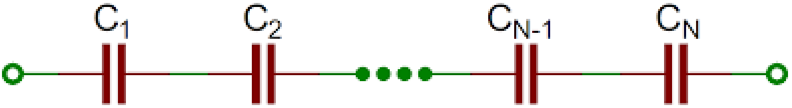
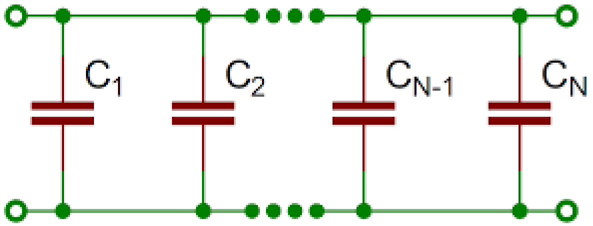
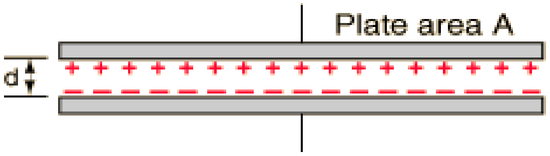
Unit-3
Electrostatic



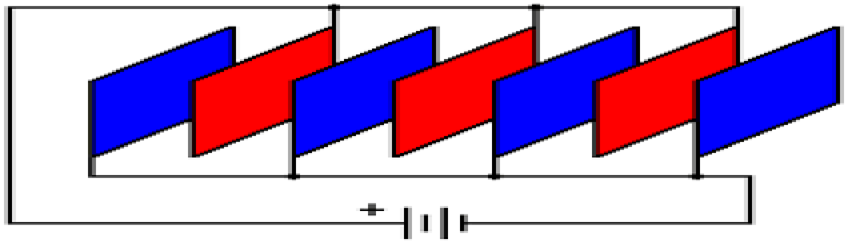
$$\frac{1}{C_{Tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_{N-1}} + \frac{1}{C_N}$$



$$C_{Tot} = C_1 + C_2 + \ldots + C_{N-1} + C_N$$



$$C = \frac{\varepsilon A}{d} = \frac{k\varepsilon_0 A}{d}$$

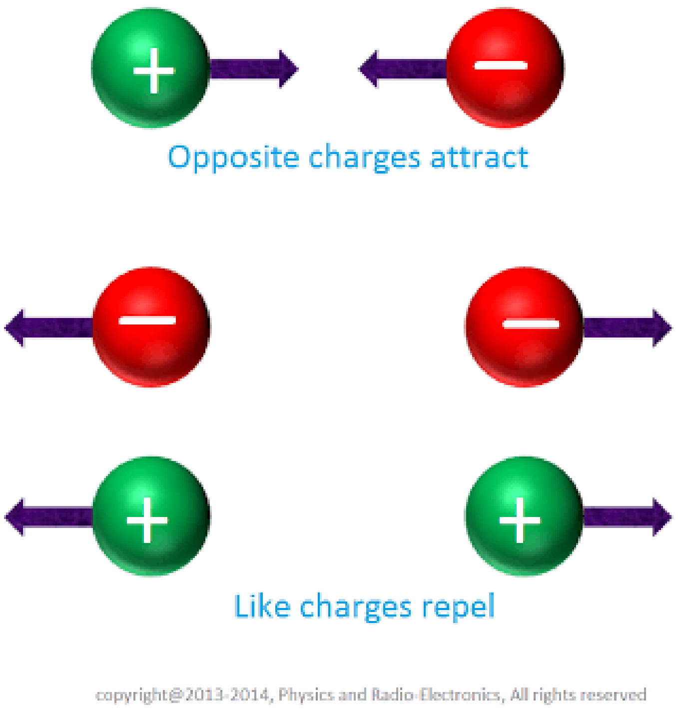


Content

- Coulomb's laws of Electrostatics
- Electric fields, Electric field strength
- Electric flux density, Relative Permittivity
- Dielectric Strength, Capacitors in Parallel and Series
- Calculation of capacitance of parallel plate
- Calculation of capacitance of multi plate Capacitor
- Examples

Coulomb's laws of Electrostatics

- **First Law:-** Like charges repel each other while unlike charges attract each other.



Coulomb's laws of Electrostatics

- **Second Law:-** Coulomb's law states that the magnitude of the electrostatic force between two point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.

$$F = k\frac{q_1 \times q_2}{d^2} \Rightarrow$$

$$F = \frac{1}{4\pi\varepsilon o\varepsilon r}\frac{Q_1 Q_2}{d^2}$$

$$K = \frac{1}{4\pi\varepsilon o\varepsilon r}$$

$$K = \frac{1}{4 \times 3.14 \times 8.854 \times 10^{-12}}$$

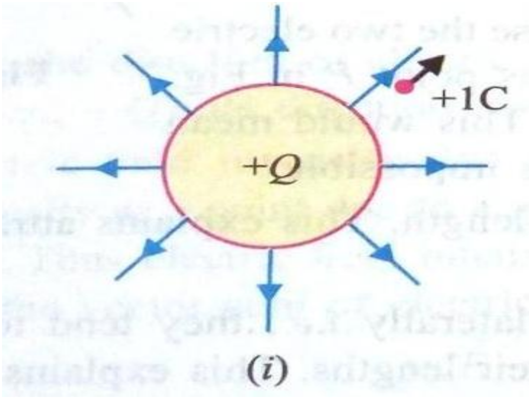
$$K = 9 \times 10^9 \text{ approx}$$

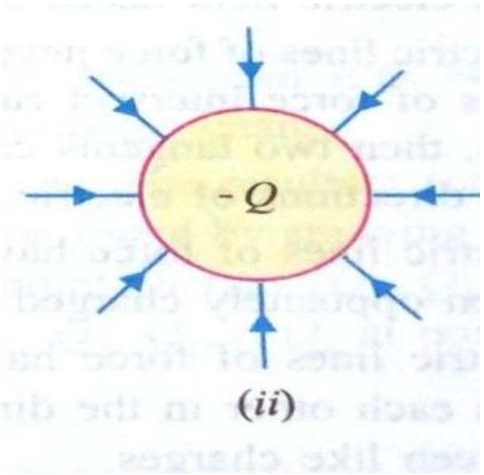
$$F = 9 \times 10^9 \frac{Q_1 Q_2}{d^2} N\ (air\ or\ vaccum)$$

$$F = 9 \times 10^9 \frac{Q_1 Q_2}{\varepsilon r d^2} N\ (in\ a\ medium)$$

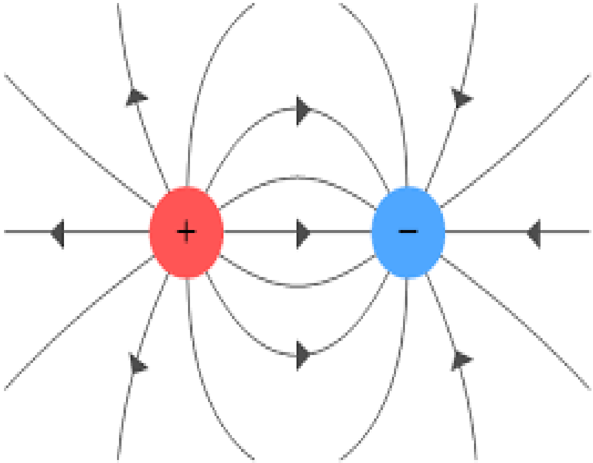
Electric Field

- The space or field in which a charge experiences a force is called an **electric field or electrostatic field.**

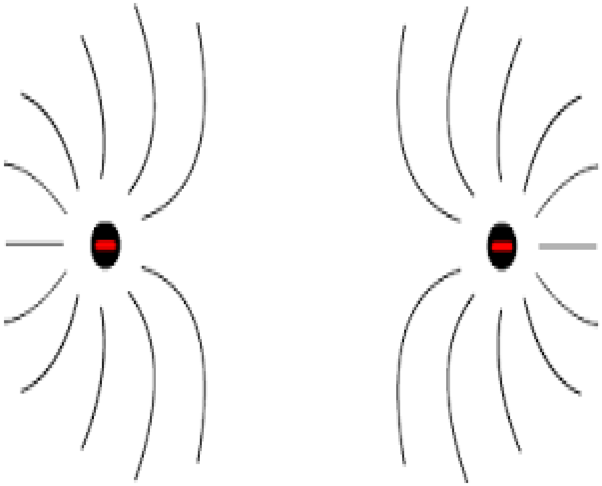




- Fig (i) shows electric field due to an isolated positively charged sphere. A unit positive charge placed near it will experience a force directed radially away from the sphere.
- Therefore, the direction of electric field will be radially outward.
- For the negatively charged sphere shows in Fig (ii), the force acting on the unit positive charge would be directed radially towards the sphere.



