

Graded Homework 3 - Section  
2.3

③

$$\left[ \begin{array}{cc|c} 7 & 5 & 0 \\ 1 & -3 & 0 \\ -13 & 2 & 0 \end{array} \right] \xrightarrow{-\frac{1}{7}R_1 + R_2 \rightarrow R_2} \sim$$

$$\left[ \begin{array}{cc|c} 7 & 5 & 0 \\ 0 & -\frac{24}{7} & 0 \\ -13 & 2 & 0 \end{array} \right] \xrightarrow{\frac{13}{7}R_1 + R_3 \rightarrow R_3} \sim \left[ \begin{array}{cc|c} 7 & 5 & 0 \\ 0 & -\frac{24}{7} & 0 \\ 0 & \frac{79}{7} & 0 \end{array} \right]$$

$$-\frac{24}{7}x = -\frac{79}{7} \quad x = \frac{79}{24} \rightarrow \xrightarrow{\frac{79}{24}R_2 + R_3 \rightarrow R_3} \sim$$

$$\left[ \begin{array}{cc|c} 7 & 5 & 0 \\ 0 & -\frac{24}{7} & 0 \\ 0 & 0 & 0 \end{array} \right] \equiv -\frac{24}{7}x_2 = 0 \quad x_2 = 0 \rightarrow 7x_1 + 5(0) = 0 \rightarrow x_1 = 0$$

As  $x_1 = 0$  and  $x_2 = 0$ , these are linearly independent.

$$\textcircled{5} \quad \left[ \begin{array}{ccc|c} 3 & 0 & 2 & 0 \\ -1 & 4 & 4 & 0 \\ 2 & 1 & 7 & 0 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_3} \left[ \begin{array}{ccc|c} -1 & 4 & 4 & 0 \\ 2 & 1 & 7 & 0 \\ 3 & 0 & 2 & 0 \end{array} \right] \xrightarrow{2R_1 + R_2 \rightarrow R_2} \sim \xrightarrow{3R_1 + R_3 \rightarrow R_3}$$

$$\left[ \begin{array}{ccc|c} -1 & 4 & 4 & 0 \\ 0 & 9 & 15 & 0 \\ 0 & 12 & 14 & 0 \end{array} \right] \xrightarrow{\sim} \left[ \begin{array}{ccc|c} -1 & 4 & 4 & 0 \\ 0 & 9 & 15 & 0 \\ 0 & 0 & -6 & 0 \end{array} \right] \rightarrow$$

$$-6x_3 = 0 \rightarrow x_3 = 0 \rightarrow 9x_2 + 15(0) = 0 \rightarrow x_2 = 0$$

$$\rightarrow -1x_1 + 4(0) + 4(0) = 0 \rightarrow x_1 = 0$$

Linearly independent as ~~except~~ only the trivial solution.

$$\textcircled{11} \quad \left[ \begin{array}{ccc|c} 3 & 1 & 0 & 0 \\ 5 & -2 & -1 & 0 \\ 4 & -4 & -3 & 0 \end{array} \right] \xrightarrow{\sim} \left[ \begin{array}{ccc|c} 3 & 1 & 0 & 0 \\ 0 & -\frac{11}{3} & -1 & 0 \\ 0 & -\frac{16}{3} & -3 & 0 \end{array} \right]$$

$-\frac{5}{3}R_1 + R_2 \rightarrow R_2$   
 $-\frac{4}{3}R_1 + R_3 \rightarrow R_3$

$$-\frac{11}{3}a = \frac{16}{3} \rightarrow a = -\frac{16}{11} \rightarrow -\frac{16}{11}R_2 + R_3 \rightarrow R_3$$

$$\left[ \begin{array}{ccc|c} 3 & 1 & 0 & 0 \\ 0 & -\frac{11}{3} & -1 & 0 \\ 0 & 0 & -\frac{17}{11} & 0 \end{array} \right]$$

$x_1, x_2, \text{ and } x_3 = 0$  so  
 linearly independent as trivial solution

$$\textcircled{15} \quad \left[ \begin{array}{cc|c} 8 & 1 & 0 \\ 0 & -1 & 0 \\ -3 & 2 & 0 \end{array} \right] \xrightarrow{\sim} \left[ \begin{array}{cc|c} 8 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 2 & 0 \end{array} \right]$$

