

~~Graded Homework 2 - Section 1.3~~ A

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(3)

$$A: x_4 + 30 + 40 = x_1 + 50$$

$$B: x_1 + x_3 + 25 = 55 + 40 + x_2$$

$$C: x_2 + 50 = x_4 + 25$$

↓  
~~↓~~

$$A: x_1 - x_4 = 20$$

$$B: x_1 - x_2 + x_3 = 70$$

$$C: x_2 - x_4 = -25$$

↓

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & -1 & 20 \\ 1 & -1 & 1 & 0 & 70 \\ 0 & 1 & 0 & -1 & -25 \end{array} \right] \xrightarrow[-1R_1 + R_2 \rightarrow R_2]{\quad\quad\quad} \quad\quad\quad$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & -1 & 20 \\ 0 & -1 & 1 & 1 & 50 \\ 0 & 1 & 0 & -1 & -25 \end{array} \right] \xrightarrow[R_2 + R_3 \rightarrow R_3]{\quad\quad\quad} \quad\quad\quad$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & -1 & 20 \\ 0 & -1 & 1 & 1 & 50 \\ 0 & 0 & 0 & 0 & +25 \end{array} \right] \Rightarrow \quad\quad\quad$$

let  $x_4 = s_1$

$$\begin{aligned}x_1 + -x_4 &= 20 \\-x_2 + x_3 + x_4 &= 50 \Rightarrow \\x_3 &= 25\end{aligned}$$

~~all cars equal~~

$$-x_2 + (25) + s_1 = 50 \Rightarrow -x_2 = 25 - s_1$$

$$\Rightarrow x_2 = s_1 - 25 \Rightarrow x_1 - s_1 = 20 \Rightarrow x_1 = 20 + s_1$$

$$x_1 = s_1 + 20$$

$$x_2 = s_1 - 25$$

$$x_3 = 25$$

$$x_4 = s_1$$

25 cars minimum

(7)  $x_1 = \frac{50 + x_2 + x_3}{3}$

$$x_2 = \frac{90 + x_3 + x_1}{3}$$

multiply  
all by 3

$$x_3 = \frac{30 + x_2 + x_1}{3}$$

$$3x_1 = 50 + x_2 + x_3$$

$$3x_1 - x_2 - x_3 = 50$$

$$3x_2 = 90 + x_3 + x_1$$

$$\Rightarrow -x_1 + 3x_2 - x_3 = 90$$

$$3x_3 = 30 + x_2 + x_1$$

$$-x_1 - x_2 + 3x_3 = 30$$

$$\left[ \begin{array}{ccc|c} 3 & -1 & -1 & 50 \\ -1 & 3 & -1 & 90 \\ -1 & -1 & 3 & 30 \end{array} \right] \xrightarrow{\text{R}_2 \leftrightarrow \text{R}_1} \left[ \begin{array}{ccc|c} -1 & 3 & -1 & 90 \\ 3 & -1 & -1 & 50 \\ -1 & -1 & 3 & 30 \end{array} \right]$$

$$3R_1 + R_2 \rightarrow R_2 \quad \left[ \begin{array}{ccc|c} -1 & 3 & -1 & 90 \\ 0 & 8 & -4 & 330 \\ -1 & -1 & 3 & 30 \end{array} \right] \xrightarrow{-R_1 + R_3 \rightarrow R_3} \sim$$

$$\left[ \begin{array}{ccc|c} -1 & 3 & -1 & 90 \\ 0 & 8 & -4 & 330 \\ 0 & -4 & 4 & -60 \end{array} \right] \xrightarrow{\text{R}_2 \rightarrow R_3} \left[ \begin{array}{ccc|c} -1 & 3 & -1 & 90 \\ 0 & -4 & 4 & -60 \\ 0 & 8 & -4 & 330 \end{array} \right]$$

$$2R_2 + R_3 \rightarrow R_3 \quad \left[ \begin{array}{ccc|c} -1 & 3 & -1 & 90 \\ 0 & -4 & 4 & -60 \\ 0 & 0 & 4 & 200 \end{array} \right] \Rightarrow$$

$$\begin{aligned} -x_1 + 3x_2 - x_3 &= 90 \\ -4x_2 + 4x_3 &= -60 \Rightarrow x_3 = 50 \Rightarrow \\ 4x_3 &= 200 \end{aligned}$$

$$-4x_2 + 4(50) = -60 \Rightarrow -4x_2 = -260 \Rightarrow x_2 = 65 \Rightarrow$$

$$-x_1 + 3(65) - (50) = 90 \Rightarrow -x_1 + 195 - 50 = 90$$

$$\Rightarrow -x_1 = -55 \Rightarrow x_1 = 55$$

$x_1 = 55$

$x_2 = 65$

$x_3 = 50$

⑨

$$(a = 60 + .2b)$$

$$(b = 40 + .3a)$$

~~equation 1~~

~~equation 2~~

~~equation 3~~

$$a - .2b = 60 \Rightarrow$$

$$-.3a + b = 40$$

$$\left[ \begin{array}{cc|c} a & b & \\ 1 & -.2 & 60 \\ -.3 & 1 & 40 \end{array} \right]$$

$$5R_1 + R_2 \rightarrow R_2$$

$\sim$

$$\left[ \begin{array}{cc|c} 1 & -.2 & 60 \\ 4.7 & 0 & 340 \end{array} \right] \Rightarrow 4.7a = 340$$

$$\Rightarrow a \approx 72.34 \Rightarrow (72.34) -.2b = 60 \Rightarrow b \approx 61.7$$

$$a = 72.34, b = 61.7$$

⑬

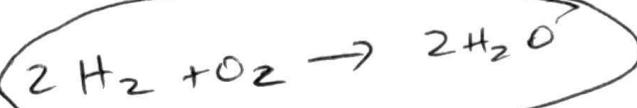


$$\left\{ \begin{array}{l} 2x_1 = 2x_3 \\ 2x_2 = x_3 \end{array} \right. \Rightarrow \begin{array}{l} 2x_1 - 2x_3 = 0 \\ 2x_2 - x_3 = 0 \end{array} \Rightarrow$$