- Fig. (iii) shows the electric field between a positive charge and a negative charge.



- Fig. (iv) shows the electric field between two similarly charged bodies.

Electric Field Strength (E)

- Electric field strength at a point in an electric field is the force acting on a unit positive charge placed at that point.

- Its direction is the direction along which the force acts.

- Electric field strength at a point, $E = \frac{F}{+Q}$ N/C

Where Q= Charge in coulomb placed at that point

F= Force in newton's acting on Q coulomb

Electric flux density (D)

- The flux per unit area measured at right angles to the direction of the electric flux is known as electric flux density.
- It is denoted by the D and its unit is coulomb per square meter (C/m2).

$$D = \frac{\psi}{A}$$

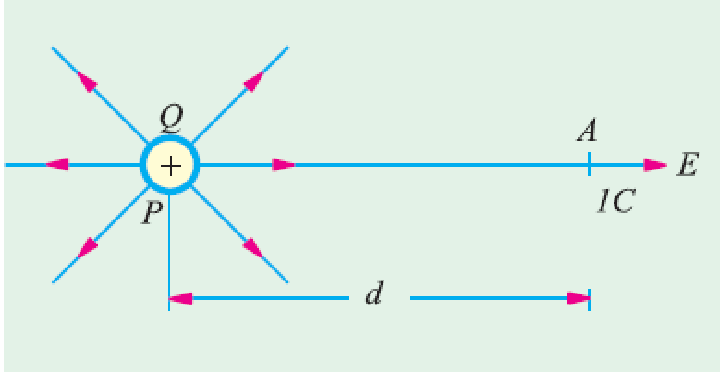
where,

Ψ= electric flux in coulomb

A= area in m2 through which flux is passing.

Relation between D and E

- Consider a charge of (+Q) coulombs placed in a medium of relative permittivity ε_r as shown in fig.



- To determine electric flux density at a point A consider and imaginary sphere passing through point A and having radius d.
- The electric flux emanated by the charge in all directions = Q coulombs.
- The surface area of the imaginary sphere = $4 * \pi * d^2$
- Flux density at A = $\frac{flux}{area} = \frac{Q}{4 * \pi * d^2}$

Relation between D and E

- Electric field intensity at point A,

  = the force acting on a unit positive charge when placed at A.

$$E = \frac{Q * 1}{4 * \pi * \varepsilon_o * \varepsilon_r * d^2}$$

$$= \frac{1}{\varepsilon_o * \varepsilon_r} * \frac{Q}{4 * \pi * d^2}$$

$$= \frac{1}{\varepsilon_o * \varepsilon_r} * D \quad [D = \frac{Q}{4 * \pi * d^2}]$$

- Hence, flux density (D) at any point in an electric field is $\varepsilon_o \varepsilon_r$ times the electric field intensity (E) at that point.

Absolute and Relative Permittivity

- If an insulating material is placed between the two parallel conducting plates and charge is placed on the plate, electric field is produced in the medium between the plates.
- The intensity of field so produced depends on the dielectric material placed between the plates.
- The ability of dielectric material to permit the electric field to be passed through it is called the permittivity.
- It is denoted by **ε.**
- **This is called the absolute permittivity.**
- Vacuum and air are also dielectric materials and they too have permittivity.
- It is denoted by $\boldsymbol{\varepsilon_0}$ and its value is **8.854*10e-12** farad/metre.

- When permittivity of air or vacuum is taken as reference and the permittivity of other dielectric material is compared with it, we get the relative permittivity of that material.
- It is denoted by $\boldsymbol{\varepsilon_r}$.

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

where

ε = absolute permittivity of a material

ε_o = absolute permittivity of air or vacuum

ε_r = relative permittivity of a material

1. **Electric Potential:-** In electric field, electric potential at a point is the work done to bring unit positive charge from infinity to that point against the filed.

- Its unit is volt.

Electric Potential= Work / Charge = W/Q

2. **Electric Potential Difference:-**

$$V_A - V_B = \frac{Q}{4*\pi*\varepsilon_o*\varepsilon_r} * [\frac{1}{d_1} - \frac{1}{d_2}]$$

3. **Potential Gradient:-** The change in potential due to the charge in distance in the direction of electric force is called the potential gradient.

- It is denoted by g.

$$g = \frac{dv}{dx}$$

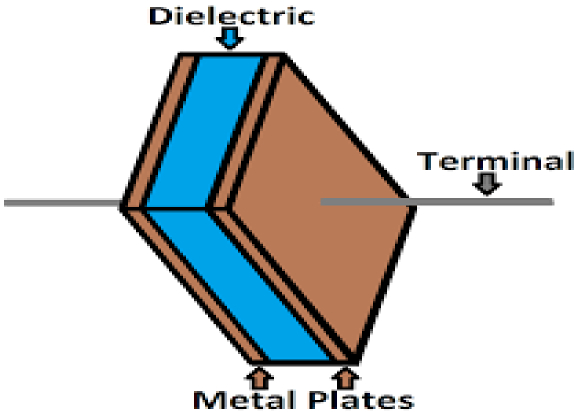
dv= change in potential

dx= change in distance

Dielectric Strength

- The maximum voltage which a unit thickness of a dielectric can withstand without being punctured by the spark is called its breakdown potential or dielectric strength.
- In other word, it is the minimum potential gradient necessary to cause breakdown of an insulator.
- Dielectric strength decreases with increase in the thickness of the insulator i.e. doubling the thickness of the insulator does not double the safe working voltage.

1. **Capacitor:-** When dielectric material is placed between the two conducting plates, capacitor is formed.



2. **Capacitance:-** The property of a capacitor to store charge (electricity) is called its capacitance.

- It is the capability of a capacitor to store charge on it.
- The charge on the plates is proportional to the potential difference applied across it.

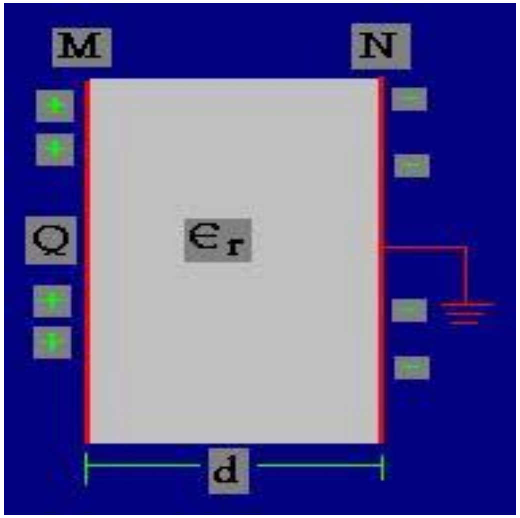
$$Q \propto V \quad \Longrightarrow \quad Q = CV$$

Where C= Capacitance

Capacitance of parallel plate capacitor

a. Uniform dielectric medium:-

- Consider a capacitor having two parallel plates of area A m2 separated by a dielectric of thickness d meters with relative permittivity ε_r as shown in figure.



