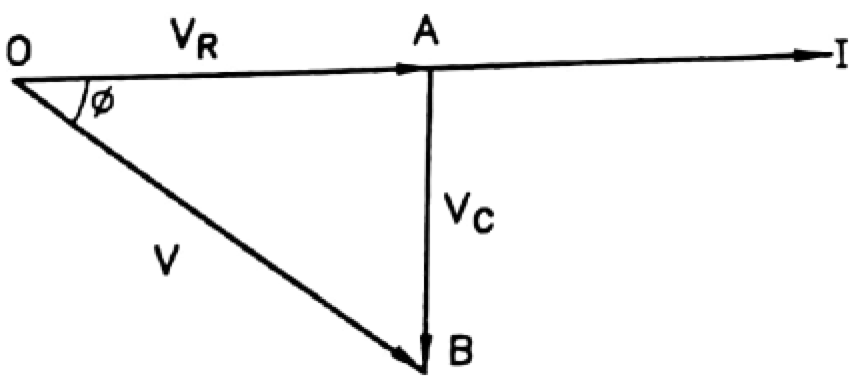
V_C = Potential difference across $C = IX_C$

f = Supply frequency



This is series connection. So we take current as a refrence.

Phasor diagram :



$$V = \sqrt{V_R^2 + V_C^2}$$

$$= \sqrt{(IR)^2 + (I\,X_C)^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}}$$

$$\text{Impedance} = Z = \sqrt{R^2 + X_C^2}$$

Phase angle

$$\tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R}$$

$$\therefore \quad \phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

Equation for current :

$$i = I_m \sin(\omega t + \phi)$$

Current I leads behind the applied voltage V by an angle ϕ.

Instantaneous Power :

$$= v \times i$$

$$= V_m \sin \omega t \times I_m \sin (\omega t + \phi)$$

$$= \frac{V_m I_m}{2} [\cos \phi - \cos (2\omega t - \phi)]$$

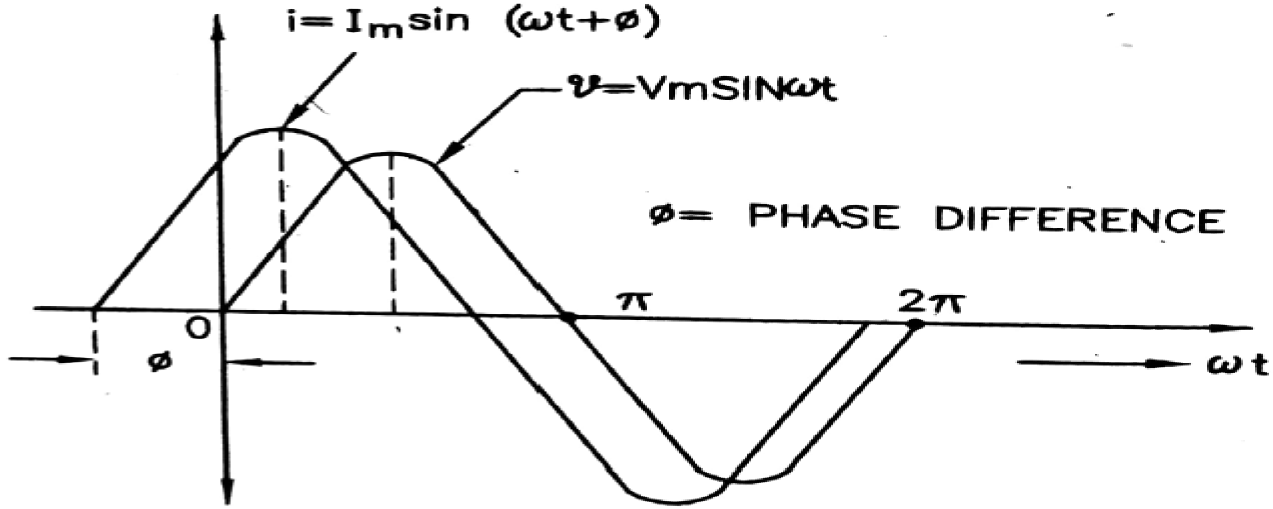
$$= \frac{1}{2} V_m I_m \cos \phi - \frac{1}{2} V_m I_m \cos (2\omega t - \phi)$$

Average Power :

$$\therefore \quad \text{Average Power} = \frac{V_m I_m}{2} \cos \phi$$

$$= V I \cos \phi$$

Wave form :



Power factor :

$$\cos\phi = \frac{VR}{V}$$

$$= \frac{IR}{IZ}$$

$$= \frac{R}{Z}$$

1. A resistor & a capacitor are connected in series across 230 V, ac supply. The current taken by the circuit is 6A for 50 Hz frequency. The current is reduced to 5 A., when the frequency of supply is decreased to 40 Hz. Determine the value of resistor & the capacitor.

