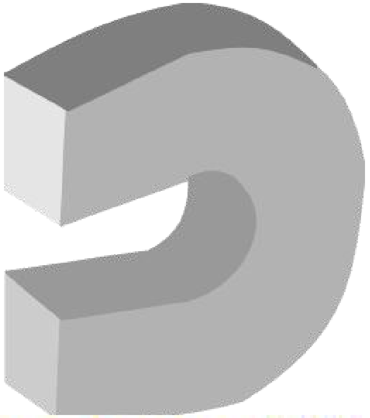
Chapter-4

Electromagnetism

MAGNETIZATION



Magnetization (**M**) = magnetic moment (**m**) per unit volume (V).

$$M = \frac{m}{V}$$

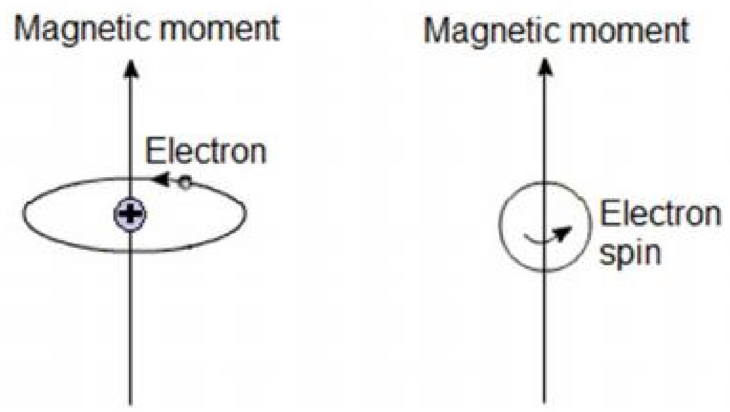
- Force of attraction or repulsion cause by the movement of charged participles

Magnetic moments

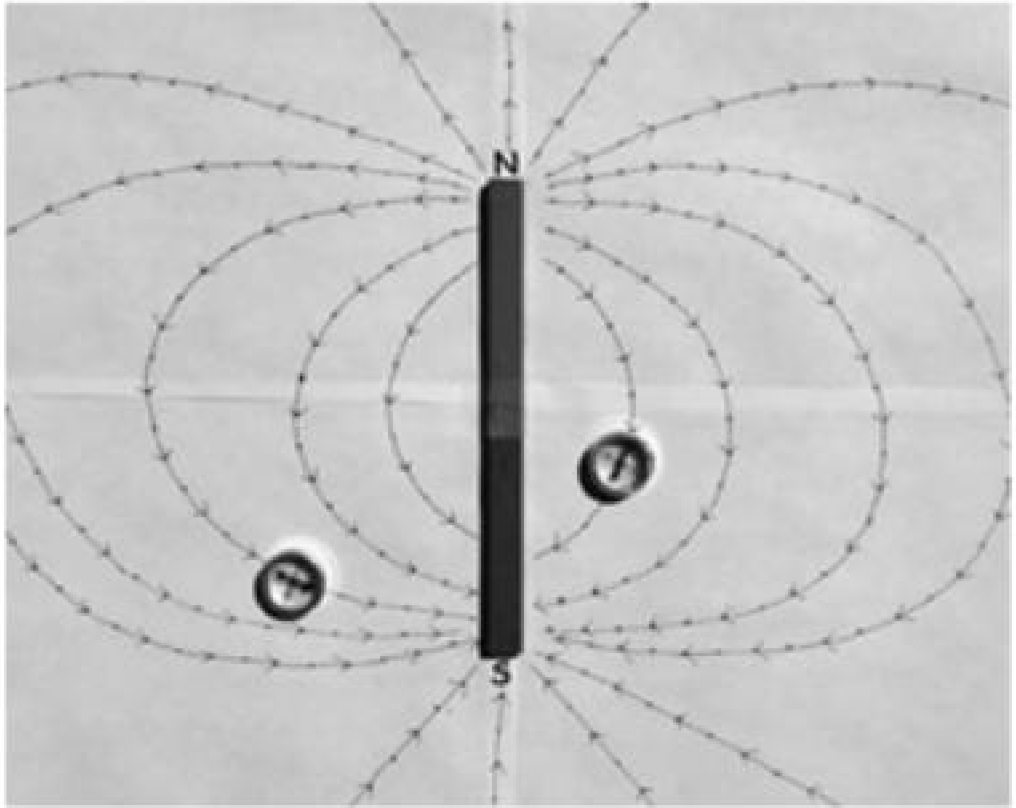
➢ Being a moving charge, electrons produce a small magnetic field having a magnetic moment along the axis of rotation.

➢ The spin of electrons also produces a magnetic moment along the spin axis.

➢ Magnetism in a material arises due to alignment of magnetic moments.



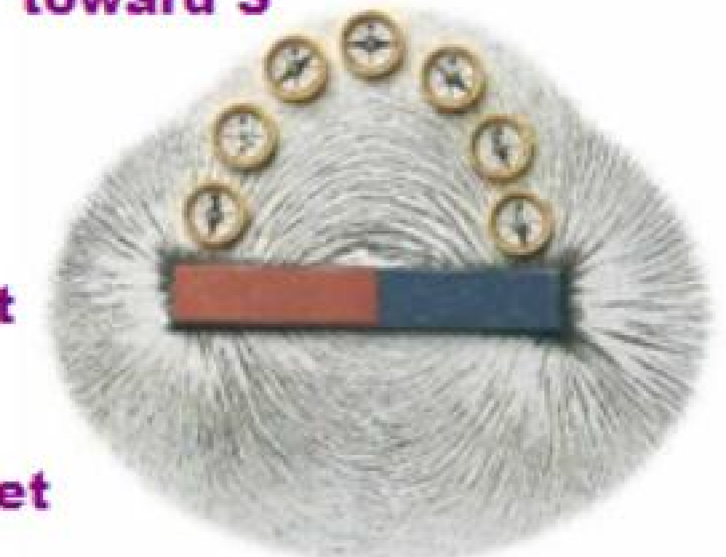
Magnetic Field



Magnetic field of a Bar Magnet: 2 poles, called North and South

Dipole field

Field has direction: lines point away from N and toward S



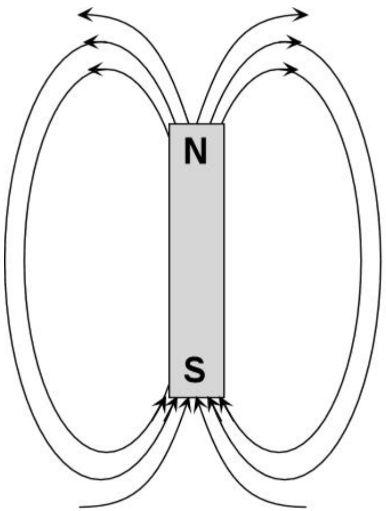
Definition of a Pole: Where lines meet (converge)

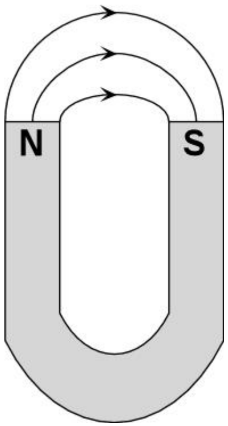
e.g.: lines of longitude on a globe meet at poles

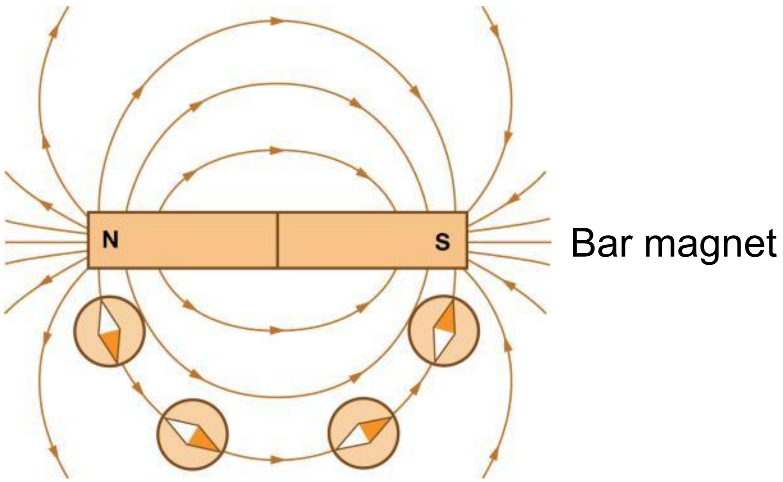
MAGNETIC FIELD

- The area around a magnet where the effect of the magnetic force produced by the magnet can be detected is called magnetic field.

- Field lines indicate both **direction** and **magnitude** (strength) of a magnetic field. They end at **poles**.







Electromagnet

- **Electromagnetism** is the phenomenon which deals with the interaction between an Electric field and a magnetic Field.
- Stationary charges in a system lead to an electric field and moving charges in a system lead to a magnetic field.
- The direction of electric field and Magnetic field is always perpendicular to each other, and the wave travels at the speed of light.
- An electromagnet is a core of iron or steel around which a coil is wound. When a current is passed through the coil, the core is magnetized temporarily.

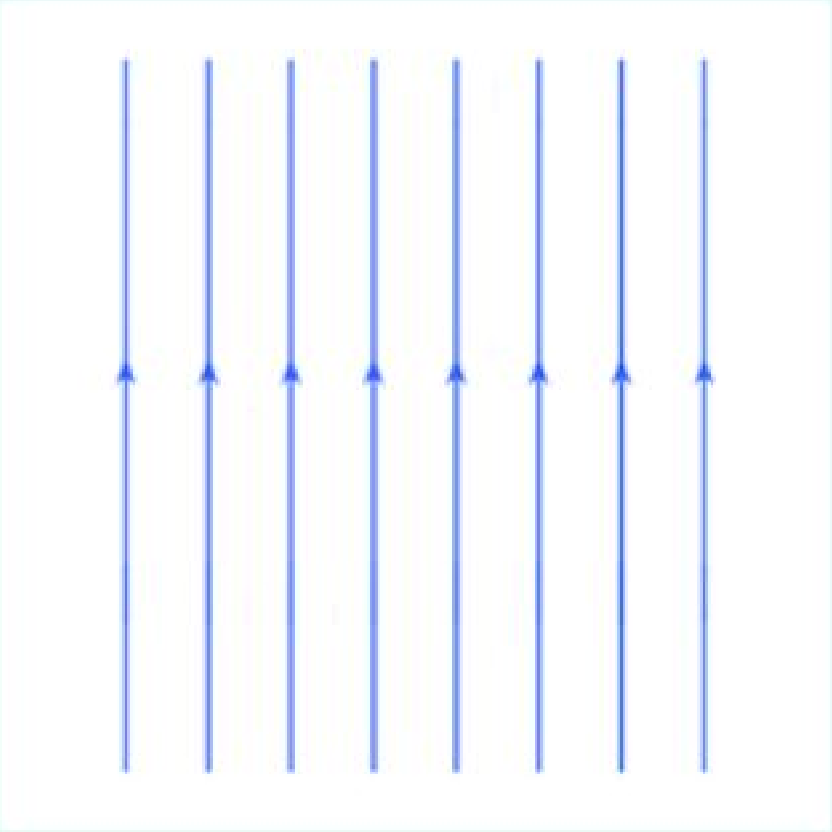
Magnetic flux

- The total number of magnetic lines of force in a magnetic field is called the magnetic flux.
- It is denoted by symbol φ
- The unit of magnetic flux is weber(Wb).

Characteristics of magnetic flux lines.