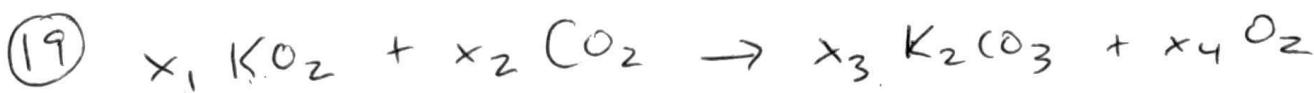
$$\left[ \begin{array}{ccc|c} 2 & 0 & -2 & 0 \\ 0 & 2 & -1 & 0 \end{array} \right] \Rightarrow \begin{array}{l} 1st \ x_3 = s_1 \\ 2nd \ x_2 - s_1 = 0 \end{array} \Rightarrow$$

$$x_2 = \frac{1}{2}s_1 \Rightarrow$$

$$2x_1 - 2s_1 = 0 \Rightarrow x_1 = s_1 \Rightarrow$$



$$\begin{aligned}x_1 &= s_1 \\x_2 &= \frac{1}{2}s_1 \\x_3 &= s_1\end{aligned}$$



$$\text{K: } x_1 = 2x_3$$

$$\text{O: } \cancel{2x_1} + 2x_2 = 3x_3 + 2x_4 \Rightarrow$$

$$\text{C: } x_2 = x_3$$

$$x_1 - 2x_3 = 0$$

$$x_2 - x_3 = 0$$

$$2x_1 + 2x_2 - 3x_3 - 2x_4 = 0$$

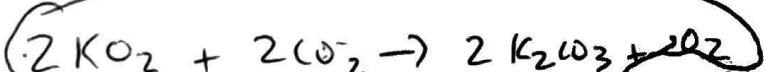
$$\Rightarrow \left[ \begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 2 & 2 & -3 & -2 & 0 \end{array} \right]$$

$$-2R_1 + R_3 \rightarrow R_3 \sim \left[ \begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & -1 & -2 & 0 \end{array} \right] -2R_2 + R_3 \rightarrow R_3$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 \end{array} \right] \Rightarrow \text{let } x_4 = s_1 \Rightarrow x_3 - 2s_1 = 0 \Rightarrow$$

$$x_3 = 2s_1 \Rightarrow x_2 - (2s_1) = 0 \Rightarrow x_2 = 2s_1 \Rightarrow$$

$$x_1 - (2s_1) = 0 \Rightarrow x_1 = 2s_1$$



$$\begin{aligned}x_1 &= 2s_1 \\x_2 &= 2s_1 \\x_3 &= 2s_1 \\x_4 &= s_1\end{aligned}$$

$$\textcircled{21} \quad \ln(p) = \ln(a) + b \ln(d) \Rightarrow \text{let } \ln(a) = a,$$

$$\text{Earth: } a_1 + b \ln(149,6) = \ln(365,2) \Rightarrow$$

$$\text{Mars: } a_1 + b \ln(227,9) = \ln(687)$$

$$\left[ \begin{array}{cc|c} a_1 & b & \\ 1 & \ln(149,6) & \ln(365,2) \\ 1 & \ln(227,9) & \ln(687) \end{array} \right] \xrightarrow{-1R_1 + R_2 \rightarrow R_2}$$

$$\left[ \begin{array}{cc|c} 1 & \ln(149,6) & \ln(365,2) \\ 0 & \ln(227,9) - \ln(149,6) & \ln(687) - \ln(365,2) \end{array} \right]$$

$$\left\{ \ln(227,9) - \ln(149,6) \right\}(b) = \left\{ \ln(687) - \ln(365,2) \right\}$$

$$b = \frac{0.631889}{-420941} \approx 1.5011 \Rightarrow$$

$$a_1 + \ln(149,6)(1.5011) = \ln(365,2)$$

$$a_1 + 7.517456 = 5,9004 \Rightarrow a_1 = -1,61705$$

$$\ln(a) = -1,61705 \Rightarrow a = \cancel{0.000000000000000}$$

$$P = \cancel{0.000000000000000} \cdot \overset{(.98147)}{(d)}^{1.5011}$$

$$\textcircled{27} \quad \left[ \frac{1}{(x)(x+1)} = \frac{A}{x} + \frac{B}{x+1} \right] (x)(x+1)$$

$$1 = A(x+1) + B(x) \quad \cancel{\text{elde aussortieren}}$$

$$\Rightarrow \text{set } x=0 \Rightarrow 1 = A(0+1) + B(0) \Rightarrow A = 1 \Rightarrow$$

$$\text{set } x = -1 \Rightarrow 1 = 1(-1+1) + B(-1) \Rightarrow B = -1$$

$$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}; A=1, B=-1$$

$$29 \left[ \frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} \right] x^2(x-1) \Rightarrow$$

