Charotar University of Science and Technology [CHARUSAT]
Faculty of Technology and Engineering
Department of Mathematical Sciences
MA 143 Engineering Mathematics-I

First Internal Exam

Semester: 1ˢᵗ Sem B. Tech. (CE/IT/ME/EE/EC/CL/CSE)
Date: 09/09/2019 (Monday)

Maximum Marks: 30
Time: 11:10am to 12:10pm

Instructions:
(i) All questions are compulsory.
(ii) Figures to the right indicate *full* marks.
(iii) Draw figure where it is required.

Q.1 Choose correct answer from the given options in the following :   03

(a) The Mean Value Theorem does not apply to $f(x) = |x-3|$ on $[1, 4]$ because

    a) f(x) is not continuous on $[1, 4]$
    b) f(x) is not differentiable on $(1, 4)$
    c) $f(1) \neq f(4)$
    d) None of these

(b) The Maclaurin's series of the function $e^x \sinh x = $ _____.

    a) $x + x^2 + \dfrac{x^3}{3} + \dfrac{x^4}{3} + \ldots$ b) $x - x^2 + \dfrac{2x^3}{3} - \dfrac{x^4}{3} + \ldots$

    c) $x + x^2 + \dfrac{2x^3}{3} + \dfrac{x^4}{3} + \ldots$ d) $1 + x^2 + \dfrac{x^3}{3} + \dfrac{x^4}{3} + \ldots$

(c) If $f(x) = \int_0^\pi \sin(ax)\,da$, then the value of $f^{(60)}(0)$ is _____, where $f^{(60)}(0)$ is 60ᵗʰ derivative of f(x) at x=0.

    a) 1   b) 0   c) 60!   d) 59!

Q-2 Attempt any three from the following questions.   12

(a) If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$

(b) Obtain the nᵗʰ derivative of the function $f(x) = \dfrac{3x+1}{(x+1)^2(x-2)}$.

(c) Expand the polynomial $f(x) = x^5 + 2x^4 - x^2 + x + 1$ in powers of $x+1$.

(d) Find the nᵗʰ derivative of the functions $(i)\, e^{2x}\cos 2x \sin x$ $(ii)\, \cos^2 2x \sin 2x$.

(e) Find the extreme value of the function $y = \left(\dfrac{1}{x}\right)^x, x > 0$

Q.3 **Choose correct answer from the given options in the following :**

(a) A set of n linear equations in n unknowns is represented by the matrix equation

Ax = b. The necessary condition for the system has unique solution if _____.

    a) A is singular.

    b) nullity(A)=1.

    c) A must be invertible.

    d) None of these.

(b) If A is 5×8 matrix and nullity of $A^T A$ is 3, then the rank of A^T is _____.

    a) 3   b) 2   c) 5   d) 0

(c) Let S be the set of 2×2 real matrices A with $A^T A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. Then the set S

contains _____.

    a)   a symmetric matrix   b)   a matrix of rank one

    c)   a matrix of rank two   d)   a skew-symmetric matrix

Q-4 **Attempt any three from the following questions.**   12

(a) Find rank of $\begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ using row echelon form.

(b) Using Gauss- Jordan method find the inverse of the matrix A= $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 2 & 2 & 3 \end{bmatrix}$, if exists.

(c) Investigate for what values of μ and λ the equations

$x + 2y + z = 8,$

$2x + 2y + 2z = 13,$

$3x + 4y + \lambda z = \mu$

have (i) no solution (ii) unique solution (iii) infinite number of solutions.

(d) Discuss the consistency of the system and if consistent, solve the equations

$4x - 2y + 6z = 8,$

$x + y - 3z = -1,$

$15x - 3y + 9z = 21.$

(e) Find rank of the following matrices (i) $\begin{bmatrix} 2 & 1 & 5 & -1 \\ -1 & 2 & 5 & 3 \\ 3 & 2 & 9 & -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 4 & 2 & 3 \\ 8 & 4 & 6 \\ -2 & -1 & -\dfrac{3}{2} \end{bmatrix}$

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