1. They have no physical existence.
2. They form closed path.
3. They never intersect each other.
4. Lines of magnetic flux closer to each other and having the same direction repel each other.
5. Lines of magnetic flux closer to each other and having the opposite direction attract each other.

Flux density, $\boldsymbol{B}$

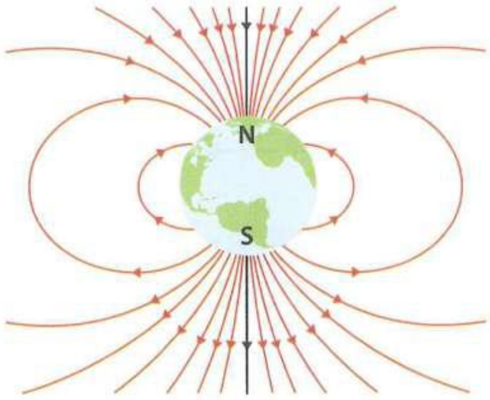


- Density of flux (or field) lines determines forces on magnetic poles
- Direction of flux indicates direction of force on a North pole

$$B = \phi / A$$

Magnetic flux density

- It is defined as the flux per unit area at right angle to the flux it is denoted by B. and unit is Wb/m^2
- The unit of **Magnetic Flux Density** (B) is the **Tesla (T)** and like the other field strengths it is a **Vector.**
- There is a stronger field at the poles where there are more field lines.
- $B = \varphi/a$
- Φ = flux in Wb
- A = area of cross section (m^2)



Magnetic induction/Magnetic flux density (B) = Magnetic flux per unit area. Units: Tesla = Weber/m²

(*i*) When the plane of the coil is perpendicular to the flux direction [See Fig. 7.3], maximum flux will pass through the coil *i.e.*

$$\text{Maximum flux, } \phi_m = B\,A \text{ Wb}$$

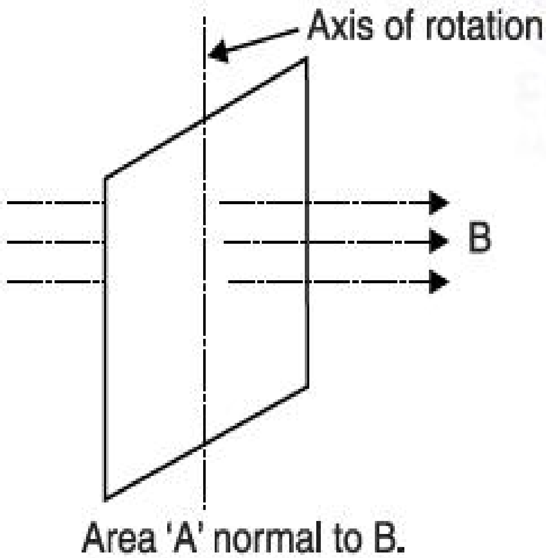


Fig. 7.3

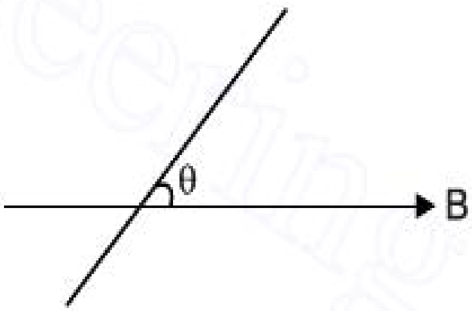


Fig. 7.4

(*ii*) When the plane of the coil is inclined at an angle θ to the flux direction [See Fig. 7.4], then flux ϕ through the coil is

$$\phi = B\,A \sin\theta \text{ Wb}$$

(*iii*) When the plane of the coil is parallel to the flux direction, $\theta = 0°$ so that no flux will pass through the coil ($\phi = BA \sin 0° = 0$).

A circular coil of 100 turns and diameter 3·18 cm is mounted on an axle through a diameter and placed in a uniform magnetic field, where the flux density is 0·01 Wb/m², in such a manner that axle is normal to the field direction. Calculate :

(*i*) *the maximum flux through the coil and the coil position at which it occurs.*

(*ii*) *the minimum flux and the coil position at which it occurs.*

(*iii*) *the flux through the coil when its plane is inclined at 60° to the flux direction.*

Solution. Fig. 7.5 shows the conditions of the problem.

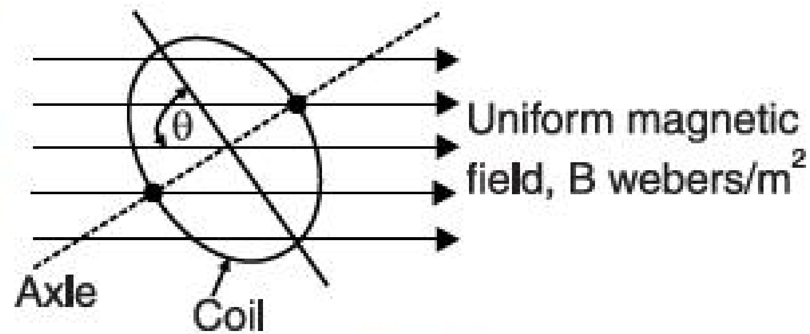


Fig. 7.5

(*i*) The maximum flux will pass through the coil when the plane of the coil is perpendicular to the flux direction.

$$\therefore \quad \text{Maximum flux, } \phi_m = B \times \text{Total coil area}$$

$$= (0{\cdot}01) \times \pi r^2$$

$$= 0{\cdot}01 \times \pi \times \left(\frac{3{\cdot}18}{2}\right)^2 \times 10^{-4} = \mathbf{0{\cdot}795 \times 10^{-5}\ Wb}$$

(*ii*) When the plane of the coil is parallel to the flux direction, no flux will pass through the coil. This is the minimum flux coil position and the minimum flux is **zero.**

(*iii*) When the plane of the coil is inclined at an angle θ to the flux direction, the flux φ through the coil is

$$\phi = B\,A \sin\theta = (B\,A)\sin\theta = (0{\cdot}795 \times 10^{-5}) \times \sin 60°$$

$$= \mathbf{0{\cdot}69 \times 10^{-5}\ Wb}$$

Magnetomotive force

- The current flowing in an electric circuit is due to the existence of electromotive force similarly **magnetomotive force** (MMF) is required to drive the magnetic flux in the magnetic circuit.

- The magnetic pressure, which sets up the magnetic flux in a magnetic circuit is called Magnetomotive Force. The SI unit of MMF is Ampere-turn (AT)

- MMF is equal to the product of current (I) flowing through coil and number of turns (N) of the coil.

$$\text{m.m.f} = NI$$

MAGNETIC FIELD INTENSITY

- Magnetic field intensity is defined as a the magnetomotive force per unit length of the magnetic flux path.

It is denoted by H.

Its unit is AT/m

H = mmf/L

= NI/L

PERMEABILITY :

It is the ability of a magnetic material to create the magnetic flux through it.

When m.m.f is applied to a ferromagnetic material, the flux produced is very large as compared to the flux produced in air, vacuum or non-magnetic material.

It is given by the product of permeability of a vacuum and relative permeability of a magnetic material.

i.e. $\boxed{\mu = \mu_o \mu_r}$

where μ_o = permeability of a vacuum

= 4×10^{-7} H/m

$\boxed{\mu_r = \text{relative permeability of a material}}$

Relative permeability :

When the magnetomotive force is applied to a ferromagnetic material, the flux produced is very large compared with the flux produced in air, vacuum or non-magnetic material.

The relative permeability of a medium or material is defined as the ratio of the flux density produced in the material to the flux density produced in vacuum (or air) by the same magnetic field intensity.

It is denoted by μ_r. Mathematically,

$$\text{Relative permeability} = \frac{\text{Flux density in the medium}}{\text{Flux density in vacuum}}$$

$$\therefore \quad \boxed{\mu_r = \frac{\text{Flux density in the medium}}{\text{Flux density in vacuum}}}$$

For air or non-magnetic material $\mu_r = 1$

A toroidal coil has a magnetic path length of 33 cm and a magnetic field strength of 650 A/m. The coil current is 250 mA. Determine the number of coil turns.

Solution. $$H = \frac{NI}{l}$$

Here, $H = 650$ A/m ; $I = 250$ mA $= 0.25$A ; $l = 33$ cm $= 0.33$ m

$$\therefore \quad 650 = \frac{N \times 0.25}{0.33} \quad \text{or} \quad N = \frac{650 \times 0.33}{0.25} = \mathbf{858\ turns}$$

Example 7.8. *Determine the m.m.f. required to generate a total flux of 100μWb in an air gap 0.2 cm long. The cross-sectional area of the air gap is 25 cm².*

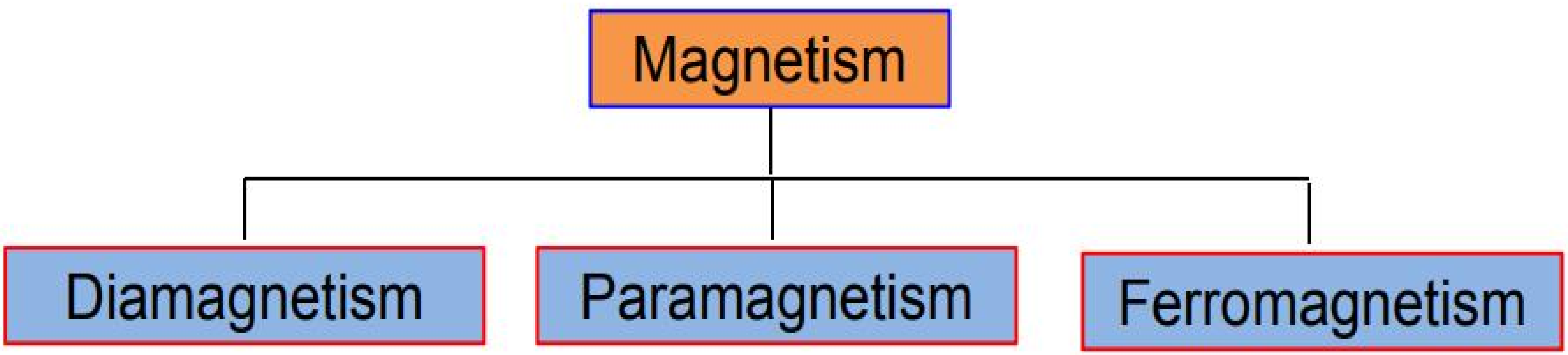
Solution. $\phi = 100\ \mu\text{Wb} = 100 \times 10^{-6}\ \text{Wb}$; $l = 0.2 \times 10^{-2}\ \text{m}$; $A = 25 \times 10^{-4}\ \text{m}^2$

$$\text{Flux density, } B = \frac{\phi}{A} = \frac{100\times10^{-6}}{25\times10^{-4}} = 4 \times 10^{-2}\ \text{Wb/m}^2$$

$$\text{Magnetising force, } H = \frac{B}{\mu_0} = \frac{4\times10^{-2}}{4\pi\times10^{-7}} = 3.18 \times 10^{4}\ \text{AT/m}$$

Magnetism

➢ Depending on the existence and alignment of magnetic moments with or without application of magnetic field, three types of magnetism can be defined.



Classification of magnetic material

- Magnetic materials are classified according to their relative permeability

Ferromagnetic materials

Their relative permeability is greater than unity and dependent on field strength. They are easily magnetized.