$$1 = A(x)(x-1) + B(x-1) + C(x^2) \Rightarrow \cancel{\text{let}}$$

$$x=0 \Rightarrow 1 = \cancel{A(0)(0-1)} + B(0-1) + \cancel{C(0)} \Rightarrow$$

$$1 = -B \Rightarrow B = -1 \Rightarrow \text{let } x=1 \Rightarrow$$

$$1 = \cancel{A(1)(1-1)} + \cancel{-1(1-1)} + C(1^2) \Rightarrow C = 1$$

$$\Rightarrow \text{let } x=2 \Rightarrow 1 = A(2)(2-1) + (-1)(2-1) +$$

$$1(2^2) \Rightarrow 1 = 2A - 1 + 4 \Rightarrow A = -1$$

$$A = -1, B = -1, C = 1; \frac{-1}{x} - \frac{1}{x^2} + \frac{1}{x-1} = \frac{1}{x^2(x-1)}$$

$$37 f(x) = ax^3 + bx^2 + cx + d, f(0) = -3, f(1) = 2$$

$$f(3) = 5, f(4) = 0$$

$$-3 = a(0)^3 + b(0)^2 + c(0) + d \Rightarrow d = -3 \Rightarrow$$

$$2 = a(1)^3 + b(1)^2 + c(1) + d \Rightarrow 2 = a + b + c - 3 \Rightarrow$$

$$5 = a(3)^3 + b(3)^2 + c(3) + d \Rightarrow 0 = a(4)^3 + b(4)^2 + c(4) + d - 3$$

$$\begin{cases} a+b+c = 5 \\ 64a+16b+4c = 8 \\ 64a+16b+4c = 3 \end{cases} \Rightarrow \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 27 & 9 & 3 & 8 \\ 64 & 16 & 4 & 3 \end{array} \right]$$

$$-2R_1 + R_2 \rightarrow R_2$$

$$\sim$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & -18 & -24 & -127 \\ 0 & 16 & 4 & 3 \end{array} \right] \xrightarrow{\text{Row operations}} \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & -18 & -24 & -127 \\ 0 & 0 & 4 & 3 \end{array} \right] \xrightarrow{-64R_1 + R_3 \rightarrow R_3}$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & -18 & -24 & -127 \\ 0 & -48 & -60 & -317 \end{array} \right] \xrightarrow{\text{Row operations}} \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & -18 & -24 & -127 \\ 0 & 0 & 4 & 3 \end{array} \right]$$

$$4c = \frac{65}{3} \Rightarrow c = \frac{65}{12} \Rightarrow -18b - 24\left(\frac{65}{12}\right) = -127 \Rightarrow$$

$$-18b - 130 = -127 \Rightarrow -18b = 3 \Rightarrow b = \frac{3}{18} \Rightarrow a + \left(\frac{3}{18}\right) + \left(\frac{65}{12}\right) =$$

$$\Rightarrow a = -\frac{1}{4} \Rightarrow f(x) = -\frac{1}{4}x^3 - \frac{3}{18}x^2 + \frac{65}{12}x - 3$$

Graded homework 2 - Section 1.4

⑨  $\begin{aligned} -5x_1 + 2x_2 &= 4 \\ 3x_1 + 10x_2 &= 2 \end{aligned} \rightarrow \begin{aligned} x_1 &= \frac{2}{5}x_2 + \frac{6}{5} \\ x_2 &= -\frac{3}{10}x_1 + \frac{1}{5} \end{aligned}$

$\begin{aligned} x_1 &= \frac{2}{5}(0) + \frac{6}{5} \rightarrow x_1 = \frac{6}{5} \\ x_2 &= -\frac{3}{10}(0) + \frac{1}{5} \rightarrow x_2 = \frac{1}{5} \end{aligned} \rightarrow \begin{aligned} x_1 &= \frac{2}{5}\left(\frac{6}{5}\right) + \frac{6}{5} \\ x_2 &= -\frac{3}{10}\left(\frac{1}{5}\right) + \frac{1}{5} \end{aligned}$

$\rightarrow \begin{aligned} x_1 &= \frac{12}{25} + \frac{6}{5} \rightarrow x_1 = \frac{42}{25} \\ x_2 &= -\frac{3}{50} + \frac{1}{5} \rightarrow x_2 = -\frac{3}{50} \end{aligned} \rightarrow \begin{aligned} x_1 &= \frac{2}{5}\left(\frac{42}{25}\right) + \frac{6}{5} \\ x_2 &= -\frac{3}{10}\left(\frac{7}{50}\right) + \frac{1}{5} \end{aligned}$

$\rightarrow \begin{aligned} x_1 &= \frac{84}{125} + \frac{6}{5} \rightarrow x_1 = \frac{234}{125} \\ x_2 &= \frac{-21}{500} + \frac{1}{5} \rightarrow x_2 = \frac{79}{500} \end{aligned}$

CONTINUED →

$$\begin{array}{l} -5x_1 + 2x_2 = 4 \\ 3x_1 + 10x_2 = 2 \end{array} \quad \xrightarrow{\frac{3}{5}E_{x_1} + E_{x_2} \rightarrow E_{x_2}} \quad \begin{array}{l} -5x_1 + 2x_2 = 6 \\ \frac{56}{5}x_2 = \frac{28}{5} \end{array}$$

$$\rightarrow x_2 = \frac{1}{2} \quad \rightarrow -5x_1 + 2\left(\frac{1}{2}\right) = 6 \quad \rightarrow$$

$$x_1 = -1$$

Solution by backsubstitution:

$$x_1 = -1$$

$$x_2 = \frac{1}{2}$$

Jacobi 3rd iteration

$$x_1 = \frac{234}{125}$$

$$x_2 = \frac{79}{500}$$

$$\text{(11)} \quad \begin{array}{l} 20x_1 + 3x_2 + 5x_3 = -26 \\ -2x_1 - 10x_2 + 3x_3 = -23 \\ x_1 - 2x_2 - 5x_3 = -13 \end{array} \quad \begin{array}{l} x_1 = -\frac{3}{20}x_2 - \frac{1}{4}x_3 - \frac{26}{20} \\ x_2 = -\frac{1}{5}x_1 + \frac{3}{10}x_3 + \frac{23}{10} \\ x_3 = \frac{1}{5}x_1 - \frac{2}{5}x_2 + \frac{13}{5} \end{array}$$

$$\rightarrow x_1 = -\frac{26}{20} \quad x_1 = -\frac{3}{20}\left(\frac{23}{10}\right) - \frac{1}{4}\left(\frac{13}{5}\right) - \frac{26}{20}$$

$$x_2 = \frac{23}{10} \quad \rightarrow \quad x_2 = -\frac{1}{5}\left(-\frac{26}{20}\right) + \frac{3}{10}\left(\frac{13}{5}\right) + \frac{23}{10}$$

$$x_3 = \frac{13}{5} \quad x_3 = \frac{1}{5}\left(-\frac{26}{20}\right) - \frac{2}{5}\left(\frac{23}{10}\right) + \frac{13}{5}$$

$$\rightarrow x_1 = -\frac{459}{200} \quad x_1 = -\frac{3}{20}\left(\frac{167}{50}\right) - \frac{1}{4}\left(\frac{71}{50}\right) - \frac{26}{20}$$

$$x_2 = \frac{167}{50} \quad \rightarrow \quad x_2 = -\frac{1}{5}\left(-\frac{459}{200}\right) + \frac{3}{10}\left(\frac{71}{50}\right) + \frac{23}{10}$$

$$x_3 = \frac{71}{50} \quad x_3 = \frac{1}{5}\left(-\frac{459}{200}\right) - \frac{2}{5}\left(\frac{167}{50}\right) + \frac{13}{5}$$

$$\rightarrow x_1 = -\frac{539}{250} \quad x_3 = \frac{161}{200} \quad \text{CONTINUED} \rightarrow$$

$$x_2 = \frac{637}{200}$$

$$20x_1 + 3x_2 + 5x_3 = -26$$

$$-2x_1 - 10x_2 + 3x_3 = -23 \quad \text{Eq}_3 \leftrightarrow \text{Eq}_1$$

$$x_1 - 2x_2 - 5x_3 = -13$$

$$x_1 - 2x_2 - 5x_3 = -13$$

$$2\text{Eq}_1 + \text{Eq}_2 \rightarrow \text{Eq}_2$$

$$-2x_1 - 10x_2 + 3x_3 = -23$$

$$20x_1 + 3x_2 + 5x_3 = -24$$

$$x_1 - 2x_2 - 5x_3 = -13$$

$$-20\text{Eq}_1 + \text{Eq}_3 \rightarrow \text{Eq}_3$$

$$-14x_2 - 7x_3 = -49$$

$$20x_1 + 3x_2 + 5x_3 = -24$$

$$x_1 - 2x_2 - 5x_3 = -13$$

$$-14x_2 - 7x_3 = -49$$

$$\frac{43}{14}R_2 + R_3 \rightarrow R_3$$

~~200x1 + 3x2 + 5x3 = -24~~

$$43x_2 + 105x_3 = 234$$

$$x_1 - 2x_2 - 5x_3 = -13$$

$$\rightarrow x_3 = 1 \rightarrow$$

$$-14x_2 - 7x_3 = -49$$

$$\frac{167}{2}x_3 = \frac{167}{2}$$

$$-14x_2 - 7(1) = -49 \rightarrow x_2 = 3 \rightarrow x_1 - 2(3) - 5(1) = -13$$

$$\rightarrow x_1 = -2$$

General solution:	Jacobi Iteration 3
$x_1 = -2$	$x_1 = -\frac{539}{200}$
$x_2 = 3$	$x_2 = \frac{637}{200}$
$x_3 = 1$	$x_3 = \frac{161}{200}$

$$(13) \begin{aligned} -5x_1 + 2x_2 &= 4 \rightarrow x_1 = \frac{2}{5}x_2 + \frac{4}{5} \\ 3x_1 + 10x_2 &= 2 \quad x_2 = -\frac{3}{10}x_1 + \frac{1}{5} \end{aligned} \rightarrow x_1 = \frac{2}{5}(\textcircled{4}) + \frac{4}{5}$$

$$x_1 = \frac{6}{5} \rightarrow x_2 = -\frac{3}{10}\left(\frac{6}{5}\right) + \frac{1}{5} \rightarrow x_2 = -\frac{4}{25}$$

$$\rightarrow x_1 = \frac{2}{5}\left(-\frac{4}{25}\right) + \frac{6}{5} \rightarrow x_1 = \frac{142}{125} \rightarrow x_2 = -\frac{3}{10}\left(\frac{142}{125}\right) + \frac{1}{5}$$

~~$$\rightarrow x_2 = -\frac{88}{125} \rightarrow x_1 = \frac{2}{5}\left(-\frac{88}{125}\right) + \frac{6}{5} \rightarrow x_1 = \frac{3574}{3125}$$~~

$$\rightarrow x_2 = -\frac{3}{10}\left(\frac{3574}{3125}\right) + \frac{1}{5} \rightarrow x_2 = \frac{-2234}{15625}$$