- Let the charge Q be established on application of V volts across the plates.
- Since the electric field between the plates is uniform.

- Electric flux density $D = \frac{Q}{A}$ (1)
- Electric field intensity $E = \frac{V}{d}$ (2)

Also as we know that $D = \varepsilon_o \varepsilon_r E$ (3)

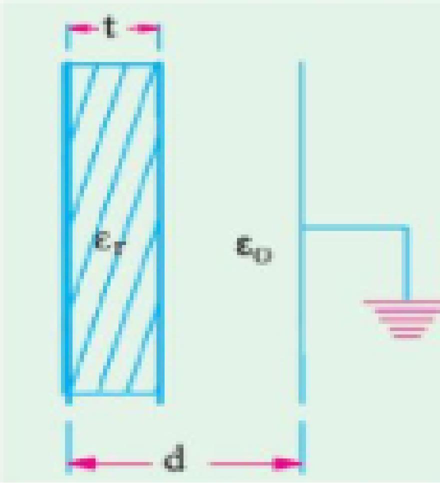
$$\frac{Q}{A} = \varepsilon_o \varepsilon_r \frac{V}{d}$$

$$\frac{Q}{V} = \varepsilon_o \varepsilon_r \frac{A}{d} \qquad (4)$$

According to definition

$$C = \frac{Q}{V} \Rightarrow \boxed{C = \frac{\varepsilon_o \varepsilon_r A}{d}}$$

b. Partly air medium:-



- As shown in fig, the medium consists partly of air and partly of parallel-sided di-electric slab of thickness t and relative permittivity ε_r.
- The dielectric flux density $D = \frac{Q}{A}$ the same in both media.
- $E_1 = \frac{D}{\varepsilon_0 * \varepsilon_r}$ in the medium
- $E_2 = \frac{D}{\varepsilon_0}$ in the air
- Total potential difference between plates,

$$V = E_1 * t + E_2 * (d-t)$$

$$= \frac{D}{\varepsilon_0 * \varepsilon_r} * t + \frac{D}{\varepsilon_0} * (d-t)$$

$$= \frac{D}{\varepsilon_0} \left(\frac{t}{\varepsilon_r} + d - t\right)$$

- $$V = \frac{Q}{\varepsilon_0 * A} * [d - (t - \frac{t}{\varepsilon_r})]$$

$$\frac{Q}{V} = \frac{\varepsilon_0 * A}{[d - (t - \frac{t}{\varepsilon_r})]}$$

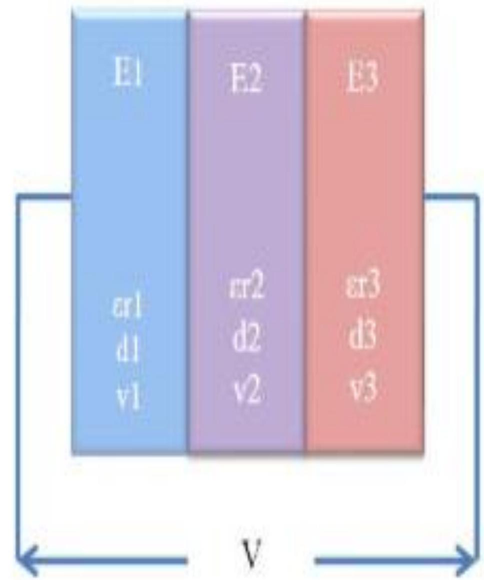
$$C = \frac{\varepsilon_0 * A}{[d - (t - \frac{t}{\varepsilon_r})]}$$

- If the medium were totally air i.e. t= 0 then capacitance would have been

$$\boxed{C = \frac{\varepsilon_0 * A}{d}}$$

C. Composite dielectric medium

- If V is the total potential difference across the capacitor plates and V1, V2, V3 the potential differences across the three dielectric slabs, then
- $$V = V_1 + V_2 + V_3$$



$$= E_1 * d_1 + E_2 * d_2 + E_3 * d_3$$

$$= \frac{D}{\varepsilon_0 * \varepsilon_{r1}} * d_1 + \frac{D}{\varepsilon_0 * \varepsilon_{r2}} * d_2 + \frac{D}{\varepsilon_0 * \varepsilon_{r3}} * d_3$$

$$= \frac{D}{\varepsilon_0} * \left(\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right) = \frac{Q}{\varepsilon_0 * A} * \left(\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right)$$

$$\boxed{C = \frac{Q}{V} = \frac{\varepsilon_0 * A}{\left(\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right)}}$$

1. A parallel plate capacitor has plate area 4 cm2. The plates are separated by three slabs of different dielectric materials of thickness 0.3, 0.4 & 0.3 mm with relative permittivity 3, 2.5 & 2. Calculate combine capacitance and dielectric stress.

Solution:- $A = 4\ cm^2$

$= 4 \times 10^{-4}\ m^2$

$d_1 = 0.3\ mm \qquad \varepsilon_{r1} = 3$

$d^2 = 0.4\ mm \qquad \varepsilon_{r2} = 2.5$

$d_3 = 0.3\ mm \qquad \varepsilon_{r3} = 2$

$$C = \frac{\varepsilon_0 A}{\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}}$$

$$= \frac{8.854 \times 10^{-12} \times 4 \times 10^{-4}}{\frac{0.3 \times 10^{-3}}{3} + \frac{0.4 \times 10^{-3}}{2.5} + \frac{0.3 \times 10^{-3}}{2}}$$

$$= \boxed{8.64\ pF}$$

Assuming supply voltage = 1000 V

Charge on the plates,

$$Q = CV$$

$$= 8.64 \times 10^{-12} \times 1000$$

$$= 8.64 \times 10^{-9} \text{ coulomb}$$

Electric flux density, $D = \dfrac{Q}{A}$

$$= \frac{8.64 \times 10^{-9}}{4 \times 10^{-4}}$$

$$= \boxed{2.16 \times 10^{5} \text{ C/m}^2}$$

Electric intensity in first dielectric $E_1 = \dfrac{D}{\varepsilon_0 \varepsilon_{r1}} = \dfrac{2.16 \times 10^{-5}}{8.854 \times 10^{-12} \times 3}$

$$= \boxed{813.2 \text{ kV/m}}$$

2. A parallel plate capacitor has plates of area 2 meter2 spaced by the three slabs of different dielectrics. The relative permittivities are 2,3 and 6 and the thickness 0.4, 0.6 and 1.2 mm. respectively. Calculate the combined capacitance and the dielectric stress in each material. When the applied voltage is 1000 V.

