

★ [Graded Homework 4 - Section 3.3] ★



①  $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$  and  $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$

$$A^{-1} = \frac{1}{(1)(1)-(3)(2)} \begin{bmatrix} 1 & -3 \\ -2 & 1 \end{bmatrix} \rightarrow A^{-1} = \boxed{\begin{bmatrix} 1 & -3 \\ -2 & 1 \end{bmatrix}}$$

③  $\begin{bmatrix} 2 & -5 \\ -4 & 10 \end{bmatrix} = A \rightarrow A^{-1} = \frac{1}{(2)(10) - (-5)(-4)} \begin{bmatrix} 2 & -5 \\ -4 & 10 \end{bmatrix}$

$\rightarrow A^{-1}$  does not exist.

⑨ ~~(\*)~~  $\left[ \begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{-3R_3 + R_2 \rightarrow R_2} \xrightarrow{R_3 + R_2 \rightarrow R_1} \rightarrow$

$$\left[ \begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{-2R_2 + R_1 \rightarrow R_1} \sim \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 7 \\ 0 & 1 & 0 & 0 & 1 & -3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

$\rightarrow A^{-1} = \boxed{\begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}}$

⑬  $\left[ \begin{array}{cccc|cccc} 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right]$   ~~$R_2 \leftrightarrow R_1$~~   
 ~~$R_4 \leftrightarrow R_2$~~   
 ~~$R_3 \leftrightarrow R_4$~~

$$\left[ \begin{array}{ccccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \rightarrow A^{-1} = \left[ \begin{array}{ccccc|ccc} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right]$$

(17)  $\begin{bmatrix} 4 & 13 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix} \rightarrow x = A^{-1}\vec{b}$

$A \quad \vec{x} \quad \vec{b}$

$$A^{-1} = \frac{1}{(4)(3) - (13)(1)} \begin{bmatrix} 3 & -13 \\ -1 & 4 \end{bmatrix} \rightarrow A^{-1} = \begin{bmatrix} -3 & 13 \\ 1 & -4 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} -3 & 13 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \end{bmatrix} \rightarrow \vec{x} = \begin{bmatrix} 35 \\ -11 \end{bmatrix} \quad x_1 = 35 \quad x_2 = -11$$

(2)  $A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} \sim \begin{bmatrix} 4 & 3 \\ 0 & -\frac{1}{4} \end{bmatrix} \Rightarrow T \text{ is bijective}$

$\Rightarrow A^{-1} \text{ exists} \rightarrow \begin{bmatrix} 4 & 3 & 1 & 0 \\ 0 & -\frac{1}{4} & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 4 & 3 & 1 & 0 \\ 0 & 1 & 0 & -4 \end{bmatrix} \sim \begin{bmatrix} 4 & 0 & -8 & 12 \\ 0 & 1 & 3 & -4 \end{bmatrix} \xrightarrow{\text{interchange } R_1, R_2} \begin{bmatrix} 4 & 0 & -8 & 12 \\ 0 & 1 & 3 & -4 \end{bmatrix}$

$$\begin{bmatrix} 4 & 3 & 1 & 0 \\ 0 & -\frac{1}{4} & 0 & 1 \end{bmatrix} \xrightarrow{\begin{array}{l} 12R_2 + R_1 \rightarrow R_1 \\ -4R_2 \rightarrow R_2 \end{array}} \begin{bmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & -4 \end{bmatrix} \rightarrow T^{-1} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

(25)  $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & -1 \end{bmatrix}$  ~~so~~  $\rightarrow A^{-1}$  does not exist  
 as  $A^{-1}$  does not exist since  $A$  is NOT BOTH  
 one-to-one and onto.

(35)  $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(39)  $A \in \mathbb{R}^{2 \times 3}$ ,  $B \in \mathbb{R}^{3 \times 2}$  s.t.  $AB = I_2 \wedge BA \neq I_3$

~~$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = B$~~

(43) False as  $A$  being invertible means it's bijective

(b) True as only square matrices can be bijective

(47) True by definition

(d) False as it also needs pivots in columns.

