

★ Graded Homework 6 - Section 5.1 ★

⑤ $A = \begin{bmatrix} 4 & 3 & 2 & 1 & 0 \\ 6 & 1 & 2 & 0 & 5 \\ 3 & 2 & 2 & 4 & 4 \\ 5 & 1 & 0 & 0 & 3 \\ 2 & 2 & 4 & 1 & 0 \end{bmatrix}$

$$M_{2,3} = \begin{bmatrix} 4 & 3 & 1 & 0 \\ 3 & 2 & 4 & 4 \\ 5 & 1 & 0 & 3 \\ 2 & 2 & 1 & 0 \end{bmatrix}$$

$$M_{3,1} = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 2 & 0 & 5 \\ 1 & 0 & 0 & 3 \\ 2 & 4 & 1 & 0 \end{bmatrix}$$

⑨ ~~$A = \begin{bmatrix} 6 & 1 & 2 \\ 4 & 3 & 0 \\ 1 & 1 & 1 \end{bmatrix}$~~ $C_{13} = (-1)^{1+3} \begin{vmatrix} 4 & 3 \\ 1 & 1 \end{vmatrix} \rightarrow$

$$(1)[(4)(1) - (3)(1)] = 1 = C_{13} \rightarrow C_{22} = \dots$$

$$(-1)^{2+2} \begin{vmatrix} 6 & 2 \\ 1 & 1 \end{vmatrix} = (1)[(6)(1) - (2)(1)] = 4 = C_{22}$$

⑬ $A = \begin{bmatrix} -1 & 1 & -1 & 2 \\ 0 & 3 & 2 & 0 \\ 1 & 4 & 0 & 1 \\ 0 & -1 & 3 & -1 \end{bmatrix} \rightarrow \det(A) = a_{21}C_{21} + a_{22}C_{22} + a_{23}C_{23} + a_{24}C_{24}$

$$= a_{22} C_{22} + a_{23} C_{23} \quad \text{because } a_{21} \text{ and } a_{24} = 0$$

~~$a_{24} = 0$~~

$$C_{22} = (-1)^{2+2} \begin{vmatrix} -1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 3 & -1 \end{vmatrix}$$

$$= (-1)(-1) + (-1)(0) + (2)(1)(3) - (2)(0)(0)$$

$$- (-1)(1)(3) - (-1)(1)(-1) = 6 + 3 - 1 = 4 = C_{22}$$

$$\rightarrow C_{23} = (-1)^{2+3} \begin{vmatrix} -1 & 1 & 2 \\ 1 & 1 & 1 \\ 0 & -1 & 1 \end{vmatrix} = -1 [(-1)(4)(-1)]$$

$$+ (1)(1)(0) + (2)(1)(-1) - (2)(4)(0) - (-1)(1)(-1)$$

$$- (1)(1)(-1)] = -1 [4 - 2 - 1 + 1] = -2 = C_{23}$$

$$\rightarrow \det(A) = (3)(4) + (2)(-2) = 8$$

$$\rightarrow \det(A) = 8 \quad \text{row method}$$

$$(b) \det(A) = a_{11} C_{11} + a_{21} C_{21} + a_{31} C_{31} + a_{41} C_{41}$$

$$\rightarrow \det(A) = a_{11} C_{11} + a_{31} C_{31} \quad \text{because } a_{21} \text{ and } a_{41} \text{ are}$$

$$0. \rightarrow C_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 8 & 0 & 3 & 2 \\ 4 & 0 & 1 & 4 & 0 \\ -1 & 3 & -1 & 1 & 3 \end{vmatrix} = (3)(0)(-1)$$

$$+ (2)(1)(-1) + (0)(4)(3) - (0)(0)(-1) - (3)(1)(3) -$$

$$(2)(4)(-1) = -2 - 9 + 8 = -3 = C_{11} \rightarrow$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 1 & -1 & 2 & 1 & -1 \\ 3 & 2 & 0 & 3 & 2 \\ -1 & 3 & -1 & 1 & 3 \end{vmatrix} = (1)(2)(-1) + (-1)(0)(-1)$$

$$\begin{aligned}
 & + (2)(3)(3) - (2)(2)(-1) - (1)(-2)(3) - (-1)(3)(-1) \\
 & = -2 + 18 + 4 - 3 = 17 = C_3, \rightarrow (-1)(-3) + (1)(17) \\
 & = \textcircled{20} = \det(A) \text{ by column}.
 \end{aligned}$$

As $\det(A) \neq \textcircled{0} \Rightarrow T$ is invertible.