Ans:- $I_1 = 6\ A$ $\quad I_2 = 5\ A$ $\quad R = ?$ $\quad C = ?$

$V = 230$ $\quad f_1 = 50\ Hz$ $\quad f_2 = 40\ Hz$

$$Z_1 = \frac{V}{I_1} = \frac{230}{6} = 38.3\ \Omega$$

$$X_{C_1} = \frac{1}{2\pi f_1 C} = \frac{1}{2\pi 50\, C}$$

$$X_{C_2} = \frac{1}{2\pi f_2 C} = \frac{1}{2\pi 40\, C}$$

$$Z_2 = \frac{230}{5} = 46\ \Omega$$

$$Z_1^2 = R^2 + X_{C_1}^2$$

$$(38.3)^2 = R^2 + X_{C1}^2 \qquad \ldots \text{(i)}$$

$$Z_2^2 = R^2 + X_{C2}^2$$

$$(46)^2 = R^2 + X_{C2}^2$$

$$2116 = R^2 + X_{C2}^2 \qquad \ldots \text{(ii)}$$

(ii) − (i)

$$649.1 = X_{C2}^2 - X_{C1}^2$$

$$649.1 = \left(\frac{1}{2\pi 40\, C}\right)^2 - \left(\frac{1}{2\pi 50\, C}\right)^2$$

$$\therefore \quad C = 0.0939 \times 10^{-3}\ \text{F}$$

$$\boxed{\mathbf{C = 93.9\ \mu F}}$$

$$2116 = R^2 + X_{C2}^{\;2}$$

$$2116 = R^2 + \left(\frac{1}{2\pi\, 40 \times 93.9 \times 10^{-6}} \right)^2$$

$$R^2 = 2116 - 1795.5$$

$$= 320.5$$

$$\boxed{\boldsymbol{R = 17.9\ \Omega}}$$

2. A capacitor and a resistor are connected in series to an ac supply of 50 V, 50 Hz. The current is 2A and the power dissipated in the circuit is 80 W. Determine the value of resistor & the capacitor.

Ans:-

$V = 50$ V, $\quad f = 50$ Hz, $\quad \omega = 2\pi f = 314$ rad/sec,

$I = 2$ A, $\quad P = 80$ W,

We know, $P = I^2 R$

$$\therefore \quad R = \frac{P}{I^2} = \frac{80}{(2)^2} = 20$$

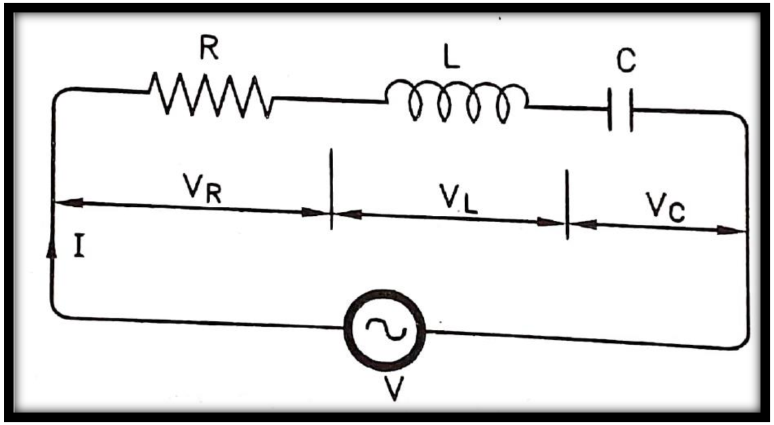
$$Z = \frac{V}{I} = \frac{50}{2} = 25\,\Omega$$

