Graded Homework 2 - Section 1.3

(3)

A: $x_4 + 30 + 40 = x_1 + 50$

B: $x_1 + x_3 + 25 = 55 + 40 + x_2$

C: $x_2 + 50 = x_4 + 25$

$$\Downarrow$$

A: $x_1 - x_4 = 20$

B: $x_1 - x_2 + x_3 = 70$

C: $x_2 - x_4 = -25$

$$\Downarrow$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 20 \\ 1 & -1 & 1 & 0 & 70 \\ 0 & 1 & 0 & -1 & -25 \end{array}\right] \xrightarrow{-1R_1 + R_2 \to R_2}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 20 \\ 0 & -1 & 1 & 1 & 50 \\ 0 & 1 & 0 & -1 & -25 \end{array}\right] \xrightarrow{R_2 + R_3 \to R_3}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 20 \\ 0 & -1 & 1 & 1 & 50 \\ 0 & 0 & 1 & 0 & +25 \end{array}\right] \Longrightarrow$$

let $x_4 = s_1$

$$\begin{aligned} x_1 + \qquad\qquad - x_4 &= 20 \\ -x_2 + x_3 + x_4 &= 50 \\ x_3 \qquad &= 25 \end{aligned} \quad \Longrightarrow$$

$$-x_2 + (25) + s_1 = 50 \Rightarrow -x_2 = 25 - s_1$$

$$\Rightarrow x_1 - s_1 = 20 \Rightarrow x_1 = 20 + s_1$$

$$\Rightarrow x_2 = s_1 - 25$$

$$\begin{aligned} x_1 &= s_1 + 20 \\ x_2 &= s_1 - 25 \\ x_3 &= 25 \\ x_4 &= s_1 \end{aligned}$$

25 cars minimum

(7)

$$x_1 = \frac{50 + x_2 + x_3}{3}$$

$$x_2 = \frac{90 + x_3 + x_1}{3} \qquad \xrightarrow{\text{multiply all by}} 3$$

$$x_3 = \frac{30 + x_2 + x_1}{3}$$

$$\begin{aligned} 3x_1 &= 50 + x_2 + x_3 \\ 3x_2 &= 90 + x_3 + x_1 \\ 3x_3 &= 30 + x_2 + x_1 \end{aligned} \quad \Rightarrow \quad \begin{aligned} 3x_1 - x_2 - x_3 &= 50 \\ -x_1 + 3x_2 - x_3 &= 90 \\ -x_1 - x_2 + 3x_3 &= 30 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 3 & -1 & -1 & 50 \\ -1 & 3 & -1 & 90 \\ -1 & -1 & 3 & 30 \end{array}\right] \quad R_2 \longleftrightarrow R_1 \quad \left[\begin{array}{ccc|c} -1 & 3 & -1 & 90 \\ 3 & -1 & -1 & 50 \\ -1 & -1 & 3 & 30 \end{array}\right]$$

$$3R_1 + R_2 \to R_2 \quad \left[\begin{array}{ccc|c} -1 & 3 & -1 & 90 \\ 0 & 8 & -4 & 330 \\ -1 & -1 & 3 & 30 \end{array}\right] \quad -R_1 + R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} -1 & 3 & -1 & 90 \\ 0 & 8 & -4 & 330 \\ 0 & -4 & 4 & -60 \end{array}\right] \quad R_2 \longleftrightarrow R_3 \quad \left[\begin{array}{ccc|c} -1 & 3 & -1 & 90 \\ 0 & -4 & 4 & -60 \\ 0 & 8 & -4 & 330 \end{array}\right]$$

$$2R_2 + R_3 \to R_3 \quad \left[\begin{array}{ccc|c} -1 & 3 & -1 & 90 \\ 0 & -4 & 4 & -60 \\ 0 & 0 & 4 & 210 \end{array}\right] \Rightarrow$$

$$\begin{aligned} -x_1 + 3x_2 - x_3 &= 90 \\ -4x_2 + 4x_3 &= -60 \\ 4x_3 &= 200 \end{aligned} \quad \Rightarrow x_3 = 50 \Rightarrow$$

$$-4x_2 + 4(50) = -60 \Rightarrow -4x_2 = -260 \Rightarrow x_2 = 65 \Rightarrow$$

$$-x_1 + 3(65) - (50) = 90 \Rightarrow -x_1 + 195 - 50 = 90$$

$$\Rightarrow -x_1 = -55 \Rightarrow x_1 = 55$$

$$x_1 = 55$$
$$x_2 = 65$$
$$x_3 = 50$$

MISTAKENLY

FORGOT TO

USE

(9) $\begin{cases} a = 60 + .2b \\ b = 40 + .3a \end{cases}$

$a - .2b = 60$

$-.3a + b = 40$

$\Rightarrow$ $\begin{array}{cc|c} a & b & \\ \end{array}$ $\left[\begin{array}{cc|c} 1 & -.2 & 60 \\ -.3 & 1 & 40 \end{array}\right]$

$5R_1 + R_2 \to R_2$

$\sim \left[\begin{array}{cc|c} 1 & -.2 & 60 \\ 4.7 & 0 & 340 \end{array}\right] \Rightarrow 4.7a = 340$

$\Rightarrow a \cong 72.34 \Rightarrow (72.34) - .2b = 60 \Rightarrow b \cong 61.7$