(17)

$$A = \begin{bmatrix} 4 & 2 & 1 & 0 & 1 \\ 0 & 3 & -1 & 1 & 2 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

@ row method:

$$|A| = a_{51}C_{51} + a_{52}C_{52} + a_{53}C_{53} + a_{54}C_{54} + a_{55}C_{55}$$

$$\rightarrow |A| = a_{53}C_{53} + a_{55}C_{55} \text{ because } a_{51}, a_{52}$$

and a_{54} are 0. $\rightarrow C_{53} = (-1)^{5+3} \begin{vmatrix} 4 & 2 & 0 & 1 \\ 0 & 3 & 1 & 2 \\ 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 3 \end{vmatrix}$

~~•~~ ~~Calculation~~ Let this be Z $\rightarrow |Z| = a_{11}C_{11} + a_{21}C_{21}$

$$+ a_{31}C_{31} + a_{41}C_{41} \rightarrow |Z| = a_{11}C_{11} \text{ because } a_{21}, a_{31}, a_{41}$$

$$= 0 \rightarrow |Z| = a_{11}C_{11} \rightarrow C_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 1 & 2 & 3 & 1 \\ -1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 3 & 1 & 0 \end{vmatrix}$$

$$= \textcircled{(3)(1)(2)(3)} + (1)(1)(1) + (2)(-1)(0) - (2)(-1)(1) - (3)(1)$$

$$\rightarrow - (1)(-1)(3) = 4 \rightarrow |Z| = (4)(4) = 16$$

~~•~~ $\rightarrow C_{53} = (+1)(16) = +16 \rightarrow \textcircled{Calculation}$

$$C_{55} = (-1)^{5+5} \begin{vmatrix} 4 & 2 & 1 & 0 \\ 0 & 3 & -1 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 2 & 0 \end{vmatrix} \rightarrow \text{let this be } Y$$

$$\rightarrow |Y| = a_{11} C_{11} + a_{21} C_{21} + a_{31} C_{31} + a_{41} C_{41} \rightarrow$$

$$|Y| = a_{11} C_{11} \text{ as } a_{21}, a_{31}, a_{41} = 0. \rightarrow C_{11} = (-1)^{1+1}$$

$$\begin{vmatrix} 3 & -1 & 1 & 3 & -1 \\ -1 & 0 & 0 & -1 & 0 \\ 1 & 2 & 0 & 1 & 2 \end{vmatrix} = (3)(0)(0) + (-1)(0)(1) + (1)(-1)(2) - (1)(0)(1) - (3)(0)(2) - (-1)(-1)(0) = -2 \rightarrow$$

$$|Y| = (-1)(-2) = -8 \rightarrow C_{55} = (-1)(-8) = -8$$

$$\therefore \rightarrow \det(A) = (1)(+16) + (1)(-8) = \boxed{-8} \text{ by now}$$

⑤ Column of A:

$$|A| = a_{11} C_{11} + a_{21} C_{21} + a_{31} C_{31} + a_{41} C_{41} + a_{51} C_{51} \rightarrow$$

$$|A| = a_{11} C_{11} \text{ as } a_{21}, a_{31}, a_{41}, a_{51} = 0 \rightarrow C_{11} =$$

$$(-1)^{1+1} \begin{vmatrix} 3 & -1 & 1 & 2 \\ -1 & 0 & 0 & 1 \\ 1 & 2 & 0 & 3 \\ 0 & 1 & 0 & 1 \end{vmatrix} \rightarrow \text{let this be } Z$$

$$|Z| = a_{13} C_{13} + a_{23} C_{23} + a_{33} C_{33} + a_{43} C_{43} \rightarrow$$

$$|Z| = a_{13} C_{13} \text{ because } a_{23} = a_{33} = a_{43} = 0.$$

$$\rightarrow C_{13} = (-1)^{1+3} \begin{vmatrix} -1 & 0 & 1 & -1 \\ 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{vmatrix} \rightarrow$$

$$(-1)(2)(1) + (\cancel{0})(3)(\cancel{0}) + (1)(1)(1) - (\cancel{1})(\cancel{2})(\cancel{0}) -$$

$$(-1)(3)(1) - (\cancel{0})(\cancel{1})(1) = -2 + 1 + 3 = 2$$

$$\rightarrow |Z| = (\overset{\wedge}{2})(2) = 2 \rightarrow \cancel{\text{row 1}} \cancel{\text{row 2}} \cancel{\text{row 3}} \text{ } C_{11} =$$