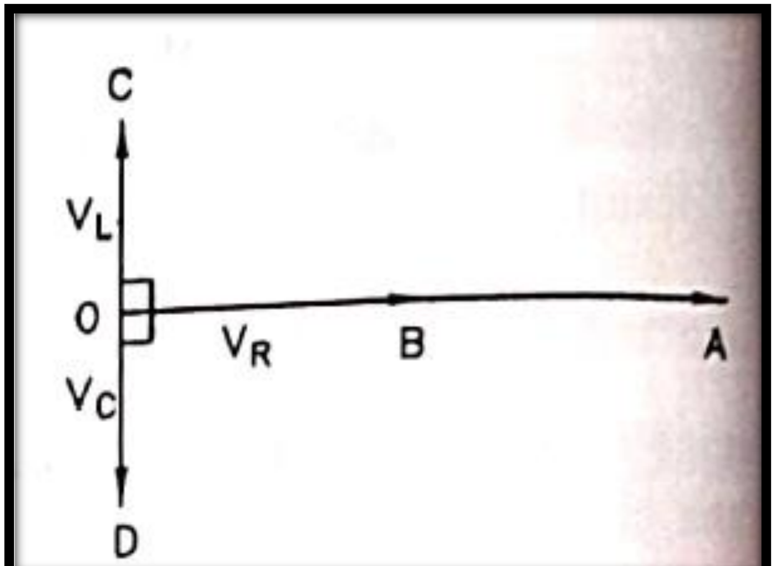
$$X_C = \sqrt{Z^2 - R^2} = \sqrt{25^2 - 20^2} = 15\,\Omega$$

$$X_C = \frac{1}{\omega C}$$

$$C = \frac{1}{\omega \times X_C} = \frac{1}{314 \times 15} = \boxed{2\ 12.3\ \mu\text{F}}$$

Series R-L-C Circuit





↳ It is shown in Fig.

V = Applied voltage

V_R = Potential difference across resistance $R = IR$

V_L = Potential difference across inductance $L = IX_L$

V_C = Potential difference across capacitance $C = IX_C$.

The phasor diagram is shown in fig.

The current (I) is taken as a refrence phasor.

(i) V_R is in phase with I.

(ii) V_L leads I by 90°

(iii) V_C lags I by 90°

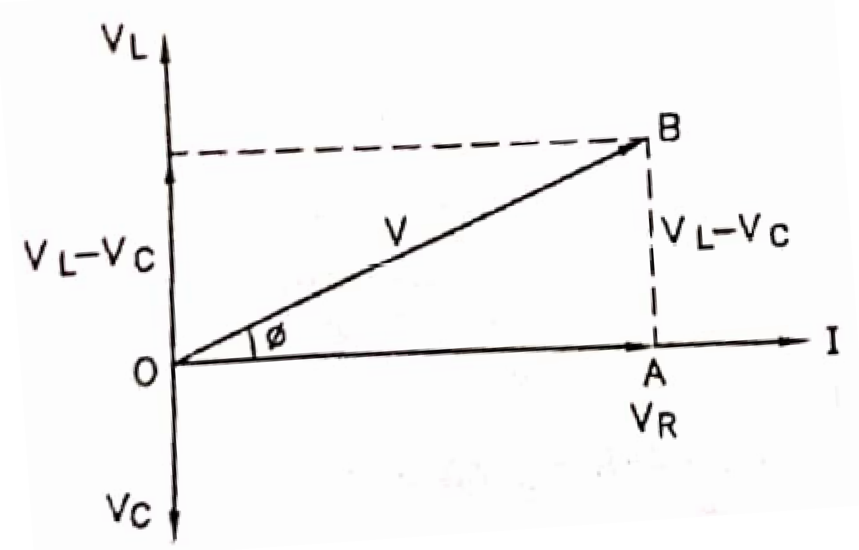
The equation for applied voltage $V = V_m \sin \omega t$

the equation for resultance current $i = I_m \sin (\omega t + *)$

If (i) $V_L > V_C$

↳ The circuit behaves as an inductive circuit

↳ The phyasor diagram is given below.



From diagram

$$OB^2 = OA^2 + AB^2$$

$$OB = \sqrt{OA^2 + AB^2}$$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$

. $Z = \sqrt{R^2 + (X_L - X_C)^2}$ is the oppositon offered to the flow of current is called impedance.