8 ★ Threaded Homework 4 - Section 3.4 ★

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$$(5) \begin{matrix} L \vec{y} = \vec{5} \\ \cancel{y_1} = 2 \\ \cancel{y_2} = 6 \\ \rightarrow x_1 = 3 \end{matrix} \rightarrow y_1 = 2 \rightarrow -2(2) + y_2 = 2 \rightarrow 3x_2 = 6 \rightarrow x_2 = 2 \rightarrow 2x_1 - 2(2) = 2$$

$$\rightarrow \vec{x} = \boxed{\begin{bmatrix} 3 \\ 2 \end{bmatrix}}$$

$$(7) \begin{matrix} y_1 = 4 \\ -1(4) + y_2 = 0 \\ -2(4) + x_3 = -4 \\ x_3 = 2 \\ -1(0) + 3(2) = 4 \end{matrix} \rightarrow y_2 = 4 \rightarrow 2(4) +$$

$$-2(4) + x_3 = -4 \rightarrow y_3 = -4 \rightarrow -2x_3 = -4 \rightarrow$$

$$x_2 + 2(2) = 4 \rightarrow x_2 = 0 \rightarrow 2x_1 = 0$$

$$\rightarrow \vec{x} = \boxed{\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}}$$

$$(13) \begin{bmatrix} 1 & -4 \\ -2 & 9 \end{bmatrix} \rightarrow L = \begin{bmatrix} 1 & : & : \\ -2 & : & : \end{bmatrix} \xrightarrow{2R_1 + R_2 \rightarrow R_2} \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} \rightarrow L = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$\rightarrow \boxed{L = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}, U = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}}$$

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$$(15) \begin{bmatrix} -2 & -1 & 1 \\ -6 & 0 & 4 \\ 2 & -2 & -1 \end{bmatrix} \xrightarrow{\begin{matrix} -3R_1 + R_2 \rightarrow R_2 \\ R_1 + R_3 \rightarrow R_3 \end{matrix}} \begin{bmatrix} -2 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & -3 & 0 \end{bmatrix} \rightarrow L = \begin{bmatrix} 1 & : & : \\ 3 & : & : \\ -1 & : & : \end{bmatrix}$$

$$\rightarrow R_2 + R_3 \rightarrow R_3 \quad \left[ \begin{array}{ccc|c} -2 & -1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{array} \right] \rightarrow L = \left[ \begin{array}{ccc|c} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1 & -1 & 1 \end{array} \right]$$

$$L = \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1 & -1 & 1 \end{array} \right], U = \left[ \begin{array}{ccc|c} -2 & -1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{array} \right]$$

(17)  $A = \left[ \begin{array}{cccc} -1 & 0 & -1 & 2 \\ 1 & 3 & 2 & -2 \\ -2 & -9 & -3 & 3 \\ -1 & 9 & -2 & 5 \end{array} \right]$

$R_1 + R_2 \rightarrow R_2$   
 $-2R_1 + R_3 \rightarrow R_3$   
 $-R_1 + R_4 \rightarrow R_4$

$\sim$

$L = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 2 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot \end{array} \right]$

$3R_2 + R_3 \rightarrow R_3$   
 $-3R_2 + R_4 \rightarrow R_4$

$\sim$

$L = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & -3 & 0 & 0 \\ 1 & 3 & \cdot & \cdot \end{array} \right]$

$2R_3 + R_4 \rightarrow R_4$

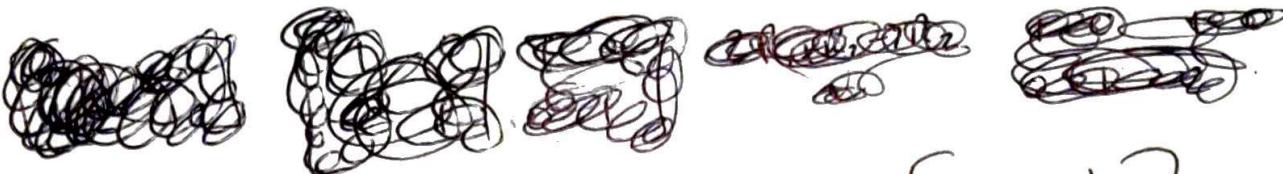
$\sim$

$L = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & -3 & 1 & 0 \\ 1 & 3 & -2 & 1 \end{array} \right], U = \left[ \begin{array}{cccc} -1 & 0 & -1 & 2 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{array} \right]$

(31)