$a = 72.34, \; b = 61.7$

(13) $x_1 H_2 + x_2 O_2 \to x_3 H_2O$

$\begin{cases} 2x_1 = 2x_3 \\ 2x_2 = x_3 \end{cases} \Rightarrow \begin{array}{l} 2x_1 - 2x_3 = 0 \\ 2x_2 - x_3 = 0 \end{array} \Rightarrow$

$\left[\begin{array}{ccc|c} 2 & 0 & -2 & 0 \\ 0 & 2 & -1 & 0 \end{array}\right] \Rightarrow$ let $x_3 = s_1 \Rightarrow$

$2x_2 - s_1 = 0$

$x_2 = \frac{1}{2} s_1 \Rightarrow$

$2x_1 - 2s_1 = 0 \Rightarrow x_1 = s_1 \Rightarrow$ $x_1 = s_1$

$x_2 = \frac{1}{2} s_1$

$x_3 = s_1$

$2H_2 + O_2 \rightarrow 2H_2O$

(19) $x_1 KO_2 + x_2 CO_2 \rightarrow x_3 K_2CO_3 + x_4 O_2$

K: $x_1 = 2x_3$

O: $2x_1 + 2x_2 = 3x_3 + 2x_4$ $\Rightarrow$

C: $x_2 = x_3$

$x_1 - 2x_3 = 0$

$x_2 - x_3 = 0$

$2x_1 + 2x_2 - 3x_3 - 2x_4 = 0$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 2 & 2 & -3 & -2 & 0 \end{array}\right]$

$-2R_1 + R_3 \rightarrow R_3$ $\sim$ $\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & -1 & -2 & 0 \end{array}\right]$ $-2R_2 + R_3 \rightarrow R_3$ $\sim$

$\left[\begin{array}{ccccc} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 \end{array}\right] \Rightarrow$ let $x_4 = s_1 \Rightarrow x_3 - 2s_1 = 0 \Rightarrow$

$x_3 = 2s_1 \Rightarrow x_2 - (2s_1) = 0 \Rightarrow x_2 = 2s_1 \Rightarrow$

$x_1 - (2s_1) = 0 \Rightarrow x_1 = 2s_1$

$x_1 = 2s_1$

$x_2 = 2s_1$

$x_3 = 2s_1$

$x_4 = s_1$

$2KO_2 + 2CO_2 \rightarrow 2K_2CO_3 + 2O_2$

(21) $\ln(p) = \ln(a) + b\ln(d) \Rightarrow$ let $\ln(a) = a_1$

Earth: $a_1 + b\ln(149.6) = \ln(365.2) \Rightarrow$

Mars: $a_1 + b\ln(227.9) = \ln(687)$

$$\begin{array}{cc} a_1 & b \end{array}$$
$$\left[\begin{array}{cc|c} 1 & \ln(149.6) & \ln(365.2) \\ 1 & \ln(227.9) & \ln(687) \end{array}\right] \quad -1R_1 + R_2 \to R_2$$

$$\left[\begin{array}{cc|c} 1 & \ln(149.6) & \ln(365.2) \\ 0 & \ln(227.9) - \ln(149.6) & \ln(687) - \ln(365.2) \end{array}\right]$$

$\{\ln(227.9) - \ln(149.6)\}(b) = \{\ln(687) - \ln(365.2)\}$

$$b = \frac{0.631889}{.420941} \approx 1.5011 \Rightarrow$$

$a_1 + \ln(149.6)(1.5011) = \ln(365.2)$

$a_1 + 7.517456 = 5.9004 \Rightarrow a_1 = -1.61705$

$\ln(a) = -1.61705 \Rightarrow a = .98147 \Rightarrow$ $\boxed{p = (.98147)d^{1.5011}}$

(27) $\left[\frac{1}{(x)(x+1)} = \frac{A}{x} + \frac{B}{x+1}\right](x)(x+1)$

$1 = A(x+1) + B(x)$

$\Rightarrow$ set $x = 0 \Rightarrow 1 = A(0+1) + B(0) \Rightarrow A = 1 \Rightarrow$

set $x = -1 \Rightarrow 1 = 1(-1+1) + B(-1) \Rightarrow B = -1$

$$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1} \; ; \; A = 1, \; B = -1$$

(29) $\left[\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}\right] x^2(x-1) \Rightarrow$

$1 = A(x)(x-1) + B(x-1) + C(x^2) \Rightarrow$ let

$x = 0 \Rightarrow 1 = \cancel{A(0)(0-1)} + B(0-1) \cancel{+ C(0)} \Rightarrow$

$1 = -B \Rightarrow B = -1 \Rightarrow$ let $x = 1 \Rightarrow$

$1 = \cancel{A(1)(1-1)} + \cancel{-1(1-1)} + C(1^2) \Rightarrow C = 1$

$\Rightarrow$ let $x = 2 \Rightarrow 1 = A(2)(2-1) + (-1)(2-1) +$

$1(2^2) \Rightarrow 1 = 2A - 1 + 4 \Rightarrow A = -1$

$$A = -1, \; B = -1, \; C = 1 \; ; \; \frac{-1}{x} - \frac{1}{x^2} + \frac{1}{x-1} = \frac{1}{x^2(x-1)}$$

