Question Paper - Evaluator view

Exam Date & Time: 16-Dec-2019 (01:30 PM - 04:30 PM)

CHAROTAR UNIVERSITY OF SCIENCE AND TECHNOLOGY

(CE/IT/CSE/ME/CL/EC/EE)

ENGINEERING MATHEMATICS-I [MA143]

Marks: 70 **Duration: 180 mins.**

Multiple choice questions

Answer all the questions.

1) The series $a + ar + ar^2 + ar^3 + \cdots$ converges if _______.

(1)

1) $|r| < 1$ 2) $r \geq 1$ 3) $r \leq -1$ 4) $r = 1$

2) The principal argument of a complex number $z = 1 - i$ is _______.

(1)

1) $-\frac{\pi}{3}$ 2) $\frac{\pi}{4}$ 3) $-\frac{\pi}{4}$ 4) $\frac{\pi}{6}$

3) If $z = 1 + i\sqrt{3}$ then $z^6 + z^3 + 1 =$_______.

(1)

1) 73 2) 9 3) 513 4) 57

4) The power series $\sum_{n=1}^{\infty}(3x)^n$ is convergent if ________.

(1)

1) $x > \frac{1}{3}$ 2) $x = \frac{1}{3}$ 3) $\frac{1}{3} < x < 1$ 4) $-\frac{1}{3} < x < \frac{1}{3}$

5) The Maclaurin's series expansion of $\log(1+x)$ up to x^4 is________.

(1)

1) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ 2) $1 + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$ 3) $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$ 4) $1 - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

6) Let $f(x) = \ln(x^2 + 5x + 6)$ then the value of $f^{(30)}(1)$ is given by_______.

(1)

1) $(-29!)(\frac{1}{3^{30}} + \frac{1}{4^{30}})$ 2) $30!\,(\frac{1}{3^{30}} + \frac{1}{4^{30}})$ 3) $(-30!)(\frac{1}{3^{30}} + \frac{1}{4^{30}})$ 4) $29!\,(\frac{1}{3^{30}} + \frac{1}{4^{30}})$

7) Which of the following statement is true?

(1)

1) $\cosh^2(z) + \sinh^2(z) = 1$ 2) $\cosh(iz) = i\cos(z)$ 3) $\sinh(iz) = i\sin(z)$ 4) $\tanh(iz) = \tan(z)$

8) If f is a function of u, v, w and u, v, w are functions of x, y and z then $\frac{\partial f}{\partial y}$ is_______.

(1)

1) $\frac{\partial f}{\partial u}.\frac{\partial u}{\partial z} + \frac{\partial f}{\partial v}.\frac{\partial v}{\partial z} + \frac{\partial f}{\partial w}.\frac{\partial w}{\partial z}$ 2) $\frac{\partial f}{\partial x}.\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}.\frac{\partial y}{\partial u} + \frac{\partial f}{\partial z}.\frac{\partial z}{\partial v}$ 3) $\frac{\partial f}{\partial x}.\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}.\frac{\partial y}{\partial v} + \frac{\partial f}{\partial z}.\frac{\partial z}{\partial v}$ 4) $\frac{\partial f}{\partial u}.\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}.\frac{\partial v}{\partial y} + \frac{\partial f}{\partial w}.\frac{\partial w}{\partial y}$

9) For the function $u = x^3 + y^3 - 3xy$ if $r = 6, s = -3, t = 6$ at point (1, 1), then at this point function u is______.

(1)

1) no conclusion 2) maximum 3) saddle point 4) minimum

10) For the function $F(x,y,z) = f(x,y,z) + \lambda\,\phi(x,y,z)$, the Lagrange's equations are_______.

(1)

1) $\frac{\partial F}{\partial x} = 0,\ \frac{\partial F}{\partial y} = 0,\ \frac{\partial F}{\partial z} = 0$ 2) $\frac{\partial f}{\partial x} = 0,\ \frac{\partial f}{\partial y} = 0,\ \frac{\partial f}{\partial z} = 0$ 3) $\frac{\partial \phi}{\partial x} = 0,\ \frac{\partial \phi}{\partial y} = 0,\ \frac{\partial \phi}{\partial z} = 0$ 4) $\frac{\partial f}{\partial x} = 0,\ \frac{\partial f}{\partial \phi} = 0,\ \frac{\partial f}{\partial z} = 0$

11) The function $z = x^4 y^2 \operatorname{cosec}^{-1}\left(\frac{x^2+y^2}{xy}\right)$ is homogeneous of degree _______.