$\frac{3}{8}R_1 + R_3 \rightarrow R_3$

$$2R_2 + R_3 \rightarrow R_3 \quad \left[ \begin{array}{cc|c} 8 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{array} \right] \rightarrow -x_2 = 0 \rightarrow x_2 = 0$$

$$\rightarrow 8x_1 + 1(0) = 0 \rightarrow x_1 = 0$$

$x_1$  and  $x_2$  are zero so no non-trivial solutions

(17)

$$\left[ \begin{array}{ccc|c} -1 & 3 & 1 & 6 \\ 4 & -3 & -1 & 0 \\ 3 & 0 & 5 & 0 \end{array} \right]$$

$$4R_1 + R_2 \rightarrow R_2$$

$$3R_1 + R_3 \rightarrow R_3$$

$$\sim \left[ \begin{array}{ccc|c} -1 & 3 & 1 & 0 \\ 0 & 9 & 2 & 0 \\ 0 & 9 & 8 & 0 \end{array} \right]$$

$$-R_2 + R_3 \rightarrow R_3$$

$\sim$

$$\left[ \begin{array}{ccc|c} -1 & 3 & 1 & 0 \\ 0 & 9 & 2 & 0 \\ 0 & 0 & 6 & 0 \end{array} \right]$$

$$\rightarrow 6x_3 = 0 \rightarrow x_3 = 0$$

$$\rightarrow 9x_2 + 2(0) = 0 \rightarrow x_2 = 0 \rightarrow -1x_1 + 3(0) + 1(0) = 0$$

$$\rightarrow x_1 = 0$$

~~crossed out~~  $x_1, x_2, x_3 = 0$  so only trivial solution

(19)

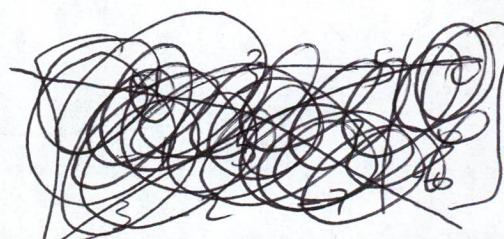
$V = \frac{1}{2} \cup$  so linearly dependent

(23)

Linearly dependent as  $\vec{0}$  is included

(27)

$$\left[ \begin{array}{ccc|c} 4 & 3 & -5 & 0 \\ -1 & 5 & 7 & 0 \\ 3 & -2 & -7 & 0 \end{array} \right] \text{ } R_2 \leftrightarrow R_1$$



$$\left[ \begin{array}{ccc|c} -1 & 5 & 7 & 0 \\ 4 & 3 & -5 & 0 \\ 3 & -2 & -7 & 0 \end{array} \right] \xrightarrow{\begin{array}{l} 4R_1 + R_2 \rightarrow R_2 \\ 3R_1 + R_3 \rightarrow R_3 \end{array}} \sim \left[ \begin{array}{ccc|c} 1 & 5 & 7 & 0 \\ 0 & 23 & 23 & 0 \\ 0 & 13 & 14 & 0 \end{array} \right]$$

$$- \frac{13}{23} R_2 + R_3 \rightarrow R_3 \sim \left[ \begin{array}{ccc|c} 1 & 5 & 7 & 0 \\ 0 & 23 & 23 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \rightarrow x_3 = 0 \rightarrow$$

~~Also~~  $23x_2 + 23(0) = 0 \rightarrow x_2 = 0 \rightarrow x_1 + 5(0) + 7(0) = 0$

$$\rightarrow x_1 = 0$$

These vectors are linearly independent and ~~also~~ so it is not possible that any of the vectors are in the span of the others.

(31)  $\left[ \begin{array}{cc|c} 6 & -9 & 0 \\ -4 & 6 & 0 \end{array} \right] \xrightarrow{\frac{4}{6} R_1 + R_2 \rightarrow R_2} \sim \left[ \begin{array}{cc|c} 6 & -9 & 0 \\ 0 & 0 & 0 \end{array} \right]$

$$\rightarrow \text{let } x_2 = s, \rightarrow 6x_1 - 9s_2 = 0 \rightarrow x_1 = \frac{9}{6}s_2$$