$$(1)(2) = 2 \rightarrow |A| = a_{11} C_{11} = (4)(2) = 8$$

by column method

$T(x)$ invertible as $|A| \neq 0$

23

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 4 \\ 1 & 6 & 1 \end{bmatrix} \rightarrow |A| = \begin{vmatrix} 3 & 1 & -1 & 3 \\ 2 & 0 & 4 & 2 \\ 1 & 6 & 1 & 6 \end{vmatrix} \rightarrow$$

$$(\cancel{3})(\cancel{0})(\cancel{1}) + (1)(4)(1) + (-1)(2)(6) - (\cancel{(-1)}(\cancel{0})(\cancel{1}))$$

$$- (3)(4)(6) - (1)(2)(1) = 4 - 12 - 72 - 2$$

$$= -82 = |A|$$

(25)

$$A = \begin{bmatrix} 6 & 1 & 2 & 1 \\ 3 & 1 & 0 & 0 \\ 0 & 2 & 2 & 1 \\ 1 & 2 & 3 & -1 \end{bmatrix}$$



Shortcut (Rule of Sarrus)
not possible as $A \in \mathbb{R}_{n \times n}$
 $n > 3$

(31)

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & a & -2 \\ 2 & 4 & 3 \end{bmatrix}, \quad \{a \mid \exists A^{-1}\}$$

$$|A| = (1)(a)(3) + (-1)(-2)(2) + (3)(0)(4) -$$

$$(3)(a)(2) - (1)(-2)(4) - (3)(0)(-1)$$

$$= 3a + 4 - 6a + 8 \rightarrow 3a + 4 - 6a + 8 = 0 \rightarrow$$

$$-3a + 12 = 0 \rightarrow a = 4 \rightarrow a = \{4\}$$

(37) No determinant of A as there is a column of 0's which means the set of the columns of A are linearly dependent.

(47) $\det(A - \lambda I_2) = 0$, find all possible λ .

$$A = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix} \rightarrow \det \left(\begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) = 0$$

$$\det \begin{pmatrix} 1-\lambda & 0 \\ -5 & 1-\lambda \end{pmatrix} = 0 \rightarrow (1-\lambda)(1-\lambda) - (-5)(-5) = 0$$

$$\rightarrow (1-\lambda)^2 = 0 \rightarrow \lambda = 1$$

(55)

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix} \rightarrow |A| = \begin{vmatrix} 3 & 1 & 0 & 3 & 1 \\ 1 & 2 & 3 & 1 & 2 \\ 0 & 2 & 1 & 0 & 2 \end{vmatrix} \rightarrow$$

$$(3)(2)(1) + (1)(3)(0) + (0)(1)(2) - (0)(2)(0) - (3)(2)(1) + (1)(3)(0) + (0)(1)(2) - (0)(2)(0) - (3)(3)(2) - (1)(1)(1) = 6 - 18 - 1 = -13$$

$$A \xrightarrow[R_2 \leftrightarrow R_3]{\sim} B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \rightarrow |B| =$$

$$\begin{vmatrix} 1 & 2 & 3 & 1 & 2 \\ 3 & 1 & 0 & 3 & 1 \\ 0 & 2 & 1 & 0 & 2 \end{vmatrix} \rightarrow |B| = (1)(1)(1) + (2)(0)(0) + (3)(3)(1) - (3)(1)(0) - (1)(0)(2) - (2)(3)(1) = 1 + 18 - 6 = 13$$

Conjecture: interchanging a row from A to get B means $|A| = -|B|$ and equivalently $|B| = -|A|$

(61)

$A \in \mathbb{R}^{2 \times 2}$ s.t. $|A| = 12$

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$$

(65)

$A \in \mathbb{R}^{3 \times 3}$ s.t. $M_{11} = \begin{bmatrix} 0 & 4 \\ 6 & -3 \end{bmatrix}$ ^

$$M_{23} = \begin{bmatrix} 5 & -1 \\ 2 & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & -1 & 10 \\ 2 & 0 & 4 \\ 2 & 6 & -3 \end{bmatrix}$$

(69)

a) False as determinants are only for square matrices.

b) False as the cofactor is also multiplied by 1 or -1 to an $(n-1) \times (n-1)$ matrix

(71)

a) False as one of the diagonal entries may be 0 and hence $\det(A) = 0$.