(1)

1) -2 2) 3 3) 6 4) 4

12) The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ is _______. (1)

1) 2 2) 3 3) 1 4) 0

13) If $f(x,y) = \log(x\tan^{-1} y),$ then $\frac{\partial^2 f}{\partial x \partial y}$ is _______. (1)

1) 0 2) $\frac{1}{x^2}$ 3) $-\frac{1}{x^2}$ 4) $\frac{1}{x}$

14) Let A be a square matrix. If the homogeneous system $AX = O$ has unique solution, then the nullity (A) is _______. (1)

1) n 2) 0 3) n-1 4) n+1

PART-A

Answer 4 out of 6 questions.

15) Obtain the n^{th} derivative of the function $\frac{x}{(x-1)(x-2)(x-3)}$. (4)

16) Test the convergence of the series $\frac{1\cdot 2}{3\cdot 4\cdot 5} + \frac{2\cdot 3}{4\cdot 5\cdot 6} + \frac{3\cdot 4}{5\cdot 6\cdot 7} + \cdots$. (4)

17) If $y = (\sin^{-1} x)^2$, then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$. (4)

18) Test the convergence of the series $\frac{1!}{3} + \frac{2!}{3^2} + \frac{3!}{3^3} + \cdots$. (4)

19) State the De Moiver's formula and hence simplify the expression $\frac{(cos2\theta + i\ sin2\theta)^6\ (cos\theta - isin\theta)^3}{(cos3\theta - i\ sin3\theta)^4\ (sin\theta + icos\theta)^3}$. (4)

20) Use De Moiver's formula to solve the equation $x^4 - x^3 + x^2 - x + 1 = 0$. (4)

PART-B

Answer 2 out of 3 questions.

21) Using Taylor's series, expand the function $x^5 - x^4 + x^3 - x^2 + x - 1$ in powers of x-1. (3)

22) Evaluate $\lim_{x \to 0} [\frac{1}{x^2} - \frac{1}{\sin^2 x}]$. (3)

23) Prove that the series $\sum_{n=1}^{\infty} \frac{2n^2+3n}{5+n^5}$ is convergent . (3)

PART-C

Answer 1 out of 2 questions.

24) Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ by Cardan's method. (6)

25) Solve the equation $x^4 - 12x^3 + 41x^2 - 18x - 72 = 0$ by Ferrari's method. (6)

PART-D

Answer 4 out of 6 questions.

26) State Euler's theorem for the homogeneous function of two independent variables.

If $u(x,y) = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. (4)

27) If $u(x,y,z) = xyz,\ \ v(x,y,z) = x^2 + y^2 + z^2,\ \ w(x,y,z) = x + y + z$, then find

$J = \frac{\partial(u,v,w)}{\partial(x,y,z)}$. (4)

28) Find the Extreme values of the function $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. (4)

29) Find the rank and the nullity of the matrix $\begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ using row echelon form. (4)

30) Using Gauss- Jordan method, find the inverse of the matrix A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$, if exists. (4)

31) Investigate for what values of α and β the system of equations

$$x + 2y + z = 8; \qquad 2x + 2y + 2z = 13; \qquad 3x + 4y + \alpha z = \beta$$ (4)

has (i) no solution (ii) unique solution (iii) infinite solutions.

PART-E

Answer 2 out of 3 questions.

32) If $u(x,y) = \sin(x^2 + y^2)$ and $a^2x^2 + b^2y^2 = c^2$, then find $\frac{du}{dx}$. Where a, b and c are constants. (3)

33) If the kinetic energy $T = \frac{1}{2}mv^2$, find approximately the change in T as mass changes from 49 to 49.5 and velocity changes from 1600 to 1590. (3)

34) Find the equation of the tangent plane and the normal line to the surface $x^2 + y^2 + z^2 = 3$ at the point $(1,1,1)$. (3)

PART-F

Answer 1 out of 2 questions.

35) Find the maximum value of $x^m y^n z^p$ when the variable x, y, z are subject to the condition $x + y + z = 2$. (6)

36) Using Gauss Elimination method, solve the following system of linear equations:

$$4x - 2y + 6z = 8; \qquad x + y - 3z = -1; \qquad 15x - 3y + 9z = 21.$$ (6)

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