Ex: Iron, Cobalt, Nickel

Paramagnetic material

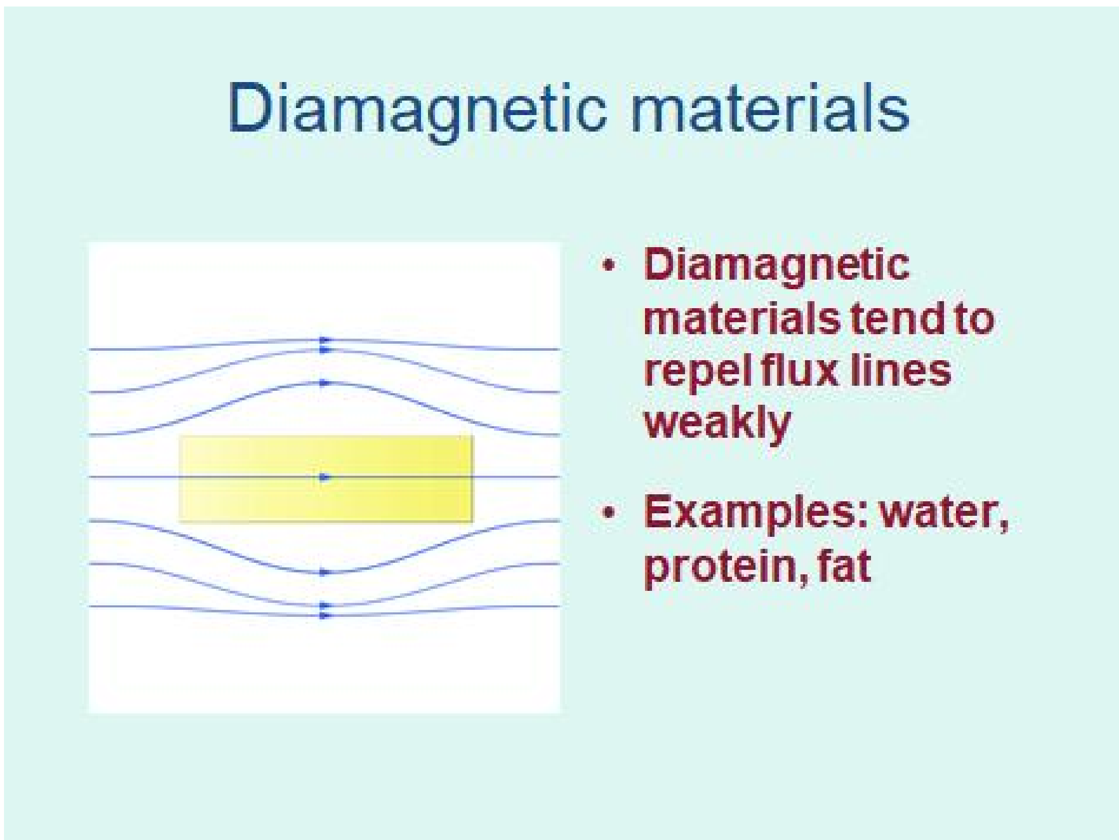
Relative permeability is slightly greater than unity and They are slightly magnetized.

Ex: Aluminium, Platinum

Diamagnetic material

Relative permeability is slightly less than unity and have the opposite effect of paramagnetic materials.

Ex: Silver, copper, hydrogen



Reluctance

- It is defined as the opposition offered by a magnetic circuit to the establishment of magnetic flux.
- It is directly proportional to the length of the magnetic circuit and inversely proportional the area of cross-section of the magnetic path.

$$\boxed{\text{Mathematically, } S = \frac{l}{\mu_o \mu_r A}}$$

Where l = length of the magnetic path

μ_o = $4\pi \times 10^{-7}$ H/m

μ_r = relative permebaility of a material

A = cross sectional area

- Its unit is AT/Wb

PERMEANCE

- Permeance is defined as the reciprocal of reluctance.
- Thus it is a measure of ease with which flux can be set up in the magnetic material.
- It is denoted by Λ. Its unit is $\frac{\text{Wb}}{\text{AT}}$.
- It is analogous to conductance in an electric circuit.

i.e. $\boxed{\text{Permeance} = \frac{1}{\text{reluctance}}}$

It is analogous to conductance in an electric circuit.

RELUCTIVITY

Reluctivity or specific reluctance is defined as the reluctance offered by a magnetic circuit of a unit length and unit cross-section.

We know that the reluctance is given by

$$S = \frac{1}{\mu} * \frac{l}{A}$$

When l = 1 m and A = 1 m^2 we have

$$\therefore \quad S = \frac{1}{\mu} \times \frac{1}{1} = \frac{1}{\mu}$$

According to the definition, the above value of reluctance is known as reluctivity or specific reluctance.

$$\therefore \text{ Specific reluctance or reluctivity } = \frac{1}{\mu}$$

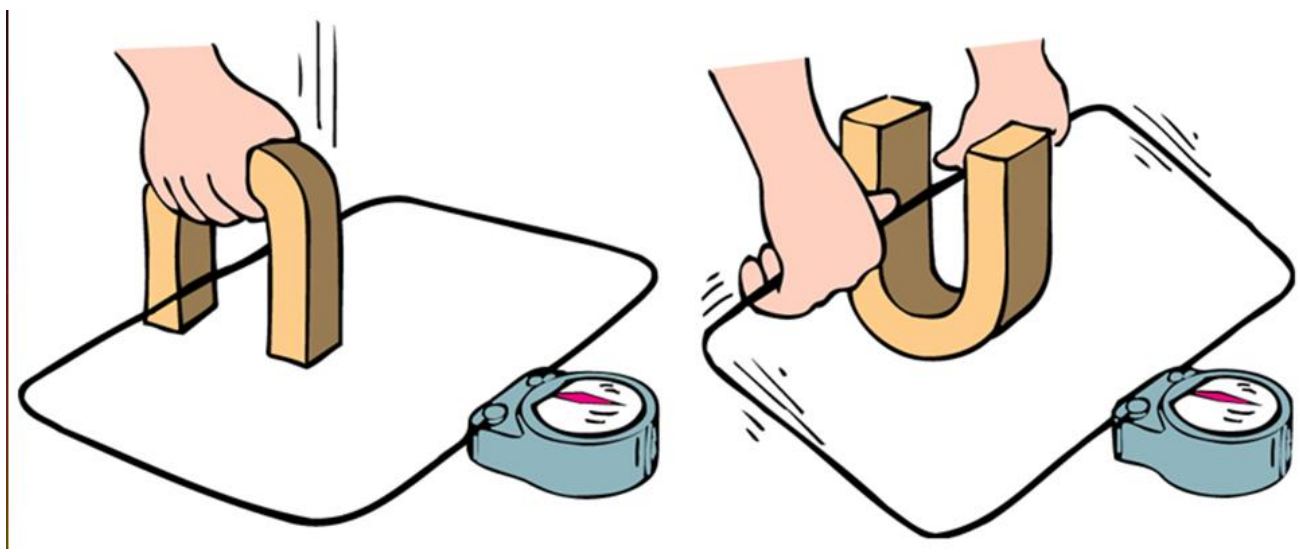
$$= \frac{1}{\text{absolute permeability}}$$

Thus, reluctivity is equal to the reciprocal of the absolute permeability. Its unit is metre/henry.

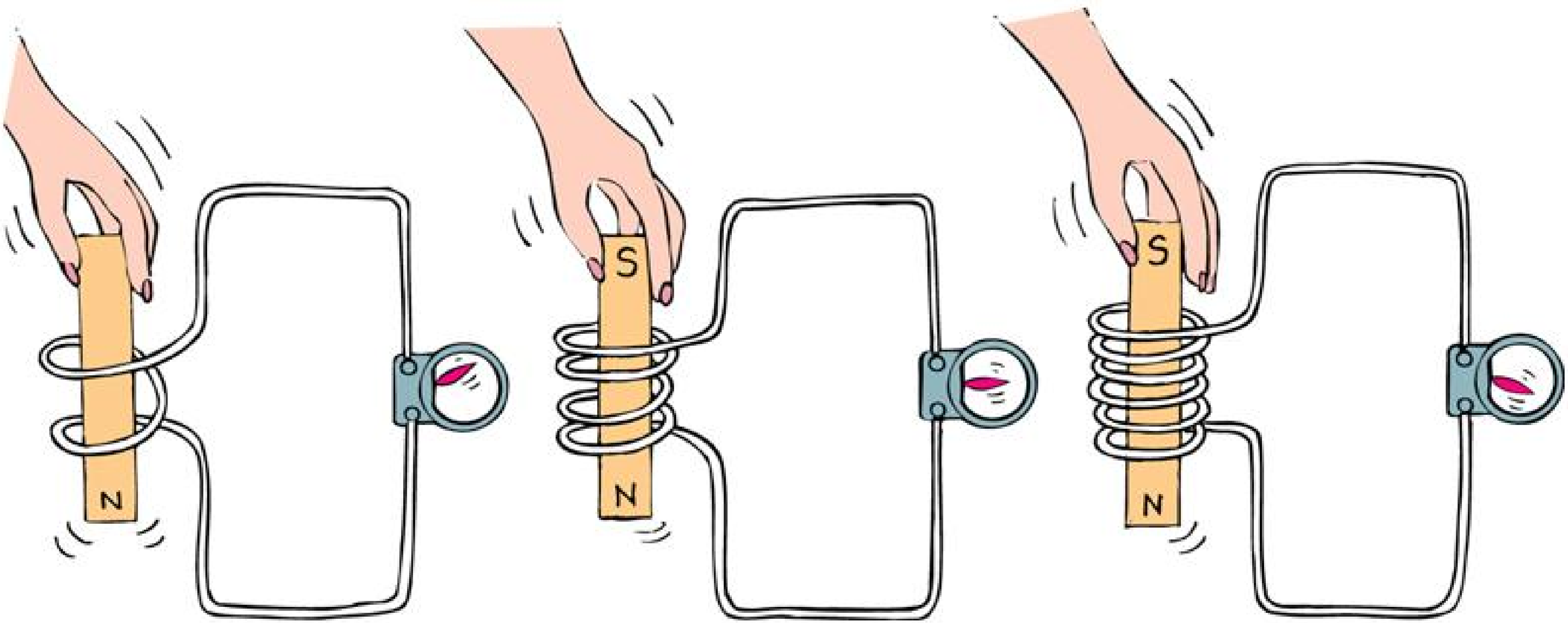
It is analogous to resistivity (specific resistance) in an electric circuit.

Electromagnetic Induction

- Process of generating a current in a circuit by changing a magnetic field.
- Magnetic field lines must be cut perpendicular by relative motion of magnet and conductor.
  - Either the magnet can move or the wire can move.



It is more difficult to push the magnet into a coil with more loops because the high current generates a stronger magnetic field which acts against the magnet.



Electromagnetic induction

Faraday's 1st Laws of Electromagnetic Induction

Faraday's law: Relative motion of a wire and a magnetic field will induce an e.m.f. (voltage). <u>If there is a complete circuit</u>, a current will be induced too.

– magnet stationary, coil moves

– coil stationary, magnet moves,

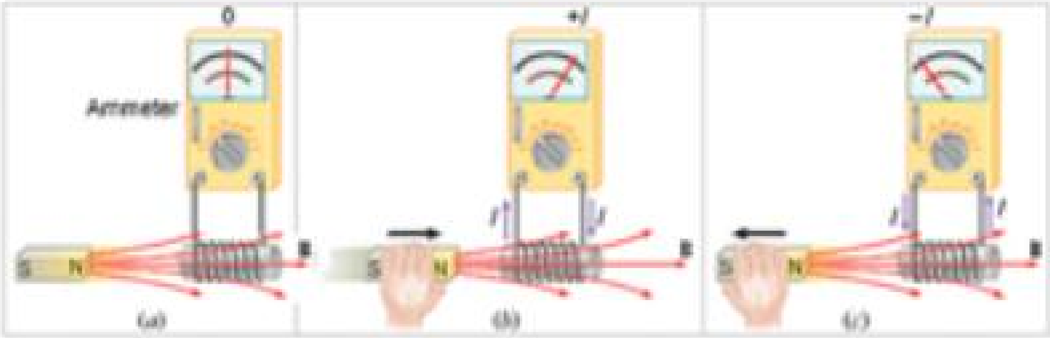
– coil stationary, magnetic field lines changing

(By rotating the coil relative to magnetic field)

Induced EMF is proportional to 'the rate at which field lines are cut'.

