Candidate's seat no.: ____________

CHAROTAR UNIVERSITY OF SCIENCE AND TECHNOLOGY

First Semester of B. Tech. (All branches) Theory Examination (March 2021)

MA143 Engineering Mathematics-I

Date: 05/03/2021 (Friday) **Time: 10:00 a.m. to 01:00 p.m.** **Maximum Marks: 70**

Instructions:

i. The question paper contains two sections.
ii. Section I and II must be attempted in separate answer sheets.
iii. Figures to the right indicate marks.
iv. Make suitable assumptions and draw neat figure where it is required.
v. All notations and terminologies are standard.

Section-I

Q-1 Choose the correct answer from the given options in the followings: **[10]**

1) $\lim_{x\to\frac{\pi}{2}}(\sin x)^{\tan x} =$ ____________.

(A) -2 (B) -1 (C) 0 (D) 1

2) The Mean value theorem is not applicable to the function ________ in the interval $[0, 2]$.

(A) $f(x)=x^3+2x+1$ (B) $f(x)=\sin x$ (C) $f(x)=e^x$ (D) $f(x)=\begin{cases} x^2+1, & x\neq 1 \\ 0, & x=1 \end{cases}$

3) The n^{th} order derivative of the function $y=5^x e^{2x}$ is $y_n =$ ________ at $x=0$.

(A) $(\log 5+2)^n$ (B) $(2\log 5)^n$ (C) 2^n (D) $(\log 5-2)^n$

4) The series $\sum_{n=1}^{\infty}\frac{1}{n^{(2p)}}$ converges, if ____________.

(A) $p>0.5$ (B) $0<p<0.5$ (C) $p=0.5$ (D) $p<0$

5) The series $\sum_{n=2}^{\infty}\left(\frac{1}{2}\right)^n$ converges to ________.

(A) 0.5 (B) 1 (C) 1.5 (D) 2

6) The series $\sum_{n=1}^{\infty}(1)^n$ is ________.

(A) a divergent series (B) a convergent series

(C) an oscillating series (D) a positive term series

7) The value of $Log(2+i)+Log(2-i)$ is ________.

(A) 0.69897 (B) 1.60944 (C) 0.30103 (D) 0.69312

8) The real part of the complex number $z=\frac{1}{1+i}$ is ______.

(A) -1 (B) -0.5 (C) 0.5 (D) 1

9) If the polynomial equation $7x^n-8x+5=0$ has only five distinct roots, then n=________.

(A) 3 (B) 4 (C) 5 (D) 6

10) $\frac{d}{dx}\tanh(ix)=$ ________.

(A) $\sec^2 x$ (B) $i\sec^2 x$ (C) $\sec x\tan x$ (D) $i\sec x\tan x$

Q-2 Attempt any *Four* of the followings: **[16]**

1) State Leibnitz's theorem for the n^{th} order derivative of product of two functions and using it prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$, where $y=(\sin^{-1}x)^2$.

2) Expand the function $f(x)=\log(x+1)$ about x=0 using Taylor's series up to first four terms and hence using it, find the approximate value of $\log(1.2)$ correct up to 4 decimal places.

3) (i) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{n^5+2n+1}{n^6+4n-1}$.

(ii) If $y=x\log\left(\frac{x-1}{x+1}\right)$, then show that $y_n=(-1)^{n-2}(n-2)!\left[\frac{x-n}{(x-1)^n}-\frac{x+n}{(x+1)^n}\right]$.

4) Find the roots of the equation $x^3-18x+35=0$ using Cardon's method.

5) Find the roots of the equation $x^4-3x^2-42x-40=0$ using Ferrari's method.

Q-3 Attempt any *Three* of the followings: **[09]**

1) Using Mean Value Theorem for the function $f(x)=\log x$, $x\in[a,b]$ and $1<a$, show that $\frac{b-a}{b}<\log\left(\frac{b}{a}\right)<\frac{b-a}{a}$.

2) (i) Test the convergence of the series $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots$

(ii) Show that $\sin(ix)=i\sinh x$ and $\cos(ix)=\cosh x$.

3) Find all fourth roots of unity using De Moivre's theorem.

4) If $\cosh(u+iv)=x+iy$, then prove that $\frac{x^2}{\cosh^2 u}+\frac{y^2}{\sinh^2 u}=1$ and $\frac{x^2}{\cos^2 v}-\frac{y^2}{\sin^2 v}=1$.

Section-II

Q-4 Choose correct answer from the given options in the followings: **[10]**

1) If $A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, then A is ________ matrix.
   (A) a unit (B) a scalar (C) a diagonal (D) a null

2) If $A = \begin{pmatrix} a & -1 & 0 \\ 0 & a & -1 \\ -1 & 0 & a \end{pmatrix}$ and $r(A) = 2$, then $a =$ ________.
   (A) 0 (B) 1 (C) 2 (D) 3

3) The rank of any 2×2 nonsingular matrix is________.
   (A) 0 (B) 1 (C) 2 (D) 3

4) The system of equations $4x + 6y = 5, 6x + 9y = 7$ has ______.
   (A) no solution (B) infinitely many solutions (C) unique solution (D) none of these

5) Which of the following matrices is not in row-echelon form with leading "1"?
   (A) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 2 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$ (D) $\begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

6) If $f(x, y) = x^3 + y^3 - 2x^2y^2$, then $\left(\frac{\partial^2 f}{\partial x^2}\right)_{(1,1)} =$ ________.
   (A) -1 (B) 0 (C) 1 (D) 2

7) If $z = \frac{1}{2}(x^2 - y^2)$, then dz is equal to________.
   (A) $x\,dx + y\,dy$ (B) $dx + y\,dy$ (C) $x\,dx - y\,dy$ (D) $x\,dx - dy$

8) The tangent plane to the surface $z = x^2 + y^2$ at $(1, -1, 2)$ is________.
   (A) $2x + 2y - z = 2$ (B) $2x - 2y - z = 2$
   (C) $2x - 2y + z = 2$ (D) $2x - y - 2z = 2$

9) If $x = r\cos\theta, y = r\sin\theta$, then $\frac{\partial(x,y)}{\partial(r,\theta)}$ is ________.
   (A) $-r$ (B) 0 (C) r^{-1} (D) r

10) An approximate value of $(4.1)^2 + (2.9)^2$ is ________using the theory of approximations.
   (A) 25.02 (B) 20.02 (C) 26.02 (D) 2.02

Q-5 Attempt any *Four* of the followings: **[16]**

1) Reduce the matrix $\begin{pmatrix} 3 & 1 & 7 \\ 1 & 2 & 4 \\ 4 & -1 & 7 \\ 4 & -1 & 5 \end{pmatrix}$ to reduced row-echelon form and hence determine the rank.

2) Find the inverse of $\begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ by Gauss-Jordan method, if exists.

3) Solve the system

$$\begin{aligned} x + 2y - 2z &= 1 \\ 2x - 3y + z &= 0 \\ 5x + y - 5z &= 1 \\ 3x + 14y - 12z &= 5 \end{aligned}$$

by Gauss elimination method, if it is consistent.

4) Find the extreme values of the function

$f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$

5) Find the maximum value of $x^l y^m z^n$ subject to $x + y + z = a.$

Q-6 Attempt any *Three* of the followings: [09]

1) If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, then find

(*i*) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

(*ii*) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$.

2) If $u = f(x - y, y - z, z - x)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$

3) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at (π, π, π), if $\sin(x + y) + \sin(y + z) + \sin(x + z) = 0.$

4) Expand $f(x,y) = xe^y + 1$ in the powers of $(x - 1)$ and y using Taylor's series expansion.

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