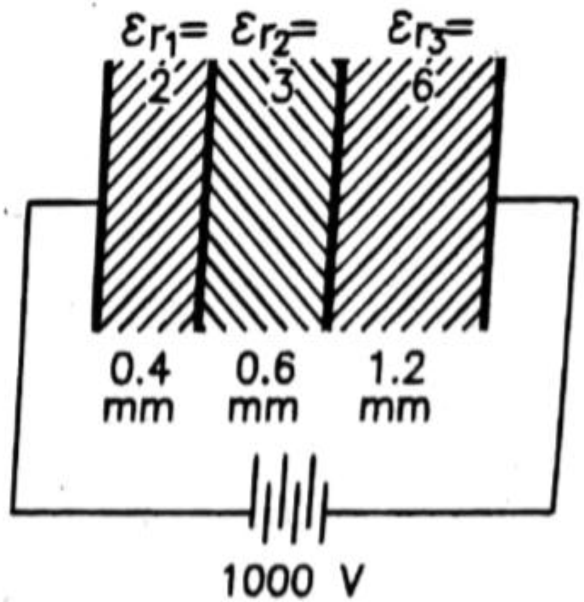
Solution:-

$A = 2\text{m}^2$

$\varepsilon_{r_1} = 2 \qquad d_1 = 0.4 \times 10^{-3}\ \text{m} \quad V = 1000\ \text{V}$

$\varepsilon_{r_2} = 3 \qquad d_2 = 0.6 \times 10^{-3}\ \text{m}$

$\varepsilon_{r_3} = 6 \qquad d_3 = 1.2 \times 10^{-3}\ \text{m}$



$$\text{Capacitance, } C = \frac{\varepsilon_0 A}{\dfrac{d_1}{\varepsilon_{r1}} + \dfrac{d_2}{\varepsilon_{r2}} + \dfrac{d_3}{\varepsilon_{r3}}}$$

$$= \frac{8.854 \times 10^{-12} \times 2}{\dfrac{0.4 \times 10^{-3}}{2} + \dfrac{0.6 \times 10^{-3}}{3} + \dfrac{1.2 \times 10^{-3}}{6}} \text{ F}$$

$$= \boxed{0.0295\ \mu\text{F}}$$

Charge on the plates, $Q = CV$

$$= 0.0295 \times 10^{-6} \times 1000$$

$$= 29.5 \times 10^{-6} \text{ C}$$

$$= \boxed{29.5\ \mu\text{C}}$$

Electric flux density, $D = \frac{Q}{A}$

$$= \frac{29.5 \times 10^{-6}}{2}$$

$$= \boxed{14.75 \times 10^{-6}\ \text{C/m}^2}$$

Electric field intensity in first dielectric, $E_1 = \dfrac{D}{\varepsilon_0 \, \varepsilon_{r_1}}$

$$= \frac{14.75 \times 10^{-6}}{8.854 \times 10^{-12} \times 2}$$

$$= \boxed{\mathbf{832.956\ kV/m}}$$

Electric field intensity in second dielectric $E_2 = \dfrac{D}{\varepsilon_0 \, \varepsilon_{r_2}}$

$$= \frac{14.75 \times 10^{-6}}{8.854 \times 10^{-12} \times 3}$$

$$= \boxed{\mathbf{555.304\ kV/m}}$$

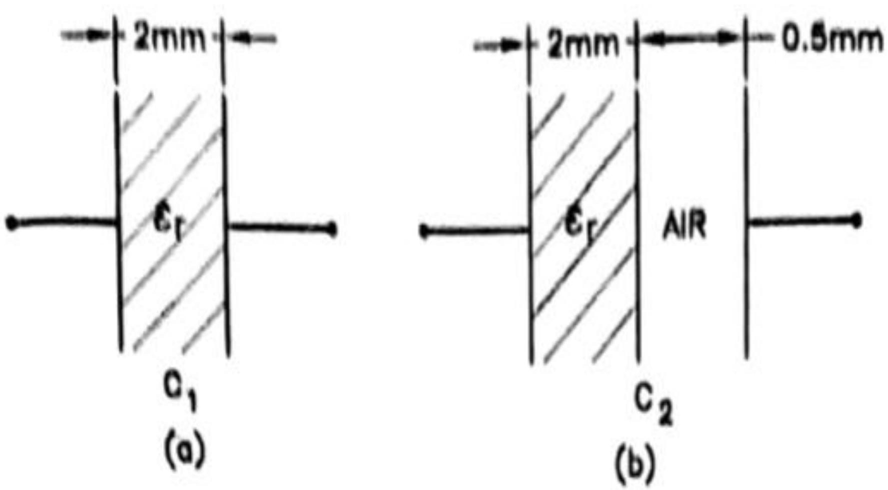
Electric field intensity in the third dielectric, $E_3 = \dfrac{D}{\varepsilon_0 \, \varepsilon_{r_3}}$

$$= \frac{14.75 \times 10^{-6}}{8.854 \times 10^{-12} \times 6}$$

$$= \boxed{\mathbf{277.652\ kV/m}}$$

3. A capacitor is made of two plates with an area of 11 cm2 which are separated by a Mica sheet 2 mm thick. If relative permittivity of Mica is 6, find its capacitance. If now one plate is moved further to give an air gap 0.5 mm wide between the plate and mica, find the change in capacitance.

Solution:-



In first case

$$C_1 = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$= \frac{8.854 \times 10^{-12} \times 6 \times 11 \times 10^{-4}}{2 \times 10^{-3}}$$

$$= 29.21 \times 10^{-12}$$

$$= \boxed{29.21 \text{ PF}}$$

In second case

$$C_2 = \frac{\varepsilon_0 A}{\frac{d_1}{\varepsilon r_1} + \frac{d_2}{\varepsilon r_2}}$$

$$= \frac{8.854 \times 10^{-12} \times 11 \times 10^{-4}}{\frac{2 \times 10^{-3}}{6} + \frac{0.5 \times 10^{-3}}{1}}$$

$$= \frac{97.35 + 10^{-16}}{\frac{2 \times 10^{-3} + 3 \times 10^{-3}}{6}}$$

$$= \frac{584.1 \times 10^{-16}}{5 \times 10^{-3}}$$

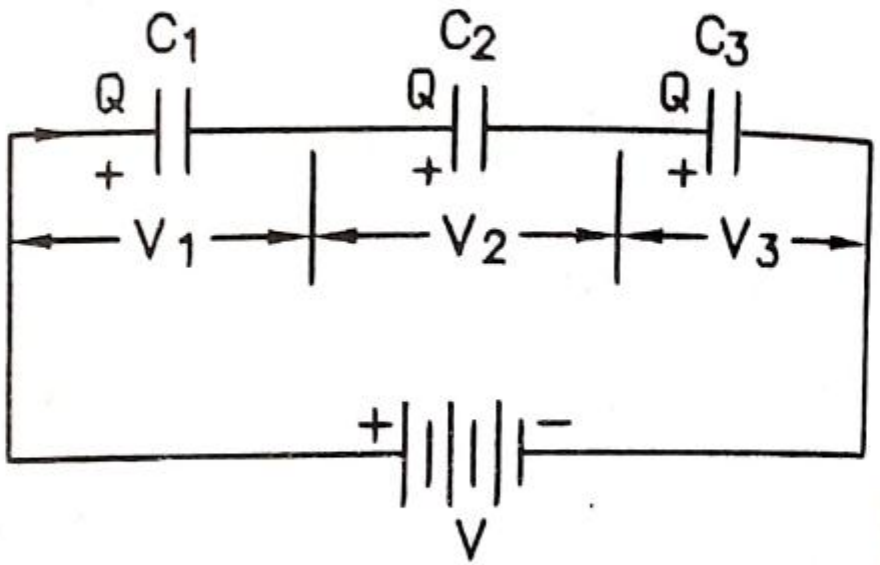
$$= 116.82 \times 10^{-13}$$

$$= 11.68 \times 10^{-12}$$

$$C_2 = \boxed{11.68 \text{ PF}}$$

CAPACITOR IN SERIES

- C1, C2, C3 = Capacitance of three capacitors
- V1, V2, V3 = Potential Difference across three capacitors
- V = applied voltage across the combination
- C_{eq} = equivalent capacitance of the combination



- In series connection as current is same the change on all capacitors is the same but p.d. across each is different.

$$\therefore \quad V = V_1 + V_2 + V_3$$

But $V_1 = \dfrac{Q}{C_1}$, $\quad V_2 = \dfrac{Q}{C_2}$ and $V_3 = \dfrac{Q}{C_3}$

$$\therefore \quad V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$= Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