Lenz's Law: The induced current always flows in such a direction as to oppose the change which causes it.

Induced Emf and Induced Current



(a) When there is no relative motion between the coil of wire and the bar magnet, there is no current in the coil.

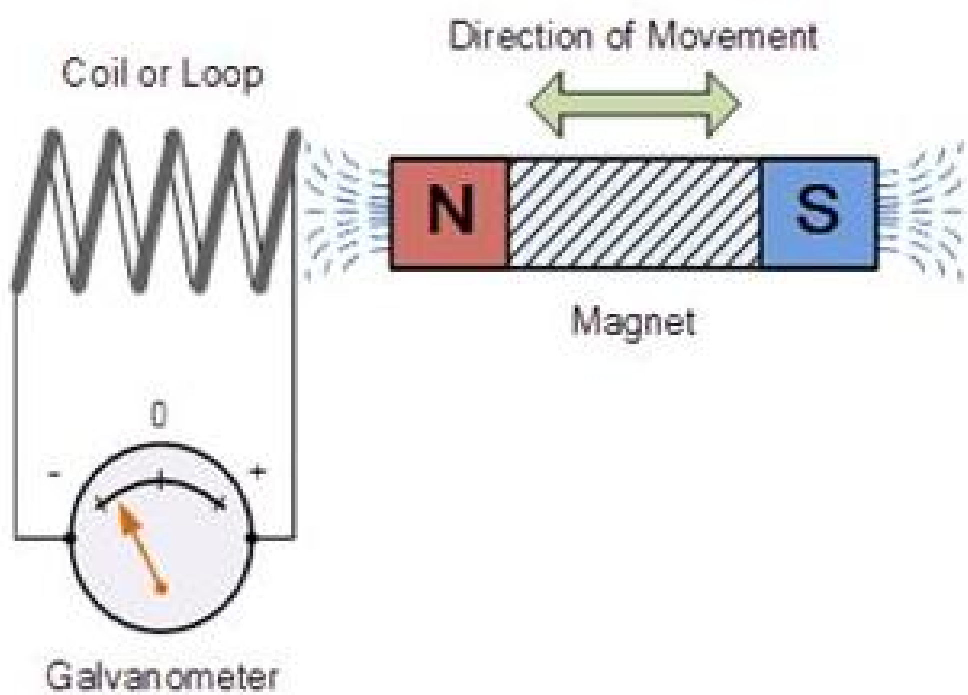
(b) A current is created in the coil when the magnet moves toward the coil.

(c) A current also exists when the magnet moves away from the coil, but the direction of the current is opposite to that in (b).

Faraday's 2nd Laws of Electromagnetic Induction

- **Faraday's second law of electromagnetic induction** states that, the magnitude of induced emf is equal to the rate of change of flux linkages with the coil. The flux linkages is the product of number of turns and the flux associated with the coil.

Faraday Law Formula



Consider, a magnet is approaching towards a coil. Here we consider two instants at time T_1 and time T_2.

Flux linkage with the coil at time,

$$T_1 = N\phi_1 \; wb$$

Flux linkage with the coil at time,

$$T_2 = N\phi_2 \; wb$$

Change in flux linkage,

$$N(\phi_2 - \phi_1)$$

Let this change in flux linkage be,

$$\phi = (\phi_2 - \phi_1)$$

So, the Change in flux linkage

$$N\phi$$

Now the rate of change of flux linkage

$$\frac{N\phi}{t}$$

Take derivative on right hand side we will get

$$N\frac{\mathrm{d}\phi}{\mathrm{d}t}$$

The rate of change of flux linkage

$$E = N\frac{d\phi}{dt}$$

But according to Faraday's law of electromagnetic induction, the rate of change of flux linkage is equal to induced emf.

$$E = -N\frac{d\phi}{dt}$$

Lenz's Law

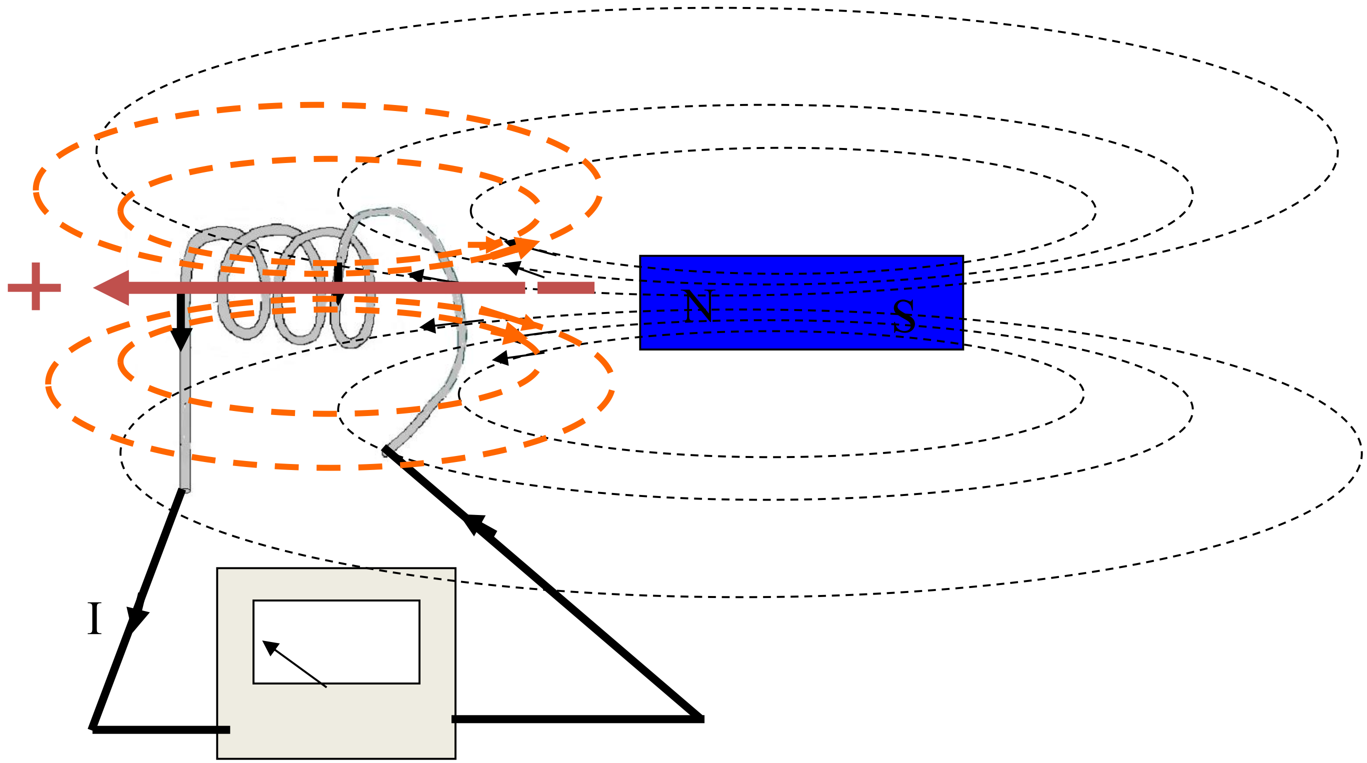
Lenz's law of electromagnetic induction states that, when an emf is induced according to Faraday's law, the polarity (direction) of that induced emf is such that it opposes the cause of its production.

Thus, considering Lenz's law

$E = -N\ (d\Phi/dt)$ (volts)

The negative sign shows that, the direction of the induced emf and the direction of change in magnetic fields have opposite signs.

Lenz's Law



Fleming's Left Hand Rule and Fleming's Right Hand Rule

Whenever a current carrying conductor comes under a magnetic field, there will be a force acting on the conductor. The direction of this force can be found using Fleming's Left Hand Rule (also known as 'Flemings left-hand rule for motors').

Similarly if a conductor is forcefully brought under a magnetic field, there will be an induced current in that conductor. The direction of this force can be found using Fleming's Right Hand Rule.

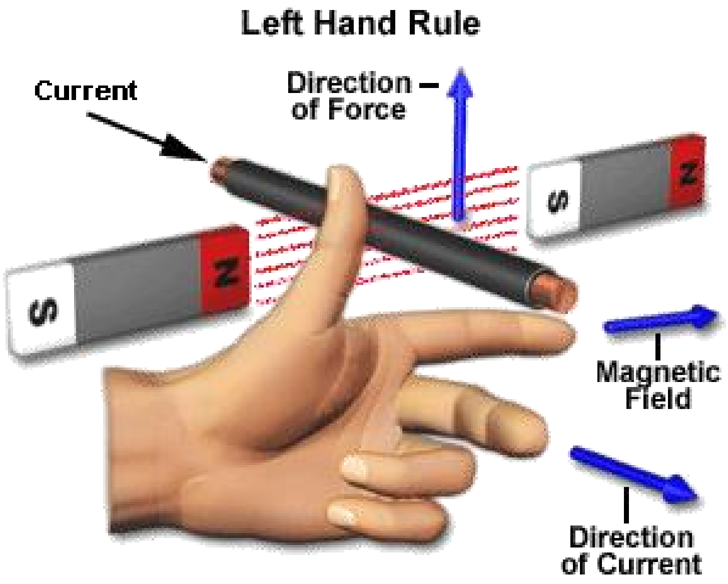
In both Fleming's left and right hand rules, there is a relation between the magnetic field, the current and force. This relation is directionally determined by **Fleming's Left Hand rule** and **Fleming's Right Hand rule** respectively.

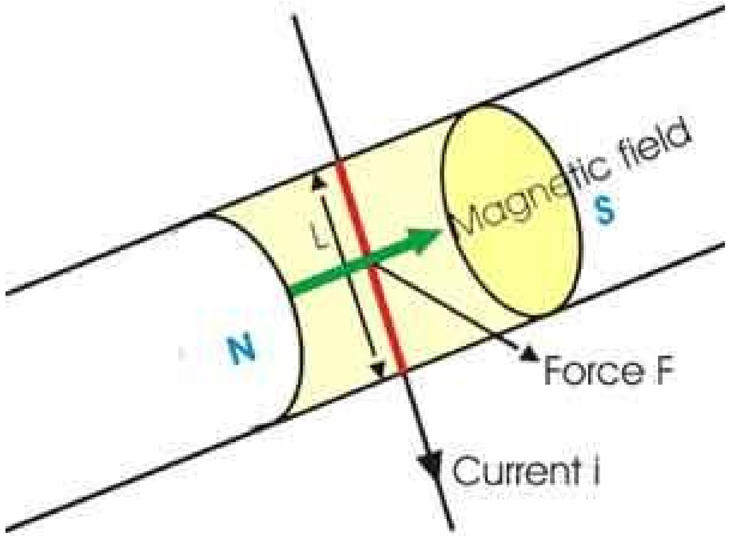
These rules do not determine the magnitude but instead show the direction of any of the three parameters (magnetic field, current, force) when the direction of the other two parameters is known.