(37) $f(x) = ax^3 + bx^2 + cx + d, \; f(0) = -3, \; f(1) = 2$

$f(3) = 5, \; f(4) = 0$

$-3 = a(0)^3 + b(0)^2 + c(0) + d \Rightarrow d = -3 \Rightarrow$

$2 = a(1)^3 + b(1^2) + c(1) - 3 \Rightarrow 2 = a + b + c - 3 \Rightarrow$

$5 = a(3)^3 + b(3)^2 + (3)c - 3 \Rightarrow 0 = a(4)^3 + b(4)^2$

$+ (4)c - 3$

$$\begin{cases} a + b + c = 5 \\ 27a + 9b + 3c = 8 \\ 64a + 16b + 4c = 3 \end{cases} \Rightarrow \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 27 & 9 & 3 & 8 \\ 64 & 16 & 4 & 3 \end{array}\right] \end{array}$$

$-27R_1 + R_2 \to R_2$ $\sim$ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & -18 & -24 & -127 \\ 64 & 16 & 4 & 3 \end{array}\right]$ $-64R_1 + R_3 \to R_3$ $\sim$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & -18 & -24 & -127 \\ 0 & -48 & -60 & -317 \end{array}\right]$ $-\frac{48}{18}R_2 + R_3 \to R_3$ $\sim$ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & -18 & -24 & -127 \\ 0 & 0 & 4 & \frac{65}{3} \end{array}\right]$

$4c = \frac{65}{3} \Rightarrow c = \frac{65}{12} \Rightarrow -18b - 24\left(\frac{65}{12}\right) = -127 \Rightarrow$

$-18b - 130 = -127 \Rightarrow -18b = 3 \Rightarrow b = \frac{-3}{18} \Rightarrow a + \left(\frac{-3}{18}\right) + \left(\frac{65}{12}\right) = 5$

$\Rightarrow a = -\frac{1}{4} \Rightarrow f(x) = -\frac{1}{4}x^3 - \frac{3}{18}x^2 + \frac{65}{12}x - 3$

Graded Homework 2 - Section 1.4

(9) $-5x_1 + 2x_2 = 6$
$3x_1 + 10x_2 = 2$
$\to$ $x_1 = \frac{2}{5}x_2 + \frac{6}{5}$
$x_2 = -\frac{3}{10}x_1 + \frac{1}{5}$ $\longrightarrow$

$x_1 = \frac{2}{5}(0) + \frac{6}{5}$
$x_2 = -\frac{3}{10}(0) + \frac{1}{5}$
$\to$ $x_1 = \frac{6}{5}$
$x_2 = \frac{1}{5}$
$\longrightarrow$ $x_1 = \frac{2}{5}\left(\frac{6}{5}\right) + \frac{6}{5}$
$x_2 = -\frac{3}{10}\left(\frac{1}{5}\right) + \frac{1}{5}$

$\to$ $x_1 = \frac{12}{25} + \frac{6}{5}$
$x_2 = -\frac{3}{50} + \frac{1}{5}$
$\to$ $x_1 = \frac{42}{25}$
$x_2 = \frac{7}{50}$
$\to$ $x_1 = \frac{2}{5}\left(\frac{42}{25}\right) + \frac{6}{5}$
$x_2 = -\frac{3}{10}\left(\frac{7}{50}\right) + \frac{1}{5}$

$\to$ $x_1 = \frac{84}{125} + \frac{6}{5}$
$x_2 = \frac{-21}{500} + \frac{1}{5}$
$\to$ $x_1 = \frac{234}{125}$
$x_2 = \frac{79}{500}$

CONTINUED $\longrightarrow$

$-5x_1 + 2x_2 = 6$ $\xrightarrow[\frac{3}{5}Eq_1 + Eq_2 \to Eq_2]{}$ $-5x_1 + 2x_2 = 6$

$3x_1 + 10x_2 = 2$ $\qquad \frac{56}{5}x_2 = \frac{28}{5}$

$\to x_2 = \frac{1}{2} \to -5x_1 + 2(\frac{1}{2}) = 6 \to$

$x_1 = -1$

Solution by backsubstitution:	Jacobi 3rd iteration
$x_1 = -1$	$x_1 = \frac{234}{125}$
$x_2 = \frac{1}{2}$	$x_2 = \frac{79}{500}$

(11) $20x_1 + 3x_2 + 5x_3 = -26$ $\qquad x_1 = -\frac{3}{20}x_2 - \frac{1}{4}x_3 - \frac{26}{20}$

$-2x_1 - 10x_2 + 3x_3 = -23 \to$ $\qquad x_2 = -\frac{1}{5}x_1 + \frac{3}{10}x_3 + \frac{23}{10}$

$x_1 - 2x_2 - 5x_3 = -13$ $\qquad x_3 = \frac{1}{5}x_1 - \frac{2}{5}x_2 + \frac{13}{5}$

$\to x_1 = -\frac{26}{20}$ $\qquad x_1 = -\frac{3}{20}\left(\frac{23}{10}\right) - \frac{1}{4}\left(\frac{13}{5}\right) - \frac{26}{20}$

$x_2 = \frac{23}{10} \to$ $\qquad x_2 = -\frac{1}{5}\left(-\frac{26}{20}\right) + \frac{3}{10}\left(\frac{13}{5}\right) + \frac{23}{10}$

