Candidate's seat No: _______________

CHAROTAR UNIVERSITY OF SCIENCE AND TECHNOLOGY
First Semester of B. Tech. (All Branches) Theory Examination
February 2022
MA143: Engineering Mathematics-I

Date: 18/02/2022 (Friday) **Time: 10:00 a.m. to 01:00 p.m.** **Maximum Marks: 70**

Instructions:

i. Figures to the right indicate marks.
ii. Make suitable assumptions and draw neat figure where it is required.
iii. All notations and terminologies are standard.

Q-1 Choose the correct answer from the given options in the followings. **[20]**

1) The maximum and minimum values of $f(x) = x^5 - 5x^4 + 5x^3 - 1$ are_______ respectively. **01**

(a) 0 and -28 **(b)** -28 and 0 **(c)** 1 and 3 **(d)** 3 and 1

2) If the third derivative of the function $y = e^x \cdot x^3$ is $e^x(x^3 + 9x^2 + 18x + A)$, then $A =$ _______. **01**

(a) 0 **(b)** 3 **(c)** 6 **(d)** 9

3) If $y = \cosh(2x)$ and n is an even natural number, then $y_n =$ _______. **01**

(a) $2^n \sinh(2x)$ **(b)** $2^n \cosh(2x)$

(c) $\cosh(2nx)$ **(d)** $\sinh(2nx)$

4) The value of $\lim_{x\to 0} x^x$ is_______. **01**

(a) -1 **(b)** 0 **(c)** 1 **(d)** ∞

5) Which of the following function is in the indeterminate form when $x = \frac{\pi}{2}$? **01**

(a) $\frac{\cos x}{2x - \frac{\pi}{4}}$ **(b)** $\frac{\cos x}{\frac{1}{2}x - \frac{\pi}{4}}$

(c) $\frac{\cos x}{\frac{1}{4}x + \frac{\pi}{2}}$ **(d)** $\frac{\cos x}{\frac{1}{4}x - \frac{\pi}{4}}$

6) Which of the following statement is false? **01**

(a) If $\lim_{n\to\infty} u_n \neq 0$, then the series $\sum u_n$ is not convergent.

(b) The geometric series with ratio r converges if $|r| < 1$.

(c) Let $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ be two positive term series. If $\lim_{n\to\infty} \frac{u_n}{v_n} = l$ with $0 \le l < \infty$, then $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ converge or diverge together.

(d) The series $\sum \frac{1}{n^p}$ is convergent if and only if $p > 1$.

7) If $|z - 1| = 2$, then the value of $z\bar{z} - z - \bar{z}$ is________. **01**

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

8) One of the values of $\sqrt{\cos\pi + i\sin\pi}$ is________. **01**

(a) -1 **(b)** 0 **(c)** 1 **(d)** i

9) Which of the following value of z satisfies the equation $Log(z^2) = 2\,Log(z)$? **01**

(a) -1 **(b)** $-i$ **(c)** i **(d)** $-1+i$

10) The imaginary part of $\sin(z)$ is________, where $z = (1+i)\pi$. **01**

(a) $\sinh(\pi)$ **(b)** $-\sinh(\pi)$ **(c)** $\sin\pi$ **(d)** $\cos\pi$

11) Which of the following is not a triangular matrix? **01**

(a) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ **(b)** $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **(d)** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

12) The number of solution(s) of the system of equations $AX = 0$, where A is a singular matrix, is/are_______. **01**

(a) 0 **(b)** 1 **(c)** 2 **(d)** infinite

13) Let A be a matrix of order $m \times n$ such that there exists a nonzero minor of order p, where $0 < p < \min\{m, n\}$, then the rank of A is_______. **01**

(a) $< p$ **(b)** $\leq p$ **(c)** $\geq p$ **(d)** $> p$

14) If a matrix A of order 4×4 is invertible, then the reduced row echelon form of A is________. **01**

(a) A **(b)** I_2 **(c)** I_3 **(d)** I_4

15) If $A = [1]$, then_______. **01**

(a) A is in row echelon form **(b)** A is in reduced row echelon form

(c) both **(a)** and **(b)** **(d)** none of these

16) If $u = x^2 + y^2$, then $\frac{\partial^2 u}{\partial x \partial y} =$ ________. **01**

(a) 0 **(b)** 2 **(c)** $2x + 2y$ **(d)** yx^{y-1}

17) If $f(x, y) = e^{xy^2}$, then the total differential of the function at point $(1, 2)$ is_______. **01**

