III	Matrix Algebra- I

Classwork Examples

3.1	Definition of Matrix, types of matrices and their properties
3.2	Determinant and their properties

Determinant:

1	Find the determinant of $\begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$. **C.W.** **Answer: 6.**
2	Find the determinant of $\begin{pmatrix} 1 & -2 & 3 \\ 1 & -1 & 2 \\ 2 & 1 & 3 \end{pmatrix}$. **C.W.** **Answer: 2.**
3	Find the determinant of $\begin{pmatrix} 2 & 0 & 0 & 3 \\ 4 & -3 & 1 & 2 \\ 3 & 1 & 2 & 1 \\ 0 & -4 & 0 & 7 \end{pmatrix}$. **C.W.** **Answer:** $\mathbf{-134}$**.**
4	Prove that the $det\begin{pmatrix} a^2 & a & bc \\ b^2 & b & ca \\ c^2 & c & ab \end{pmatrix} = -det\begin{pmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{pmatrix}$. **C.W.**
5	Find the determinant of $\begin{pmatrix} 2 & 0 & -1 & 3 \\ 0 & -3 & 0 & 2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. **H.W.** **Answer: 0.**
6	Find the determinant of $\begin{pmatrix} 9 & 9 & 12 \\ 1 & 3 & -4 \\ 1 & 9 & 12 \end{pmatrix}$. **H.W.** **Answer: 576.**

3.3	Rank and nullity of a matrix
3.4	Determination of rank

Rank using minors:

1	Find the rank of $\begin{pmatrix} 2 & 3 & 4 \\ 3 & 5 & 7 \end{pmatrix}$ using minors. **C.W.** **Answer: 2.**
2	Find the rank of $\begin{pmatrix} 1 & 3 & -4 \\ -1 & -3 & 4 \\ 2 & 6 & -8 \end{pmatrix}$ using minors. **C.W.** **Answer: 1.**
3	Find the rank of $\begin{pmatrix} 1 & 1 & a \\ 1 & a & 1 \\ a & 1 & 1 \end{pmatrix}$ using minors. **C.W.** **Answer:** **If $a \neq -2$ and $a \neq 1$ then rank is 3.** **If $a = 1$ then rank is 1.** **If $a = -2$ then rank is 2.**
4	Find the rank of $\begin{pmatrix} 2 & 1 & 5 & -1 \\ -1 & 2 & 5 & 3 \\ 3 & 2 & 9 & -1 \end{pmatrix}$ using minors. **C.W.** **Answer: 2.**
5	Find the rank of $\begin{pmatrix} 2 & 0 & 0 & 3 \\ 0 & -3 & 0 & 2 \\ 0 & 2 & 7 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ using minors. **H.W.** **Answer: 3.**
6	Find the rank of $\begin{pmatrix} 1 & -1 & 2 \\ 2 & 2 & -4 \\ -5 & 5 & -10 \\ 3 & -3 & 6 \end{pmatrix}$ using minors. **H.W.** **Answer: 1.**

Row-echelon/ Reduced row-echelon form:

1	Reduce the matrix $\begin{pmatrix} 1 & -1 & 2 & 3 & -2 \\ 0 & 1 & -1 & -6 & 6 \\ 0 & -2 & 2 & 4 & -4 \end{pmatrix}$ to row-echelon/reduced row-echelon form and hence determine the rank. **C.W.** **Answer:** $\begin{pmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{2} & \mathbf{3} & \mathbf{-2} \\ \mathbf{0} & \mathbf{1} & \mathbf{-1} & \mathbf{-6} & \mathbf{6} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{-1} \end{pmatrix}$, $\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{-1} \end{pmatrix}$**, rank is 3.**
2	Reduce the matrix $\begin{pmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \\ -2 & 8 & 2 \end{pmatrix}$ to row-echelon/reduced row-echelon form and hence determine the rank. **H.W.** **Answer:** $\begin{pmatrix} \mathbf{1} & \mathbf{3} & \mathbf{5} \\ \mathbf{0} & \mathbf{1} & \mathbf{6/7} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$, $\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{17/7} \\ \mathbf{0} & \mathbf{1} & \mathbf{6/7} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$**, rank is 2.**
3	Reduce the matrix $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{pmatrix}$ to row-echelon/reduced row-echelon form and hence determine the rank. **C.W.** **Answer:** $\begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} \\ \mathbf{0} & \mathbf{1} & \mathbf{2/3} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$, $\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{5/3} & \mathbf{2} \\ \mathbf{0} & \mathbf{1} & \mathbf{2/3} & \mathbf{-1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$**, rank is 2.**
4	Reduce the matrix $\begin{pmatrix} 3 & 1 & 7 \\ 1 & 2 & 4 \\ 4 & -1 & 7 \\ 4 & -1 & 5 \end{pmatrix}$ to row-echelon/reduced row-echelon form and hence determine the rank. **C.W.** **Answer:** $\begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{4} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$, $\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$**, rank is 3.**
5	Reduce the matrix $\begin{pmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{pmatrix}$ to row-echelon/reduced row-echelon form and hence determine the rank. **H.W.**

	Answer: $\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{2/3} & \mathbf{2/3} \\ \mathbf{0} & \mathbf{1} & \mathbf{2/3} & \mathbf{29/14} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$**, rank is 2.**
6	Reduce the matrix $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{pmatrix}$ to row-echelon/reduced row-echelon form and hence determine the rank. **H.W.** **Answer:** $\begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{2} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}, \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$**, rank is 3.**