$$\cos\phi = \frac{VR}{V}$$

$$= \frac{IR}{IZ}$$

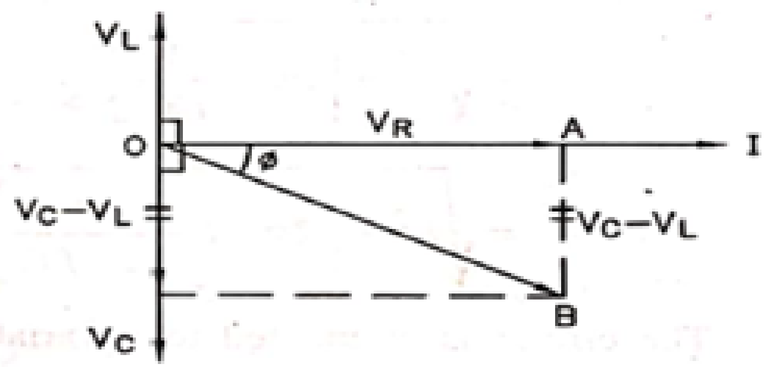
$$= \frac{R}{Z}$$

$$= \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

If (ii) $V_C > V_L$

↳ The circuit behaves as a capacitive circuit

↳ The phasor diagram is given below



$$OB^2 = OA^2 + AB^2$$

$$V^2 = V_R^2 + (V_C - V_L)^2$$

$$V = I\sqrt{R^2 + (X_C - X_L)^2}$$

$$I = \frac{V}{\sqrt{R^2 + (X_C - X_L)^2}}$$

$$= \frac{V}{Z}$$

Where Z = Impedance of the circuit

P.F. :

$$\cos\phi = \frac{VR}{V}$$

$$= \frac{IR}{IZ}$$

$$= \frac{R}{Z}$$

$$= \frac{R}{\sqrt{R^2 + (X_C - X_L)^2}}$$

1. A series circuit has resistance of 10 ohms, inductance 200/π mH and capacitance 1000/ π micro-farad. Calculate:

(i) The current, flowing in the circuit of supply voltage 250 V, 50 Hz.

(ii) Power factor for the circuit.

(iii) Power drawn from the supply.

(iv) Also draw the phasor diagram.

Solution :

$R = 10$ ohms $\qquad V = 250$ V

$L = \dfrac{200}{\pi}$ mH $\qquad f = 50$ Hz $\qquad C = \dfrac{1000}{\pi}\ \mu$F

$$X_L = \omega L = 2\pi f L$$

$$= 2\pi \times 50 \times \frac{200}{\pi} \times 10^{-3}$$

$$= 20 \text{ ohm}$$

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi \times 50 \times \dfrac{10^3}{\pi} \times 10^{-6}}$$

$$= 10 \text{ ohm}$$

Impedance of the circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{10^2 + (20-10)^2}$

$= 14.14$ ohm

(i) Current, $I = \frac{V}{Z}$

$= \frac{250}{14.14}$

$\boxed{= 17.68 \text{ A}}$

(ii) Power factor, $(\cos \phi) = \frac{R}{Z}$

$= \frac{10}{14.14}$

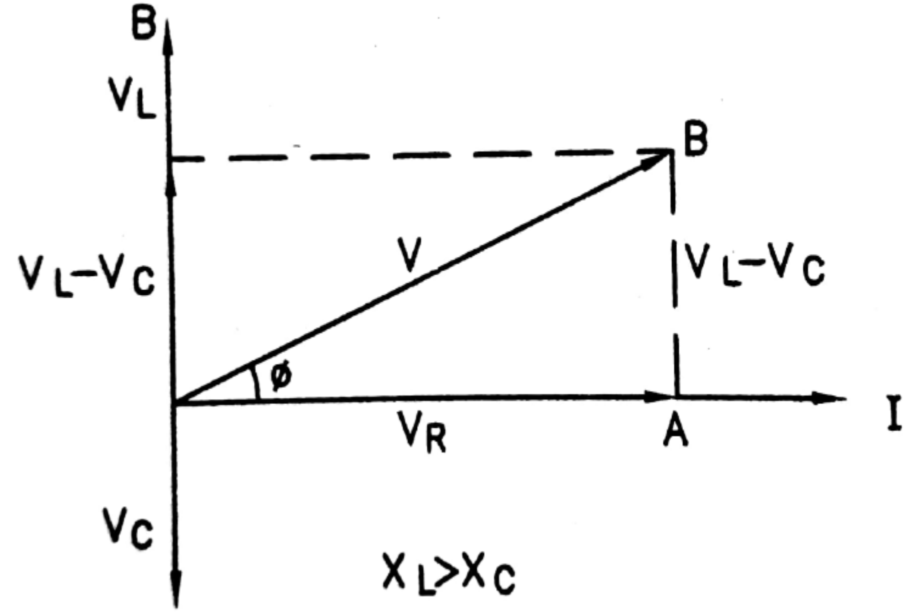
$\boxed{= 0.707 \text{ (lag)}}$ $[\because X_L > X_C]$