$x_3 = \frac{13}{5}$ $\qquad x_3 = \frac{1}{5}\left(-\frac{26}{20}\right) - \frac{2}{5}\left(\frac{23}{10}\right) + \frac{13}{5}$

$\to x_1 = \frac{-459}{200}$ $\qquad x_1 = -\frac{3}{20}\left(\frac{167}{50}\right) - \frac{1}{4}\left(\frac{71}{50}\right) - \frac{26}{20}$

$x_2 = \frac{167}{50} \to$ $\qquad x_2 = -\frac{1}{5}\left(\frac{-459}{200}\right) + \frac{3}{10}\left(\frac{71}{50}\right) + \frac{23}{10}$

$x_3 = \frac{71}{50}$ $\qquad x_3 = \frac{1}{5}\left(\frac{-459}{200}\right) - \frac{2}{5}\left(\frac{167}{50}\right) + \frac{13}{5}$

$\to x_1 = \frac{-539}{250}$ $\quad x_3 = \frac{161}{200}$

$x_2 = \frac{637}{200}$

CONTINUED $\to$

$$20x_1 + 3x_2 + 5x_3 = -26$$
$$-2x_1 - 10x_2 + 3x_3 = -23$$
$$x_1 - 2x_2 - 5x_3 = -13$$

$$\xrightarrow{Eq_3 \longleftrightarrow Eq_1}$$

$$x_1 - 2x_2 - 5x_3 = -13$$
$$-2x_1 - 10x_2 + 3x_3 = -23$$
$$20x_1 + 3x_2 + 5x_3 = -24$$

$$\xrightarrow{2Eq_1 + Eq_2 \to Eq_2}$$

$$x_1 - 2x_2 - 5x_3 = -13$$
$$-14x_2 - 7x_3 = -49$$
$$20x_1 + 3x_2 + 5x_3 = -24$$

$$\xrightarrow{-20Eq_1 + Eq_3 \to Eq_3}$$

$$x_1 - 2x_2 - 5x_3 = -13$$
$$-14x_2 - 7x_3 = -49$$
$$43x_2 + 105x_3 = 234$$

$$\xrightarrow{\frac{43}{14}R_2 + R_3 \to R_3}$$

$$x_1 - 2x_2 - 5x_3 = -13$$
$$-14x_2 - 7x_3 = -49$$
$$\frac{167}{2}x_3 = \frac{167}{2}$$

$$\longrightarrow x_3 = 1 \longrightarrow$$

$$-14x_2 - 7(1) = -49 \longrightarrow x_2 = 3 \longrightarrow x_1 - 2(3) - 5(1) = -13$$

$$\longrightarrow x_1 = -2$$

General solution:	Jacobi iteration 3
$x_1 = -2$	$x_1 = -\frac{539}{250}$
$x_2 = 3$	$x_2 = \frac{637}{200}$
$x_3 = 1$	$x_3 = \frac{161}{200}$

(13) $-5x_1 + 2x_2 = 6$ $\rightarrow$ $x_1 = \frac{2}{5}x_2 + \frac{6}{5}$ $\rightarrow$ $x_1 = \frac{2}{5}(0) + \frac{6}{5}$

$3x_1 + 10x_2 = 2$ $\quad$ $x_2 = -\frac{3}{10}x_1 + \frac{1}{5}$

$x_1 = \frac{6}{5} \rightarrow x_2 = -\frac{3}{10}\left(\frac{6}{5}\right) + \frac{1}{5} \rightarrow x_2 = -\frac{4}{25}$

$\rightarrow x_1 = \frac{2}{5}\left(-\frac{4}{25}\right) + \frac{6}{5} \rightarrow x_1 = \frac{142}{125} \rightarrow x_2 = -\frac{3}{10}\left(\frac{142}{125}\right) + \frac{1}{5}$

$\rightarrow x_2 = -\frac{88}{625} \rightarrow x_1 = \frac{2}{5}\left(\frac{-88}{625}\right) + \frac{6}{5} \rightarrow x_1 = \frac{3574}{3125}$

$\rightarrow x_2 = -\frac{3}{10}\left(\frac{3574}{3125}\right) + \frac{1}{5} \rightarrow x_2 = \frac{-2236}{15625}$

Solution from 9	Gauss-Seidel 3rd Iteration
$x_1 = -1$	$x_1 = \frac{3574}{3125}$
$x_2 = \frac{1}{2}$	$x_2 = -\frac{2236}{15625}$

(15) $x_1 = -\frac{3}{20}x_2 - \frac{1}{4}x_3 - \frac{24}{20}$ $\rightarrow$ $x_1 = -\frac{24}{20}$

$x_2 = -\frac{1}{5}x_1 + \frac{3}{10}x_3 + \frac{23}{10}$ $\quad$ $x_2 = -\frac{1}{5}\left(-\frac{24}{20}\right) + \frac{3}{10}(0) + \frac{23}{10}$

$x_3 = \frac{1}{5}x_1 - \frac{2}{5}x_2 + \frac{13}{5}$ $\quad$ $x_2 = \frac{64}{25}$