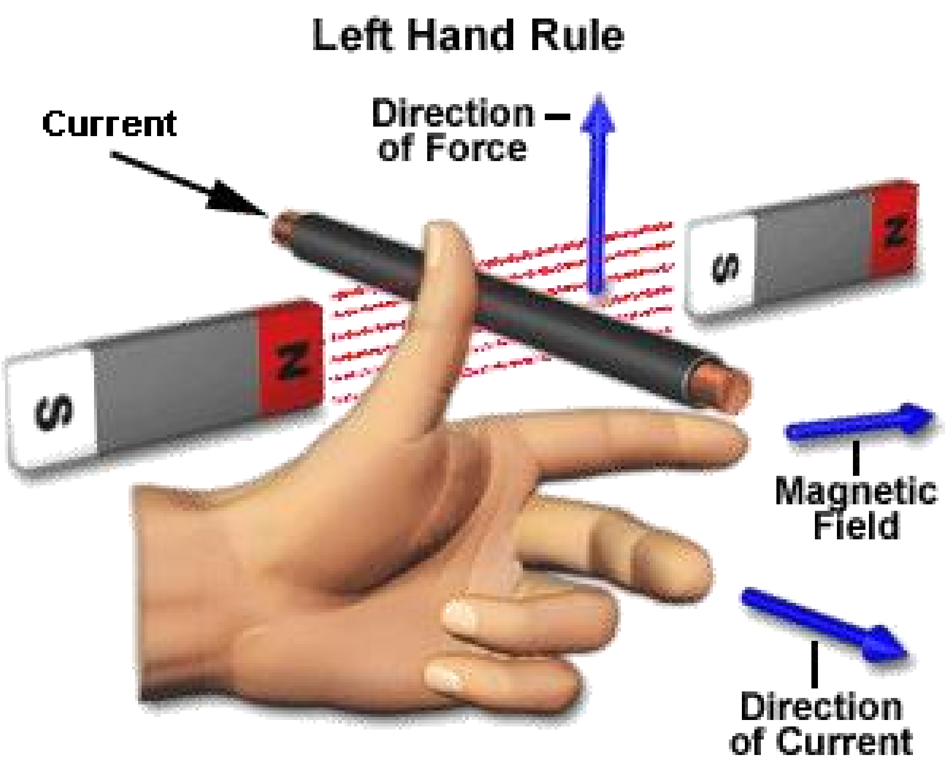
Fleming's Left-Hand rule is mainly applicable to electric motors and **Fleming's Right-Hand rule** is mainly applicable to electric generators.

Fleming's Left Hand Rule

- It is found that whenever a current carrying conductor is placed inside a magnetic field, a force acts on the conductor, in a direction perpendicular to both the directions of the current and the magnetic field.
- In the figure below, a portion of a conductor of length 'L' is placed vertically in a uniform horizontal magnetic field of strength 'H', produced by two magnetic poles N and S. If the current 'I' is flowing through this conductor, the force acting on the conductor




Magnetic field
S
N
L
Force F
Current I

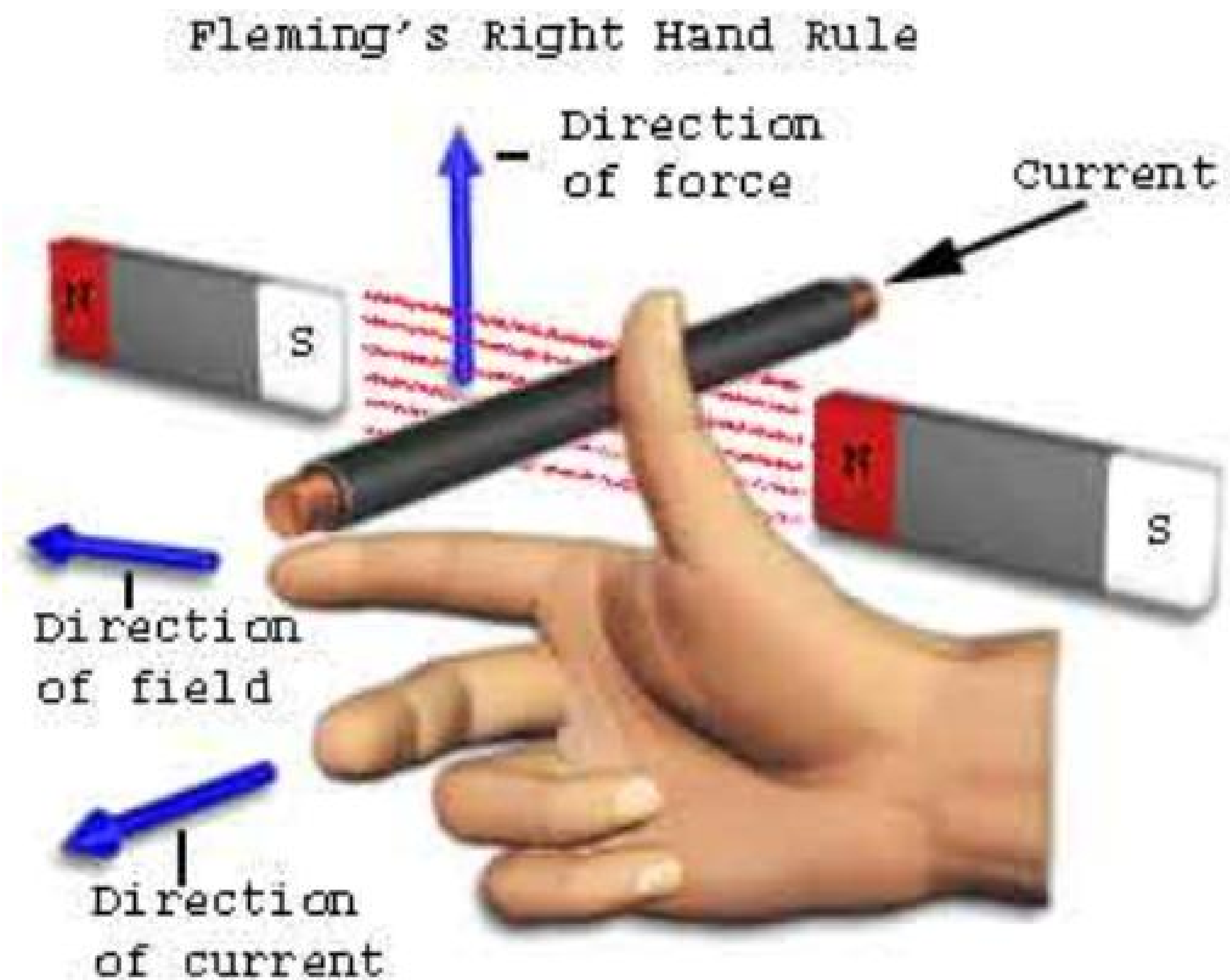

Left Hand Rule
Current
Direction of Force
S
N
S
N
Magnetic Field
Direction of Current


MOTION
FIELD
CURRENT

Fleming Right Hand Rule

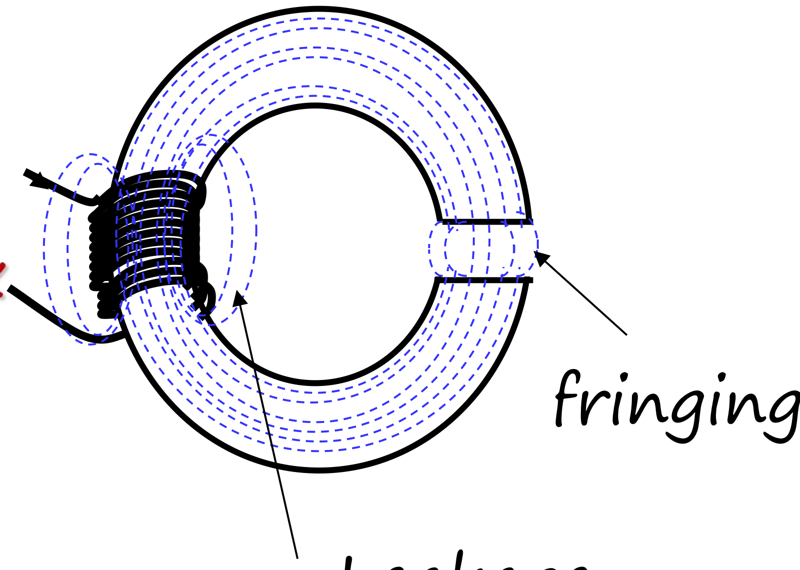
- As per Faraday's law of electromagnetic induction, whenever a conductor moves inside a magnetic field, there will be an induced current in it. If this conductor gets forcefully moved inside the magnetic field, there will be a relation between the direction of applied force, magnetic field and the current. This relation among these three directions is determined by **Fleming's right-hand Rule**.
- This rule states "Hold out the right hand with the first finger, second finger and thumb at the right angle to each other. If forefinger represents the direction of the line of force, the thumb points in the direction of motion or applied force, then second finger points in the direction of the induced current".

Fleming Right Hand Rule



Leakage Flux and Fringing

- It is found that it is impossible to confine all the flux to the iron path only. Some of the flux leaks through air surrounding the iron ring.
- Spreading of lines of flux at the edges of the air-gap. Reduces the flux density in the air-gap.



$$\text{Leakage coefficient } \lambda = \frac{\text{Total flux produced}}{\text{Useful flux available}}$$

Relation between B and H

The flux density B produced in a material is directly proportional to the applied magnetising force H. In other words, the greater the magnetising force, the greater is the flux density and *vice-versa i.e.*

$$B \propto H$$

or
$$\frac{B}{H} = \text{Constant} = \mu$$

The ratio B/H in a material is always constant and is equal to the absolute permeability μ ($= \mu_0 \mu_r$) of the material. This relation gives yet another definition of absolute permeability of a material.

$$\text{Obviously,} \quad B = \mu_0 \mu_r H \qquad \text{...in a medium}$$

$$= \mu_0 H \qquad \text{...in air}$$

Suppose a magnetising force H produces a flux density B_0 in air. Clearly, $B_0 = \mu_0 H$. If air is replaced by some other material (relative permeability μ_r) and the same magnetising force H is applied, then flux density in the material will be $B_{mat} = \mu_0 \mu_r H$.

$$\therefore \qquad \frac{B_{mat}}{B_0} = \frac{\mu_0 \mu_r H}{\mu_0 H} = \mu_r$$

Hence **relative permeability** *of a material is equal to the ratio of flux density produced in that material to the flux density produced in air by the same magnetising force.*

Thus when we say that μ_r of soft iron is 8000, it means that for the same magnetising force, flux density in soft iron will be 8000 times its value in air. In other words, for the same cross-sectional area and H, the magnetic lines of force will be 8000 times greater in soft iron than in air.

Magnetic circuit

as air. The combinatio...

Analysis of simple magnetic circuit :

- Consider a circular solenoid or a toroidal ring having N turns wound on an iron core (ring) as shown in Fig. 3.9. When current I amperes is flowing, magnetic flux is set up in the core.