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Graded Homework 6 - Section 5.2

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(3) $A = \begin{bmatrix} 1 & -1 & -3 \\ -2 & 2 & 6 \\ -3 & -3 & 10 \end{bmatrix}$

\sim $\begin{array}{l} 2R_1 + R_2 \rightarrow R_2 \\ 3R_1 + R_3 \rightarrow R_3 \end{array}$

$\sim \begin{bmatrix} 1 & -1 & -3 \\ 0 & 0 & 0 \\ 0 & -6 & 1 \end{bmatrix}$

$\rightarrow |A| = 0$ because there is a 0 in the diagonal as we convert A to R.E.F.

(5) $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$\sim \begin{array}{l} R_1 \leftrightarrow R_2 \\ R_3 \leftrightarrow R_4 \end{array}$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$|A| = (-1)(-1)(1)(1)(1)(1) = 1 = |A|$

(7) ~~$|A| =$~~ $= (-1)[(1)(-4) - (-5)(0)] = 4 \Rightarrow A$ is invertible.

~~10/10~~

(11) $B = \begin{bmatrix} -6 & 2 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & -4 \end{bmatrix} \rightarrow |B| = 0 \rightarrow |A| = (-\frac{1}{7})(\det(B))$

$|A| = 0$. A is not invertible.

(13) $B = \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix} \rightarrow |B| = (1)(1)(2)(4) = 8 \rightarrow$

$|A| = (-1)(-1)(|B|) = 8 = |A|$

A is invertible.

(15) $\det(A) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$

~~\det~~ $\begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix} = -3$

(19) $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 3 & 7 \end{bmatrix} \rightarrow |A| = -11 \rightarrow$

$|B| = -3 \rightarrow AB = \begin{bmatrix} 9 & 19 \\ -12 & -29 \end{bmatrix} \rightarrow |AB| = -33$

$\rightarrow (-11)(-3) = -33 \rightarrow A+B = \begin{bmatrix} 2 & 2 \\ 4 & 3 \end{bmatrix}$

$$|A+B| = -2 \neq -33 = |AB|$$

(25) $A, B \in \mathbb{R}^{n \times n} \wedge |A|=3 \wedge |B|=-2$

a) $|A^2 B^3| = |A| \cdot |A| \cdot |B| \cdot |B| \cdot |B| = -72 = |A^2 B^3|$

b) $|AB^{-1}| = |A| \cdot \frac{1}{|B|} = -\frac{3}{2} = |AB^{-1}|$

c) $|B^3 A^T| = |B| \cdot |B| \cdot |B| \cdot |A^T| =$
 $|B| \cdot |B| \cdot |B| \cdot |A| = -24 = |B^3 A^T|$

d) $|A^2 B^3 B^T| = |A| \cdot |A| \cdot |B| \cdot |B| \cdot |B| \cdot |B^T|$
 $= |A| \cdot |A| \cdot |B| \cdot |B| \cdot |B| \cdot |B| = 144$
 $= |A^2 B^3 B^T|$

(39) $A = \begin{bmatrix} 1 & 1 & -2 \\ 3 & -2 & 2 \\ 6 & 7 & -1 \end{bmatrix}, b = \begin{bmatrix} -3 \\ 9 \\ 4 \end{bmatrix}$, $\vec{Ax} = \vec{b}$ has a unique solution?

$$|A| = (1)(-2)(-1) + (1)(2)(6) + (-2)(3)(-7) - (-2)(-2)(1)$$

$$- (1)(2)(-7) - (-1)(3)(1) = 2 + 12 + 42 - 24$$

$$+ 14 + 3 = 49 = |A| \Rightarrow \text{unique solution exists}$$

④) $A \in \mathbb{R}^{2 \times 2}$ s.t. $A \neq 0^{2 \times 2}$ \wedge $3|A| = |3A|$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \rightarrow$$

$$|A| = (1)(4) - (2)(2) = 0 \quad \text{and} \rightarrow 3|A| = (3)(0) = 0$$

$$\rightarrow 3A = \begin{bmatrix} 3 & 6 \\ 6 & 12 \end{bmatrix} \rightarrow |3A| = (3)(12) - (6)(6) = 0$$

④) a) False, interchanging the rows of a matrix means that the computed determinant must be multiplied by -1 .

b) True as $\det(A) \neq 0 \Rightarrow A^{-1}$ exists which requires linear independence in the columns of A .