Solution from 9

$$x_1 = -1 \\ x_2 = \frac{1}{2}$$

~~Gauss-Seidel 3rd Iteration~~

$$x_1 = \frac{3574}{3125}$$

$$x_2 = \frac{-2234}{15625}$$

$$(15) \begin{aligned} x_1 &= -\frac{3}{20}x_2 - \frac{1}{4}x_3 - \frac{24}{20} \rightarrow x_1 = -\frac{24}{20} \quad \text{del} \\ x_2 &= \frac{1}{5}x_1 + \frac{3}{10}x_3 + \frac{23}{10} \quad \text{del} \quad \downarrow \\ x_3 &= \frac{1}{5}x_1 - \frac{2}{5}x_2 + \frac{13}{5} \quad x_2 = -\frac{1}{5}\left(-\frac{24}{20}\right) + \frac{3}{10}(0) + \frac{23}{10} \\ &\quad \downarrow \\ x_2 &= \frac{64}{25} \end{aligned}$$

$$\rightarrow x_3 = \frac{1}{5}\left(-\frac{24}{20}\right) - \frac{2}{5}\left(\frac{64}{25}\right) + \frac{13}{5} \rightarrow x_3 = \frac{329}{250}$$

$$x_1 = -\frac{3}{20} \left( \frac{64}{25} \right) - \frac{1}{4} \left( \frac{329}{250} \right) - \frac{24}{20} \rightarrow x_1 = -\frac{2013}{1000} \rightarrow$$

$$\textcircled{1} \quad x_2 = -\frac{1}{5} \left( -\frac{2013}{1000} \right) + \frac{3}{10} \left( \frac{329}{250} \right) + \frac{23}{10} \rightarrow x_2 = \frac{15487}{5000}$$

$$\rightarrow x_3 = \frac{1}{5} \left( -\frac{2013}{1000} \right) - \frac{2}{5} \left( \frac{15487}{5000} \right) + \frac{13}{5} \rightarrow x_3 = \frac{23961}{25000}$$

$$\rightarrow x_1 = -\frac{3}{20} \left( \frac{15487}{5000} \right) - \frac{1}{4} \left( \frac{23961}{25000} \right) - \frac{24}{20} \rightarrow x_1 = -\frac{100211}{50000}$$

$$\rightarrow x_2 = -\frac{1}{5} \left( -\frac{100211}{50000} \right) + \frac{3}{10} \left( \frac{23961}{25000} \right) + \frac{23}{10} \rightarrow x_2 = \frac{1648993}{250000}$$

$$\rightarrow x_3 = \frac{1}{5} \left( -\frac{100211}{50000} \right) - \frac{2}{5} \left( \frac{1648993}{250000} \right) + \frac{13}{5} \rightarrow x_3 = -\frac{632317}{156250}$$

Solution from (1)

$$x_1 = -2$$

$$x_2 = 3$$

$$x_3 = 1$$

Gauss-Seidel iteration 3

$$x_1 = -\frac{100211}{50000}$$

$$x_2 = \frac{1648993}{250000}$$

$$x_3 = -\frac{632317}{156250}$$

(19)

Not diagonally dominant. Also not possible to make diagonally dominant.

(23)

$$x_1 - 2x_2 + 5x_3 = -1$$

$$x_1 = 2x_2 - 5x_3 - 1$$

$$5x_1 + x_2 - 2x_3 = 8 \rightarrow$$

$$x_2 = -5x_1 + 2x_3 + 8$$

$$2x_1 - 10x_2 + 3x_3 = -1$$

$$x_3 = -\frac{2}{3}x_1 + \frac{10}{3}x_2 - \frac{1}{3}$$

CONTINUED



$$x_1 = 2(0) - 5(0) - 1$$

$$x_2 = -5(0) + 2(0) + 8$$

$$x_3 = -\frac{2}{3}(0) + \frac{10}{3}(0) - \frac{1}{3}$$

$$\rightarrow x_1 = -1 \quad x_1 = 2(8) - 5(-\frac{1}{3}) - 1$$

$$\rightarrow x_2 = 8 \quad \rightarrow x_2 = -5(-1) + 2(\frac{1}{3}) + 8$$

$$\rightarrow x_3 = -\frac{1}{3} \quad x_3 = -\frac{2}{3}(-1) + \frac{10}{3}(8) - \frac{1}{3}$$

$$\rightarrow x_1 = \frac{50}{3} \quad x_1 = 2(\frac{41}{3}) - 5(27) - 1 \quad x_1 = \frac{-324}{3}$$

$$x_2 = \frac{41}{3} \quad \rightarrow x_2 = -5(\frac{50}{3}) + 2(27) + 8 \quad \rightarrow x_2 = -\frac{64}{3}$$

$$x_3 = 27 \quad x_3 = -\frac{2}{3}(\frac{50}{3}) + \frac{10}{3}(\frac{41}{3}) - \frac{1}{3} \quad x_3 = \frac{307}{9}$$

$$\rightarrow x_1 = 2(-\frac{64}{3}) - 5(\frac{307}{9}) - 1$$

$$x_2 = -5(-\frac{324}{3}) + 2(\frac{307}{9}) + 8 \rightarrow$$

$$x_3 = -\frac{2}{3}(-\frac{324}{3}) + \frac{10}{3}(-\frac{64}{3}) - \frac{1}{3}$$

