Candidate seat No: _______________

Charotar University of Science and Technology [CHARUSAT]
Faculty of Technology and Engineering
Department of Mathematical Sciences
MA143 Engineering Mathematics I
First Internal Exam

Semester: 1st Semester B. Tech. (All Branch) **Maximum Marks: 30**

Date: 29/11/2021 (Monday) **Time: 11:10 am to 12:10 pm**

Instructions:

(i) Figures to the right indicate ***full*** marks.
(ii) Use of scientific calculator is allowed.
(iii) Draw figure where it is required.

Q-1 Choose the correct answer from the given options in the following: **[06]**

1. If $y(x) = e^x + 6^x$, then $y_n(0) =$ _______.

a) $1 - (\log 6)^n$ b) $(\log 6)^n$
c) 0 d) $(\log e)^n + (\log 6)^n$

2. On which of the following functions the Lagrange's mean value theorem is applicable in the given interval?

a) $f(x) = |x|$ in $[-1, 1]$ b) $f(x) = \frac{6}{x} - 3$ in $[-1, 2]$
c) $f(x) = \sin x$ in $[-1, 1]$ d) $f(x) = \begin{cases} x^2 + 1, x \neq 0 \\ 0, \quad x = 0 \end{cases}$ in $[0, 2]$

3. The coefficient of x^2 in the Taylor's series expansion of $e^x \cdot \cos x$, in powers of x, is_______.

a) 0 b) $-\frac{1}{2}$ c) 1 d) $\frac{1}{2}$

4. If the matrix $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then $[\det(A)]^{2021} =$ _______.

a) 0 b) 1 c) $-\det(A)$ d) A

5. Let A be a square matrix of order n, then nullity of A is_______.

a) $n - r(A)$ b) $n + r(A)$ c) $r(A) - n$ d) n

6. Which of the following matrices is not in reduced row-echelon form?

a) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

Q-2 Attempt any Three. **[12]**

(a) Find the extreme values of the function $f(x) = x^5 - 5x^4 + 5x^3 - 1$.

(b) Find the n[th] order derivative of the function $y = \frac{x^2+4x+1}{(x+2)(x+1)(x-1)}$.

(c) If $y = \sin log(x^2 + 2x + 1)$, then prove that

$$(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0.$$

(d) **(i)** A thermometer was taken from a freezer and placed in a boiling water. It took 22 seconds for the thermometer to raise from $-10°\text{C}\ to\ 100°\text{C}$. Show that the rate of change of temperature at some time t is $5°C$ per second. (Use the Mean Value Theorem to solve the problem.)

(ii) Find the Maclaurin's series expansion of $f(x) = \sinh x$ up to x^7.

Q-3 Attempt any Three. **[12]**

(a) Determine the rank of $A = \begin{bmatrix} 1 & 1 & 1 \\ a & 1 & 1 \\ 1 & b & 1 \end{bmatrix}$ using minors.

(b) Find the rank and nullity of the matrix $A = \begin{bmatrix} 3 & 1 & 7 \\ 1 & 2 & 4 \\ 4 & -1 & 7 \\ 4 & -1 & 5 \end{bmatrix}$ using row echelon form.

(c) Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 8; 2x + 2y + 2z = 13; 3x + 4y + \lambda z = \mu$ has (i) a unique solution, (ii) infinite solutions, and (iii) no solution.

(d) Decrypt the received encoded message $[2 \quad -3\ ]\ [20 \quad 4]$; where the encryption matrix is $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$ and the decryption matrix is its inverse. Here, the system of codes are represented as follows; the numbers $1 - 26$ by the letters $A - Z$ respectively, and the number 0 by the blank space. (Use Gauss Jordan method to find the inverse.)

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