(a) $e(dx + dy)$ **(b)** $e^4(dx + dy)$

(c) $e^4(4\,dx + dy)$ **(d)** $4e^4(dx + dy)$

18) In the Taylor's series expansion of $e^x \cdot \sin y$ about the point $(1, \pi)$, the coefficient of $(x-1)(y-\pi)$ is_______. **01**

(a) $-2e$ **(b)** $-e$ **(c)** 0 **(d)** e

19) The equation of the tangent plane to the surface $x^2 + y^2 + z^2 = 14$ at point $(1, 2, 3)$ is________. **01**

(a) $2x + 4y + 6z = 14$ **(b)** $x + 2y + 3z = 0$

(c) $x + 2y + 3z = 1$ **(d)** $x + 2y + 3z = 14$

20) The function $f(x, y) = y^2 - x^2$ has________. **01**

(a) minimum at (0,0)

(b) maximum at (1,1)

(c) neither minimum nor maximum at (0, 0)

(d) neither minimum nor maximum at (1, 1)

Q-2 Attempt any <u>four</u> of the following. **[16]**

1) **(a)** A truck travels on a toll road with a speed limit of $80\ km/hr$. The truck completes a $164\ km$ journey in $2\ hours$. At the end of the toll road the trucker is issued with a speed violation notice. Justify this using the Mean Value Theorem.

(b) Find the nature of the series $\sum_{n=1}^{\infty} \frac{4\cdot 7\cdot 10\cdot \ldots \cdot (3n+1)}{1\cdot 2\cdot 3\cdot \ldots \cdot n}$.

2) State Leibnitz's theorem for the n^{th} order derivative of product of two functions and using it show that $(1 + x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$, where $y = \tan^{-1} x$.

3) **(a)** Simplify $(1 + cos\theta + isin\theta)^n + (1 + cos\theta - isin\theta)^n$, where n is an integer.

(b) Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\log(n+1)}$ is convergent.

4) Solve the equation $x^4 + 2x^3 - 7x^2 - 8x + 12 = 0$ using Ferrari's method.

5) **(a)** Find the values of $\log(1 + i) + \log(1 - i)$.

(b) If $cosh(u + iv) = x + iy$, then prove that $\frac{x^2}{cosh^2 u} + \frac{y^2}{sinh^2 u} = 1$.

Q-3 Attempt any <u>three</u> of the following. **[09]**

1) If $y = \frac{2}{(x-1)(x-2)(x-3)}$, then find y_n.

2) Prove that $\tan^{-1} x = \tan^{-1}\frac{\pi}{4} + \frac{\left(x-\frac{\pi}{4}\right)}{1+\frac{\pi^2}{16}} - \frac{\pi\left(x-\frac{\pi}{4}\right)^2}{4\left(1+\frac{\pi^2}{16}\right)^2} + \cdots$.

3) Use De Moivre's theorem to solve $x^4 + x^3 + x^2 + x + 1 = 0, x \in \mathbb{C}$.

4) Solve the equation $x^3 - 27x + 54 = 0$ using Cardon's method.

Q-4 Attempt any <u>four</u> of the following. **[16]**

1) **(a)** Determine the rank of $\begin{bmatrix} 1 & 2 & -1 \\ 5 & 10 & -5 \end{bmatrix}$ using minors.

(b) Find $\frac{dy}{dx}$ at $(-1, 1)$ if $xy + y^2 - 3x - 3 = 0$.

2) Use Gauss elimination method to solve the system of linear equations

$x - 2y + z - w = 0; x + y - 2z + 3w = 0; 4x + y - 5z + 8w = 0;$

$5x - 7y + 2z - w = 0.$

3) If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, then prove that $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$.

4) Find the minimum value of $x^2 + y^2 + z^2$ subject to $xyz = a^3$, where a is constant.

5) **(a)** Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ for $u = x^2 - y^2, v = 2xy$.

(b) Find an approximate value of $(1.04)^{3.01}$ using theory of approximation**.**

Q-5 Attempt any <u>three</u> of the following. **[09]**

1) Reduce the matrix $\begin{bmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \\ -2 & 8 & 2 \end{bmatrix}$ to row echelon form and determine the rank.

2) Decrypt the received encoded message $[30 \quad 9]$ $[19 \quad 8]$; where the encryption matrix is $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$ and the decryption matrix is its inverse. Here, the system of codes are represented as follows; the numbers $1 - 26$ by the letters $A - Z$ respectively, and the number 0 by the blank space. (Use Gauss Jordan method to find the inverse.)

3) If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\cot u$, using Euler's theorem.

4) Show that the minimum value of the function $x^2 + y^2 + 6x + 12$ is 3.