$x_1 = -\frac{1928}{9}$   
 $x_2 = \frac{5576}{9}$   
 $x_3 = 1$

$$5x_1 + x_2 - 2x_3 = 8 \quad x_1 = -\frac{1}{5}x_2 + \frac{2}{5}x_3 + \frac{8}{5}$$

$$2x_1 - 10x_2 + 3x_3 = -1 \rightarrow x_2 = -\frac{1}{5}x_1 + \frac{3}{10}x_3 + \frac{1}{10}$$

$$x_1 - 2x_2 + 5x_3 = -1 \quad x_3 = -\frac{1}{5}x_1 + \frac{2}{5}x_2 - \frac{1}{5}$$

$$\rightarrow x_1 = -\frac{1}{5}(0) + \frac{2}{5}(0) + \frac{8}{5} \rightarrow x_1 = \frac{8}{5}$$

$$x_2 = \frac{1}{5}(0) + \frac{3}{10}(0) + \frac{1}{10} \quad x_2 = \frac{1}{10} \rightarrow$$

$$x_3 = -\frac{1}{5}(0) + \frac{2}{5}(0) - \frac{1}{5} \quad x_3 = -\frac{1}{5}$$

$$x_1 = -\frac{1}{5}(\frac{1}{10}) + \frac{2}{5}(-\frac{1}{5}) + \frac{8}{5}$$

$$x_2 = \frac{1}{5}(\frac{8}{5}) + \frac{3}{10}(-\frac{1}{5}) + \frac{1}{10} \rightarrow x_2 = \frac{9}{25}$$

$$x_3 = -\frac{1}{5}(\frac{8}{5}) + \frac{2}{5}(\frac{1}{10}) - \frac{1}{5} \quad x_3 = -\frac{12}{25}$$

CONTINUED

$$x_1 = -\frac{1}{5} \left( \frac{9}{25} \right) + \frac{2}{5} \left( -\frac{12}{25} \right) + \frac{8}{5} \quad x_1 = \frac{167}{125}$$

$$x_2 = \frac{1}{5} \left( \frac{3}{2} \right) + \frac{3}{10} \left( -\frac{12}{25} \right) + \frac{1}{10} \rightarrow x_2 = \frac{32}{125} \rightarrow$$

$$x_3 = -\frac{1}{5} \left( \frac{3}{2} \right) + \frac{2}{5} \left( \frac{9}{25} \right) - \frac{1}{5} \quad x_3 = -\frac{89}{250}$$

$$x_1 = -\frac{1}{5} \left( \frac{3^2}{125} \right) + \frac{2}{5} \left( -\frac{89}{250} \right) + \frac{8}{5}$$

$$x_2 = \frac{1}{5} \left( \frac{167}{125} \right) + \frac{3}{10} \left( -\frac{89}{250} \right) + \frac{1}{10} \rightarrow$$

$$x_3 = -\frac{1}{5} \left( \frac{167}{125} \right) + \frac{2}{5} \left( \frac{3^2}{125} \right) - \frac{1}{5}$$

$$\begin{aligned} x_1 &= \frac{879}{625} \\ x_2 &= \frac{651}{2500} \\ x_3 &= -\frac{228}{625} \end{aligned}$$

(27)  $x_1 = 2x_2 - 5x_3 - 1$

$$x_2 = -5x_1 + 2x_3 + 8 \rightarrow x_1 = 2(0) - 5(0) - 1 \rightarrow$$

$\bullet$   $x_3 = -\frac{2}{3}x_1 + \frac{10}{3}x_2 - \frac{1}{3}$

$$x_1 = -1 \rightarrow x_2 = -5(-1) + 2(0) + 8 \rightarrow x_2 = 13 \rightarrow$$

$$x_3 = -\frac{2}{3}(-1) + \frac{10}{3}(13) - \frac{1}{3} \rightarrow x_3 = \frac{131}{3} \rightarrow x_1 = 2(13) - 5\left(\frac{131}{3}\right)$$

$$\rightarrow x_1 = -\frac{580}{3} \rightarrow x_2 = -5\left(-\frac{580}{3}\right) + 2\left(\frac{131}{3}\right) + 8 \rightarrow x_2 = 1062$$

$$\rightarrow x_3 = -\frac{2}{3}\left(-\frac{580}{3}\right) + \frac{10}{3}(1062) - \frac{1}{3} \rightarrow x_3 = \frac{33017}{9} \rightarrow$$

$$x_1 = 2(1062) - 5\left(\frac{33017}{9}\right) - 1 \rightarrow x_1 = \frac{145978}{9} \rightarrow x_2 = -5\left(\frac{-145978}{9}\right)$$

$\bullet$   $+ 2\left(\frac{33017}{9}\right) + 8 \rightarrow x_2 = 88444 \rightarrow x_3 = -\frac{2}{3}\left(-\frac{145978}{9}\right) +$

$$\frac{10}{3}(88444) - \frac{1}{3} \rightarrow x_3 = \frac{7667995}{27}$$

★ [Section -2.1 : Graded Homework #2] ★

(5)

$$u = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}, v = \begin{bmatrix} -4 \\ 1 \\ 5 \end{bmatrix}, w = \begin{bmatrix} 2 \\ -7 \\ -1 \end{bmatrix} \rightarrow -v + v + w$$

$$\rightarrow \begin{bmatrix} -5 \\ -4 \\ 4 \end{bmatrix}$$

$$\rightarrow 2u - v + 3w \rightarrow$$

$$\begin{bmatrix} 16 \\ -26 \\ -8 \end{bmatrix}$$

(9)

$$-6x_1 + 5x_2 = 4$$

$$5x_1 - 3x_2 + 2x_3 = 16$$

(13)

$$x_1 - x_2 - 3x_3 - x_4 = -1$$

$$-2x_1 + 2x_2 + 6x_3 + 2x_4 = -1$$

$$-3x_1 - 3x_2 + 10x_3 = 5$$

$$x_1 \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix}$$

$$+ x_3 \begin{bmatrix} -3 \\ 6 \\ 10 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 5 \end{bmatrix}$$

