III	Matrix Algebra- I

Practice Examples

3.1	Definition of Matrix, types of matrices and their properties
3.2	Determinant and their properties

Determinant:

1	Find the determinant of $\begin{pmatrix} 0 & 2 & 7 \\ -1 & 5 & 0 \\ 8 & -3 & 2 \end{pmatrix}$. **Answer:** -255.
2	Find the determinant of $\begin{pmatrix} \sin\theta & -\cos\theta \\ \cos\theta & -\sin\theta \end{pmatrix}$. **Answer:** $\cos 2\theta$.
3	Find the determinant of $\begin{pmatrix} 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 7 \\ -2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{pmatrix}$. **Answer:** -210.
4	Show that the $det\begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix} = (a-b)(b-c)(c-a)$.
5	Prove that the $det\begin{pmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ac & bc & -c^2 \end{pmatrix} = 4a^2b^2c^2$.
6	Show that the $det\begin{pmatrix} 3a & b-a & c-a \\ a-b & 3b & c-b \\ a-c & b-c & 3c \end{pmatrix} = 3(a+b+c)(ab+bc+ca)$.

3.3	Rank and nullity of a matrix
3.4	Determination of rank

Rank using minors:

1	Determine the rank of $\begin{pmatrix} 3 & 5 & 1 \\ 2 & -2 & 4 \\ 7 & 1 & 9 \end{pmatrix}$ using min1ors. **Answer:** 2.
2	Determine the rank of $\begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{pmatrix}$ using minors. **Answer:** (i) If $a = b = c$, then the rank is 1. (ii) If exactly two of a, b and c are equal, then the rank is 2. (iii) If a,b, c are different and $a + b + c = 0$, then the rank is 2. (iv) If a, b, c are different and $a + b + c \neq 0$, then the rank is 3.
3	Determine the rank of $\begin{pmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -3 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ using minors. **Answer:** 4.
4	Determine the rank of $\begin{pmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{pmatrix}$ using minors. **Answer:** 1.
5	Determine the rank of $\begin{pmatrix} 1 & 2 & -1 \\ 5 & 10 & -5 \end{pmatrix}$ using minors. **Answer:** 1.
6	Determine the rank of $\begin{pmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{pmatrix}$ using minors. **Answer:** 4.

Row-echelon/ reduced row-echelon form:

1	Reduce the matrix $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 1 & 2 \end{pmatrix}$ to row-echelon/reduced row-echelon form and hence determine the rank. **Answer:** 3.
2	Reduce the matrix $\begin{pmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{pmatrix}$ to row-echelon/reduced row-echelon form and hence determine the rank. **Answer:** 3.
3	Reduce the matrix $\begin{pmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{pmatrix}$ to row-echelon/reduced row-echelon form and hence determine the rank. **Answer:** 3.
4	Reduce the matrix $\begin{pmatrix} 3 & 2 & 5 & 7 & 12 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{pmatrix}$ to row-echelon/reduced row-echelon form and hence determine the rank. **Answer:** 2.
5	Reduce the matrix $\begin{pmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{pmatrix}$ to row-echelon/reduced row-echelon form and hence determine the rank. **Answer:** 4.
6	Reduce the matrix $\begin{pmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{pmatrix}$ to row-echelon/reduced row-echelon form and hence determine the rank. **Answer:** 3.

3.5	Solution of a system of linear equations by Gauss elimination and Gauss Jordan Methods.

System of linear equations:

1	Solve the system: $$3x - 11y + 5z = 0$$ $$4x + y - 10z = 0$$ $$5x + 13y - 6z = 0$$ by Gauss elimination/Gauss-Jordan method. **Answer:** $(0, 0, 0)$
2	Identify the conditions on a, b and c so that the system: $$x + 2y + 3z = a$$ $$2x + 5y + 3z = b$$ $$x + 8y = c$$ is consistent. **Answer:** a, b, $c \in \mathbb{R}$.
3	Solve the system: $$x + y + 2z = 9$$ $$2x + 4y - 3z = 1$$ $$3x + 6y - 5z = 0$$ by Gauss elimination/Gauss-Jordan method, if it is consistent. **Answer:** $(1, 2, 3)$
4	Solve the system: $$x - 2y + z - w = 0$$ $$x + y - 2z + 3w = 0$$ $$4x + y - 5z + 8w = 0$$ $$5x - 7y + 2z - w = 0$$ by Gauss elimination/Gauss-Jordan method, if it is consistent. **Answer:** $\left\{\left(-\frac{5}{3}k_1 + k_2, -\frac{4}{3}k_1 + k_2, k_2, k_1\right) \middle\vert\ k_1, k_2 \in \mathbb{R}\right\}$.

5	Establish the conditions under which the system of linear equations $ax + by + cz = 0$ $bx + cy + az = 0$ $cx + ay + bz = 0$ has infinitely many solutions. **Answer:** $a + b + c = 0$ or $a = b = c$.
6	Solve the system: $x - 4y - 3z = -16$ $2x + 7y + 12z = 48$ $4x - y + 6z = 16$ $5x - 5y + 3z = 0$ by Gauss elimination/Gauss-Jordan method if it is consistent. **Answer:** $\left\{\left(-\frac{16}{3} - \frac{9}{5}k, -\frac{16}{3} - \frac{6}{5}k, k\right) \middle\vert\ k \in \mathbb{R}\right\}$