Unit I

1.	**First order and First-degree Ordinary Differential Equations:**
1.1	Formation of Ordinary Differential Equation
1.2	Concept of general and particular solutions
1.3	Initial value problems
1.4	Solutions of first order and first degree differential equations: Linear, Bernoulli, Exact and non-exact differential equations

1.1 Formation of Ordinary differential equation

Examples

1.	Form the differential equation for a relation $y = Ax + B$, where A and B are arbitrary constants.
2.	Form the differential equation satisfied by $x^2 + y^2 = c$, where c is an arbitrary constant.
3.	Form the differential equation of the simple harmonic motion given by $x = \beta \sin(\omega t + \alpha)$, where α and β are an arbitrary constants and ω is fixed constant.
4.	Form the differential equation satisfied by $y = Ae^{\alpha x} + Be^{\beta x}$, where A and B are arbitrary constants, α and β are some fixed real numbers.
5.	Find the differential equation of the family of circles whose center is $(a, 0)$ and radius is a.

1.2 Concept of general and particular solutions

Examples

1.	Show that $\sin y + \cos x = 2$ is a particular solution of the differential equation $\frac{dy}{dx} = \frac{\sin x}{\cos y}$.
2.	Show that $y = Ae^{-3x} + Be^{-2x}$ is a general solution of the differential equation $y'' + 5y' + 6y = 0$, where A and B are arbitrary constants.
3.	Show that $x^2 + y^2 = 4$ is a particular solution of the differential equation

	$\frac{dy}{dx} = -\frac{x}{y}$.

1.3 Initial value problems

1.4 Solutions of first order and first degree differential equations

Following are the methods to find solution of 1st ordinary differential equation.

1.4.1 Linear differential equation or Leibnitz's differential equation
1.4.2 Bernoulli's differential equation
1.4.3 Exact and non-exact differential equation

Examples

Ex.	**Solve the following differential equations:**
1.	$\frac{dy}{dx} + 2xy = 2e^{-x^2}$
2.	$\frac{dy}{dx} + 2y\tan x = \sin x\,, y\left(\frac{\pi}{3}\right) = 0.$
3.	$(1+y^2)dx = (\tan^{-1} y - x)dy.$
4.	$dr + (2r\cot\theta - \sin 2\theta)d\theta = 0.$
5.	$\frac{dy}{dx} - (1+3x^{-1})y = x+2, y(1) = e-1.$
6.	$\frac{dx}{dy} - \frac{1}{y} = \frac{e^{3x}}{y^3}.$
7.	$\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2.$
8.	$\cos y\frac{dy}{dx} + x\sin y = 2x.$
9.	$(2xy)dx + (1+x^2)dy = 0.$
10.	$\left(y^2 e^{xy^2} + 4x^3\right)dx + \left(2xy\, e^{xy^2} - 3y^2\right)dy = 0.$
11.	$(xy - 2y^2)dx - (x^2 - 3xy)dy = 0.$
12.	$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0.$
13.	$(1+2xy)ydx + (1-xy)xdy = 0.$
14.	$(xy\sin xy + \cos xy)ydx + (xy\sin xy - \cos xy)xdy = 0.$

15.	$(2x\log x - xy)dy + 2ydx = 0.$
16.	$(x\sec^2 y - x^2\cos y)dy = (\tan y - 3x^4)dx.$
17.	$(xy^3 + y)dx + 2(x + x^2y^2 + y^4)dy = 0.$
18.	$(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0.$

Applications:

1	The potential difference E across an element of inductance L is equal to the product of L and the time rate of change of current I in the inductor. Form the differential equation. **Ans:** $\boldsymbol{L\frac{di}{dt} = E}$
2	A particle moves along the x- axis such that its velocity is inversely proportional to time. Form the differential equation. **Ans:** $\boldsymbol{\frac{dx}{dt} = \frac{k}{t}}$
3	The current $i(t)$ flowing in an R-L circuit is governed by the equation $L\frac{di}{dt} + Ri = E_0 sin(\omega t)$, where R is the constant resistance , L is the constant inductance and $E_0 sin(\omega t)$ is the voltage at time t, E_0 and ω being constants.Find the current at any time t assuming that initially it is zero. **Ans:** $\boldsymbol{i(t) = \frac{E_0}{R^2+w^2L^2}[R\sin(\omega t) - \omega L\, cos(\omega t)] + \frac{\omega L E_0}{R^2+w^2L^2} e^{-\frac{Rt}{L}}}$
4	Let F be the constant force generated by the motor of an automobile of mass M, and its velocity be given by $M\frac{dV}{dt} = F - kV$, where k is a constant. Find V in terms of t given that $V = 0$ at $t = 0$. Ans: $\boldsymbol{V = \frac{F}{k}\left(1 - e^{-\frac{kt}{M}}\right)}$
5	A chain coiled up near the edge of a smooth table begins to fall over the edge. When a length x of the chain has fallen ,the equation of th motion is given by $$\frac{d}{dt}(m\,x\,v) = mxg$$ where m is the mass of the chain per unit length , v is the speed , g is the acceleration due to gravity and t is the time. Find the speed v at time t depending on the length x. **Ans:** $\boldsymbol{v = \sqrt{\frac{2g}{3}x}}$