~~Question~~

•  $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = L$ ,  $U = \begin{bmatrix} 2 & -2 \\ 0 & 3 \end{bmatrix}$ , find  $U^{-1}L^{-1} = A^{-1}$



$$U^{-1} = \frac{1}{(2)(3) - (-2)(0)} \begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{1}{3} \end{bmatrix} \rightarrow$$

$$L^{-1} = \frac{1}{(1)(1) - (0)(-2)} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \rightarrow$$

•  $\begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} \frac{7}{6} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}}$

~~A~~ Graded Homework 4 - Section 35 ~~A~~

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③

A is stochastic

⑦

$$\text{A is stochastic} \quad A = \begin{bmatrix} \frac{2}{13} & \frac{3}{7} & c \\ a & \frac{3}{7} & \frac{1}{5} \\ \frac{3}{13} & b & \frac{7}{10} \end{bmatrix},$$

$$a = \frac{8}{13}, b = \frac{1}{7}, c = \frac{1}{10}$$

15

$$A = \begin{bmatrix} \frac{1}{3} & \frac{2}{5} \\ \frac{2}{3} & \frac{3}{5} \end{bmatrix}, X_0 = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \rightarrow X_1 = \begin{bmatrix} \frac{1}{6} + \frac{2}{10} \\ \frac{2}{6} + \frac{3}{10} \end{bmatrix} = \begin{bmatrix} \frac{11}{30} \\ \frac{19}{30} \end{bmatrix} \rightarrow$$

$$X_2 = \begin{bmatrix} \frac{11}{90} + \frac{38}{150} \\ \frac{22}{90} + \frac{57}{150} \end{bmatrix} = \begin{bmatrix} \frac{55 + 114}{450} \\ \frac{110 + 171}{450} \end{bmatrix} = \begin{bmatrix} \frac{169}{450} \\ \frac{281}{450} \end{bmatrix} \rightarrow X_3 = \begin{bmatrix} \frac{169}{1350} + \frac{281}{1125} \\ \frac{169}{675} + \frac{281}{750} \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} \frac{2531}{6750} \\ \frac{4219}{6750} \end{bmatrix}} \approx \boxed{\begin{bmatrix} 0.374942 \\ 0.625037 \end{bmatrix}}$$

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17 A is a regular stochastic matrix as the values will always be positive, non-zero

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~~(A - I)x = 0~~  $\rightarrow \left[ \begin{array}{cc|c} -0.2 & 0.5 & 0 \\ 0.2 & -0.5 & 0 \end{array} \right] \xrightarrow{R_1 + R_2 \rightarrow R_2}$

~~8~~  $\left[ \begin{array}{cc|c} -0.2 & 0.5 & 0 \\ 0 & 0 & 0 \end{array} \right] \rightarrow \text{let } x_2 = s_1 \rightarrow -0.2 + 0.5s_1 = 0$

~~$\rightarrow x_1 = \frac{5}{2}s_1 \rightarrow x = s_1 \begin{bmatrix} \frac{5}{2} \\ 1 \end{bmatrix} \rightarrow s_1 = \frac{1}{\frac{5}{2}+1} = \frac{2}{7}$~~

~~$\rightarrow x = \begin{bmatrix} 10/14 \\ 2/7 \end{bmatrix}$~~

19 A is regular as there will always be positive entries  
and it is stochastic

$(A - I)x = 0 \rightarrow \left[ \begin{array}{ccc|c} -0.6 & 0.5 & 0.3 & 0 \\ 0.2 & -0.7 & 0.4 & 0 \\ 0.4 & 0.2 & -0.7 & 0 \end{array} \right] \xrightarrow{\frac{1}{3}R_1 + R_2 \rightarrow R_2} \sim \xrightarrow{\frac{2}{3}R_2 + R_3 \rightarrow R_3}$

$\rightarrow \left[ \begin{array}{ccc|c} -0.6 & 0.5 & 0.3 & 0 \\ 0 & -8/15 & 0.5 & 0 \\ 0 & \frac{16}{30} & -0.5 & 0 \end{array} \right] \xrightarrow{R_2 + R_3 \rightarrow R_3} \sim$

$\left[ \begin{array}{ccc|c} -0.6 & 0.5 & 0.3 & 0 \\ 0 & -8/15 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \text{let } x_3 = s_1 \rightarrow -\frac{8}{15}x_2 + 0.5s_1 = 0$