~~One solution for all b~~

Infinite solutions

so linearly dependent and cannot find solutions for all b

(33)

$$\left[ \begin{array}{ccc|c} 2 & -1 & 0 & 0 \\ 1 & 0 & 1 & 6 \\ -3 & 4 & 5 & 6 \end{array} \right] \quad \begin{matrix} -\frac{1}{2} R_1 + R_2 \rightarrow R_2 \\ \frac{3}{2} R_1 + R_3 \rightarrow R_3 \end{matrix}$$

~

$$\left[ \begin{array}{ccc|c} 2 & -1 & 0 & 0 \\ 0 & \frac{1}{2} & 1 & 0 \\ 0 & 5/2 & 5 & 0 \end{array} \right] \quad \begin{matrix} -5 R_2 + R_3 \rightarrow R_3 \\ ~ \end{matrix}$$

$$\left[ \begin{array}{ccc|c} 2 & -1 & 0 & 0 \\ 0 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Not a unique solution  
as the set of equations  
are not linearly independent  
and do not span  $\mathbb{R}^3$ .

(39)

$$\left[ \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right] \left[ \begin{array}{c} 2 \\ 2 \\ 2 \end{array} \right] \left[ \begin{array}{c} 3 \\ 3 \\ 3 \end{array} \right]$$

(43)

~~False~~

@ False, a set of vectors that are multiples  
of each other (an example of linear dependence)  
would not span  $\mathbb{R}^n$

(b) False, you can have a set of vectors that span a space and add a linearly dependent set to the aforementioned and you would get ~~a~~ a linearly ~~independent~~ dependent set that still spans  $\mathbb{R}^n$

(45) (a) False, we just know that this ~~set~~ set spans the space.

(b) True by definition

(47) (a) False as they could be multiples of each other

(b) False for the same reason as above

(49) (a) False as  $\vec{v}_4$  could be a linear combination

(b) True no matter what as any ~~vector~~ vector being a combination of others is a linearly dependent set.

(51)

- @ False by definition of linear independence
- b True by definition

(53)

~~One set of linearly independent vectors are a set of linearly dependent vectors.~~

a, b, c are the only possible vectors that can be linearly independent.



~~A~~ Graded Homework #3 - Section 3.1

~~A~~

~~A~~

~~A~~

~~A~~

~~A~~

~~A~~

(3)

$$A = \begin{bmatrix} 0 & -4 & 2 \\ 3 & 1 & -2 \end{bmatrix}, \vec{u}_1 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 4 \\ -5 \\ -2 \end{bmatrix}$$

$$T(\vec{u}_1) = \begin{bmatrix} -6 \\ 9 \end{bmatrix}$$

$$T(\vec{u}_2) = \begin{bmatrix} 16 \\ 11 \end{bmatrix}$$

(5)

$$\left[ \begin{array}{ccc|c} 1 & -2 & 0 \\ 3 & 2 & 1 \end{array} \right] \left[ \begin{array}{c} -3 \\ 6 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 1 & -2 & 0 & -3 \\ 3 & 2 & 1 & 6 \end{array} \right]$$

$\rightarrow -3R_1 + R_2 \rightarrow R_2$

$$\rightarrow \left[ \begin{array}{ccc|c} 1 & -2 & 0 & -3 \\ 0 & 8 & 1 & 15 \end{array} \right] \xrightarrow{\text{let } x_3 = s} \text{let } x_3 = s, \rightarrow$$

$$x_2 = -\frac{1}{8}s, x_1 - 2(-\frac{1}{8}s) + \frac{15}{8} = -3 \rightarrow$$

$$x_1 = -\frac{1}{4}s, \quad y \text{ is in the range of } T$$

(1)  $T(u_1) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, T(u_2) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ , And  $T(-2\vec{u}_1 + 3\vec{u}_2) =$

$$T(-2\vec{u}_1 + 3\vec{u}_2) = -2T(\vec{u}_1) + 3T(\vec{u}_2) =$$

$$-2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 3 \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \boxed{\begin{bmatrix} -13 \\ 4 \end{bmatrix}}$$

(13)

$$A = \begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Is a linear transformation

(15)