Let

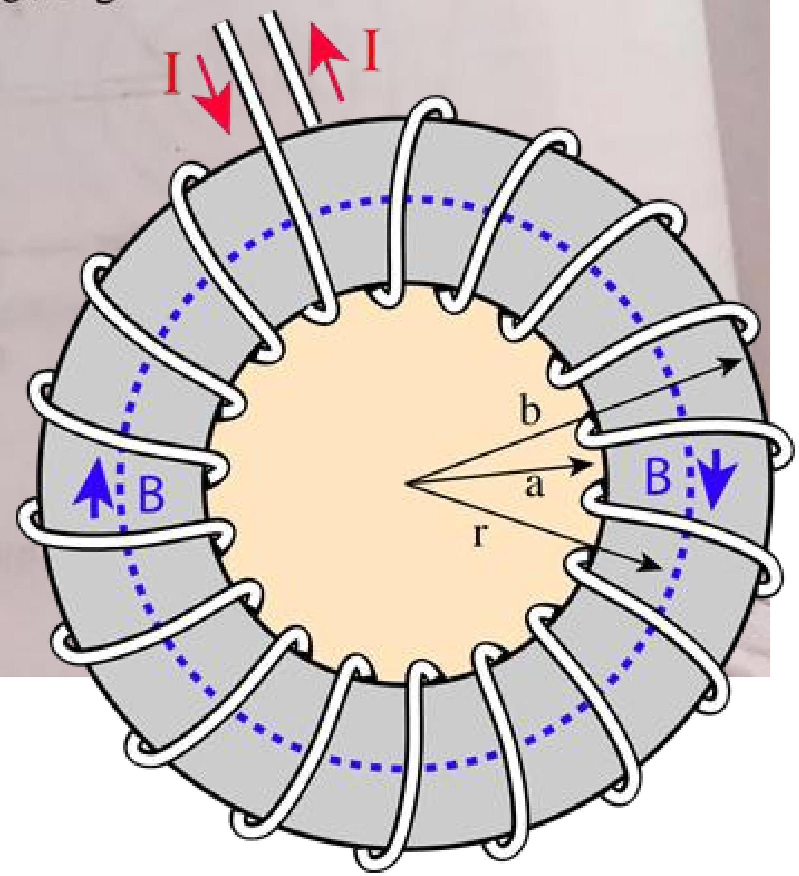
l = mean length of magnetic path in m

A = area of cross section of core in m^2

μ_r = relative permeability of core material

. According to the definition of field strength.

$$H = \frac{NI}{l}$$



Flux density in the core material is given by

$$B = \frac{\phi}{A} \text{ Wb/m}^2$$

Now $B = \mu_0 \mu_r H$

$$= \mu_0 \mu_r \frac{NI}{l} \text{ Wb / m}^2$$

Total flux produced

$$\phi = B \times A$$

$$= \mu_0 \mu_r \frac{NI}{l} A$$

$$= \frac{NI}{(l / \mu_0 \mu_r A)}$$

or $\phi = \frac{NI}{S}$

Where $S = \frac{l}{\mu_0 \mu_r A}$

$$\therefore \text{ flux} = \frac{\text{m.m.f.}}{\text{reluctance}}$$

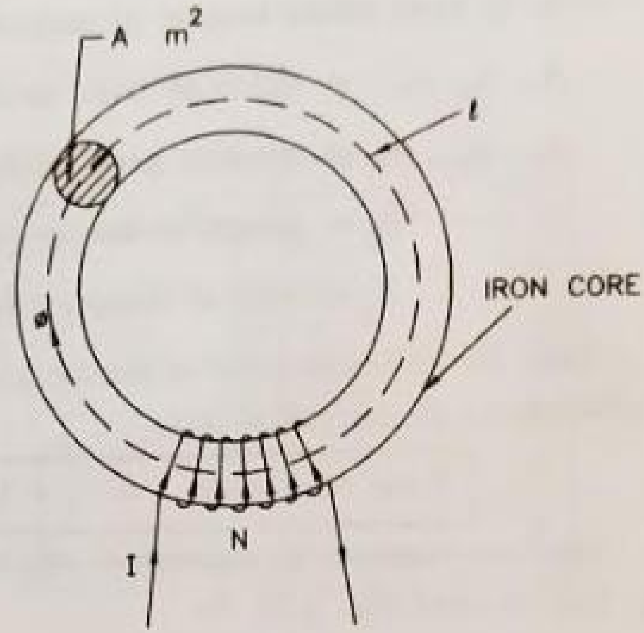


FIG. 3.9

It may be noted that the above expression has a strong resemblance to Ohm's law for electric circuit given below.

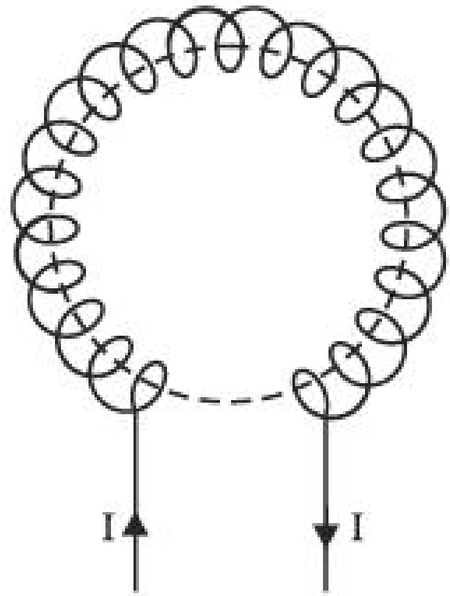
$$\text{Current} = \frac{\text{e.m.f.}}{\text{resistance}}$$

The m.m.f. is analogous to e.m.f. in electric circuit, reluctance is analogous to resistance and flux is analogous to current. Because of this similarity, the above expression i.e.

$$\boxed{\text{flux} = \frac{\text{m.m.f.}}{\text{reluctance}}}$$

is also known as "Ohm's law of magnetic circuit."

Example 7.12. *An air-cored solenoid has length of 15 cm and inside diameter of 1.5 cm. If the coil has 900 turns, determine the total flux within the solenoid when the coil current is 100 mA.*



Solution. For a solenoid, the length of the magnetic circuit, l = coil length $= 15 \times 10^{-2}$ m.

$$D = 1.5 \times 10^{-2}\text{ m} \;;\; N = 900\text{ turns} \;;\; I = 100 \times 10^{-3}\text{ A}$$

$$\therefore \quad \text{m.m.f.} = NI = 900 \times 100 \times 10^{-3} = 90\text{ AT}$$

$$\text{Magnetising force, } H = \frac{\text{m.m.f.}}{l} = \frac{90}{15 \times 10^{-2}} = 600\text{ AT/m}$$

$$\text{Magnetic flux density, } B = \mu_0 H = 4\pi \times 10^{-7} \times 600 = 24\pi \times 10^{-5} \text{ Wb/m}^2$$

$$\therefore \quad \text{Total flux, } \phi = BA = 24\pi \times 10^{-5} \times \pi \frac{D^2}{4}$$

$$= 24\pi \times 10^{-5} \times \pi \times \frac{(1.5 \times 10^{-2})^2}{4} = \mathbf{1.33 \times 10^{-7} \text{ Wb}}$$

Comparison Between Magnetic and Electric Circuits

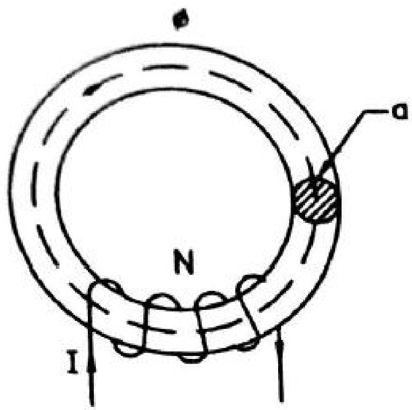


FIG. 3.18

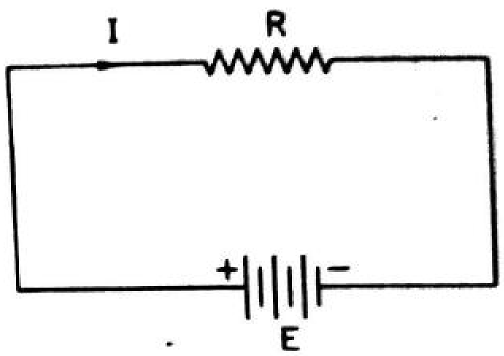


FIG. 3.19

Similarities

Magnetic circuit	Electric circuit
1. The closed path for magnetic flux is called magnetic circuit.	1. The closed path for electric current is called electric circuit.
2. Flux $= \dfrac{\text{m.m.f}}{\text{Reluctance}}$	2. Current $= \dfrac{\text{e.m.f}}{\text{resistance}}$

3.	Flux, ϕ in Weber	3.	Current, I in ampere
4.	m.m.f. in Amp Turn (AT)	4.	e.m.f. in volts (V)
5.	Reluctance $S = \dfrac{l}{\mu_o \mu_r A}$ AT / Wb	5.	Resistance $R = \varrho \dfrac{1}{a}$ ohms
6.	Permeance $= \dfrac{1}{\text{Reluctance}}$	6.	Conductance $= \dfrac{1}{\text{resistance}}$
7.	Permeability μ	7.	Conductivity $\sigma = \dfrac{1}{\varrho}$
8.	Reluctivity	8.	Resistivity
9.	Flux–density, $B = \dfrac{\phi}{A}$ Wb / m^2	9.	Current density, $J = \dfrac{I}{A}$ A/m^2
10.	Magnetic intensity $H = \dfrac{NI}{l}$ AT / m	10.	Electric intensity, $E = \dfrac{V}{d}$

Dissimilarities

1. Truly speaking, magnetic flux does not flow.	1. The electric current acutally flows in an electric circuit.
2. There is no magnetic insulator. For example, flux can be set up even in air (the best known magnetic insulator) with reasonable m.m.f.	2. There are a number of electric insulators. For instance, air is a very good insulator and current cannot pass through it.
3. The value of μ_r is not constant for a given magnetic material. It varies considerably with flux density (B) in the material. This implies that reluctance of a magnetic circuit is not constant rather it depends upon B.	3. The value of resistivity (ρ) varies very slightly with temperature. Therefore, the resistance of an electric circuit is practically constant. This salient feature calls for different approach to the solution of magnetic and electric circuits.
4. No energy is expended in a magnetic circuit. In other words, energy is required in creating the flux, and not in maintaining it.	4. When current flows through an electric circuit, energy is expended so long as the current flows. The expended energy is dissipated in the form of heat.

Statically induced EMF and Dynamically Induced EMF

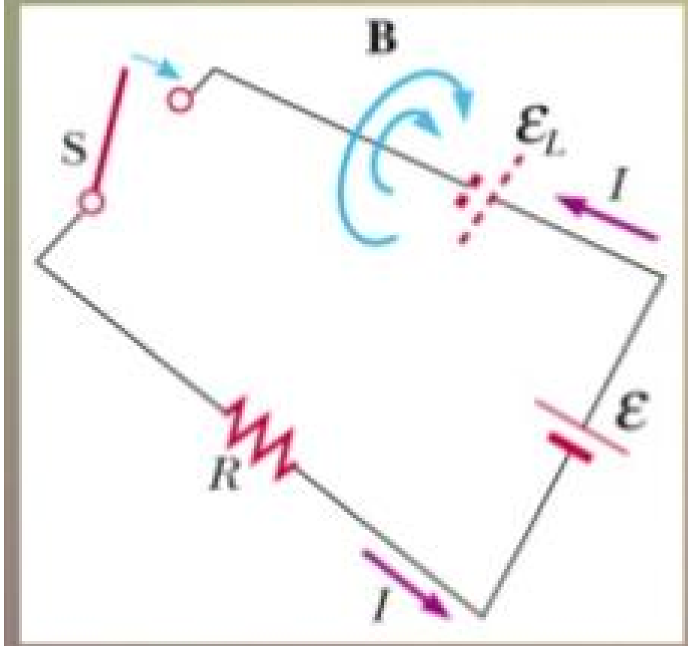
When the magnetic flux linking a conductor (or coil) changes, an e.m.f. is induced in it. This change in flux linkages can be brought about in the following two ways :

(*i*) The conductor is moved in a stationary magnetic field in such a way that the flux linking it changes in magnitude. The e.m.f. induced in this way is called **dynamically induced e.m.f.** (as in a d.c. generator). It is so called because e.m.f. is induced in the conductor which is in motion.

(*ii*) The conductor is stationary and the magnetic field is moving or changing. The e.m.f. induced in this way is called **statically induced e.m.f.** (as in a transformer). It is so called because the e.m.f. is induced in a conductor which is stationary.