(iii) Power, $P = V I \cos \phi$

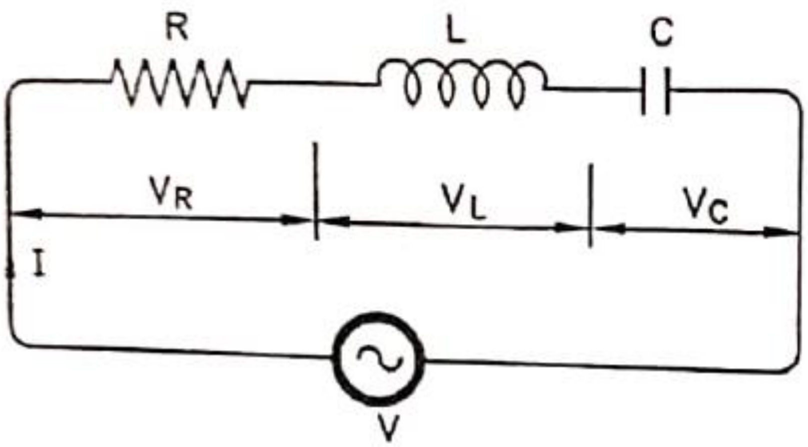
$= 250 \times 17.68 \times 0.707$

$= 3124.94$ W

$\boxed{= 3.125 \text{ kW}}$



Resonance in R-L-C Series Circuit



↳ Let us consider R-L-C series circuit :

↳ The applied voltage

$$V = V_R + V_L + V_C$$

$$= I_R + j\,IX_L - j\,IX_C$$

$$= I\,(R + j\,X_L - j\,X_C)$$

$\therefore$ Impedance $Z = (R + j\,X_L - j\,X_C)$

the magnitude of the impendance is given by

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2}$$

$$= \sqrt{R^2 + \left[2\pi f L - \frac{1}{2\pi f C}\right]^2}$$

- The circuit is connected to a variable frequency a.c. source
- We know that inductive reactance X_L is directly proportional to the frequency and capacitive reactance X_C is inversely proportional to frequency.
- So if frequency increase, X_L increase while X_C decrease.
- At one frequency the value of inductive reactance (X_L) becomes equal to capacitive reactance (X_C) is called the **resonant frequency**.

So, at resonance

$$X = 0$$

$$X_L - X_C = 0$$

$$X_L = X_C$$

$$\omega_r L = \frac{1}{\omega_r C}$$

$$\omega_r^2 = \frac{1}{LC}$$

$$\therefore \omega_r = \frac{1}{\sqrt{LC}}$$

$$2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

under this condition

$$Z = \sqrt{R^2 + X^2}$$

$$= R \quad [\because X = X_L - X_C = 0]$$

↳ The impedance becomes minimum and equals R and the current becomes maximum.

$$\boxed{I_{max} = \frac{V}{Z} = \frac{V}{R} \quad [\because Z = R]}$$

Q-FACTOR

↳ The Q-factor of an R-L-C series circuit is given below.

↳ It is given by the voltage magnification produced in the circuit at resonance.

↳ We have seen that at resonance, current has maximum value.

$$I_m = \frac{V}{R}, \quad \text{Supply voltage } V = I_m R$$

↳ Voltage across inductor or capacitor is given by

$$V_C = I_m X_L \quad [\text{or } V_C = I_m X_C]$$

$$\text{Q-factor} = \text{Voltage magnification} = \frac{\text{Voltage across } L\,[\text{or } C]}{\text{Applied voltage}}$$