If C_{eq} is the equivalent capacitance, then

$$V = \frac{Q}{Ceq}$$

- Substituting the value of V in above equation, we get,

$$\frac{Q}{C_{eq}} = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

$$\therefore \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

- Similarly, when 'n' capacitors are connected in series,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots + \frac{1}{C_n}$$

$$= \sum_{n=1}^{n} \frac{1}{C_n}$$

CAPACITORS IN PARALLEL

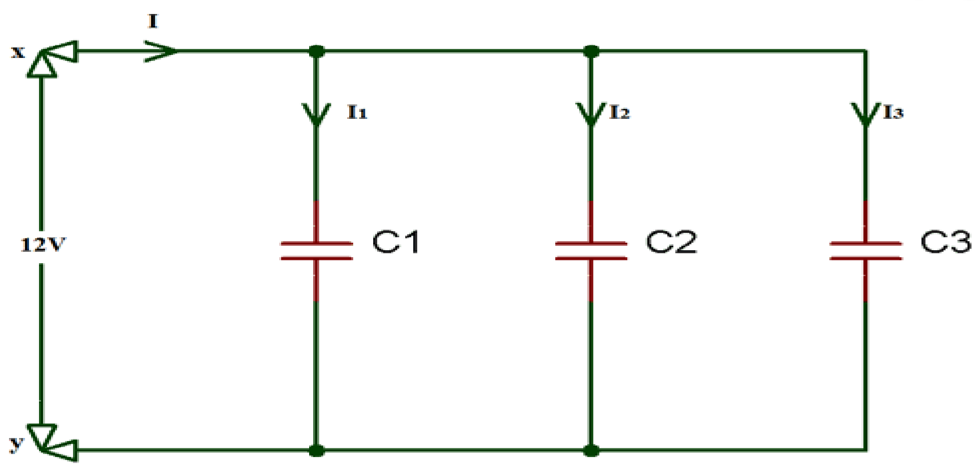
C_1, C_2, C_3 = capacitances of capacitors

Q_1, Q_2, Q_3 = charges on the capacitors

V = applied voltage across the combination

C_{eq} = equivalent capacitance of the combination

$$I = I_1 + I_2 + I_3 \rightarrow I\,t = I_1\,t + I_2\,t + I_3\,t$$



- In this case,

$$Q = Q_1 + Q_2 + Q_3 \qquad [\because Q = It]$$

But $Q_1 = C_1 V$. $\quad Q_2 = C_2 V$ and $Q_3 = C_3 V$

- Substituting the values of Q1, Q2 and Q3 in above equation, we get,

$$Q = C_1 V + C_2 V + C_3 V$$
$$= V [C_1 + C_2 + C_3]$$

- If C_{eq} is equivalent capacitance of the combination, then,

$$Q = C_{eq} V$$

- Thus,

$$C_{eq} V = V (C_1 + C_2 + C_3)$$

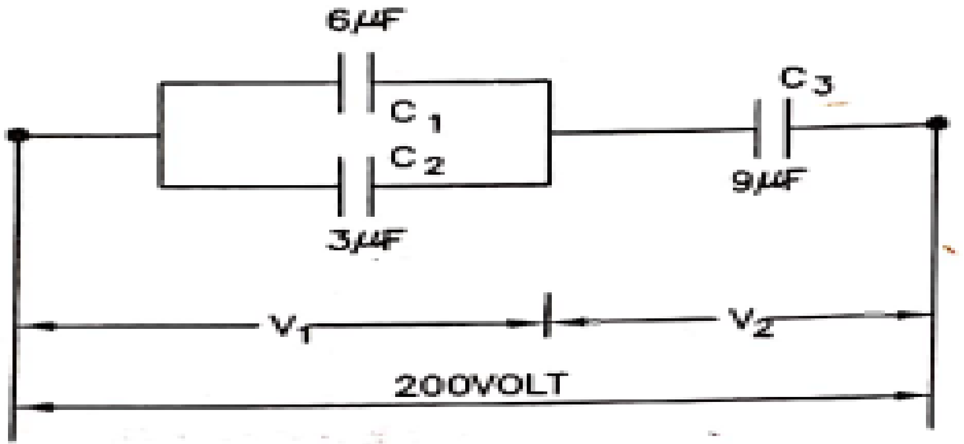
$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$

- Similarly, if there are 'n' capacitors in parallel, then equivalent value of the capacitances is given by,

$$C_{eq} = C_1 + C_2 + C_3 + \dots + C_n$$

$$= \sum_{n=1}^{n} C_n$$

1. Calculate charge and voltage of each capacitor.



Equivalent capacitance of C_1 and C_2

$$C_p = C_1 + C_2$$

$$= 6 + 3$$

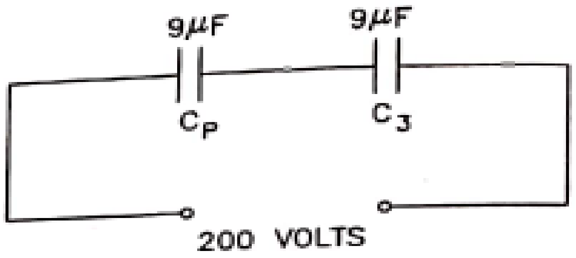
$$= 9\ \mu F$$

C_p and C_3 are in series, so total capacitance of the circuit.

$$C_{eq} = \frac{C_p \times C_3}{C_p + C_3}$$

$$= \frac{9 \times 9}{9+9}$$

$$= 4.5\ \mu F$$



Total capacitance $C_{eq} = 4.5\ \mu F$

$\therefore$ Total charge $Q = CV$

$= 4.5 \times 10^{-6} \times 200$

$\boxed{= 900 \times 10^{-6}\ C}$

$$\therefore \quad Q = C_p V_1 = C_3 V_2$$

$$\frac{V_1}{V_2} = \frac{C_3}{C_p} = \frac{9}{9} = 1$$

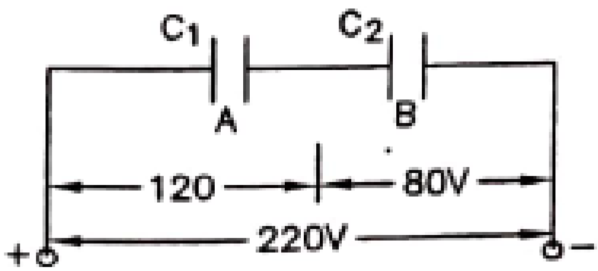
$\therefore \quad V_1 = V_2$

Also $\quad V_1 + V_2 = 200$

$\therefore \quad \boxed{V_1 = 100}$

and $\quad \boxed{V_2 = 100}$

2. When two capacitors A and B are connected across 200 volt d.c. supply, the potential difference across A is 120 volt and that across B is 80 volts. The p.d. across A is rises to 140 volts when B is shunted by 3 microfarad capacitor . Calculate capacitance of A and B.



CASE−1

Case-I

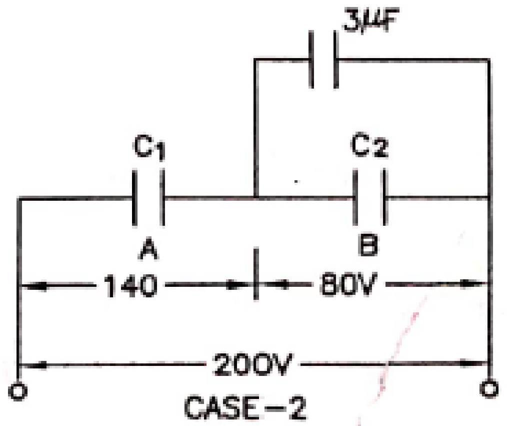
A & B in series

$\therefore$ The charge across each is same.