It may be noted that in either case, the magnitude of induced e.m.f. is given by $N d\phi/dt$ *or derivable from this relation.*

Self-Inductance



When the switch is closed, the battery (source emf) starts pushing electrons around the circuit.

The current starts to rise, creating an increasing magnetic flux through the circuit.

This increasing flux creates an induced emf in the circuit.

The induced emf will create a flux to oppose the increasing flux.

The direction of this induced emf will be opposite the source emf.

This results in a gradual increase in the current rather than an instantaneous one.

The induced emf is called the **self-induced emf** or **back emf**.

Self Induced emf & Self Inductance

- The induced emf, e, in a coil is proportional to the rate of the change of the magnetic flux passing through it due to its own current. This emf is termed as **Self Induced EMF**
- The induced emf e is proportional to the rate of change of current through coil and this proportionality constant is called the **self inductance**, L.

$$e_1 = -L \frac{di}{dt}$$

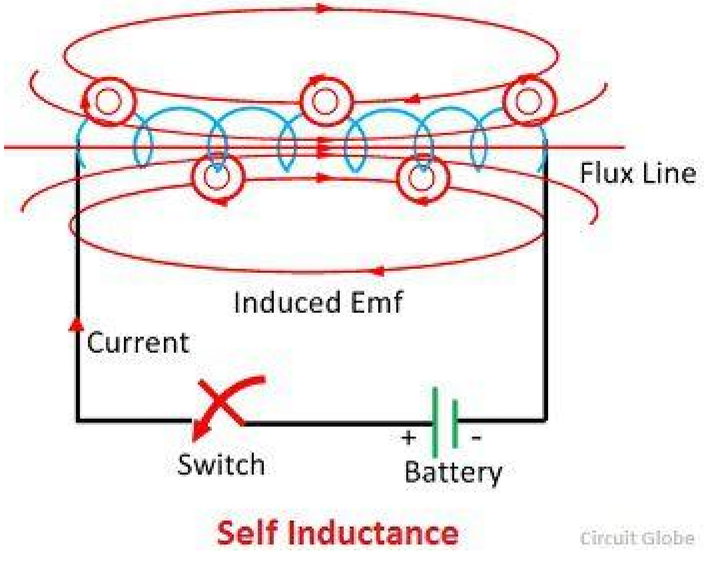
- The negative sign is used to indicate that EMF is opposing the cause producing it

Self Inductance

- The property of a coil due to which it opposes the change of current flowing through itself is called self inductance or inductance of the coil.

- Inductance is attained by a coil due to the self-induced emf produced in the coil itself by changing the current flowing through it.

- If the current in the coil is increasing, the self-induced emf produced in the coil will oppose the rise of current, that means the direction of the induced emf is opposite to the applied voltage.



- If the current in the coil is decreasing, the emf induced in the coil is in such a direction as to oppose the fall of current; this means that the direction of the self-induced emf is same as that of the applied voltage.

- Self-inductance does not prevent the change of current, but it delays the change of current flowing through it.

- This property of the coil only opposes the changing current (alternating current) and does not affect the steady current that is (direct current) when flows through it. The unit of inductance is Henry (H).

Expression For Self Inductance

You can determine the self-inductance of a coil by the following expression

$$e = L\,\frac{dI}{dt}$$

Or

$$L = \frac{e}{{dI}/{dt}}$$

The above expression is used when the magnitude of self-induced emf (e) in the coil and the rate of change of current (dI/dt) is known.

Putting the following values in the above equations as e = 1 V, and dI/dt = 1 A/s then the value of Inductance will be L = 1 H.

Hence, from the above derivation, a statement can be given that a coil is said to have an inductance of 1 Henry if an emf of 1 volts is induced in it when the current flowing through it changes at the rate of 1 Ampere/second.

The expression for Self Inductance can also be given as

$$e = L\frac{dI}{dt} = \frac{d}{dt}(LI) \text{ also } e = N\frac{d\varphi}{dt} = \frac{d}{dt}(N\varphi)$$

$$LI = N\varphi \text{ or } L = \frac{N\varphi}{I} \text{ Henry}$$

where,

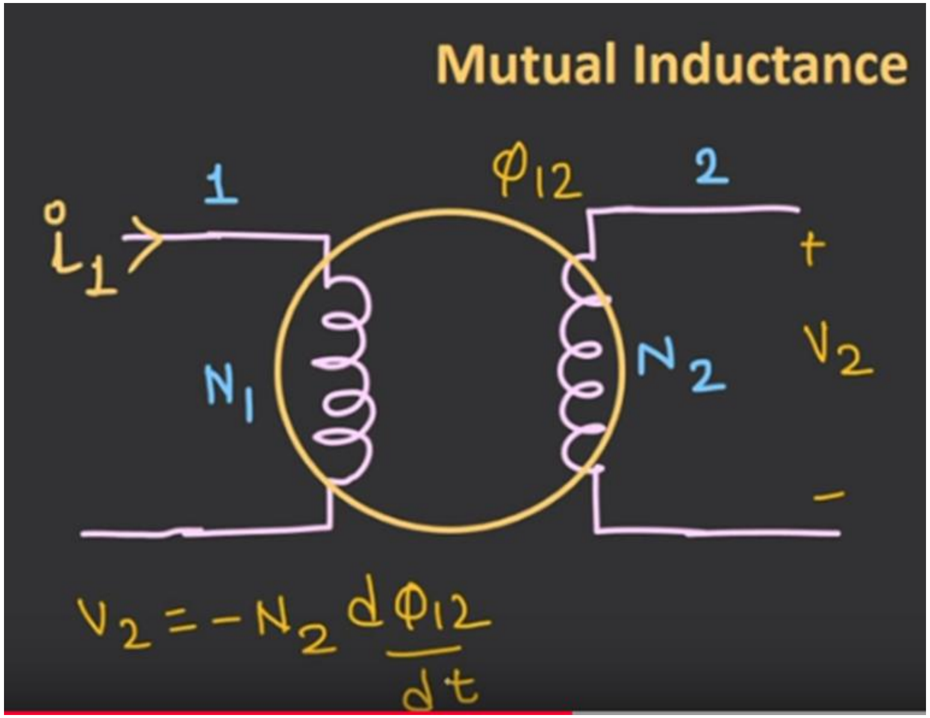
N – number of turns in the coil

Φ – magnetic flux

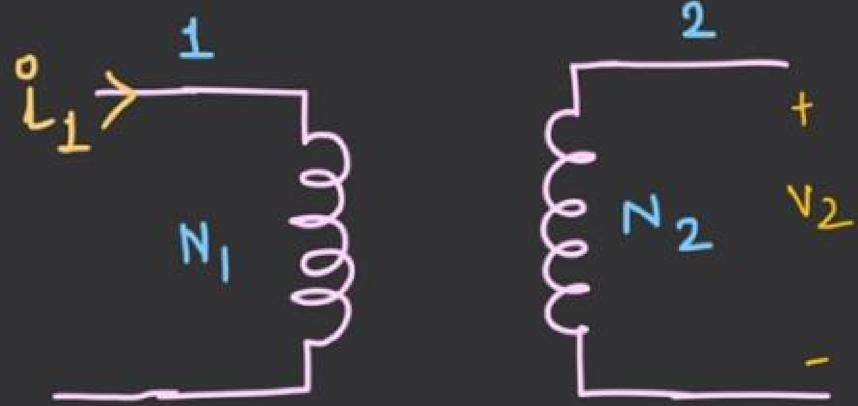
I – current flowing through the coil

Mutual Inductance

- When two coils are placed very close to each other if the current is passed through one coil (primary) a magnetic field is set around this coil. Now the second coil (secondary) is kept in the magnetic field created by the primary coil. Thus magnetic flux is linked with the secondary.
- When a current flowing through primary changes the magnetic flux linked with the secondary changes. Due to which an induced e.m.f. is generated in the secondary coil. This property of producing induced e.m.f. in secondary due to change in the current



Mutual Inductance



$V_2 \propto \frac{di_1}{dt}$

$V_2 = -M\frac{di_1}{dt}$

$$M = \frac{N_2 \phi_{12}}{i_1} = \frac{N_1 \phi_{21}}{i_2}$$

Mutual Inductance

$$\boxed{V_2 = -M\frac{di_1}{dt}} \quad \text{---}(3)$$

$$\boxed{V_2 = -N_2\frac{d\phi_{12}}{dt}} \quad \cdots$$

$$-M\frac{di_1}{dt} = -N_2\frac{d\phi_{12}}{dt}$$

$$\Rightarrow M\,di_1 = N_2\,d\phi_{12}$$

$$M i_1 = N_2 \phi_{12}$$

$$\boxed{M = \frac{N_2 \phi_{12}}{i_1}}$$

Coefficient of coupling

[GTU, Sept. 2009, June 2011]

When current flows through one coil, it produces flux ϕ_1. The whole of this flux may not be linking with the other coil magnetically coupled to it as shown in Fig. 3.46 It may be reduced because of leakage flux ϕ_ℓ.

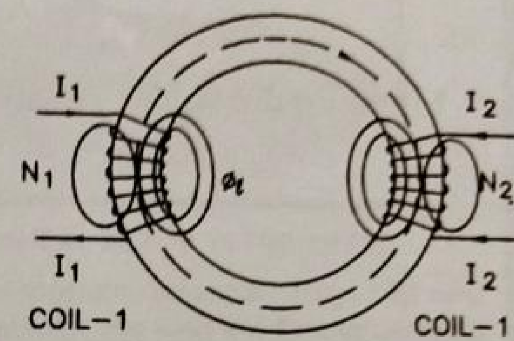


FIG. 3.46

Let N_1 = no. of turns on coil 1

N_2 = no. of turns on coil 2

I_1 = current flowing through coil 1

I_2 = current flowing through coil 2

S = reluctance of the magnetic circuit

ϕ_1 = flux produced by coil 1 due to I_1

ϕ_2 = flux produced by coil 2 due to I_2

The individual self–inductances of coil 1 and coil 2 are given by

$$L_1 = \frac{N_1^2}{S}, \quad L_2 = \frac{N_2^2}{S}, \quad \text{where } S = \frac{\ell}{\mu_0 \mu_r A} \qquad \dots (1)$$

The flux ϕ_1 is produced by coil 1 due to the current I_1

$$\therefore \phi_1 = \frac{N_1 I_1}{S}$$

Suppose a fraction K_1 of this flux ϕ_1 links the coil 2,

$\therefore$ flux linking the coil 2 $= \phi_{12}$

$= K_1 \phi_1 \quad (\because K_1 \leq 1)$

$$\therefore \text{Mutual inductance, } M = \frac{N_2 \phi_{12}}{I_1}$$

$\therefore$ Mutual inductance, $M = \dfrac{N_2 \phi_{12}}{I_1}$

$$= \frac{N_2 (K_1 \phi_1)}{I_1}$$

$$= \frac{N_2 K_1}{I_1} \left(\frac{N_1 I_1}{S} \right)$$

$$= \frac{K_1 N_1 N_2}{S} \qquad \ldots \quad \ldots \quad \ldots (2)$$

Similarly, the flux ϕ_2 is produced by coil 2 due to current I_2 in it

$$\therefore \phi_2 = \frac{N_2 I_2}{S}$$

Suppose a fraction K_2 of this flux links the coil 1

$\therefore$ flux linking the coil 1 $= \phi_{21}$

$$= K_2 \phi_2$$

$\therefore$ Mutual inductance, $M = \dfrac{N_1 \phi_{21}}{I_2}$

$$= \frac{N_1 K_2 \phi_2}{I_2}$$

$$= \frac{N_1 K_2}{I_2} \left(\frac{N_2 I_2}{S} \right)$$

$$= \frac{K_2 N_1 N_2}{S}$$

Multiplying eqn. (2) and eqn. (3), we get

$$M^2 = \left(\frac{K_1 N_1 N_2}{S} \right) \left(\frac{K_2 N_1 N_2}{S} \right)$$

$$= K_1 K_2 \frac{N_1^2}{S} \frac{N_2^2}{S}$$

$$= K_1 K_2 L_1 L_2 \qquad (\because \text{from eqn. 1})$$

Putting $\sqrt{K_1 K_2} = K$

$$\therefore \quad M^2 = K^2 L_1 L_2$$

$$M = K \sqrt{L_1 L_2}$$

Example 9.15. *A circuit has 1000 turns enclosing a magnetic circuit 20 cm² in section. With 4A, the flux density is 1 Wb/m² and with 9A, it is 1.4 Wb/m². Find the mean value of the inductance between these current limits and the induced e.m.f. if the current falls from 9A to 4A in 0·05 seconds.*

