Graded Homework 4 - Section 3.3

① $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ and $A = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{(7)(1)-(3)(2)}\begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix} \rightarrow A^{-1} = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$

③ $\begin{bmatrix} 2 & -5 \\ -4 & 10 \end{bmatrix} = A \rightarrow A^{-1} = \frac{1}{(2)(10)-(-5)(-4)}\begin{bmatrix} 2 & -5 \\ -4 & 10 \end{bmatrix}$

$\rightarrow$ A^{-1} does not exist.

⑨ $\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$ $-3R_3 + R_2 \rightarrow R_2$, $R_3 + R_1 \rightarrow R_1$ $\sim$ $\rightarrow$

$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$ $-2R_2 + R_1 \rightarrow R_1$ $\sim$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 7 \\ 0 & 1 & 0 & 0 & 1 & -3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

$\rightarrow A^{-1} = \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$

⑬ $\left[\begin{array}{cccc|cccc} 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$ $R_2 \leftrightarrow R_1$, $R_4 \leftrightarrow R_2$, $R_3 \leftrightarrow R_4$ $\sim$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}\right] \rightarrow A^{-1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(17)

$$\underbrace{\begin{bmatrix} 4 & 13 \\ 1 & 3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} -3 \\ 2 \end{bmatrix}}_{\vec{b}} \rightarrow x = A^{-1}\vec{b} \rightarrow$$

$$A^{-1} = \frac{1}{(4)(3)-(13)(1)} \begin{bmatrix} 3 & -13 \\ -1 & 4 \end{bmatrix} \rightarrow A^{-1} = \begin{bmatrix} -3 & 13 \\ 1 & -4 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} -3 & 13 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \end{bmatrix} \rightarrow \vec{x} = \boxed{\begin{bmatrix} 35 \\ -11 \end{bmatrix}} \quad x_1 = 35 \quad x_2 = -11$$

(21)

$$A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} \underset{\sim}{-\tfrac{3}{4}R_1 + R_2 \to R_2} \begin{bmatrix} 4 & 3 \\ 0 & -\tfrac{1}{4} \end{bmatrix} \Rightarrow T \text{ is bijective}$$

$$\Rightarrow A^{-1} \text{ exists} \longrightarrow \left[\begin{array}{cc|cc} 4 & 3 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{array}\right] \underset{\sim}{-\tfrac{3}{4}R_1 + R_2 \to R_2}$$

$$\left[\begin{array}{cc|cc} 4 & 3 & 1 & 0 \\ 0 & -\tfrac{1}{4} & -\tfrac{3}{4} & 1 \end{array}\right] \underset{\sim}{\begin{array}{c} 12R_2 + R_1 \to R_1 \\ -4R_2 \to R_2 \end{array}} \left[\begin{array}{cc|cc} 4 & 0 & -8 & 12 \\ 0 & 1 & 3 & -4 \end{array}\right] \underset{\sim}{\tfrac{1}{4}R_1 \to R_1}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & -4 \end{array}\right] \longrightarrow T^{-1}\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(25) $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & -1 \end{bmatrix} \to T^{-1}$ does not exist as A^{-1} does not exist since A is NOT BOTH one-to-one and onto.

(35) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(39) $A \in \mathbb{R}^{2\times 3}$, $B \in \mathbb{R}^{3\times 2}$ s.t. $AB = I_2 \wedge BA \neq I_3$

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = B$

(43)
a) False as A being invertible means it's bijective
b) True as only square matrices can be bijective

(47)
a) True by definition
b) False as it also needs pivots in columns.