$\therefore \quad C_1V_1 = C_2V_2$

$\therefore \quad 120\,C_1 = 80\,C_2$

$\therefore \quad C_1 = \frac{80}{120}\,C_2$ ....(i)



3μF and C_2 are in parallel so total Capacitance of parallel circuit is

$$C_{eq} = C_2 + 3\mu F$$

But this combination is in series with C_1, So charge across each must be same. i.e.

$$140\ C_1 = 80\ (C_2 + 3) \qquad \text{..........} (2)$$

Substitute the value of C_1 in eqn. (2)

$$C_1 = \frac{80}{120} C_2$$

$$= \frac{80}{120} \times 18$$

$$C_1 = \boxed{12\ \mu F}$$

$$\therefore \quad 140 \left[\frac{80}{120} C_2\right] = 80 (C_2 + 3)$$

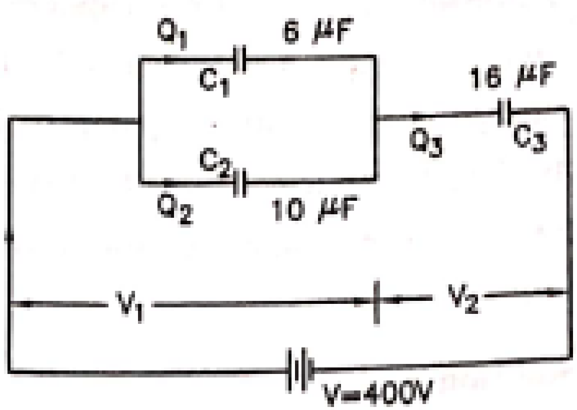
$$\therefore \quad 93.33\ C_2 = 80\ C_2 + 240$$

$$\therefore \quad 13.33\ C_2 = 240$$

$$\therefore \quad C_2 = \frac{240}{13.33}$$

18 microfarad

2. Two capacitors having capacitance of 6 microfarad and 10 microfarad are connected in parallel. A 16 microfarad capacitor is connected in series. With this combination and complete circuit is connected across 400 V. Calculate: (i) total capacitance of circuit (ii) voltage across each capacitor (iii) total charge in the circuit and (iv) the charge on each capacitor



Let $C_1 = 6\ \mu F$

$C_2 = 10\ \mu F$

$C_3 = 16\ \mu F$

(i) Equivalent capacitance

of C_1 and C_2, $C_{12} = C_1 + C_2$

$= 6 + 10$

$= 16\ \mu F$

Total capacitance Ceq $= \dfrac{C_{12} \times C_3}{C_{12} + C_3}$

$$= \frac{16 \times 16}{16 + 16}$$

$$\boxed{= 8\ \mu F}$$

(ii) In series, the charge is same

$V_1\ C_{12} = V_2\ C_3$

$$\frac{V_1}{V_2} = \frac{C_3}{C_{12}} = \frac{16}{16} = 1$$

$\therefore \quad V_1 = V_2$

Again $V = V_1 + V_2 = 400$

$$V_1 = V_2 = \frac{400}{2} = \boxed{200\ V}$$

(iii) Total capacitance, Ceq = 8 μF

$\therefore$ Total charge, $Q = Ceq\ V$

$$= 8 \times 10^{-6} \times 400$$

$$\boxed{= 32 \times 10^{-4}\ C = 3.2\ mC}$$

(iv) Charge on capacitor $C_1, Q_1 = C_1 V_1$

$$= 6 \times 10^{-6} \times 200$$

$$= 12 \times 10^{-4}\ C = \boxed{1.2\ mC}$$

Charge on capacitor C_2, $Q_2 = C_2\ V_1$

$$= 10 \times 10^{-6} \times 200$$

$$= 2 \times 10^{-3}\ C = \boxed{2\ mC}$$

Charge on capacitor C_3, $Q_3 = C_3\ V_2$

$$= 16 \times 10^{-6} \times 200$$

$$= 32 \times 10^{-4}\ C = \boxed{3.2\ mC}$$

OR $\quad Q_3 = Q_1 + Q_2$

$$= (1.2 + 2)\ mC$$

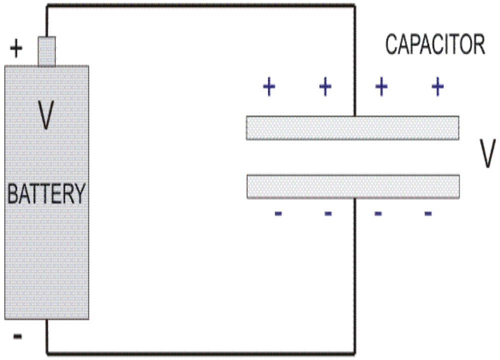
$$\boxed{= 3.2\ mC}$$

ENERGY STORED IN CAPACITOR

- When a p.d. is applied across a capacitor, the electrons are transferred from one plate to the other through the source and capacitor is charged.

- This involves expenditure of energy because electrons are moved against the opposing forces.

- This energy is stored in the electrostatic field set up between the plates of the capacitor in the dielectric medium.



- Consider a capacitor of C farad connected across a source of V volts for charging.
- During charging work is done in shifting the charge from one plate to the other.
- Let any instant, the charge on the capacitor is q coulomb and p.d. across it is v volts.

$$\text{Then } C = \frac{q}{v}$$

$$\therefore \quad q = Cv$$

- If further charge dq is shifted, then amount of work is given by,

$$dw = v\,dq$$

$$dq = C\,dv$$

$$dw = Cv\,dv$$

- Total work done in raising the potential of uncharged capacitor will be

$$W = \int_0^V Cv\,dv \quad = C\int_0^V v\,dv \quad = C\left[\frac{v^2}{2}\right]_0^V \quad = \frac{1}{2}CV^2$$

$\therefore$ Energy stored in the capacitor $= \frac{1}{2} CV^2$

$$= \frac{1}{2} CV.V$$

$$= \frac{1}{2} QV \qquad (\because Q = CV)$$

$$= \frac{1}{2} Q \cdot \frac{Q}{C} \quad \left(\because V = \frac{Q}{C}\right)$$

$$\boxed{= \frac{Q^2}{2C} \text{ joules}}$$

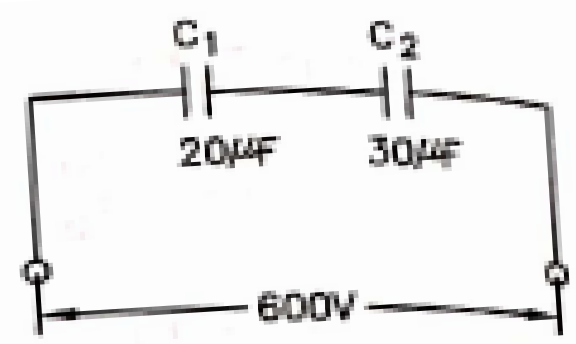
4. Two capacitors having capacitance of 20 μF and 30 μF are connected in series across a 600 V d.c. supply. Calculate the potential difference across each capacitor. If a third capacitor of unknown capacitance is now connected in parallel with the 20 μF capacitor such that the potential difference across 30 μF capacitor is 400 V, calculate (i) the value of unknown capacitance and (ii) energy stored in the third capacitor.

