Graded Homework 1

Section 1.1

(5)

$(1,2,3)$: $\begin{cases} -2(1) + 9(2) - (3) = -10 \\ (1) - 5(2) + 2(3) = 4 \end{cases}$

X

$\begin{cases} 13 \neq -10 \\ -3 \neq 4 \end{cases}$

$(1,-1,1)$: $\begin{cases} -2(1) + 9(-1) - (1) = -10 \\ (1) - 5(-1) + 2(1) = 4 \end{cases}$

X

$\begin{cases} -12 \neq -10 \\ 8 \neq 4 \end{cases}$

$(-1,-2,-6)$: $\begin{cases} -2(-1) + 9(-2) - (-6) = -10 \\ -1 - 5(-2) + 2(-6) = -4 \end{cases}$

X

$\begin{cases} -10 = -10 \\ -3 \neq 4 \end{cases}$

None of these satisfy the system of linear equations

(11)

$-10x_1 + 4x_2 = 2$

$\frac{2}{3}(15x_1 - 6x_2 = -3) \Rightarrow$

$-10x_1 + 4x_2 = 2$

$10x_1 - 4x_2 = -2$

$0 = 0$

$-10x_1 + 4x_2 = 2$

$-10x_1 = -4x_2 + 2 \Rightarrow$

$x_1 = \frac{2}{5}s_1 - \frac{1}{5}, \quad s_1 = x_2$

(17) Is in echelon form; x_1 and x_3 are leading variables whereas x_2 is a free variable.

(21) Is in echelon form; x_1 and x_3 are leading variables whereas x_2 and x_4 are free variables

(23) $-5x_1 - 3x_2 = 4 \Rightarrow x_2 = 5$

$2x_2 = 10$

$-5x_1 - 3(5) = 4 \Rightarrow x_1 = -\frac{19}{5}$

$x_1 = -\frac{19}{5}, x_2 = 5$

(25) $-3x_1 + 4x_2 = 2 \rightarrow x_2 = s_1 \Rightarrow -3x_1 + 4s_1 = 2$

$x_1 = \frac{4}{3}s_1 - \frac{2}{3}$

(27) $x_1 + 5x_2 - 2x_3 = 0$

$-2x_2 + x_3 - x_4 = -1$

$x_4 = 5$

$\Rightarrow -2x_2 + x_3 - (5) = -1$

$\Downarrow$

let $x_3 = s_1$

$\Downarrow$

$-2x_2 + s_1 - 5 = -1$

$\Downarrow$

$-2x_2 = -s_1 + 4$

$x_2 = \frac{1}{2}s_1 - 2$

$x_1 + 5(\frac{1}{2}s_1 - 2) - 2s_1 = 0$

$x_1 + \frac{5}{2}s_1 - 10 - 2s_1 = 0$

$x_1 + \frac{1}{2}s_1 - 10 = 0$

$x_1 = -\frac{1}{2}s_1 + 10$

$x_1 = -\frac{1}{2}s_1 + 10$

$x_2 = \frac{1}{2}s_1 - 2$

$x_3 = s_1$

$x_4 = 5$

(29) $-2x_1 + x_2 + 2x_3 = 1$
$-3x_3 + x_4 = -4$ $\Rightarrow$ let $x_4 = s_1$

$-3x_3 + s_1 = -4 \Rightarrow x_3 = \frac{4}{3} - \frac{1}{3}s_1 \Rightarrow$ let $x_2 = s_2$

$\Rightarrow -2x_1 + s_2 + 2\left(\frac{4}{3} - \frac{1}{3}s_1\right) = 1$

$\Downarrow$

$-2x_1 + s_2 + \frac{8}{3} - \frac{2}{3}s_1 = 1$

$\Downarrow$

$-2x_1 = -\frac{5}{3} - s_2 + \frac{2}{3}s_1$

$\Downarrow$

$x_1 = \frac{5}{6} + \frac{1}{2}s_2 - \frac{1}{3}s_1$

$x_1 = \frac{5}{6} + \frac{1}{2}s_2 - \frac{1}{3}s_1$
$x_2 = s_2$
$x_3 = \frac{4}{3} - \frac{1}{3}s_1$
$x_4 = s_1$

(33) $x_1 + 2x_2 - x_3 + x_4 = 1$
$x_2 + 2x_3 - 2x_4 = 2$
$x_4 = 0$ $\Rightarrow$ let $x_3 = s_1$

$\Downarrow$

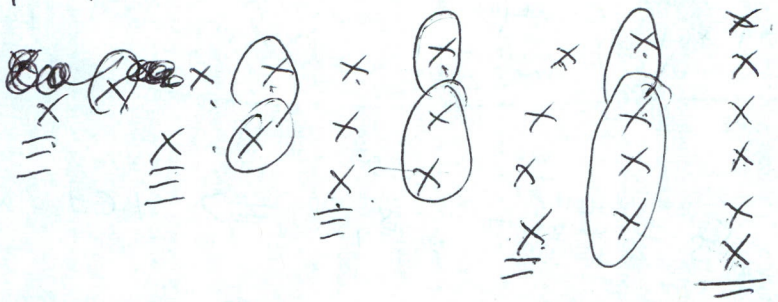
$x_2 + 2s_1 - 2(0) = 2$

$x_2 = 2 - 2s_1 \Rightarrow x_1 + 2(2 - 2s_1) - s_1 + 0 = 1 \Rightarrow$

$x_1 + 4 - 4s_1 - s_1 = 1 \Rightarrow x_1 = -3 + 5s_1$

$x_1 = -3 + 5s_1$
$x_2 = 2 - 2s_1$
$x_3 = s_1$
$x_4 = 0$

(39) 4 free variables and 5 leading variables



9 variables

(51) 3 equations and 2 variables

(a)

$$x_1 + x_2 = c_1$$
$$2x_1 + 2x_2 = c_2$$
$$4x_1 + 4x_2 = c_3$$

False; can have the case where some of the [illegible] equations are actualy equivalent in space.

(b) False as they could be parallel

(55)

(a) True as we don't have squared terms.

(b) True by the fact that you can have free variables.

(57) let x_1 = adults and x_2 = children

$$x_1 + x_2 = 385$$
$$11x_1 + 8x_2 = 3974$$

[illegible] $\Rightarrow x_1 = 385 - x_2$

$\hookrightarrow 11(385 - x_2) + 8x_2 = 3974 \Rightarrow 4235 - 11x_2 + 8x_2 = 3974$

$-3x_2 = -261 \Rightarrow x_2 = 87 \Rightarrow x_1 + 87 = 385$

$\Rightarrow x_1 = 298$

298 adults and 87 children