Not a linear transformation

(17)

$$A = \begin{bmatrix} -4 & 0 & 1 \\ 6 & 5 & 0 \end{bmatrix}$$

linear transformation

(27)

$$\left[ \begin{array}{ccc|c} 2 & 8 & 4 & 0 \\ 3 & 2 & 3 & 0 \\ 1 & 4 & 5 & 0 \end{array} \right] \xrightarrow{-\frac{3}{2}R_1 + R_2 \rightarrow R_2} \left[ \begin{array}{ccc|c} 2 & 8 & 4 & 0 \\ 0 & -10 & -9 & 0 \\ 1 & 4 & 5 & 0 \end{array} \right] \xrightarrow{-\frac{1}{2}R_1 + R_3 \rightarrow R_3} \sim \left[ \begin{array}{ccc|c} 2 & 8 & 4 & 0 \\ 0 & 10 & 9 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 2 & 8 & 4 & 0 \\ 0 & 10 & -3 & 0 \\ 0 & 10 & 3 & 0 \end{array} \right] \xrightarrow{R_2 + R_3 \rightarrow R_3} \sim \left[ \begin{array}{ccc|c} 2 & 8 & 4 & 0 \\ 0 & 10 & -3 & 0 \\ 0 & 0 & 6 & 0 \end{array} \right]$$

→ A lacks column and row pivots. It does not span  $\mathbb{R}^3$  and ~~its~~ its columns are not linearly independent.

T is neither one-to-one nor onto

(35)

~~Set A~~  
~~Set B~~

~~Set C~~  
~~Set D~~

$$T([3]) = Ax \xrightarrow{?} [1 \ 0]$$

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} 2 & 1 \\ 2 & -6 \end{bmatrix}$$

(39)

(a) False as ~~onto~~ codomain will always be equal to or larger than the range.

(b) True by definition

(43)

(a) True by definition

(b) True as the entire space needs to be available for an onto relationship.

45

a) True as each  $T_1$  and  $T_2$  output is used for  $\vec{x}$

b) True as you would need equivalent points in columns and rows and the only way to do that is a square matrix.

# ★ Graded Homework #3 - Section 3.2



~~A A A A A A A~~

(5)

⑤ a)  $\left( \begin{bmatrix} 5 & 0 \\ -1 & 4 \\ 3 & 3 \end{bmatrix} + \begin{bmatrix} 1 & -4 & -5 \\ -2 & 1 & -3 \\ 0 & 2 & 6 \end{bmatrix} \right) \begin{bmatrix} 0 & 4 \\ -2 & 5 \end{bmatrix}$

Not possible to add ~~as~~ due to  
differing dimensions

b)  $\begin{bmatrix} 0 & 4 \\ -2 & 5 \end{bmatrix} \left( \begin{bmatrix} 5 & 0 \\ -1 & 4 \\ 3 & 3 \end{bmatrix}^T + \begin{bmatrix} 1 & 0 & -3 \\ -2 & 5 & -1 \end{bmatrix} \right) =$

$$\begin{bmatrix} 0 & 4 \\ -2 & 5 \end{bmatrix} \left( \begin{bmatrix} 5 & -1 & 3 \\ 0 & 4 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -3 \\ -2 & 5 & -1 \end{bmatrix} \right) =$$

$$\begin{bmatrix} 0 & 4 \\ -2 & 5 \end{bmatrix} \left( \begin{bmatrix} 6 & -1 & 0 \\ -2 & 9 & 2 \end{bmatrix} \right) = \boxed{\begin{bmatrix} -8 & 36 & 8 \\ -22 & 47 & 10 \end{bmatrix}}$$

c)  $\begin{bmatrix} 1 & 4 & -5 \\ -2 & 1 & -3 \\ 0 & 2 & 6 \end{bmatrix} + \left( \begin{bmatrix} 5 & 0 \\ -1 & 4 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ -2 & 5 & -1 \end{bmatrix} \right) =$

$$\begin{bmatrix} 1 & 4 & -5 \\ 2 & 1 & -3 \\ 0 & 2 & 6 \end{bmatrix} + \begin{bmatrix} 5 & 0 & -15 \\ -9 & 20 & -1 \\ -3 & 15 & -12 \end{bmatrix} = \boxed{\begin{bmatrix} 6 & 4 & -20 \\ -11 & 21 & -4 \\ -3 & 17 & -6 \end{bmatrix}}$$