$x_3 = \frac{1}{5}\left(-\frac{24}{20}\right) - \frac{2}{5}\left(\frac{64}{25}\right) + \frac{13}{5} \rightarrow x_3 = \frac{329}{250}$

$$x_1 = -\frac{3}{20}\left(\frac{64}{25}\right) - \frac{1}{4}\left(\frac{329}{250}\right) - \frac{24}{20} \rightarrow x_1 = \frac{-2013}{1000} \longrightarrow$$

$$x_2 = -\frac{1}{5}\left(\frac{-2013}{1000}\right) + \frac{3}{10}\left(\frac{329}{250}\right) + \frac{23}{10} \rightarrow x_2 = \frac{15487}{5000}$$

$$\rightarrow x_3 = \frac{1}{5}\left(\frac{-2013}{1000}\right) - \frac{2}{5}\left(\frac{15487}{5000}\right) + \frac{13}{5} \rightarrow x_3 = \frac{23961}{25000}$$

$$\rightarrow x_1 = -\frac{3}{20}\left(\frac{15487}{5000}\right) - \frac{1}{4}\left(\frac{23961}{25000}\right) - \frac{24}{20} \rightarrow x_1 = \frac{-100211}{50000}$$

$$\rightarrow x_2 = -\frac{1}{5}\left(\frac{-100211}{5000}\right) + \frac{3}{10}\left(\frac{23961}{25000}\right) + \frac{23}{10} \rightarrow x_2 = \frac{1648993}{250000}$$

$$\rightarrow x_3 = \frac{1}{5}\left(\frac{-100211}{5000}\right) - \frac{2}{5}\left(\frac{1648993}{250000}\right) + \frac{13}{5} \rightarrow x_3 = -\frac{632317}{156250}$$

Solution from 11	Gauss-Seidel iteration 3
$x_1 = -2$	$x_1 = \frac{-100211}{5000}$
$x_2 = 3$	$x_2 = \frac{1648993}{250000}$
$x_3 = 1$	$x_3 = \frac{-632317}{156250}$

(19) Not diagonally dominant. Also not possible to make diagonally dominant.

(23)

$$x_1 - 2x_2 + 5x_3 = -1$$
$$5x_1 + x_2 - 2x_3 = 8 \longrightarrow$$
$$2x_1 - 10x_2 + 3x_3 = -1$$

$$x_1 = 2x_2 - 5x_3 - 1$$
$$x_2 = -5x_1 + 2x_3 + 8$$
$$x_3 = -\frac{2}{3}x_1 + \frac{10}{3}x_2 - \frac{1}{3}$$

CONTINUED $\longrightarrow$

$$
\begin{aligned}
x_1 &= 2(0) - 5(0) - 1 \\
x_2 &= -5(0) + 2(0) + 8 \\
x_3 &= -\tfrac{2}{3}(0) + \tfrac{10}{3}(0) - \tfrac{1}{3}
\end{aligned}
\rightarrow
\begin{aligned}
x_1 &= -1 \\
x_2 &= 8 \\
x_3 &= -\tfrac{1}{3}
\end{aligned}
\rightarrow
\begin{aligned}
x_1 &= 2(8) - 5(-\tfrac{1}{3}) - 1 \\
x_2 &= -5(-1) + 2(-\tfrac{1}{3}) + 8 \\
x_3 &= -\tfrac{2}{3}(-1) + \tfrac{10}{3}(8) - \tfrac{1}{3}
\end{aligned}
$$

$$
\rightarrow
\begin{aligned}
x_1 &= \tfrac{50}{3} \\
x_2 &= \tfrac{41}{3} \\
x_3 &= 27
\end{aligned}
\rightarrow
\begin{aligned}
x_1 &= 2(\tfrac{41}{3}) - 5(27) - 1 \\
x_2 &= -5(\tfrac{50}{3}) + 2(27) + 8 \\
x_3 &= -\tfrac{2}{3}(\tfrac{50}{3}) + \tfrac{10}{3}(\tfrac{41}{3}) - \tfrac{1}{3}
\end{aligned}
\rightarrow
\begin{aligned}
x_1 &= \tfrac{-326}{3} \\
x_2 &= \tfrac{-64}{3} \\
x_3 &= \tfrac{307}{9}
\end{aligned}
$$

$$
\rightarrow
\begin{aligned}
x_1 &= 2(\tfrac{-64}{3}) - 5(\tfrac{307}{9}) - 1 \\
x_2 &= -5(\tfrac{-326}{3}) + 2(\tfrac{307}{9}) + 8 \\
x_3 &= -\tfrac{2}{3}(\tfrac{-326}{3}) + \tfrac{10}{3}(-\tfrac{64}{3}) - \tfrac{1}{3}
\end{aligned}
\rightarrow
\begin{aligned}
x_1 &= -\tfrac{1928}{9} \\
x_2 &= \tfrac{5576}{9} \\
x_3 &= 1
\end{aligned}
\longrightarrow
$$

$$
\begin{aligned}
5x_1 + x_2 - 2x_3 &= 8 \\
2x_1 - 10x_2 + 3x_3 &= -1 \\
x_1 - 2x_2 + 5x_3 &= -1
\end{aligned}
\longrightarrow
\begin{aligned}
x_1 &= -\tfrac{1}{5}x_2 + \tfrac{2}{5}x_3 + \tfrac{8}{5} \\
x_2 &= \tfrac{1}{5}x_1 + \tfrac{3}{10}x_3 + \tfrac{1}{10} \\
x_3 &= -\tfrac{1}{5}x_1 + \tfrac{2}{5}x_2 - \tfrac{1}{5}
\end{aligned}
$$