$$= \frac{V_L}{V} \quad \text{or} \quad \frac{V_C}{V}$$

$$= \frac{I_m X_L}{I_m R} \quad \text{or} \quad \frac{I_m X_C}{I_m R}$$

$$= \frac{X_L}{R} \quad \text{or} \quad \frac{X_C}{R}$$

$$= \frac{2\pi f_r L}{R} \quad \text{or} \quad \frac{1}{2\pi f_r CR}$$

$$= \frac{\omega_r \cdot L}{R} \quad \text{or} \quad \frac{1}{\omega_r CR}$$

but at resonance

$$\omega_r^2 = \frac{1}{LC}$$

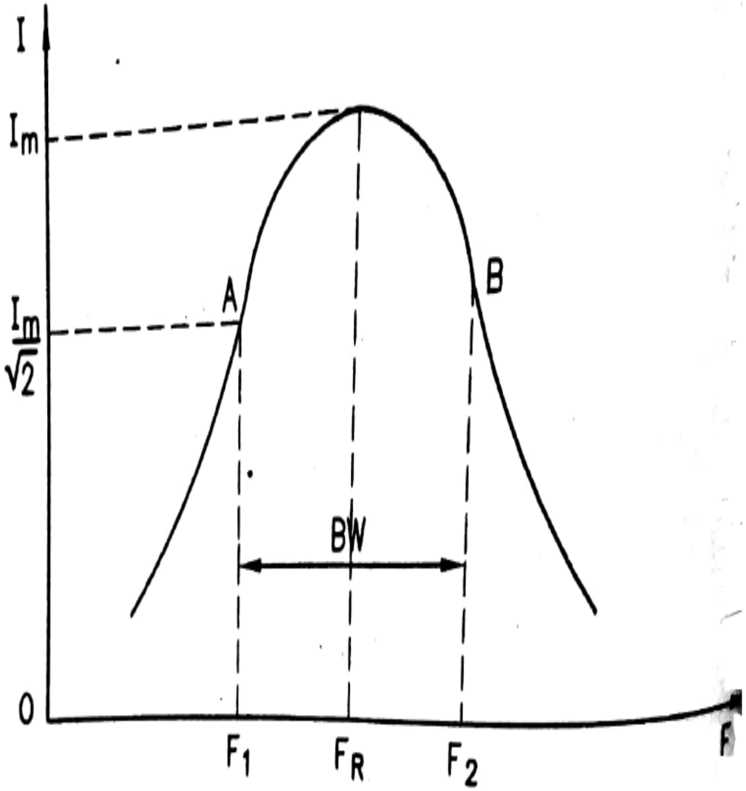
$$\therefore \quad \omega_r = \frac{1}{\sqrt{LC}}$$

$$\text{So} = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} \quad \text{or} \quad \frac{\sqrt{LC}}{CR}$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}} \qquad = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\therefore \quad \boxed{\text{Q - factor} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Bandwidth of Resonant circuit



↳ Bandwith of a circuit is given by the band of frequencies with lies between two points on either side of the resonant frequency. Where current falls to $\frac{1}{\sqrt{2}}$ of its maximum value.

$$\therefore \text{B.W.} = \frac{\text{Resonant frequency}}{\text{Q-factor}}$$

1. An RLC series circuit has R=10 Ω, L=10 mH and C= 1μF has an applied voltage of 230 V at resonant frequency. Calculate the resonant frequency, the current in the circuit and voltages across the each elements at resonance. Find also the Q-factor and bandwidth.

Solution :

$R = 10\ \Omega$

$L = 10\ \text{mH} = 10 \times 10^{-3}\ \text{H} = 10^{-2}\ \text{H}$

$C = 1\ \mu\text{F} = 1 \times 10^{-6}$ F

$V = 230\ \text{V}$

Resonant frequency, $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{10^{-2} \times 10^{-6}}}$$

$$= \frac{10^4}{2\pi}$$

$$\boxed{= 1591.55\ \text{Hz}}$$

Current at resonance, $I = \dfrac{V}{R}$

$$= \frac{230}{10}$$

$$\boxed{= 23\ \text{A}}$$

$$X_L = 2\pi f_r L$$

$$= 2 \times \pi \times 1591.55 \times 10^{-2} = 100\ \Omega$$

$$X_C = \frac{1}{\omega C}$$

$$= \frac{1}{2 \times \pi \times 1591.55 \times 10^{-6}} = 100\ \Omega$$

$$\text{Voltage across } R = IR$$

$$= 23 \times 10 = 230 \text{ V}$$

$$\text{Voltage across } L = I\, X_L$$

$$= 23 \times 100 = 2300 \text{ volt}$$

$$\text{Voltage across } C = I\, X_C$$

$$= 23 \times 100 = 2300 \text{ volt}$$

$$\text{Q–factor } \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{10^{2}}{10^{6}}} = \boxed{10}$$

$$\text{Band width} = \frac{f_r}{Q} = \frac{1591.55}{10} = \boxed{159.155 \text{ Hz}}$$