(17)

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ 0 \end{bmatrix} + s_1 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

(25) 
$$\begin{aligned} -3a - 4 &= -10 \\ -9 + 4b &= 19 \end{aligned} \rightarrow \begin{cases} a = 2 \\ b = \cancel{\frac{28}{4}} = 7 \end{cases}$$

(28) 
$$\begin{aligned} -1 + 2b - 2 &= -3 \\ -2 + 2 - c &= -4 \\ -a - 4 - 5 &= 3 \\ -1 + b - 0 &= d \end{aligned} \rightarrow \begin{cases} b = 0 \\ c = 4 \\ a = -12 \\ d = 5 \end{cases}$$

(33)  $a_1 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, a_2 = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix}, b = \begin{bmatrix} 1 \\ -5 \\ -2 \end{bmatrix}$

Only possibility to get  $b_1 = 1$  is to multiply  $\frac{1}{2}$  to  $a_1$ 's 1st component

$$\frac{1}{2}a_1 + a_2 = \begin{bmatrix} (\frac{1}{2})(2) + 0 \\ (\frac{1}{2})(-3) + 3 \\ \frac{1}{2}(1) - 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \\ -\frac{5}{2} \end{bmatrix} \neq \begin{bmatrix} 1 \\ -5 \\ -2 \end{bmatrix}$$

$b$  is not a linear combination of  $a_1$  and  $a_2$

(37)  $2 \begin{bmatrix} 28 \\ 34 \end{bmatrix} + \begin{bmatrix} 18 \\ 25 \end{bmatrix} = \begin{bmatrix} 76 \\ 31 \\ 14 \end{bmatrix}$

(39)  $x_1 \begin{bmatrix} 28 \\ 34 \end{bmatrix} + x_2 \begin{bmatrix} 18 \\ 25 \end{bmatrix} = \begin{bmatrix} 112 \\ 81 \\ 26 \end{bmatrix} \rightarrow$

$28x_1 + 18x_2 = 112$

$3x_1 + 25x_2 = 81$

$4x_1 + 6x_2 = 26$

$$\left[ \begin{array}{cc|c} 28 & 18 & 112 \\ 3 & 25 & 81 \\ 4 & 6 & 26 \end{array} \right]$$

$$-\frac{3}{29} R_1 + R_2 \rightarrow R_2$$

$$\left[ \begin{array}{ccc|c} 29 & 18 & 112 & \\ 0 & 671/29 & 2013/29 & \\ 0 & 6 & 26 & \end{array} \right] \xrightarrow{-\frac{4}{29}R_1 + R_3 \rightarrow R_3}$$

$$\left[ \begin{array}{ccc|c} 29 & 18 & 112 & \\ 0 & 671/29 & 2013/29 & \\ 0 & 6 & 26 & \end{array} \right] \rightarrow \frac{671}{29}x = \frac{-102}{29} \rightarrow x = -\frac{102}{671}$$

$$-\frac{102}{671} R_2 + R_3 \rightarrow R_3$$

$$\left[ \begin{array}{ccc|c} 29 & 18 & 112 & \\ 0 & 671/29 & 2013/29 & \\ 0 & 0 & 0 & \end{array} \right] \rightarrow$$

$$\frac{671}{29}x_2 = \frac{2013}{29} \rightarrow x_2 = \frac{2013}{671} = 3 \rightarrow$$

$$29x_1 + 18(3) = 112 \rightarrow 29x_1 = 112 - 54 \rightarrow x_1 = 2$$

200 pounds of Vigeno and 300 pounds of Parker's

(61)  $U + V = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} \rightarrow U = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}, V = \begin{bmatrix} -3 \\ -2 \\ -3 \end{bmatrix}$

(67)  $\begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$

(71)  $U = \begin{bmatrix} -3 \\ 5 \end{bmatrix} \rightarrow -2U = \begin{bmatrix} -3 \cdot -2 \\ -2 \cdot 5 \end{bmatrix} = \begin{bmatrix} 6 \\ -10 \end{bmatrix}$

True

73 Does  $(\vec{u} + \vec{v}) = \vec{u} + \vec{v}$ ?

a)  $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, c = 2$  ~~scrambled~~  $\rightarrow$

$$2 \left( \begin{bmatrix} 1+2 \\ 2+3 \\ 3+4 \end{bmatrix} \right) = \begin{bmatrix} 6 \\ 10 \\ 14 \end{bmatrix} \rightarrow \begin{bmatrix} 2(1)+2(2) \\ 2(2)+3(2) \\ 2(3)+2(4) \end{bmatrix} \rightarrow \begin{bmatrix} 6 \\ 10 \\ 14 \end{bmatrix}$$

↓

Equivalent and true

b)  $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, c_1 = 2, c_2 = 3 \rightarrow (2)(3) + 3 \left( \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right)$

$$\rightarrow 6 + \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \rightarrow ((2) + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix})^3 \rightarrow 6 + \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$$

↓

Equivalent and true

75

a) True

b) True as you can multiply any vector by the zero vector to get the 0 vector.