$$
\longrightarrow
\begin{aligned}
x_1 &= -\tfrac{1}{5}(0) + \tfrac{2}{5}(0) + \tfrac{8}{5} \\
x_2 &= \tfrac{1}{5}(0) + \tfrac{3}{10}(0) + \tfrac{1}{10} \\
x_3 &= -\tfrac{1}{5}(0) + \tfrac{2}{5}(0) - \tfrac{1}{5}
\end{aligned}
\rightarrow
\begin{aligned}
x_1 &= \tfrac{8}{5} \\
x_2 &= \tfrac{1}{10} \\
x_3 &= -\tfrac{1}{5}
\end{aligned}
\longrightarrow
$$

$$
\begin{aligned}
x_1 &= -\tfrac{1}{5}(\tfrac{1}{10}) + \tfrac{2}{5}(-\tfrac{1}{5}) + \tfrac{8}{5} \\
x_2 &= \tfrac{1}{5}(\tfrac{8}{5}) + \tfrac{3}{10}(-\tfrac{1}{5}) + \tfrac{1}{10} \\
x_3 &= -\tfrac{1}{5}(\tfrac{8}{5}) + \tfrac{2}{5}(\tfrac{1}{10}) - \tfrac{1}{5}
\end{aligned}
\longrightarrow
\begin{aligned}
x_1 &= \tfrac{3}{2} \\
x_2 &= \tfrac{9}{25} \\
x_3 &= -\tfrac{12}{25}
\end{aligned}
$$

CONTINUED →

$x_1 = -\frac{1}{5}\left(\frac{9}{25}\right) + \frac{2}{5}\left(-\frac{12}{25}\right) + \frac{8}{5}$ $x_1 = \frac{167}{125}$

$x_2 = \frac{1}{5}\left(\frac{3}{2}\right) + \frac{3}{10}\left(-\frac{12}{25}\right) + \frac{1}{10} \longrightarrow x_2 = \frac{32}{125} \longrightarrow$

$x_3 = -\frac{1}{5}\left(\frac{3}{2}\right) + \frac{2}{5}\left(\frac{9}{25}\right) - \frac{1}{5}$ $x_3 = \frac{-89}{250}$

$x_1 = -\frac{1}{5}\left(\frac{32}{125}\right) + \frac{2}{5}\left(\frac{-89}{250}\right) + \frac{8}{5}$

$x_2 = \frac{1}{5}\left(\frac{167}{125}\right) + \frac{3}{10}\left(\frac{-89}{250}\right) + \frac{1}{10} \longrightarrow$ $x_1 = \frac{879}{625}$, $x_2 = \frac{651}{2500}$, $x_3 = \frac{-228}{625}$

$x_3 = -\frac{1}{5}\left(\frac{167}{125}\right) + \frac{2}{5}\left(\frac{32}{125}\right) - \frac{1}{5}$

27 $x_1 = 2x_2 - 5x_3 - 1$

$x_2 = -5x_1 + 2x_3 + 8 \longrightarrow x_1 = 2(0) - 5(0) - 1 \longrightarrow$

$x_3 = -\frac{2}{3}x_1 + \frac{10}{3}x_2 - \frac{1}{3}$

$x_1 = -1 \longrightarrow x_2 = -5(-1) + 2(0) + 8 \longrightarrow x_2 = 13 \longrightarrow$

$x_3 = -\frac{2}{3}(-1) + \frac{10}{3}(13) - \frac{1}{3} \longrightarrow x_3 = \frac{131}{3} \longrightarrow x_1 = 2(13) - 5\left(\frac{131}{3}\right) - 1$

$\longrightarrow x_1 = \frac{-580}{3} \longrightarrow x_2 = -5\left(-\frac{580}{3}\right) + 2\left(\frac{131}{3}\right) + 8 \longrightarrow x_2 = 1062$

$\longrightarrow x_3 = -\frac{2}{3}\left(-\frac{580}{3}\right) + \frac{10}{3}(1062) - \frac{1}{3} \longrightarrow x_3 = \frac{33017}{9} \longrightarrow$

$x_1 = 2(1062) - 5\left(\frac{33017}{9}\right) - 1 \longrightarrow x_1 = -\frac{145978}{9} \longrightarrow x_2 = -5\left(-\frac{145978}{9}\right)$

$+ 2\left(\frac{33017}{9}\right) + 8 \longrightarrow x_2 = 88444 \longrightarrow x_3 = -\frac{2}{3}\left(-\frac{145978}{9}\right) +$

$\frac{10}{3}(88444) - \frac{1}{3} \longrightarrow x_3 = \frac{7667995}{27}$

Section -2.1 : Graded Homework #2

(5) $u=\begin{bmatrix}3\\-2\\0\end{bmatrix}$ $v=\begin{bmatrix}-4\\1\\5\end{bmatrix}$ $w=\begin{bmatrix}2\\-7\\-1\end{bmatrix}$ $\rightarrow -u+v+w$