~~$\rightarrow -\frac{8}{15}x_2 = -\frac{1}{2}s_1 \rightarrow x_2 = \left(-\frac{1}{2}\right)\left(-\frac{15}{8}\right)s_1 \rightarrow$~~

$x_2 = \frac{15}{16}s_1 \quad -0.6 + 0.5\left(\frac{15}{16}s_1\right) + 0.3s_1 = 0 \rightarrow$

$$-\frac{6}{10}x_1 + \left(\frac{15}{32}\right)\frac{5}{5}s_1 + \frac{3}{10}s_1 = 0 \rightarrow -\frac{6}{10}x_1 + \frac{75}{160}s_1 + \frac{48}{160}s_1 = 0$$

$$\rightarrow \left(-\frac{6}{10}x_1 = \left(-\frac{123}{160}s_1\right)^{-\frac{10}{6}}\right) \rightarrow x_1 = \frac{1230}{960}s_1 = \frac{41}{32}s_1$$

$$\rightarrow x = s_1 \begin{bmatrix} \frac{41}{32} \\ \frac{15}{16} \\ 1 \end{bmatrix} \quad s_1 = \frac{1}{41/32 + 15/16 + 1} = s_1 = \frac{32}{103}$$

$$\rightarrow x = \begin{bmatrix} 398 \\ 291 \\ 3106 \end{bmatrix}$$

(23) Let ~~i~~ denote a placeholder variable such that

$$\bullet \in \mathbb{R} \wedge \bullet > 0$$

$$A^2 = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.8 & 0.4 \\ 0 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.8 & 0.4 \\ 0 & 0 & 0.5 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ 0 & 0 & \bullet \end{bmatrix} \xrightarrow{\text{multiply by itself}} \begin{bmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ 0 & 0 & \bullet \end{bmatrix} \rightarrow A \text{ is not regular}$$

27)  $\begin{bmatrix} 1-a & b \\ a & 1-b \end{bmatrix} = A$

$$\vec{x} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

~~Check~~  $\lim_{n \rightarrow \infty} A^n = \vec{x}$

$$(A - I) \vec{x} = 0 \rightarrow \begin{bmatrix} -a & b \\ a & -b \end{bmatrix} \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} = \vec{0} \rightarrow$$

$$-\frac{2}{3}a + \frac{1}{3}b = 0 \quad \text{Eq}_1 + \text{Eq}_2 \rightarrow \text{Eq}_2 \quad -\frac{2}{3}a + \frac{1}{3}b = 0$$

$$\frac{2}{3}a - \frac{1}{3}b = 0 \quad \sim \quad 0 - 0 = 0 \rightarrow$$

$$-2a + b = 0 \rightarrow \text{let } b = s, \rightarrow \text{Eq}_2 \quad a = \frac{1}{2}s,$$

$\boxed{\begin{bmatrix} \frac{1}{2} & 1 \\ \frac{1}{2} & 0 \end{bmatrix} = A \text{ for } s_1 = 1}$

- 31)
- (a) ~~True~~ False, this would only be true if A was doubly stochastic
  - (b) False as A may not be bijective

33)

(a)

$$A = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{5} & \frac{1}{2} \\ \frac{4}{5} & \frac{1}{2} \end{bmatrix} \rightarrow AB^T = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{5} & \frac{4}{5} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} \frac{6}{20} & \frac{9}{20} \\ \frac{9}{20} & \frac{15}{20} \end{bmatrix} \quad \text{False}$$

(b) ~~False~~ False as it can have 0's

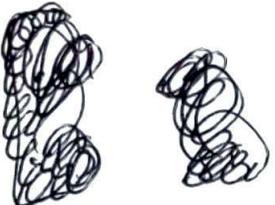
35)

a) False as it depends on initial vector and if A is regular.

b) True as the columns of A would no longer sum to 1.