Graded Homework 4 - Section 3.4

(5) $L\vec{y} = \vec{b} \rightarrow y_1 = 2 \rightarrow -2(2) + y_2 = 2 \rightarrow$
$y_2 = 6 \rightarrow 3x_2 = 6 \rightarrow x_2 = 2 \rightarrow 2x_1 - 2(2) = 2$
$\rightarrow x_1 = 3 \rightarrow \vec{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

(7) $y_1 = 4 \rightarrow -1(4) + y_2 = 0 \rightarrow y_2 = 4 \rightarrow 2(4) +$
$-2(4) + x_3 = -4 \rightarrow y_3 = -4 \rightarrow -2x_3 = -4 \rightarrow$
$x_3 = 2 \rightarrow x_2 + 2(2) = 4 \rightarrow x_2 = 0 \rightarrow 2x_1$
$-(0) + 3(2) = 4 \rightarrow x_1 = -1 \quad \vec{x} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

(13) $\begin{bmatrix} 1 & -4 \\ -2 & 9 \end{bmatrix} \rightarrow L = \begin{bmatrix} 1 & \vdots \\ -2 & \vdots \end{bmatrix} \quad \xrightarrow{2R_1 + R_2 \rightarrow R_2} \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} \rightarrow L = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

$\rightarrow L = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}, U = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$

(15) $\begin{bmatrix} -2 & -1 & 1 \\ -6 & 0 & 4 \\ 2 & -2 & -1 \end{bmatrix} \xrightarrow[R_1 + R_3 \rightarrow R_3]{-3R_1 + R_2 \rightarrow R_2} \begin{bmatrix} -2 & -1 & 1 \\ 0 & 3 & 1 \\ 0 & -3 & 0 \end{bmatrix} \rightarrow L = \begin{bmatrix} 1 & \vdots & \vdots \\ 3 & \vdots & \vdots \\ -1 & \vdots & \vdots \end{bmatrix}$

$\rightarrow \quad R_2+R_3 \to R_3 \quad \sim \quad \begin{bmatrix} -2 & -1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1 & -1 & \cdot \end{bmatrix}$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} -2 & -1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

17

$A = \begin{bmatrix} -1 & 0 & -1 & 2 \\ 1 & 3 & 2 & -2 \\ -2 & -9 & -3 & 3 \\ -1 & 9 & -2 & 5 \end{bmatrix}$ $\begin{matrix} R_1+R_2 \to R_2 \\ -2R_1+R_3 \to R_3 \\ -R_1+R_4 \to R_4 \\ \sim \end{matrix}$ $\begin{bmatrix} -1 & 0 & -1 & 2 \\ 0 & 3 & 1 & 0 \\ 0 & -9 & -1 & -1 \\ 0 & 9 & -1 & 3 \end{bmatrix}$

$\rightarrow L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & \cdot & 0 & 0 \\ 2 & \cdot & \cdot & 0 \\ 1 & \cdot & \cdot & \cdot \end{bmatrix}$ $\begin{matrix} 3R_2+R_3 \to R_3 \\ -3R_2+R_4 \to R_4 \\ \sim \end{matrix}$ $\begin{bmatrix} -1 & 0 & -1 & 2 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -4 & 3 \end{bmatrix}$

$\rightarrow L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & -3 & \cdot & 0 \\ 1 & 3 & \cdot & \cdot \end{bmatrix}$ $\begin{matrix} 2R_3+R_4 \to R_4 \\ \sim \end{matrix}$ $\begin{bmatrix} -1 & 0 & -1 & 2 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\rightarrow L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & -3 & 1 & 0 \\ 1 & 3 & -2 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} -1 & 0 & -1 & 2 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

31

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = L \quad , \quad U = \begin{bmatrix} 2 & -2 \\ 0 & 3 \end{bmatrix} , \text{ find } U^{-1}L^{-1} = A^{-1}$$

$$U^{-1} = \frac{1}{(2)(3) - (-2)(0)} \begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{1}{3} \end{bmatrix} \longrightarrow$$

$$L^{-1} = \frac{1}{(1)(1) - (0)(-2)} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \longrightarrow$$

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 7/6 & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

Graded Homework 4 - Section 3.5

③ A is stochastic

⑦ $A = \begin{bmatrix} \frac{2}{13} & \frac{3}{7} & c \\ a & \frac{3}{7} & \frac{1}{5} \\ \frac{3}{13} & b & \frac{7}{10} \end{bmatrix}$, $a = \frac{8}{13},\ b = \frac{1}{7},\ c = \frac{1}{10}$