Graded Homework 2 - Section 2.2

~~A A A A A~~

③  $u_1 = \begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix} \rightarrow c_1 u_1 + c_2 v_2 \rightarrow c_1 = 0 \text{ and } c_2 = 0$

$$\rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow C_1 = 1 \quad C_2 = 1 \quad \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} \rightarrow C_1 = 2 \quad C_2 = 2 \quad \begin{bmatrix} 6 \\ 10 \\ 2 \end{bmatrix}$$

(11)  ~~$\begin{bmatrix} -1 & 2 & -10 \\ 4 & 8 & -8 \\ -3 & -7 & 7 \end{bmatrix}$~~   $4R_1 + R_2 \rightarrow R_2$   $\sim \begin{bmatrix} -1 & 2 & -10 \\ 0 & 16 & -48 \\ -3 & -7 & 7 \end{bmatrix}$

$$-3R_1 + R_3 \rightarrow R_3 \quad \sim \quad \begin{bmatrix} -1 & 2 & -10 \\ 0 & 16 & -48 \\ 0 & -13 & 37 \end{bmatrix} \quad \frac{13}{16}R_2 + R_3 \rightarrow R_3 \quad \sim$$

~~$\begin{bmatrix} -1 & 2 & -10 \\ 0 & 16 & -48 \\ 0 & 0 & -2 \end{bmatrix}$~~

No solutions and hence,  $\vec{b}$  is not in  $\text{span}\{\vec{a}_1, \vec{a}_2\}$

(15)  $A = \begin{bmatrix} 1 & -1 & -3 & -1 \\ -2 & 2 & 6 & 2 \\ -3 & -3 & 10 & 0 \end{bmatrix}$   ~~$\vec{a}$~~   $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 5 \end{bmatrix}$

(17)  $x_1 \begin{bmatrix} 5 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ -5 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -4 \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \end{bmatrix}$

- (23) Spans  $\mathbb{R}^2$
- (25) Spans  $\mathbb{R}^3$

$$(33) \quad \begin{bmatrix} -3 & 2 & 1 \\ 1 & -1 & -1 \\ 5 & -4 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \rightarrow$$

A                            X

$$\left[ \begin{array}{ccc|c} -3 & 2 & 1 & b_1 \\ 1 & -1 & -1 & b_2 \\ 5 & -4 & -3 & b_3 \end{array} \right] \xrightarrow{\text{Row } R_2 \leftrightarrow R_1} \sim \left[ \begin{array}{ccc|c} 1 & -1 & -1 & b_2 \\ -3 & 2 & 1 & b_1 \\ 5 & -4 & -3 & b_3 \end{array} \right]$$

$$3R_1 + R_2 \rightarrow R_2 \quad \sim \quad \left[ \begin{array}{ccc|c} 1 & -1 & -1 & b_2 \\ 0 & -1 & -2 & 3b_2 + b_1 \\ 5 & -4 & -3 & b_3 \end{array} \right] \quad -5R_1 + R_3 \rightarrow R_3 \quad \sim$$

$$\left[ \begin{array}{ccc|c} 1 & -1 & -1 & b_2 \\ 0 & -1 & -2 & 3b_2 + b_1 \\ 0 & 1 & 2 & -5b_2 + b_3 \end{array} \right] \quad R_2 + R_3 \rightarrow R_3 \quad \sim \quad \left[ \begin{array}{ccc|c} 1 & -1 & -1 & b_2 \\ 0 & -1 & -2 & 3b_2 + b_1 \\ 0 & 0 & 0 & b_1 - 2b_2 + b_3 \end{array} \right]$$

No solution for all choices of  $b$

(35)  $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$(43) \quad \left[ \begin{array}{cc} 1 & 3 \\ 2 & 5 \\ -1 & 4 \end{array} \right] \quad R_2 \rightarrow R_2 \quad \left[ \begin{array}{cc} 1 & 3 \\ 0 & -1 \\ -1 & 4 \end{array} \right] \quad R_1 + R_3 \rightarrow R_3 \quad \sim$$

$$\text{Q1} \quad \left[ \begin{array}{cc|c} 1 & 3 & \\ 0 & -1 & \\ 0 & 7 & \end{array} \right] \xrightarrow{-R_2+R_3 \rightarrow R_3} \sim \left[ \begin{array}{cc|c} 1 & 3 & \\ 0 & -1 & \\ 0 & 0 & \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & 3 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{-7R_2+R_3 \rightarrow R_3} \sim \left[ \begin{array}{cc|c} 1 & 3 & 0 \\ 0 & -1 & 6 \\ 0 & 7 & 1 \end{array} \right] \xrightarrow{-R_1+R_3 \rightarrow R_3} \sim \left[ \begin{array}{cc|c} 1 & 3 & 0 \\ 0 & -1 & 6 \\ -1 & 4 & 1 \end{array} \right]$$

$$\xrightarrow{2R_1+R_2 \rightarrow R_2} \sim \left[ \begin{array}{cc|c} 1 & 3 & 0 \\ 2 & 5 & 6 \\ -1 & 4 & 1 \end{array} \right]$$

$\left[ \begin{array}{c} 0 \\ 6 \\ 1 \end{array} \right] \notin \text{span} \left\{ \left[ \begin{array}{c} 1 \\ 2 \\ -4 \end{array} \right], \left[ \begin{array}{c} 3 \\ 5 \\ 4 \end{array} \right] \right\}$

53

$$\left[ \begin{array}{c|c|c|c} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{array} \right]$$

57

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

59

(a) True

(b) False as the zero vector does not preclude other possibilities.

61

(a) False as you could have  $m$  equations  $> n$  components

(b) True as ~~the~~ the entire space can be spanned by all possible linear combinations

(63) (a) False as ~~any~~ any of the vectors could be multiples of each other.

(b) True

(65)

(a) False

(b) True

(67) c and d as ~~combinations of~~  
~~there must be at least n vectors to~~  
Span  $\mathbb{R}^n$