47)

a) From  
 To Yes  $\begin{bmatrix} .9 & .15 \\ .1 & .85 \end{bmatrix}$   
 No  $\begin{bmatrix} .9 & .15 \\ .1 & .85 \end{bmatrix}$

b)   $\begin{bmatrix} .9 & .15 \\ .1 & .85 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} .9 \\ .1 \end{bmatrix} \rightarrow$   
 $\xrightarrow{x_0 \text{ (initial state)}}$

$$\begin{bmatrix} .9 & .15 \\ .1 & .85 \end{bmatrix} \begin{bmatrix} .9 \\ .1 \end{bmatrix} = \begin{bmatrix} .825 \\ .175 \end{bmatrix} \rightarrow \begin{bmatrix} .9 & .15 \\ .1 & .85 \end{bmatrix} \begin{bmatrix} .825 \\ .175 \end{bmatrix} = \begin{bmatrix} .76875 \\ .23125 \end{bmatrix} \xrightarrow{x_1}$$

$$\rightarrow \begin{bmatrix} .9 & .15 \\ .1 & .85 \end{bmatrix} \begin{bmatrix} .76875 \\ .23125 \end{bmatrix} \approx \begin{bmatrix} .7266 \\ .2734 \end{bmatrix} \rightarrow \begin{bmatrix} .9 & .15 \\ .1 & .85 \end{bmatrix} \begin{bmatrix} .7266 \\ .2734 \end{bmatrix} \xrightarrow{x_2}$$

$$\approx \begin{bmatrix} .6950 \\ .3051 \end{bmatrix} \rightarrow \begin{bmatrix} .9 & .15 \\ .1 & .85 \end{bmatrix} \begin{bmatrix} .6950 \\ .3051 \end{bmatrix} \approx \begin{bmatrix} .6713 \\ .3288 \end{bmatrix} \xrightarrow{x_3}$$

Probability that 6<sup>th</sup> person hears wrong news:  
3288

c)  $(A - I)\vec{x} = \vec{0} \rightarrow \left[ \begin{array}{cc|c} -.1 & .15 & 0 \\ .1 & -.15 & 0 \end{array} \right] \xrightarrow{\sim R_1 + R_2 \rightarrow R_2}$

$\left[ \begin{array}{cc|c} -.1 & .15 & 0 \\ 0 & 0 & 0 \end{array} \right] \rightarrow \text{let } s_2 = s_1 \rightarrow -.1s_1 + .15s_1 = 0$

~~also~~  $-.1s_1 = -.15s_1 \rightarrow s_1 = \frac{-15}{100}s_1 \rightarrow -\frac{10}{100}s_1$

$s_1 = \frac{3}{2}s_1 \rightarrow \vec{x} = s_1 \begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix} \rightarrow s_1 = \frac{1}{1 + \frac{3}{2}} = \frac{2}{5}$

$\rightarrow \vec{x} = \frac{2}{5} \begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix} \rightarrow \vec{x} = \begin{bmatrix} \frac{3}{5} \\ \frac{2}{5} \end{bmatrix}$

51

a)  $\begin{bmatrix} .4 & .1 & .2 \\ .3 & .7 & .7 \\ .3 & .2 & .1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} .4 \\ .3 \\ .3 \end{bmatrix} \rightarrow \text{also}$

$\begin{bmatrix} .4 & .1 & .2 \\ .3 & .7 & .7 \\ .3 & .2 & .1 \end{bmatrix} \begin{bmatrix} .4 \\ .3 \\ .3 \end{bmatrix} = \begin{bmatrix} .25 \\ .54 \\ .21 \end{bmatrix} \rightarrow \vec{x}_2$

.21 chance that it ends up at C after two circulations

b)

$$\begin{bmatrix} .4 & .1 & .2 \\ .3 & .7 & .7 \\ .3 & .2 & .1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix} \rightarrow$$

$\xrightarrow{x_0}$        $\xrightarrow{x_1}$

$$\begin{bmatrix} .4 & .1 & .2 \\ .3 & .7 & .7 \\ .3 & .2 & .1 \end{bmatrix} \begin{bmatrix} .1 \\ .7 \\ .2 \end{bmatrix} = \begin{bmatrix} .15 \\ .66 \\ .19 \end{bmatrix} \rightarrow \begin{bmatrix} .4 & .1 & .2 \\ .3 & .7 & .7 \\ .3 & .2 & .1 \end{bmatrix} \begin{bmatrix} .15 \\ .66 \\ .19 \end{bmatrix}$$