⑮

$A = \begin{bmatrix} \frac{1}{3} & \frac{2}{5} \\ \frac{2}{3} & \frac{3}{5} \end{bmatrix}$, $x_0 = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \rightarrow x_1 = \begin{bmatrix} \frac{1}{6} + \frac{2}{10} \\ \frac{2}{6} + \frac{3}{10} \end{bmatrix} = \begin{bmatrix} \frac{11}{30} \\ \frac{19}{30} \end{bmatrix} \rightarrow$

$x_2 = \begin{bmatrix} \frac{11}{90} + \frac{38}{150} \\ \frac{22}{90} + \frac{57}{150} \end{bmatrix} = \begin{bmatrix} \frac{55 + 114}{450} \\ \frac{110 + 171}{450} \end{bmatrix} = \begin{bmatrix} \frac{169}{450} \\ \frac{281}{450} \end{bmatrix} \rightarrow x_3 = \begin{bmatrix} \frac{169}{1350} + \frac{281}{1125} \\ \frac{169}{675} + \frac{281}{750} \end{bmatrix}$

$= \begin{bmatrix} \frac{2531}{6750} \\ \frac{4219}{6750} \end{bmatrix} \approx \begin{bmatrix} .374\overline{962} \\ .625\overline{037} \end{bmatrix}$

⑰ A is a regular stochastic matrix as the values will always be positive, non-zero

CONTINUED →

$(A-I)x=0 \rightarrow \left[\begin{array}{cc|c} -0.2 & 0.5 & 0 \\ 0.2 & -0.5 & 0 \end{array}\right] \xrightarrow{R_1+R_2 \to R_2}$

$\left[\begin{array}{cc|c} -0.2 & 0.5 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightarrow$ let $x_2 = s_1 \rightarrow -0.2 + .5s_1 = 0$

$\rightarrow x_1 = \frac{5}{2}s_1 \rightarrow x = s_1 \begin{bmatrix} \frac{5}{2} \\ 1 \end{bmatrix} \rightarrow s_1 = \frac{1}{\frac{5}{2}+1} = \frac{2}{7}$

$\rightarrow x = \begin{bmatrix} 10/14 \\ 2/7 \end{bmatrix}$

(19) A is regular as there will always be positive entries and it is stochastic

$(A-I)x=0 \rightarrow \left[\begin{array}{ccc|c} -0.6 & 0.5 & 0.3 & 0 \\ 0.2 & -0.7 & 0.4 & 0 \\ 0.4 & 0.2 & -0.7 & 0 \end{array}\right]$ $\frac{1}{3}R_1+R_2 \to R_2$ $\sim$ $\frac{2}{3}R_1 + R_3 \to R_3$

$\rightarrow \left[\begin{array}{ccc|c} -0.6 & 0.5 & 0.3 & 0 \\ 0 & -8/15 & 0.5 & 0 \\ 0 & \frac{16}{30} & -0.5 & 0 \end{array}\right]$ $R_2+R_3 \to R_3$ $\sim$ $\longrightarrow$

$\left[\begin{array}{ccc|c} -0.6 & 0.5 & 0.3 & 0 \\ 0 & -8/15 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow$ let $x_3 = s_1 \rightarrow -\frac{8}{15}x_2 + 0.5s_1 = 0$

$\rightarrow -\frac{8}{15}x_2 = -\frac{1}{2}s_1 \rightarrow x_2 = \left(-\frac{1}{2}\right)\left(-\frac{15}{8}\right)s_1 \rightarrow$