$\rightarrow \begin{bmatrix}-5\\-4\\4\end{bmatrix} \rightarrow 2u-v+3w \rightarrow \begin{bmatrix}16\\-26\\-8\end{bmatrix}$

(9)
$$-6x_1+5x_2 = 4$$
$$5x_1-3x_2+2x_3=16$$

(13)
$$x_1-x_2-3x_3-x_4=-1$$
$$-2x_1+2x_2+6x_3+2x_4=-1$$
$$-3x_1-3x_2+10x_3=5$$

$$\rightarrow x_1\begin{bmatrix}1\\-2\\-3\end{bmatrix}+x_2\begin{bmatrix}-1\\2\\-3\end{bmatrix}+x_3\begin{bmatrix}-3\\6\\10\end{bmatrix}+x_4\begin{bmatrix}-1\\2\\0\end{bmatrix}=\begin{bmatrix}-1\\-1\\5\end{bmatrix}$$

(17) $$X=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}7\\-3\\0\end{bmatrix}+s_1\begin{bmatrix}-2\\0\\1\end{bmatrix}$$

(25) $-3a - 4 = -10$

$-a + 4b = 19$ $\rightarrow$ $a = 2$, $b = \frac{28}{4} = 7$

(29) $-1 + 2b - 2 = -3$

$-2 + 2 - c = -4$

$-a - 4 - 5 = 3$

$-1 + 6 - 0 = d$

$\rightarrow$ $b = 0$, $c = 4$, $a = -12$, $d = 5$

(33) $a_1 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$ $a_2 = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ -5 \\ -2 \end{bmatrix}$

Only possibility to get $b_1 = 1$ is to multiply $\frac{1}{2}$ to a_1's 1st component

$$\frac{1}{2} a_1 + a_2 = \begin{bmatrix} (1/2)(2) + 0 \\ (1/2)(-3) + 3 \\ \frac{1}{2}(1) - 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3/2 \\ -5/2 \end{bmatrix} \neq \begin{bmatrix} 1 \\ -5 \\ -2 \end{bmatrix}$$

b is not a linear combination of a_1 and a_2

(37) $$2\begin{bmatrix} 29 \\ 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 18 \\ 25 \\ 6 \end{bmatrix} = \begin{bmatrix} 76 \\ 31 \\ 14 \end{bmatrix}$$

(39) $$x_1\begin{bmatrix} 29 \\ 3 \\ 4 \end{bmatrix} + x_2\begin{bmatrix} 18 \\ 25 \\ 6 \end{bmatrix} = \begin{bmatrix} 112 \\ 81 \\ 26 \end{bmatrix} \rightarrow$$

$29x_1 + 18x_2 = 112$

$3x_1 + 25x_2 = 81$

$4x_1 + 6x_2 = 26$

$$\rightarrow \left[\begin{array}{cc|c} 29 & 18 & 112 \\ 3 & 25 & 81 \\ 4 & 6 & 26 \end{array}\right]$$

$-\frac{3}{29}R_1 + R_2 \to R_2 \quad \left[\begin{array}{cc|c} 29 & 18 & 112 \\ 0 & 671/29 & 2013/29 \\ 4 & 6 & 26 \end{array}\right] \quad -\frac{4}{29}R_1 + R_3 \to R_3$

$\left[\begin{array}{cc|c} 29 & 18 & 112 \\ 0 & 671/29 & 203/29 \\ 0 & 102/29 & 306/29 \end{array}\right] \to \frac{671}{29}x = \frac{-102}{29} \to x = -\frac{102}{671}$

$-\frac{102}{671}R_2 + R_3 \to R_3 \quad \left[\begin{array}{cc|c} 29 & 18 & 112 \\ 0 & 671/29 & 2013/29 \\ 0 & 0 & 0 \end{array}\right] \to$

$\frac{671}{29}x_2 = \frac{2013}{29} \to x_2 = \frac{2013}{671} = 3 \to$

$29x_1 + 18(3) = 112 \to 29x_1 = 112 - 54 \quad x_1 = 2$

200 pounds of Vigoro and 300 pounds of Parker's

(61) $u + v = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} \to u = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}, v = \begin{bmatrix} -3 \\ -2 \\ -3 \end{bmatrix}$

(67) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$

(71) $u = \begin{bmatrix} -3 \\ 5 \end{bmatrix} \to -2u = \begin{bmatrix} -3 \cdot -2 \\ -2 \cdot 5 \end{bmatrix} = \begin{bmatrix} 6 \\ -10 \end{bmatrix}$ True

(73) Does $c(\vec{u}+\vec{v}) = c\vec{u}+c\vec{v}$?

(a) $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, $c = 2$ $\longrightarrow$

$2\left(\begin{bmatrix} 1+2 \\ 2+3 \\ 3+4 \end{bmatrix}\right) = \begin{bmatrix} 6 \\ 10 \\ 14 \end{bmatrix} \rightarrow \begin{bmatrix} 2(1)+2(2) \\ 2(2)+3(2) \\ 2(3)+2(4) \end{bmatrix} \rightarrow \begin{bmatrix} 6 \\ 10 \\ 14 \end{bmatrix}$

Equivalent and true

(b) $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $c_1 = 2$, $c_2 = 3$ $\rightarrow (2)(3) + 3\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right)$

$\rightarrow 6 + \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \rightarrow \left((2) + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right)3 \rightarrow 6 + \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$

Equivalent and true

(75) (a) True

(b) True as you can multiply any vector by the zero vector to get the 0 vector.