3.5	Solution of a system of linear equations by Gauss elimination and Gauss Jordan Methods.

System of linear equations:

1	Solve the system: $2x + y - z = 4,$ $x - y + 2z = -2,$ $-x + 2y - z = 2$ by Gauss elimination/Gauss-Jordan method, if it is consistent. **H.W.** **Answer:** $(\mathbf{1}, \mathbf{1}, -\mathbf{1})$**.**
2	Solve the system: $2x + z = 3,$ $x - y + z = 1,$ $4x - 2y + 3z = 3$ by Gauss elimination/Gauss-Jordan method, if it is consistent. **C.W.** **Answer: The system is inconsistent.**
3	Solve the system: $4x - 3y - 9z + 6w = 0,$ $2x + 3y + 3z + 6w = 6,$ $4x - 21y - 39z - 6w = -24$ by Gauss elimination/Gauss-Jordan method, if it is consistent. **C.W.**

	Answer: $\left\{\left(\boldsymbol{1-2k_1+k_2,\frac{1}{3}(4-2k_1-5k_2)\,,k_1}\right)\middle|\ \boldsymbol{k_1,k_2}\in\mathbb{R}\right\}$**.**
4	Solve the system: $3x+y+2z=0,$ $x-2y+3z=0,$ $x+5y-4z=0$ by Gauss elimination/Gauss-Jordan method. **C.W.** **Answer:** $\{(\boldsymbol{-k,k,k})\mid\ \boldsymbol{k}\in\mathbb{R}\}$**.**
5	Consider the system of linear equations: $x+y+z=6,$ $x+2y+3z=10$ $x+2y+\lambda z=\mu.$ Find the values of λ and μ so that the system (a) has unique solution (b) has infinite number of solutions (c) is inconsistent. **C.W.** **Answer:** **(a) If $\boldsymbol{\lambda\neq 3}$ and $\boldsymbol{\mu}\in\mathbb{R}$, then it has unique solution.** **(b) If $\boldsymbol{\lambda=3}$ and $\boldsymbol{\mu=10}$, then it has infinitely many solutions.** **(c) If $\boldsymbol{\lambda=3}$ and $\boldsymbol{\mu\neq 10}$, then it is inconsistent.**
6	Investigate for what values of λ and μ the equations $x+2y+z=8; 2x+2y+2z=13; 3x+4y+\lambda z=\mu$ have (i) a unique solution, (ii) infinite solutions, and (iii) no solution. **H.W.** **Answer:** **(i) If $\boldsymbol{\lambda\neq 3}$ and $\boldsymbol{\mu}\in\mathbb{R}$, then it has unique solution.** **(ii) If $\boldsymbol{\lambda=3}$ and $\boldsymbol{\mu=21}$, then it has infinite solutions.** **(iii)If $\boldsymbol{\lambda=3}$ and $\boldsymbol{\mu\neq 21}$, then it has no solution.**
7	Find the value of k so that the equations $x+y+3z=0; 4x+3y+kz=0; 2x+y+2z=0$ have a non-trivial solution. **C.W.** **Answer:** $\boldsymbol{k=8}$
8	Solve the system: $x+2y-2z=1$

	$2x - 3y + z = 0$ $5x + y - 5z = 1$ $3x + 14y - 12z = 5$ by Gauss elimination/Gauss-Jordan method, if it is consistent. **H.W.** **Answer:** $(\mathbf{1}, \mathbf{1}, \mathbf{1})$
9	By applying Kirchhoff's law to a circuit we obtain the following equations: $$7i_1 + 9i_2 = 3$$ $$5i_1 + 7i_2 = 1$$ where i_1 and i_2 represents currents. Find the values of i_1 and i_2 by Gauss elimination/Gauss-Jordan method, if it is consistent. **H.W.** **Answer:** $(\mathbf{3}, \mathbf{-2})$
10	A pulley system gives the following equations $$\ddot{x}_1 + \ddot{x}_2 = 0$$ $$2\ddot{x}_1 = 20 - T$$ $$5\ddot{x}_2 = 50 - T$$ where $\ddot{x}_1, \ddot{x}_2$ represent acceleration and T represents tension in the rope. Determine $\ddot{x}_1, \ddot{x}_2$ and T by Gauss elimination/Gauss-Jordan method, if it is consistent. **C.W.** **Answer:** $\left(-\frac{\mathbf{30}}{\mathbf{7}}, \frac{\mathbf{30}}{\mathbf{7}}, \frac{\mathbf{200}}{\mathbf{7}}\right)$
11	The prices of three commodities A, B and C are ₹ x, y and z per units respectively. A person P purchases 4 units of B and sells 2 units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A and 1 unit of B. Person R purchases 1 unit of A and sells 3 unit of B and one unit of C. In the process, P, Q and R earn ₹ 15,000, ₹ 1,000 and ₹ 4,000 respectively. Find the prices per unit of A, B and C. (Use Gauss elimination/Gauss-Jordan method to solve the problem.) **C.W.** **Answer:** $(\mathbf{2000}, \mathbf{1000}, \mathbf{3000})$
12	The upward speed $v(t)$ of a rocket at time t is approximated by $v(t) = at^2 + bt + c, 0 \leq t \leq 100$ where $a, b,$ and c are constants. It has been found that the speed at times $t = 3, t = 6$, and $t = 9$ seconds are $64, 133$, and 208 miles per second respectively. Find the speed at time $t = 15$ seconds. (Use Gauss elimination/Gauss-Jordan method) **H.W.**

	Answer: $\boldsymbol{v(15) = 376}$**.**