$x_2 = \frac{15}{16}s_1 \quad -0.6 + 0.5\left(\frac{15}{16}s_1\right) + 0.3s_1 = 0 \longrightarrow$

$$-\frac{6}{10}x_1 + \left(\frac{15}{32}\right)\frac{5}{5}s_1 + \frac{3}{10}s_1 = 0 \rightarrow -\frac{6}{10}x_1 + \frac{75}{160}s_1 + \frac{48}{160}s_1 = 0$$

$$\rightarrow \frac{-10}{6}\left(-\frac{6}{10}x_1 = \left(-\frac{123}{160}s_1\right)\right)\frac{-10}{6} \rightarrow x_1 = \frac{1230}{960}s_1 = \frac{41}{32}s_1$$

$$\rightarrow x = s_1 \begin{bmatrix} \frac{41}{32} \\ \frac{15}{16} \\ 1 \end{bmatrix} \qquad s_1 = \frac{1}{41/32 + 15/16 + 1} = s_1 = \frac{32}{103}$$

$$\rightarrow x = \begin{bmatrix} .398 \\ .291 \\ .3106 \end{bmatrix}$$

(23) let $\bullet$ denote a placeholder variable such that

$\bullet \in \mathbb{R} \wedge \bullet > 0$

$$A^2 = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.8 & 0.4 \\ 0 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.8 & 0.4 \\ 0 & 0 & 0.5 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ 0 & 0 & \bullet \end{bmatrix} \xrightarrow[\text{by itself}]{\text{multiply}} \begin{bmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ 0 & 0 & \bullet \end{bmatrix} \rightarrow \text{A is not regular}$$

(27) $\begin{bmatrix} 1-a & b \\ a & 1-b \end{bmatrix} = A$

$$\lim_{n \to \infty} A^n = \vec{x}$$

$\vec{x} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$

$(A - I)\vec{x} = 0 \rightarrow \begin{bmatrix} -a & b \\ a & -b \end{bmatrix} \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} = \vec{0} \rightarrow$

$-\frac{2}{3}a + \frac{1}{3}b = 0$ $\quad Eq_1 + Eq_2 \rightarrow Eq_2 \quad$ $-\frac{2}{3}a + \frac{1}{3}b = 0$

$\frac{2}{3}a - \frac{1}{3}b = 0 \quad \sim \quad 0 - 0 = 0 \rightarrow$

$-2a + b = 0 \rightarrow$ let $b = s_1 \rightarrow$ $a = \frac{1}{2}s_1$

$\begin{bmatrix} \frac{1}{2} & 1 \\ \frac{1}{2} & 0 \end{bmatrix} = A$ for $s_1 = 1$

(31)

(a) False, this would only be true if A was doubly stochastic

(b) False as A may not be bijective

(33)

(a)

$A = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} \end{bmatrix}$ $B = \begin{bmatrix} \frac{1}{5} & \frac{1}{2} \\ \frac{4}{5} & \frac{1}{2} \end{bmatrix} \rightarrow AB^T = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{5} & \frac{4}{5} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$\rightarrow \begin{bmatrix} \frac{6}{20} & \frac{9}{20} \\ \frac{9}{20} & \frac{17}{20} \end{bmatrix}$ False

(b) False as it can have 0's

(35)

(a) False as it depends on initial vector and if A is regular.

(b) True as the columns of A would no longer sum to 1.

(47)

(a)

$$\begin{array}{cc} & \text{From} \\ & \begin{array}{cc} \text{Yes} & \text{No} \end{array} \\ \begin{array}{c} \text{To Yes} \\ \text{No} \end{array} & \begin{bmatrix} .9 & .15 \\ .1 & .85 \end{bmatrix} \end{array}$$

(b)

$$\begin{bmatrix} .9 & .15 \\ .1 & .85 \end{bmatrix} \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\vec{x}_0 \text{ (initial state)}} = \underbrace{\begin{bmatrix} .9 \\ .1 \end{bmatrix}}_{\vec{x}_1} \rightarrow$$

$$\begin{bmatrix} .9 & .15 \\ .1 & .85 \end{bmatrix} \begin{bmatrix} .9 \\ .1 \end{bmatrix} = \underbrace{\begin{bmatrix} .825 \\ .175 \end{bmatrix}}_{\vec{x}_2} \rightarrow \begin{bmatrix} .9 & .15 \\ .1 & .85 \end{bmatrix} \begin{bmatrix} .825 \\ .175 \end{bmatrix} = \underbrace{\begin{bmatrix} .76875 \\ .23125 \end{bmatrix}}_{\vec{x}_3}$$