Graded Homework 2 - Section 2.2

(3) $u_1 = \begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix}$ $u_2 = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix} \rightarrow c_1 u_1 + c_2 u_2 \rightarrow c_1 = 0$ and $c_2 = 0$

$$\rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{matrix} c_1 = 1 \\ c_2 = 1 \end{matrix} \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} \rightarrow \begin{matrix} c_1 = 2 \\ c_2 = 2 \end{matrix} \begin{bmatrix} 6 \\ 10 \\ 2 \end{bmatrix}$$

(11)

$$\left[\begin{array}{cc|c} -1 & 2 & -10 \\ 4 & 8 & -8 \\ -3 & -7 & 7 \end{array}\right] \underset{\sim}{4R_1 + R_2 \rightarrow R_2} \left[\begin{array}{cc|c} -1 & 2 & -10 \\ 0 & 16 & -48 \\ -3 & -7 & 7 \end{array}\right]$$

$$\underset{\sim}{-3R_1 + R_3 \rightarrow R_3} \left[\begin{array}{cc|c} -1 & 2 & -10 \\ 0 & 16 & -48 \\ 0 & -13 & 37 \end{array}\right] \underset{\sim}{\tfrac{13}{16} R_2 + R_3 \rightarrow R_3}$$

$$\begin{bmatrix} -1 & 2 & -10 \\ 0 & 16 & -48 \\ 0 & 0 & -2 \end{bmatrix} \rightarrow$$

No solutions and hence, $\vec{b}$ is not in span$\{\vec{a}_1, \vec{a}_2\}$

(15)

$$\underbrace{\begin{bmatrix} 1 & -1 & -3 & -1 \\ -2 & 2 & 6 & 2 \\ -3 & -3 & 10 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} -1 \\ -1 \\ 5 \end{bmatrix}}_{b}$$

(17)

$$x_1 \begin{bmatrix} 5 \\ 11 \end{bmatrix} + x_2 \begin{bmatrix} 7 \\ -5 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -4 \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \end{bmatrix}$$

(23) Spans $\mathbb{R}^2$

(25) Spans $\mathbb{R}^3$

(33)
$$\underbrace{\begin{bmatrix} -3 & 2 & 1 \\ 1 & -1 & -1 \\ 5 & -4 & -3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}}_{\vec{b}} \longrightarrow$$

$$\left[\begin{array}{ccc|c} -3 & 2 & 1 & b_1 \\ 1 & -1 & -1 & b_2 \\ 5 & -4 & -3 & b_3 \end{array}\right] \underset{\sim}{R_2 \longleftrightarrow R_1} \left[\begin{array}{ccc|c} 1 & -1 & -1 & b_2 \\ -3 & 2 & 1 & b_1 \\ 5 & -4 & -3 & b_3 \end{array}\right]$$

$$\underset{\sim}{3R_1 + R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & -1 & -1 & b_2 \\ 0 & -1 & -2 & 3b_2 + b_1 \\ 5 & -4 & -3 & b_3 \end{array}\right] \underset{\sim}{-5R_1 + R_3 \to R_3}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & b_2 \\ 0 & -1 & -2 & 3b_2 + b_1 \\ 0 & 1 & 2 & -5b_2 + b_3 \end{array}\right] \underset{\sim}{R_2 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -1 & -1 & b_2 \\ 0 & -1 & -2 & 3b_2 + b_1 \\ 0 & 0 & 0 & b_1 - 2b_2 + b_3 \end{array}\right]$$

No solution for all choices of b

(35) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(43) $\begin{bmatrix} 1 & 3 \\ 2 & 5 \\ -1 & 4 \end{bmatrix} \underset{\sim}{-2R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 3 \\ 0 & -1 \\ -1 & 4 \end{bmatrix} \underset{\sim}{R_1 + R_3 \to R_3}$

$\begin{bmatrix} 1 & 3 \\ 0 & -1 \\ 0 & 7 \end{bmatrix} \underset{\sim}{7R_2 + R_3 \to R_3} \begin{bmatrix} 1 & 3 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \to \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & -1 & 6 \\ 0 & 0 & 1 \end{array}\right]$

$\underset{\sim}{-7R_2 + R_3 \to R_3} \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & -1 & 6 \\ 0 & 7 & 1 \end{array}\right] \underset{\sim}{-R_1 + R_3 \to R_3} \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & -1 & 6 \\ -1 & 4 & 1 \end{array}\right]$

$\underset{\sim}{2R_1 + R_2 \to R_2} \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 2 & 5 & 6 \\ -1 & 4 & 1 \end{array}\right]$ $\begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix} \notin \text{Span}\left\{\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix}\right\}$

53 $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \\ 4 \end{bmatrix}$

57 $\vec{u}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ $u_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

59

a True

b False as the zero vector does not preclude other possibilities.

61

a False as you could have m equations > n components

b True as the entire space can be used by all possible linear combinations

(63) (a) False as any of of the v vectors could be multiples of each other.

(b) True

(65) (a) False

(b) True

(67) c and d as there must be at least n vectors to span $\mathbb{R}^n$