Unit IV

4	**Matrix Algebra II**
4.1	Revision of matrices and determinant.
4.2	Eigenvalues and Eigenvectors of matrices
4.3	Eigenvalues and Eigenvector of special matrices
4.4	Cayley-Hamilton's Theorem and its applications.
4.5	Crout's method of LU decomposition

Examples

1. Find eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$.

2. Find eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

3. Find eigenvalues and corresponding eigenvectors of $\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$

4. Find eigenvalues and corresponding eigenvectors of $\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$.

5. Find eigenvalues and corresponding eigenvectors of $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$.

6. Find eigenvalues and corresponding eigenvectors of $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

7. If $A = \begin{bmatrix} 3 & 2 & 3 \\ 0 & 6 & 10 \\ 0 & 0 & 2 \end{bmatrix}$, then find the eigenvalues of
$(i)\ A \quad (ii)\ A^T \quad (iii)\ A^3 \quad (iv)\ 3A \quad (v)\ A^{-1}$.

8. Find eigenvectors of the matrix $A = \begin{bmatrix} 3 & 2 & 3 \\ 0 & 6 & 10 \\ 0 & 0 & 2 \end{bmatrix}$.

9. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then find A^2, A^3, A^{-1}, A^{-2} using the Cayley-Hamilton theorem.

10. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and hence find A^{-1} and A^8.

11. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, then prove that

$$A^5 - A^4 + 3A^3 - 5A^2 + 2A - 3I_2 = 32A - 103I_2.$$

12. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and hence find A^{-1}.

13. Find the characteristic equation of a matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$ and hence express $A^9 - 6A^8 + 10A^7 - 3A^6 + A + I_3$ as a linear polynomial in A and find its value.

14. Factorize the matrix $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ into the LU form.

15. Find the LU decomposition of the matrix $B = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. **(H.W)**

16. Using Crout's LU decomposition (or factorization method), solve the following system of linear equations:

$2x + y + 4z = 12;\ 8x - 3y + 2z = 20;\ 4x + 11y - z = 33.$

17. Solve the following system of linear equations using LU decomposition method.

$4x + 3y + 2z = 16; 2x + 3y + 4z = 20; x + 2y + z = 8.$ **(H.W.)**