$$\rightarrow \begin{bmatrix} .9 & .15 \\ .1 & .85 \end{bmatrix} \begin{bmatrix} .76875 \\ .23125 \end{bmatrix} \approx \underbrace{\begin{bmatrix} .7266 \\ .2734 \end{bmatrix}}_{\vec{x}_4} \rightarrow \begin{bmatrix} .9 & .15 \\ .1 & .85 \end{bmatrix} \begin{bmatrix} .7266 \\ .2734 \end{bmatrix}$$

$$\approx \underbrace{\begin{bmatrix} .6950 \\ .3051 \end{bmatrix}}_{\vec{x}_5} \rightarrow \begin{bmatrix} .9 & .15 \\ .1 & .85 \end{bmatrix} \begin{bmatrix} .6950 \\ .3051 \end{bmatrix} \approx \underbrace{\begin{bmatrix} .6713 \\ .3288 \end{bmatrix}}_{\vec{x}_6}$$

Probability that 6^{th} person hears wrong news:
.3288

(c) $(A-I)\vec{x} = \vec{0} \rightarrow \left[\begin{array}{cc|c} -.1 & .15 & 0 \\ .1 & -.15 & 0 \end{array}\right]$ $R_1 + R_2 \rightarrow R_2$ $\sim$

$\left[\begin{array}{cc|c} -.1 & .15 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightarrow$ let $x_2 = s_1 \rightarrow -.1x_1 + .15s_1 = 0$

$-.1x_1 = -.15s_1 \rightarrow x_1 = \dfrac{-\frac{15}{100}}{-\frac{10}{100}} s_1 \rightarrow$

$x_1 = \frac{3}{2}s_1 \rightarrow \vec{x} = s_1 \begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix} \rightarrow s_1 = \dfrac{1}{1+\frac{3}{2}} = \frac{2}{5}$

$\rightarrow \vec{x} = \frac{2}{5}\begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix} \rightarrow \vec{x} = \begin{bmatrix} \frac{3}{5} \\ \frac{2}{5} \end{bmatrix}$

(51)

(a) $\begin{bmatrix} .4 & .1 & .2 \\ .3 & .7 & .7 \\ .3 & .2 & .1 \end{bmatrix} \underset{\vec{x}_0}{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}} = \underset{\vec{x}_1}{\begin{bmatrix} .4 \\ .3 \\ .3 \end{bmatrix}} \rightarrow$

$\begin{bmatrix} .4 & .1 & .2 \\ .3 & .7 & .7 \\ .3 & .2 & .1 \end{bmatrix} \begin{bmatrix} .4 \\ .3 \\ .3 \end{bmatrix} = \underset{\vec{x}_2}{\begin{bmatrix} .25 \\ .54 \\ .21 \end{bmatrix}}$

.21 chance that it ends up at C after two circulations

(b)

$$\begin{bmatrix} .4 & .1 & .2 \\ .3 & .7 & .7 \\ .3 & .2 & .1 \end{bmatrix} \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{\vec{x}_0} = \underbrace{\begin{bmatrix} .1 \\ .7 \\ .2 \end{bmatrix}}_{\vec{x}_1} \longrightarrow$$

$$\begin{bmatrix} .4 & .1 & .2 \\ .3 & .7 & .7 \\ .3 & .2 & .1 \end{bmatrix} \begin{bmatrix} .1 \\ .7 \\ .2 \end{bmatrix} = \underbrace{\begin{bmatrix} .15 \\ .66 \\ .19 \end{bmatrix}}_{\vec{x}_2} \rightarrow \begin{bmatrix} .4 & .1 & .2 \\ .3 & .7 & .7 \\ .3 & .2 & .1 \end{bmatrix} \begin{bmatrix} .15 \\ .66 \\ .19 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} .164 \\ .64 \\ .196 \end{bmatrix}}_{\vec{x}_3}$$

