Graded Homework 9 - Section 8.3

(1) Not symmetric

(3) Symmetric

(5) Not symmetric

(11) A is orthogonal

(13) A is not orthogonal

(17) $P = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & -1 \\ 1 & 0 & 2 \end{bmatrix} \xrightarrow[\text{columns}]{\text{normalize}} \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & -1/\sqrt{4} \\ 1/\sqrt{3} & -1/\sqrt{2} & -1/\sqrt{4} \\ 1/\sqrt{3} & 0 & 2/\sqrt{4} \end{bmatrix}$

$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(19) $A = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$, $\lambda = 0, 5 \rightarrow \left[\begin{array}{cc|c} 4 & 2 & 0 \\ 2 & 1 & 0 \end{array}\right] \xrightarrow{-\frac{1}{2}R_1 + R_2 \to R_2} \sim$

$\left[\begin{array}{cc|c} 4 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightarrow$ let $x_2 = s_1 \rightarrow 4x_1 + 2s_1 = 0 \rightarrow x_1 = -\frac{1}{2} s_1$

$\rightarrow \vec{x} = s_1 \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} \rightarrow (A - 5I)\vec{x} = \vec{0} \rightarrow \left[\begin{array}{cc|c} -1 & 2 & 0 \\ 2 & -4 & 0 \end{array}\right]$

$$2R_1+R_2\to R_2 \;\sim\; \left[\begin{array}{cc|c} -1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right] \to \text{let } x_2=s_1 \to -x_1+2s_1=0$$

$$\to x_1=2s_1 \to \vec{x}=s_1\begin{bmatrix}2\\1\end{bmatrix} \to P=\begin{bmatrix}-1/2 & 2\\ 1 & 1\end{bmatrix} \to$$

$$\frac{1}{\sqrt{5/4}}\begin{bmatrix}-1/2\\1\end{bmatrix}=\begin{bmatrix}-1/2\sqrt{5/4}\\ 1/\sqrt{5/4}\end{bmatrix} \to \frac{1}{\sqrt{5}}\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}2/\sqrt{5}\\ 1/\sqrt{5}\end{bmatrix}$$

$$\to P=\begin{bmatrix}\frac{-1}{2\sqrt{5/4}} & \frac{2}{\sqrt{5}}\\ \frac{1}{\sqrt{5/4}} & \frac{1}{\sqrt{5}}\end{bmatrix},\quad D=\begin{bmatrix}0 & 0\\ 0 & 5\end{bmatrix}$$

23 $(A-(-1)I)\vec{u}=\vec{0} \to \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right] \xrightarrow{-R_1+R_3\to R_3 \,\sim}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to x_2=0 \to$$

let $x_3=s_1 \to x_1+s_1=0 \to x_1=-s_1$

$$\to \vec{x}=s_1\begin{bmatrix}-1\\0\\1\end{bmatrix} \to \left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{array}\right] \; R_1+R_3\to R_3 \sim$$

$$\left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \text{let } x_3=s_1 \to \text{let } x_2=s_2 \to$$

$-x_1+s_1=0 \to x_1=s_1 \to$

$$\left\{\begin{bmatrix}1\\0\\1\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix}\right\} \to P=\begin{bmatrix}-1 & 1 & 0\\ 0 & 0 & 1\\ 1 & 1 & 0\end{bmatrix} \to$$

$$\begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} = P \qquad D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

49 (a) True as all symmetric matrices are diagonalizable.

(b) False as you can have square matrix where algebraic multiplicity is different from geometric.

Graded Homework 9 - Section 7.1

(9)
- Closed under addition
- Closed under scalar multiplication.
- Zero vector exists
- Additive inverse exists.
- Distributive, associative, commutative, and identity properties exist.

V is a vector space.

(11) No additive identity $\vec{0} + \vec{v} = \vec{0}, \forall v \in V$ so

V is not a vector space.

(15)
- Zero vector is present, $f(4) = 0$
- $f(4) = 0$ and $g(4) = 0 \Rightarrow f(x) + g(x) = 0$

$\rightarrow 0 + 0 = 0$

Subspace

- $f(4) = 0 \Rightarrow cf(4) = c(0) = 0$

(17)
- Zero vector is present, $a_1 = a_2 = 0 \Rightarrow T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = 0$

$\forall x_1, x_2 \in \mathbb{R}$

- Closed under addition: $T_1\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) + T_2\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) =$

$$\begin{bmatrix} a_1 x_1 \\ a_2 x_2 \end{bmatrix} + \begin{bmatrix} b_1 x_1 \\ b_2 x_2 \end{bmatrix} = \begin{bmatrix} (a_1 + b_1) x_1 \\ (a_2 + b_2) x_2 \end{bmatrix}$$

- Closed under multiplication:

$$c\,T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} (ca_1) x_1 \\ (ca_2) x_2 \end{bmatrix}$$

(23) Not closed under addition.
Two vectors that are opposite values of each other would sum to 0 which defies the rules.

Not a subspace.

(25) • Zero is possible if $g(x) = 0$ is included.
• Closed under addition:

$$(p+q)(2) + (p+q)(3) = 0 \rightarrow (p(2) + p(3)) + (q(2) + q(3))$$

$$= 0 + 0 = 0.$$

• Closed under multiplication:

$$c\,(g(2) + g(3)) = c\,(0) = 0.$$

Subspace

(27) Not a subspace as $0^{\mathbb{R}^{3\times 3}}$ not in set.