⑤) a) True as $|A| = \frac{1}{2} |B| \equiv |B| = 2|A|$

b) False if $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \nexists M_{ij} \neq 0 \forall i, j \in \mathbb{N}$

~~★~~ Graded Homework 6 - Section 5.3 ~~★~~



① $A = \begin{bmatrix} 6 & -5 \\ -2 & 7 \end{bmatrix}, \vec{b} = \begin{bmatrix} 12 \\ 0 \end{bmatrix} \rightarrow \text{find } \vec{x} \text{ with}$

Cramer's rule. $\rightarrow |A| = (6)(7) - (-5)(-2) = 32$

$$\rightarrow x_1 = \frac{\det(A_1)}{\det(A)} = \frac{\begin{vmatrix} 12 & -5 \\ 0 & 7 \end{vmatrix}}{32} = \frac{(12)(7) - 5(0)}{32} =$$

$$\frac{84}{32} = x_1 \rightarrow x_2 = \frac{\det(A_2)}{\det(A)} = \frac{\begin{vmatrix} 6 & 12 \\ -2 & 0 \end{vmatrix}}{32} =$$

$$\frac{(6)(0) - (12)(-2)}{32} = \frac{24}{32} = \frac{3}{4} = x_2 \rightarrow \vec{x} = \boxed{\begin{bmatrix} \frac{84}{32} \\ \frac{3}{4} \end{bmatrix}}$$

⑤ $A = \begin{bmatrix} 1 & 1 & -2 \\ 3 & -2 & 2 \\ 6 & -7 & -1 \end{bmatrix}, \vec{b} = \begin{bmatrix} -3 \\ 9 \\ 4 \end{bmatrix} \rightarrow \text{find } \vec{x} \text{ with Cramer's rule.}$

$$\rightarrow x_1 = \frac{\det(A_1)}{\det(A)} \rightarrow \det(A) = \begin{vmatrix} 1 & 1 & -2 & 1 \\ 3 & -2 & 2 & 3 \\ 6 & -7 & -1 & 6 \\ 0 & 0 & 0 & -1 \end{vmatrix} =$$

$$(1)(-2)(-1) + (1)(2)(6) + (-2)(3)(-7) - (-2)(-2)(6)$$

$$\rightarrow (1)(2)(-7) - (1)(3)(-1) = 2 + 12 + 42 - 24$$

$$+ 14 + 3 = 49 \rightarrow \boxed{49} \quad \text{CONTINUED} \rightarrow$$

$$|A_1| = \begin{vmatrix} -3 & -2 & -3 & 1 \\ 9 & 2 & 9 & -2 \\ 4 & -7 & 4 & 7 \end{vmatrix} = (-3)(-2)(1) + (1)(2)(4)$$

$$+ (-2)(9)(-7) - (-2)(-2)(4) - (-3)(2)(-7) - (1)(9)(1)$$

$$= 6 + 8 + 126 - 16 - 42 - 9 = 73 \rightarrow$$

$$x_1 = \frac{73}{49} \rightarrow x_2 = \frac{|A_2|}{|A_1|} \rightarrow |A_2| = \cancel{\text{something}} \rightarrow$$

$$\begin{vmatrix} -3 & -2 & 1 & -3 \\ 9 & 2 & 3 & 9 \\ 4 & -7 & 4 & 7 \end{vmatrix}$$

$$= (1)(9)(-1) + (-3)(2)(6) + (-2)(3)(4) - (-2)(9)(6) - (1)(2)(4) - (-3)(3)(-1) = -9 - 36 - 24 + 108 - 8 - 9 = 108 - 86$$

$$= 22 \rightarrow x_2 = \frac{22}{49} \rightarrow |A_3| = \begin{vmatrix} 1 & -3 & 1 & 1 \\ 3 & 2 & 9 & 3 & -2 \\ 6 & -7 & 4 & 6 & -7 \end{vmatrix}$$

$$= (1)(-2)(4) + (1)(9)(6) + (-3)(3)(-7) - (-3)(-2)(6) - 36$$

$$- (1)(9)(-7) - (1)(4)(3) = -8 + 54 + 63 \cancel{+} \cancel{+}$$

$$63 - 12 = \cancel{124} \rightarrow x_3 = \frac{124}{49} \rightarrow$$

$$x = \boxed{\begin{bmatrix} 73/49 \\ 22/49 \\ 124/49 \end{bmatrix}}$$

$$\textcircled{9} \quad A = \begin{bmatrix} 3 & 0 & 2 \\ 0 & 3 & 2 \\ 2 & 3 & 1 \end{bmatrix}, \vec{b} = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix} \rightarrow \text{find } x_2$$