.64 chance it ends up at B

(c) $(A - I)\vec{x} = \vec{0} \rightarrow \begin{bmatrix} -.6 & .1 & .2 & | & 0 \\ .3 & -.3 & .7 & | & 0 \\ .3 & .2 & -.9 & | & 0 \end{bmatrix}$ $\frac{1}{2}R_1 + R_3 \to R_3$, $\frac{1}{2}R_1 + R_2 \to R_2$ $\sim$

$$\rightarrow \begin{bmatrix} -.6 & .1 & .2 & | & 0 \\ 0 & -.25 & .8 & | & 0 \\ 0 & .25 & -.8 & | & 0 \end{bmatrix} \underset{\sim}{R_2 + R_3 \to R_3} \begin{bmatrix} -.6 & .1 & .2 & | & 0 \\ 0 & -.25 & .8 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$\rightarrow$ let $x_3 = s_1 \rightarrow -.25x_2 + .8s_1 = 0 \rightarrow$

$$x_2 = \frac{-\frac{80}{100}}{-\frac{25}{100}} s_1 \rightarrow x_2 = \frac{16}{5} s_1 \rightarrow -.6x_1 + .1\left(\frac{16}{5}\right)s_1 + .2s_1 = 0$$

$$\rightarrow -\frac{60}{100}x_1 + \frac{160}{500}s_1 + \frac{100}{500}s_1 = 0 \rightarrow x_1 = \frac{-\frac{260}{500}}{-\frac{60}{100}} s_1 \longrightarrow$$

$$x_1 = \frac{13}{15} s_1 \rightarrow \vec{x} = s_1 \begin{bmatrix} \frac{13}{15} \\ \frac{16}{5} \\ 1 \end{bmatrix} \rightarrow s_1 = \frac{1}{\left(\frac{13}{15}\right) + \left(\frac{16}{5}\right) + 1}$$

$$s_1 = \frac{15}{76} \rightarrow \vec{x} = \frac{15}{76} \begin{bmatrix} \frac{13}{15} \\ \frac{16}{5} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{13}{76} \\ \frac{12}{19} \\ \frac{15}{76} \end{bmatrix}$$

Graded Homework 4 - Section 4.1

① $\left\{ \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} \right\}$ s.t. $a \wedge b \in \mathbb{R} \Rightarrow a \in \{0\} \wedge b \in \{0\}$

$\Rightarrow \exists \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. let $\vec{u} = \begin{bmatrix} a_1 \\ 0 \\ b_1 \end{bmatrix}$ s.t. $a_1, b_1 \in \mathbb{R} \wedge$

let $\vec{v} = \begin{bmatrix} a_2 \\ 0 \\ b_2 \end{bmatrix}$ s.t. $a_2, b_2 \in \mathbb{R}$

$\rightarrow \exists\ \vec{u} + \vec{v} = \begin{bmatrix} a_1 + a_2 \\ 0+0 \\ b_1 + b_2 \end{bmatrix} \forall a_1, a_2, b_1, b_2 \in \mathbb{R}$.

$\rightarrow c \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} = \begin{bmatrix} ca \\ c0 \\ cb \end{bmatrix} \forall c \in \mathbb{R}$.

It is a subspace.

③ $\left\{ \begin{bmatrix} a \\ b \end{bmatrix} \right\}$ s.t. $a + b = 1 \wedge a, b \in \mathbb{R} \Rightarrow$

$a \neq 0 \wedge b \neq 0 \Rightarrow$ not a subspace.

⑤ $\left\{ \begin{bmatrix} a \\ 1 \\ 0 \\ b \end{bmatrix} \right\}$ s.t. $a \wedge b \in \mathbb{R}$.

Not a subspace as $\vec{0} \in \mathbb{R}^4$ not there.

(9) $\left\{\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right\}$ s.t. $abc = 0 \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \in \left\{\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right\}$

$\rightarrow \left\{\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}\right\} \in \left\{\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right\} \rightarrow \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} \wedge (2)(-1)(-1) = 2 \Rightarrow$ not a subspace

as it is not closed under attention.