$\xrightarrow{x_2}$

$$= \begin{bmatrix} .144 \\ .64 \\ .196 \end{bmatrix}$$

$\xrightarrow{x_3}$

.64 chance it ends up at B

c)  $(A - I)\vec{x} = \vec{0} \rightarrow$

$$\left[ \begin{array}{ccc|c} -.6 & .1 & .2 & 0 \\ .3 & -.3 & .7 & 0 \\ .3 & .2 & -.9 & 0 \end{array} \right] \xrightarrow{\frac{1}{2}R_1 + R_3 \rightarrow R_3} \sim$$

$$\xrightarrow{\frac{1}{3}R_1 + R_2 \rightarrow R_2} \sim$$

$$\rightarrow \left[ \begin{array}{ccc|c} -.6 & .1 & .2 & 0 \\ 0 & -.25 & .8 & 0 \\ 0 & .25 & -.8 & 0 \end{array} \right] \xrightarrow{R_2 + R_3 \rightarrow R_3} \sim \left[ \begin{array}{ccc|c} -.6 & .1 & .2 & 0 \\ 0 & -.25 & .8 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\rightarrow \text{let } x_3 = s_1 \rightarrow -.25x_2 + .8s_1 = 0 \rightarrow$$

$$x_2 = \frac{-80}{100}s_1 \rightarrow x_2 = \frac{14}{5}s_1 \rightarrow -.6x_1 + .1\left(\frac{14}{5}\right)s_1 + .2s_1 = 0$$

$$\rightarrow -\frac{60}{100}x_1 + \frac{160}{500}s_1 + \frac{100}{500}s_1 = 0 \rightarrow x_1 = \frac{-260}{500}s_1 \rightarrow$$

$$x_1 = \frac{13}{15} s_1 \rightarrow \vec{x} = s_1 \begin{bmatrix} \frac{13}{15} \\ \frac{16}{5} \\ 1 \end{bmatrix} \rightarrow s_1 = \frac{1}{\left(\frac{13}{15}\right) + \left(\frac{16}{5}\right) + 1}$$

(scratched)

$$s_1 = \frac{15}{76} \rightarrow \vec{x} = \frac{15}{76} \begin{bmatrix} \frac{13}{15} \\ \frac{16}{5} \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{13}{76} \\ \frac{12}{19} \\ \frac{15}{76} \end{bmatrix}$$

(scratched)

★ Graded Homework 4 - Section 4.1 ★

①  $\left\{ \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} \right\}$  s.t.  $a \wedge b \in \mathbb{R} \Rightarrow a \in \{0\} \wedge b \in \{0\}$

$\Rightarrow \exists \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ . let  $\vec{v} = \begin{bmatrix} a_1 \\ 0 \\ b_1 \end{bmatrix}$  s.t.  $a_1, b_1 \in \mathbb{R}$

let  $\vec{v} = \begin{bmatrix} a_2 \\ 0 \\ b_2 \end{bmatrix}$  s.t.  $a_2, b_2 \in \mathbb{R}$

$\rightarrow \vec{v} + \vec{v} = \begin{bmatrix} a_1 + a_2 \\ 0+0 \\ b_1 + b_2 \end{bmatrix} \quad \forall a_1, a_2, b_1, b_2 \in \mathbb{R}$ .

$\rightarrow c \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} = \begin{bmatrix} ca \\ c0 \\ cb \end{bmatrix} \quad \forall c \in \mathbb{R}$ .

(It is a subspace.)

③  $\left\{ \begin{bmatrix} a \\ b \end{bmatrix} \right\}$  s.t.  $a+b=1 \wedge a, b \in \mathbb{R} \Rightarrow$

$a \neq 0 \wedge b \neq 0 \Rightarrow$  not a subspace.

⑤  $\left\{ \begin{bmatrix} a \\ 1 \\ 0 \\ b \end{bmatrix} \right\}$  s.t.  $a \wedge b \in \mathbb{R}$ .

(Not a subspace as  $\vec{0} \in \mathbb{R}^4$  not there)

⑨  $\left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right\}$  s.t.  $abc = 0 \Rightarrow \left[ \begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right] \in \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right\}$

$\rightarrow \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\} \in \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right\} \rightarrow \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} \wedge (2)(-1)(-1) = 2 \Rightarrow$  not a subspace

as it is not closed under addition.