$$C_{eq} = \frac{C_1 \cdot C_2}{C_1 + C_2} = \frac{20 \times 30}{50} = 12\ \mu F$$

$$Q = C_{eq} \cdot V$$

$$= 12 \times 600$$

$$= 7200\ \mu\ \text{coulomb}$$



$\therefore$ Voltage across capacitor $C_1 = Vc_1 = \dfrac{Q}{C_1}$

$$= \frac{7200}{20}$$

$$\boxed{= 360 \text{ Volts}}$$

Voltage across Capacitor $C_2 = Vc_2 = \dfrac{Q}{C_2}$

$$= \frac{7200}{30}$$

$$\boxed{= 240 \text{ volts}}$$

$$Q = C_2 \cdot Vc_2$$

$$= 30 \times 400$$

$$= 12000\ \mu \text{ coulomb}$$

C_{eq} of C_1 and $C_3 = C_1 + C_3$

$$= 20 + C_3$$

Voltage across ceq $= 600 - 400 = 200$ V

$$\therefore \quad C_{eq.} = \frac{Q}{V}$$

Energy Stored in third capacitor $= \dfrac{1}{2} CV^2$

$$= \frac{1}{2} \times 40 \times 10^{-6} \times (200)^2$$

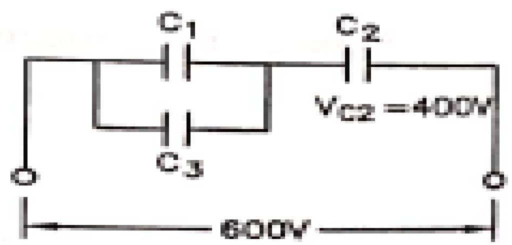
$$\boxed{= 0.8 \text{ joule}}$$

$$(20 + C_3) = \frac{1200 \times 10^{-6}}{200}$$

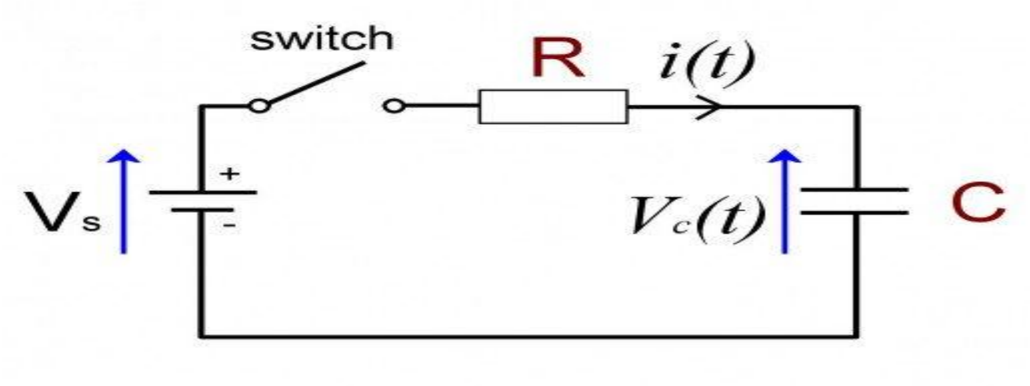
$$20 + C_3 = 60 \times 10^{-6}$$

$$\therefore \quad C_3 = 40 \times 10^{-6}$$

$$\boxed{= 40\ \mu F}$$



CHARGING OF CAPACITOR



- Figure shows a capacitor of capacitance C connected in series with a no inductive resistor R and the combination is connected to a d.c. supply of V volts through a switch S.
- For charging, the switch must be closed. At the instant of closing the switch, there is no charge on the capacitor and therefore no potential difference across it.

- As a result, the whole of the applied voltage must momentarily act across a non-inductive resistor R.
- As a result, initial value of charging current becomes equal to V/R which is maximum possible.
- The charging current gradually decreases from its maximum value till it finally becomes zero when the potential difference across the capacitor plates becomes equal and opposite to the supply voltage V.
- After closing the switch, the capacitor starts charging and voltage across it increases gradually.

Let at any instant during charging.

v_c = potential difference across the capacitor

i = charging current

q = charge on the capacitor

According to Kirchhoff's second law,

Applied voltage = p.d. across R + p.d. across C

$$\therefore \quad V = iR + v_c \qquad \text{... (i)}$$

$$= C\frac{dv_c}{dt} \times R + v_c \qquad \left[\because\ i = \frac{dq}{dt} = \frac{d}{dt}(Cv_c) = C\frac{dv_c}{dt}\right]$$

$$= v_c + RC\frac{dv_c}{dt}$$

$$V - v_c = RC\frac{dv_c}{dt}$$

$$\therefore \quad \frac{dv_c}{V - v_c} = \frac{1}{RC}dt$$

Multiplying both the sides by negative sign

$$\therefore \quad \frac{-dv_c}{V-v_c} = \frac{-1}{RC}\,dt$$

Integrating both the sides

$$\int \frac{-d\,v_c}{V-v_c} = \int \frac{-1}{RC}\,dt$$

$$\log_e (V-v_c) = \frac{-t}{RC} + \mathrm{K} \qquad \text{... (ii)}$$

where K is a constant of integration.

Its value can be determined from the initial conditions. At the instant of closing the switch, he initial conditions are,

$$t = 0 \quad \text{and} \quad v_c = 0$$

Substituting the initial conditions in eqn. (ii), we get

$$\log_e (V-0) = 0 + \mathrm{K}$$

$$\therefore \ \mathrm{K} = \log_e \mathrm{V}$$

Substituting the value of K in eqn. (ii), we get,

$$\log_e (V - v_c) = -\frac{t}{RC} + \log_e V$$

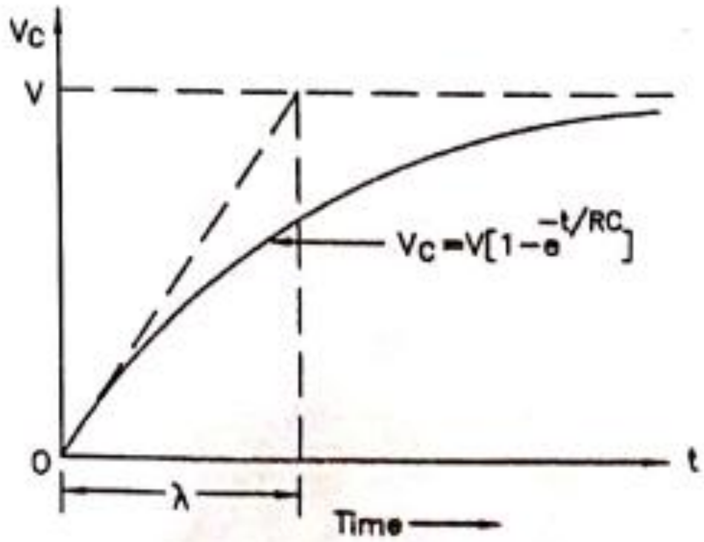
$$\therefore \quad \log_e (V - v_c) - \log_e V = -\frac{t}{RC}$$

$$\therefore \quad \log_e \left(\frac{V - v_c}{V} \right) = -\frac{t}{RC}$$

$$\frac{V - v_c}{V} = e^{-t/RC}$$

$$V - v_c = V\, e^{-t/RC}$$

$$\boxed{v_c = V\,[1 - e^{-t/RC}]}$$



5. Capacitors of $8\mu F$ is connected to a d.c. supply through a resistance of $1M\Omega$. Calculate the time taken for the capacitor to reach 95% of its final charge.

Ans:-

$R = 1\ M\Omega = 10^6\ \Omega$

$C = 8\ \mu F = 8 \times 10^{-6}\ F$

$V_C = 95\%$ of V

$= 0.95\ V$

$\lambda = RC = 10^6 \times 8 \times 10^{-6} = 8$

$V_C = V\,[1 - e^{-t/T}]$

$\therefore\ 0.95 = [1 - e^{-t/T}]$

$\therefore\ 0.95 = 1 - e^{-t/T}$

$\therefore\ e^{t/T} = 1 - 0.95 = 0.5$

$\therefore\ e^{t/T} = 20$

$\therefore\ t/T = ln\ 20 = 2.99$

$\therefore\ t = 2.99 \times T$

$= 2.99 \times 8$ $\qquad (\because\ T = RC = 1 \times 10^6 \times 8 \times 10^{-6} = 8)$

$\boxed{t = 23.96\ sec}$

6. A capacitor of 50μF is connected through 100 kilo ohms resistance to a 230V d.c. supply. Calculate the time taken to reach the capacitor voltage to 200V after closure of switch.