Solution.
$$L = N\frac{d\phi}{dI} = N\frac{d}{dI}(BA) = NA\frac{dB}{dI}$$

Here $N = 1000$; $dB = 1{\cdot}4 - 1 = 0{\cdot}4 \text{ Wb/m}^2$; $dI = 9 - 4 = 5\text{A}$

$$L = (1000) \times (20 \times 10^{-4}) \times \frac{0.4}{5} = \mathbf{0.16\ H}$$

Also
$$e = L\frac{dI}{dt} = 0{\cdot}16 \times \frac{5}{0{\cdot}05} = \mathbf{16\ V}$$

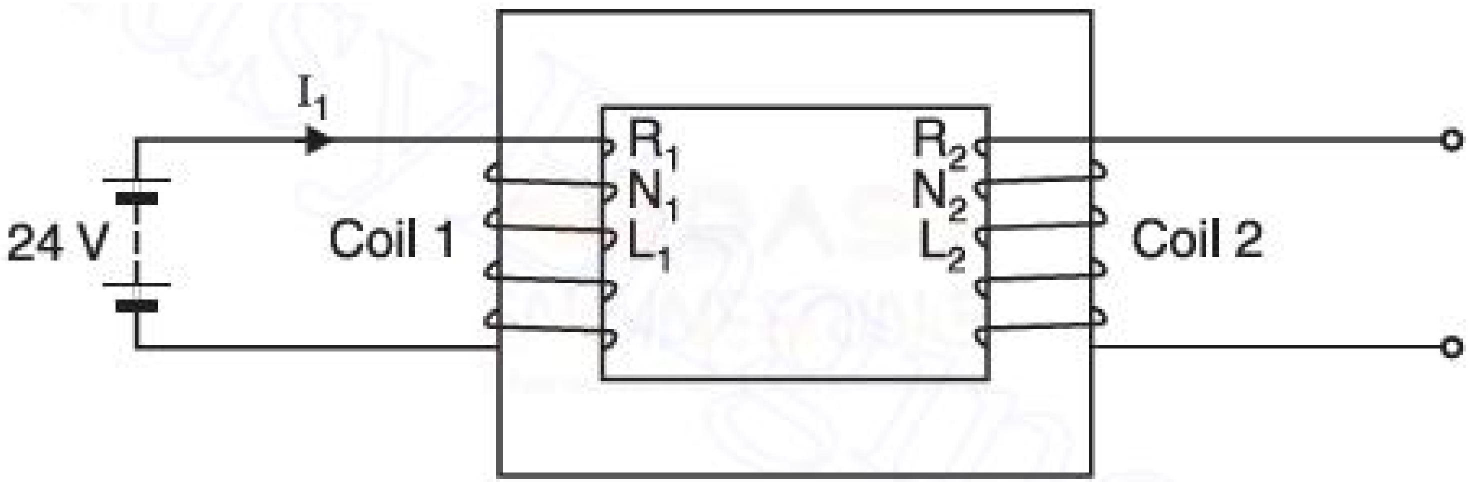
Example 9.18. *A battery of 24 V is connected to the primary (coil 1) of a two-winding transformer as shown in Fig. 9.11 and the secondary (coil 2) is open-circuited. The coil parameters are :*

$$R_1 = 10\ \Omega \qquad R_2 = 30\ \Omega$$

$$N_1 = 100 \text{ turns} \qquad N_2 = 160 \text{ turns}$$

$$\phi_1 = 0{\cdot}01 \text{ Wb} \qquad \phi_2 = 0{\cdot}008 \text{ Wb}$$

Calculate (i) the self-inductance of coil 1 (ii) the mutual inductance (iii) the coefficient of coupling and (iv) the self-inductance of coil 2.



Solution. *(i)* $I_1 = V/R_1 = 24/10 = 2{\cdot}4\text{A}$

$\therefore$ $$L_1 = \frac{N_1\,\phi_1}{I_1} = \frac{100 \times 0.01}{2{\cdot}4} = \mathbf{0.417\ H}$$

(ii) $$M = \frac{N_2\,\phi_2}{I_1} = \frac{160 \times 0.008}{2{\cdot}4} = \mathbf{0.533\ H}$$

(iii) $$k = 0{\cdot}008/0{\cdot}01 = \mathbf{0{\cdot}8}$$

(iv) $$M = k\sqrt{L_1 L_2} \quad \text{or} \quad 0{\cdot}533 = 0{\cdot}8\sqrt{0{\cdot}417 \times L_2} \quad \therefore \quad L_2 = \mathbf{1{\cdot}064\ H}$$

Flux of 1mWb is produced when a coil A having 800 turns carries a current of 4 A. The same current when flows through a coil B of 1200 turns produces a flux of 1.5mWb. The two coils are placed beside such that 80% of the flux produced by coil A links with coil B. Find (i) Self-inductance of each coil.(ii)Mutual inductance between the two coils.(ii) Co-efficient of coupling.

Ans. L1=0.2H
L2=0.45H
M=0.24H
K=0.8

Dynamically Induced EMF

Consider a single conductor of length l metres moving at *right angles to a uniform magnetic field of B Wb/m^2 with a velocity of v m/s [See Fig. 9.5 (i)]. Suppose the conductor moves through a small distance dx in dt seconds. Then area swept by the conductor is $= l \times dx$.

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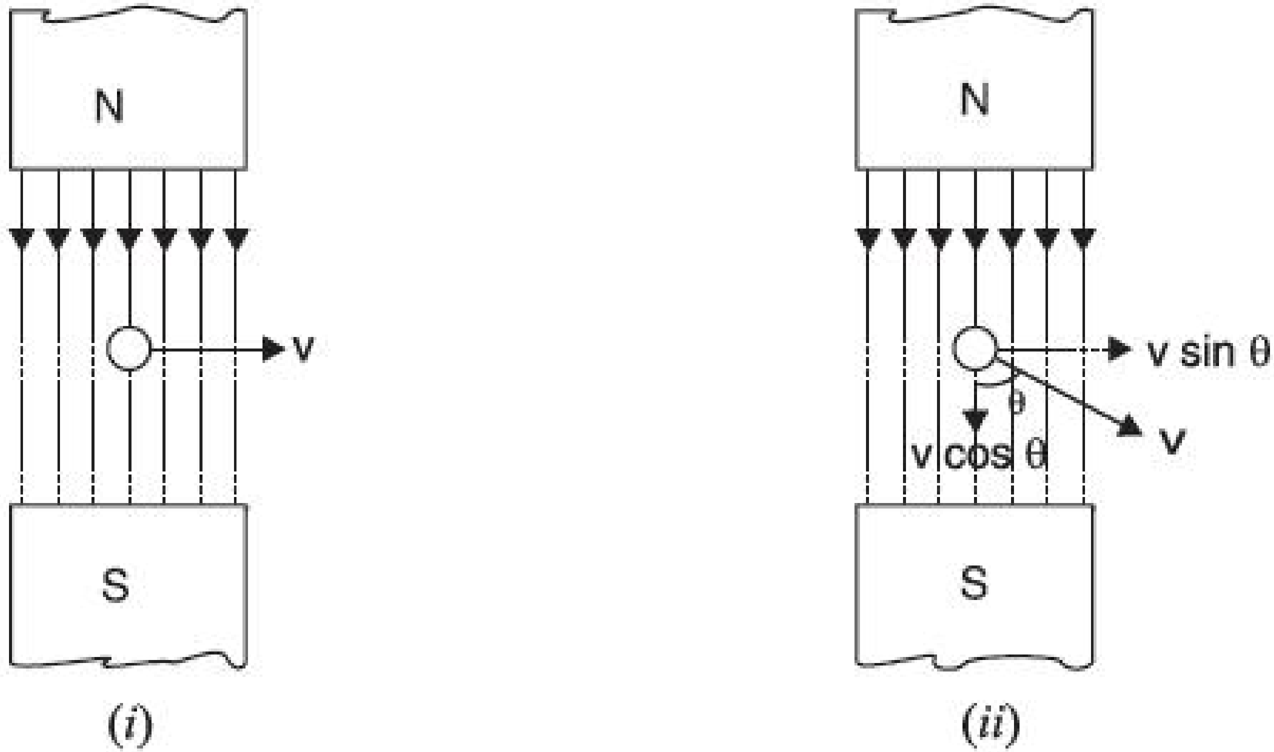
* If the conductor is moved parallel to the magnetic field, there would be no change in flux and hence no e.m.f. would be induced.



$\therefore$ Flux cut, $d\phi$ = Flux density × Area swept = $B\,l\,dx$ Wb

According to Faraday's laws of electromagnetic induction, the magnitude of e.m.f. e induced in the conductor is given by ;

$$e = N\frac{d\phi}{dt} = \frac{B\,l\,dx}{dt} \qquad (\because N = 1)$$

$$\therefore \quad e = B\,l\,v \text{ volts} \qquad (\because dx/dt = v)$$

Special case. If the conductor moves at angle θ to the magnetic field [See Fig. 9.5 (*ii*)], then the velocity at which the conductor moves across the field is *$v \sin\theta$.

$$\therefore \quad e = B\,l\,v \sin\theta$$

The direction of the induced e.m.f. can be determined by Fleming's right-hand rule.

A conductor of length 10 metres moves at right angle to a uniform magnetic field of flux density 10 Wb/m^2 with a velocity of 100 m/sec. Calculate the emf induced in it. Find also the value of emf induced when conductor moves at an angle of 40 degree to the direction of the field.

Ans. E1=10KV

E2=7451.13 V

Basis For Comparison	Electric Field	Magnetic Field
Definition	It is the force around the electrical charge particle.	The region around the magnetic where poles exhibits a force of attraction or repulsion.
Unit	Volt/meter or Newton/coulomb	Tesla, (Newton × Second) /(Coulomb × Meter)
Symbol	E	B
Formula	$E = \frac{F}{C}\ newtons/columb$	$B = \frac{\emptyset}{A}\, tesla\ or\ wb/m^2$
Measuring Instrument	Magnetometer	Electrometer
Pole	Mono pole	Dipole
Electromagnetic Field	It is perpendicular to the magnetic field.	It is perpendicular to the electric field.
Field	Vector	Vector

Basis For Comparison	Electric Field	Magnetic Field
Field Line	Induces on a positive charge and terminate on a negative charge	Generate at north pole and terminate at the south pole.
Loop	Electric field lines do not form a closed loop.	Magnetic line forms a closed loop.
Type of charge	Negative or positive charge.	North or south pole.
Force	Repulsion force on like charges and attraction force on unlike charges.	Repulsion force on like poles and attraction force on unlike poles.
Dimension	Exist in two dimensions	Remain in three dimensions
Work	Field can do work (the speed and direction of particles changes)	Magnetic field cannot do work (speed of particles remain constant)