(11) $\left\{\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right\}$ s.t. $a \geq 0,\ b \geq 0,\ c \geq 0 \longrightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \in \left\{\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right\}$

$\rightarrow \left\{\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right\} = \left\{ a\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}$

$\rightarrow$ all vectors in the positive quadrant of $\mathbb{R}^3$ are available $\rightarrow$ Closed under addition $\rightarrow$

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \in \left\{\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right\}$ but $-1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \notin \left\{\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right\} \rightarrow$ not

closed under multiplication $\rightarrow$ not a subspace

(15) $\vec{V} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ s.t. $v_1 + \ldots + v_n = 0 \longrightarrow$

$A\vec{x} = 0 \rightarrow A = [1 \ldots\ldots 1] \rightarrow A\vec{x} = 0$ so

the solution to this homogeneous system is a subspace

(25) $A = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 5 & -7 \end{bmatrix} \rightarrow null(A) = \left[\begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ -2 & 5 & -7 & 0 \end{array}\right]$

$2R_1 + R_2 \rightarrow R_2 \sim \left[\begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right] \longrightarrow$ let $x_3 = s_1 \longrightarrow$

$x_2 - 3(s_1) = 0 \longrightarrow x_2 = 3s_1 \longrightarrow x_1 - 2(3s_1) + 2(s_1) = 0$

$\longrightarrow x_1 = 4s_1 \longrightarrow$ $null(A) = \begin{bmatrix} 4s_1 \\ 3s_1 \\ s_1 \end{bmatrix} = s_1 \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$

(29) $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 3 \end{bmatrix} \rightarrow null(A) = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right] \longrightarrow$

$3x_3 = 0 \longrightarrow x_3 = 0 \longrightarrow x_2 + 3(0) = 0 \longrightarrow x_2 = 0$

$\longrightarrow x_1 - 1(0) + 1(0) = 0 \longrightarrow x_1 = 0 \longrightarrow \vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\rightarrow null(A) = Span \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

(35) $\begin{bmatrix} 4 & -2 \\ 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} -5 \\ 2 \end{bmatrix} = \begin{bmatrix} -24 \\ 1 \\ 4 \end{bmatrix} \Rightarrow b \notin Ker(T)$

C cannot be in range(T) as it has only 2 components.

(37) $\left\{ \begin{bmatrix} a \\ b \end{bmatrix} \right\}$ s.t. $a > 0 \wedge b > 0$

(45) (a) True as the $\vec{0}$ was not included

(b)

$$A = \left[\begin{array}{ccc|c} \bullet & \bullet & \bullet & 0 \\ \bullet & \bullet & \bullet & 0 \\ \bullet & \bullet & \bullet & 0 \\ \bullet & \bullet & \bullet & 0 \\ \bullet & \bullet & \bullet & 0 \end{array}\right], \quad \bullet = \text{placeholder } \mathbb{R} \text{ variable}$$

False, the set of solutions produces the trivial $\vec{0}$ subspace $\in \mathbb{R}^3$.

(47) (a)

$T(\vec{x}) = A\vec{x}$, $T: \mathbb{R}^5 \to \mathbb{R}^8 \rightsquigarrow A =$

$$\begin{bmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{bmatrix}, \quad \bullet = \text{placeholder} \in \mathbb{R}$$

$\to \ker(T) = \vec{x}$ s.t. $A\vec{x} = \vec{0} \longrightarrow \vec{x}$ is subspace of the trivial $\vec{0}$ type $\in \mathbb{R}^5$ so false

(b) $T: \mathbb{R}^2 \to \mathbb{R}^7 \rightsquigarrow T(\vec{x}) = A\vec{x} \to A = \begin{bmatrix} \cdot & \cdot \\ \vdots & \vdots \\ \cdot & \cdot \end{bmatrix} \in \mathbb{R}^{7\times 2}$

$\to$ range(T) = Span $\{\vec{a}_1, \vec{a}_2\}$ $\to$ a subspace for $\mathbb{R}^7$ $\to$ not a subspace of $\mathbb{R}^2$ - false

(51) (a) False as for any $\vec{v} \in S \Rightarrow c_1\vec{v} \in S$ where $c \in \mathbb{R}$ and hence, S must be infinite set outside of $S = \{\vec{0}\}$

(b) True as in a non-zero subspace, all points in the subspace are accessible.