⑩  $\left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right\}$  s.t.  $a \geq 0, b \geq 0, c \geq 0 \rightarrow \left[ \begin{smallmatrix} 0 \\ 0 \\ 0 \end{smallmatrix} \right] \in \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right\}$

$\rightarrow \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right\} = \left\{ a \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

~~all~~ → all vectors in the positive quadrant of

$\mathbb{R}^3$  are available → Closed under addition →

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \in \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right\}$  but  $-1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \notin \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right\}$  → not

Closed under multiplication → not a subspace

$$15 \quad \text{if } \vec{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \text{ s.t. } v_1 + \dots + v_n = 0 \rightarrow$$

$$A\vec{x} = 0 \rightarrow A = [1 \dots 1] \rightarrow A\vec{x} = 0 \text{ so}$$

the solution to this homogeneous system is a subspace.

$$25 \quad A = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 5 & -7 \end{bmatrix} \rightarrow \text{null}(A) = \left[ \begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ -2 & 5 & -7 & 0 \end{array} \right]$$

$$2R_1 + R_2 \rightarrow R_2 \sim \left[ \begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ 0 & 1 & -3 & 0 \end{array} \right] \rightarrow \text{let } x_3 = s_1 \rightarrow$$

$$x_2 - 3(s_1) = 0 \rightarrow x_2 = 3s_1 \rightarrow x_1 - 2(3s_1) + 2(s_1) = 0$$

$$\rightarrow x_1 = 4s_1 \rightarrow \text{null}(A) = \begin{bmatrix} 4s_1 \\ 3s_1 \\ s_1 \end{bmatrix} = s_1 \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

$$29 \quad A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 3 \end{bmatrix} \rightarrow \text{null}(A) = \left[ \begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 3 & 0 \end{array} \right] \rightarrow$$

$$3x_3 = 0 \rightarrow x_3 = 0 \rightarrow x_2 + 3(0) = 0 \rightarrow x_2 = 0$$

$$\rightarrow x_1 - 1(0) + 1(0) = 0 \rightarrow x_1 = 0 \rightarrow \vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \text{null}(A) = \text{Span} \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$$35 \quad \begin{bmatrix} 4 & -2 \\ 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} -5 \\ 2 \end{bmatrix} = \begin{bmatrix} -24 \\ 1 \\ 4 \end{bmatrix} \Rightarrow b \notin \text{ker}(T)$$

$C$  cannot be in range( $T$ ) as it has only 2 components.

(37)  $\left\{ \begin{bmatrix} a \\ b \end{bmatrix} \right\}$  s.t.  $a > 0 \wedge b > 0$

(45) a) True as the  $\vec{0}$  was not included

b)  $A = \begin{bmatrix} \cdot & \cdot & \cdot & | & 0 \\ \cdot & \cdot & \cdot & | & 0 \\ \cdot & \cdot & \cdot & | & 0 \\ \cdot & \cdot & \cdot & | & 0 \\ \cdot & \cdot & \cdot & | & 0 \end{bmatrix}, \cdot = \text{placeholder } \mathbb{R} \text{ variable}$

False, the set of solutions produces the trivial  $\vec{0}$  subspace  $\in \mathbb{R}^3$ .

(47) a)  $T(\vec{x}) = A\vec{x}$ ,  $T: \mathbb{R}^5 \rightarrow \mathbb{R}^8 \rightsquigarrow A =$

$$\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}, \cdot = \text{placeholder } \in \mathbb{R}$$

$\rightarrow \ker(T) = \vec{x}$  s.t.  $A\vec{x} = \vec{0} \rightarrow \vec{x}$  is subspace  
of the trivial  $\vec{0}$  type  $\in \mathbb{R}^5$  so false

⑥  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^7 \rightsquigarrow T(\vec{x}) = A\vec{x} \rightarrow A = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \in \mathbb{R}^{7 \times 2}$

$\rightarrow \text{range}(T) = \text{Span} \{ \vec{a}_1, \vec{a}_2 \} \rightarrow$  ~~a~~ a subspace

for  $\mathbb{R}^7 \rightarrow$  not a subspace of  $\mathbb{R}^2$  - false

51 a) ~~False~~ as for any  $\vec{v} \in S \Rightarrow c, \vec{v} \in S$  where  
 $c \in \mathbb{R}$  and hence,  $S$  must be infinite ~~set~~ set outside  
of  $S = \{ \vec{0} \}$

b) True as in a non-zero subspace, all points in the  
subspace are accessible.