⑦  $2b - a = 3$   
 $3b + 2 = 5 \rightarrow -6 + 2a = -8 \rightarrow a = -1$   
 $-6 + 2a = -8$   
 $\dots -13 = c \quad \hookrightarrow 3b + 2 = 5 \rightarrow b = 1 \rightarrow$   
 $2(1) - (-1) = 3 \rightarrow 3 = 3$

Solution:

$$a = -1, b = 1, c = -13$$

⑨  $2a + 0 - 2c = 4 \rightarrow c = -3 \rightarrow 2a - 2(-3) = 4$   
 $b + 0 + 4c = -6 \quad \text{and} \quad a = -1 \rightarrow b = 3$   
 $-a + 3b - 2 = d$   
 $-3 - 2b + 4 = -5 \quad \hookrightarrow \cancel{-a + 3b - 2 = d}$   
 $-(-1) + 3(3) - 2 = d \rightarrow d = 8$

Solution:

$$a = -1, b = 3, c = -3, d = 8$$

$$\textcircled{11} \quad \begin{bmatrix} 5 & -10 \\ a & -4 \end{bmatrix} \begin{bmatrix} 5 & -10 \\ a & -4 \end{bmatrix} = \begin{bmatrix} 5 & -10 \\ a & -4 \end{bmatrix} \rightarrow$$

$$\textcircled{11} \quad \begin{aligned} 25 - 10a &= 5 & a &= 2 \\ 5a - 4a &= a & a &\leq a \\ -50 + 40 &= -10 & -10 &\cancel{=} -10 \\ -10a + 16 &= -4 & a &= 2 \end{aligned}$$

$$a = 2$$

$$\textcircled{17} \quad (A + B^2)(BA - A^2 + B^3A - B^2A) \rightarrow ABA - A^2 + B^3A -$$

$$\textcircled{21} \quad (A - B)(A + B) \rightarrow A^2 - BA + AB - B^2$$

$$\neq A^2 - B^2 \Leftrightarrow -BA + AB \neq I_2$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} = B \rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 6 & 11 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 9 & 11 \end{bmatrix} = A^2 - B^2 \rightarrow$$

$$\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 6 & 11 \end{bmatrix} B^2$$

$A^2$        $BA$        $A B$        $B^2$

$$\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - \begin{bmatrix} 3 & 4 \\ 11 & 16 \end{bmatrix} + \begin{bmatrix} 4 & 7 \\ 8 & 15 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 6 & 11 \end{bmatrix} =$$

$A^2$        $BA$        $AB$        $B^2$

$$A^2 - BA + AB + B^2 = \begin{bmatrix} 6 & 10 \\ 6 & 10 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ 9 & 11 \end{bmatrix} =$$

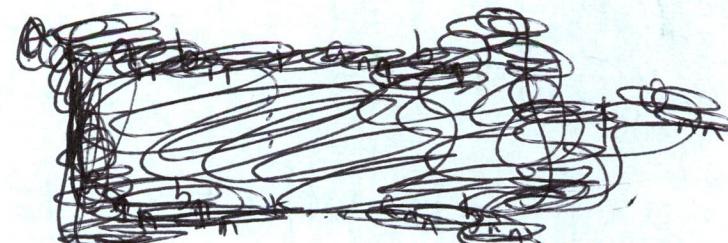
$A^2 - B^2$

(47)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$A$        $B$



(51)

@

$$AB \neq 0_{nn} \rightarrow$$

$$(a_{11}b_{11} + \dots + a_{nn}b_{nn})$$

does not necessarily mean that  $A+B$

$\neq 0_{nn}$  Example:

Let  $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} -2 & -1 \\ -3 & -4 \end{bmatrix}$

$$A+B = 0_{22} \text{ but } AB = \begin{bmatrix} -7 & -6 \\ -18 & -19 \end{bmatrix}$$



(b)

True as  $(A+B^T)^T = A^T + B^T = A^T + B$

(53)

@ True by definition?

@ False, it's possible to get a zero matrix

(57)

@ True by definition.

@

False

$$A = \begin{bmatrix} a_{11} & & & \\ 0 & a_{22} & & \\ 0 & 0 & a_{33} & \\ 0 & 0 & 0 & \ddots \\ 0 & 0 & 0 & 0 & a_{nn} \end{bmatrix} \quad \xrightarrow{\hspace{10em}}$$

$$A^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ a_{11} & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 \\ a_{31} & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & 0 & 0 & 0 \end{bmatrix}$$