$$x_2 = \frac{|A_2|}{|A|} \rightarrow |A| = \begin{vmatrix} 3 & 0 & 2 & 3 & 0 \\ 0 & 3 & 2 & 0 & 3 \\ 2 & 3 & 1 & 2 & 3 \end{vmatrix} = \rightarrow$$

$$(3)(3)(1) + (0)(2)(2) + (2)(0)(3) - (2)(3)(2)$$

$$- (3)(2)(3) - (0)(0)(1) = 9 - 12 - 18 = -21$$

$$\rightarrow |A_2| = \begin{vmatrix} 3 & 1 & 2 & 3 & 1 \\ 0 & 3 & 2 & 0 & 3 \\ 2 & -4 & 1 & 2 & -4 \end{vmatrix} = (3)(3)(1) +$$

$$(1)(2)(2) + (2)(0)(4) - (2)(3)(2) - (3)(2)(-4)$$

$$- (1)(0)(1) = 9 + 4 - 12 + 24 = 25$$

$$\rightarrow x_2 = \frac{|A_2|}{|A|} = \frac{-25}{21}$$

$$\textcircled{13} \quad A = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix} \rightarrow \text{find adj}(A) \text{ and } A^{-1} \rightarrow$$

CONTINUED

$$\textcircled{14} \quad \text{adj}(A) = C^T \rightarrow C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \rightarrow$$

$$C_{11} = (-1)^{1+1} |7| = 7 \rightarrow C_{12} = (-1)^{1+2} |3|$$

$$= -3 \rightarrow C_{21} = (-1)^{2+1} |5| = -5 \rightarrow C_{22} = (-1)^{2+2}$$

$$|2| = 2 \rightarrow C = \begin{bmatrix} 7 & -3 \\ -5 & 2 \end{bmatrix} \rightarrow C^T = \begin{bmatrix} 7 & -5 \\ -3 & 2 \end{bmatrix}$$

$\text{adj}(A)$

$$\rightarrow A^{-1} = \frac{1}{|A|} \text{adj}(A) \rightarrow$$

$$|A| = \cancel{(7)(2)} - (5)(3) = -1 \rightarrow A^{-1} = \frac{1}{-1} \text{adj}(A)$$

$$= A^{-1} = \begin{bmatrix} -7 & 5 \\ 3 & -2 \end{bmatrix}$$

$$\textcircled{15} \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \rightarrow \text{find adj}(A) \text{ and } A^{-1}$$

$$\text{adj}(A) = \cancel{C^T} \rightarrow C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \rightarrow$$

$$\textcircled{15} \quad C_{11} = (-1)^{1+1} \left| \begin{array}{cc} 0 & 1 \\ 0 & 0 \end{array} \right| = 0 \rightarrow C_{12} = (-1)^{1+2} \left| \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right|$$

$$= (-1)(-1) = 1 \rightarrow C_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$$

$$\rightarrow C_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 \rightarrow C_{22} = (-1)^{2+2} \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}$$

$$= 0 \rightarrow C_{23} = (-1)^{2+3} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 1 \rightarrow C_{31} =$$

$$(-1)^{3+1} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \rightarrow C_{32} = (-1)^{3+2} \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 \rightarrow$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0 \rightarrow C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \rightarrow$$

$$C^T = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \text{adj}(A) \rightarrow |A| = \begin{vmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

$$= \cancel{(0)} + (1)(1)(1) + \cancel{(0)} - \cancel{(0)} -$$

$$= \cancel{(0)} + (1)(1)(1) + \cancel{(0)} - \cancel{(0)} = 1 \rightarrow A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

$$\rightarrow A^{-1} = \frac{1}{1} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \rightarrow A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(37) $\begin{cases} x_1 + x_2 = 1 \\ 2x_1 + 2x_2 = 2 \end{cases} \rightarrow \text{cannot be solved}$

by cramer's rule. ~~is~~ Proven by the following-

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, \vec{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \rightarrow x_1 = \frac{|A_1|}{|A|} \rightarrow$$

However, $|A| = (1)(2) - (1)(2) = 0 \rightarrow$ Since division by zero is undefined, Cramer's rule cannot be applied.

- (43)
- (a) False as Cramer's rule assumes $\det(\text{coefficients}) \neq 0$ and hence, the columns of the coefficient matrix would need to be linearly independent which does not need to be the case.
 - (b) True because creating $\text{adj}(A)$ from A only involves subtraction, addition, and multiplication. Those operations with integers as inputs will always be outputted as integers.