Ans:-

$C = 50\ \mu F = 50 \times 10^{-6}\ F$

$R = 100\ K\Omega = 100 \times 10^{3}\ \Omega$

$V = 230\ V$

$V_c = 200\ V$

$t = ?$

Time constant $\lambda = RC$

$= 100 \times 10^{3} \times 50 \times 10^{-6}$

$= 5$ sec

$V_c = V\,[1 - e^{-t/\lambda}]$

$\therefore \quad 200 = 230\,[1 - e^{-t/5}]$

$\therefore \quad \dfrac{200}{230} = [1 - e^{-t/5}]$

$\therefore \quad 0.86 = 1 - e^{-t/5}$

$\therefore \quad e^{-t/5} = 1 - 0.86 = 0.1304$

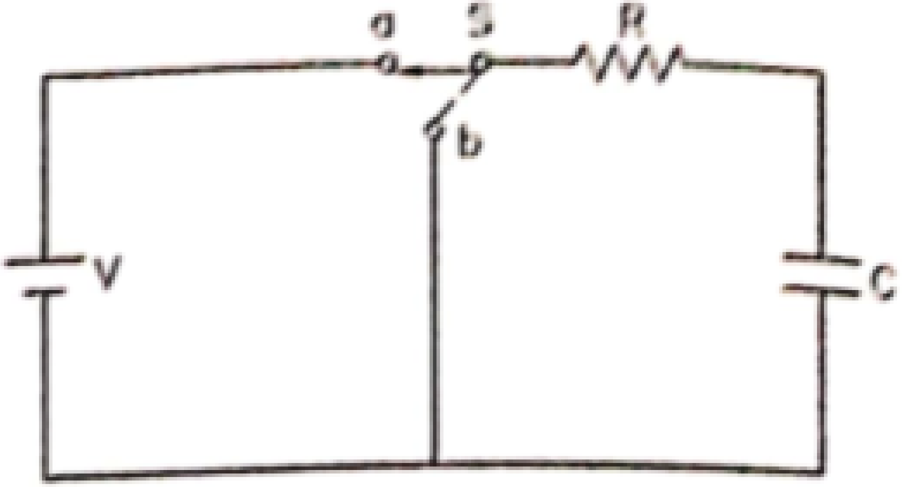
$\therefore \quad e^{t/5} = 7.6687$

$\therefore \quad \dfrac{t}{5} = \log_e 7.6687$

$\therefore \quad t = 5 \times 3.3304$

$\therefore \quad \boxed{t = 16.65\ \text{sec.}}$

Discharging of Capacitor



Consider a capacitor of C farads connected in series with a resistor of R ohms and a switch S. When the switch is in position 'a', the capacitor gets charged to V volts.

When the switch S is closed to position 'b' the charge on the capacitor starts decreasing and so does the voltage across it.

Let at any time during discharging.

v_c = p.d. across the capacitor

i = discharging current

q = charge on the capacitor

= C v_c

According to kVL,

$$0 = v_c + i\,R$$

$$\therefore \quad 0 = v_c + RC\,\frac{dv_c}{dt} \qquad \left[i = \frac{dq}{dt} = \frac{d}{dt}(Cv_c) = C\,\frac{dv_c}{dt}\right]$$

$$\therefore \quad \frac{dv_c}{v_c} = -\frac{dt}{RC}$$

Integrating both sides, we get,

$$\log_e v_c = -\frac{1}{RC}t + K_1 \qquad \text{... (i)}$$

Where K_1 is a constant of integration which can be determined from the initial conditions.

Initial conditions :

$t = 0, \quad v_c = V$

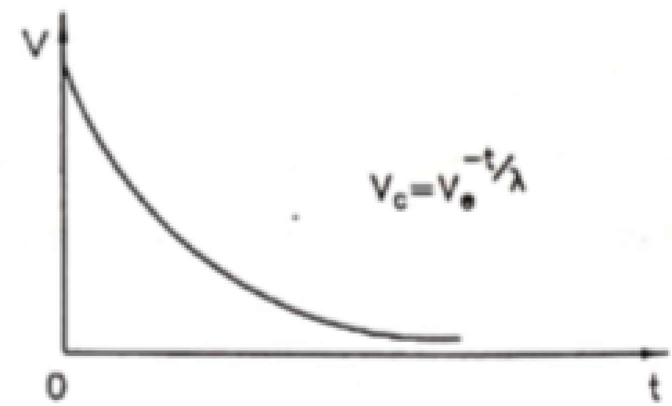
Substituting the initial conditions in equation (i)

$$\log_e V = 0 + K_1$$

$$\therefore \quad K_1 = \log_e V$$

Hence equation (i), becomes

$$\log_e v_c = \frac{-t}{RC} + \log_e V$$



or $\log_e \frac{v_c}{V} = \frac{-t}{RC}$

$$\therefore \frac{v_c}{V} = e^{-t/RC}$$

$$v_c = Ve^{-t/RC}$$

$$\boxed{= V e^{-t/\lambda}} \quad \ldots \text{(ii)} \qquad [\because \lambda = RC = \text{time constant}]$$

Variation of discharging current :

We know that

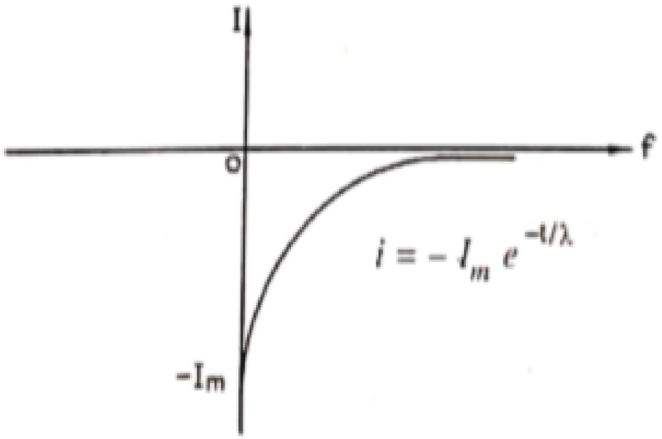
$$O = v_c + iR$$

$$\therefore \quad iR = -v_c$$

$$= -V e^{-t/\lambda}$$

$$i = \frac{-V}{R} e^{-t/\lambda}$$

$$\boxed{= -I_m e^{-t/\lambda} \quad \left[\because I_m = \frac{V}{R} = \text{initial current}\right]}$$



7. A 20 μF capacitor initially charged to a potential difference of 500 V is discharged through an unknown resistance. After one minute, the potential difference at the terminals of the capacitor is 200 V. What is the magnitude of the resistance.

$V = 500\ V$ $\quad v = 200\ V$ $\quad C = 20\ \mu F$

$t = 1\ min = 60\ sec.$ $\quad = 20 \times 10^{-6}\ F$

During discharging

$$v_c = V\, e^{-t/\lambda}$$

$$\therefore \quad 200 = 500\, e^{-60/\lambda}$$

$$\therefore \quad 0.4 = e^{-60/\lambda}$$

$$e^{60/\lambda} = 2.5$$

$$60/\lambda = \log 2.5$$

$$\lambda = \frac{60}{\log_e 2.5}$$

$$= 65.48\ sec$$

$$RC = \lambda = 65.48$$

$$R = \frac{65.48}{C} = \frac{65.48}{20 \times 10^{-6}} = 3.274 \times 10^{+6}$$

$$= \boxed{3.274\ M\Omega}$$