HW3: Image Enhancement in the Frequency Domain

DIP Teaching Stuff, *Sun Yat-sen University*

Welcome to your thrid DIP homework! Here comes the crazy Fourier transform! Are you ready for the challenge? The third assignment is more difficult than the first two, but we believe you will overcome many difficulties. When submitting a job, you need to submit a report (in **PDF** format) and all job-related code.Warning: We encourage discussions among students, but homework solutions should be written and submitted individually, without copying existed answers. Plagiarism = Fail. Besides,there may be at least 30% penalty for late homework.

Exercises

Please answer the following questions in the report.

1.1 （10pts） Show that the DFT of the discrete function f(x, y)=cos(2πu_0x+2πv_0y) is

$$F(u, v) = \frac{1}{2}[\delta(u + Mu_0,\ v + Nv_0) + \delta(u - Mu_0,\ v - Nv_0)]$$

1.2 (10pts)The two Fourier spectra shown are of the same image. The spectrum on the left corresponds to the original image, and spectrum on the right was obtained after the image was padded with zeros. Explain the significant increase in signal strength along the vertical and horizontal axes of the spectrum shown on the right.

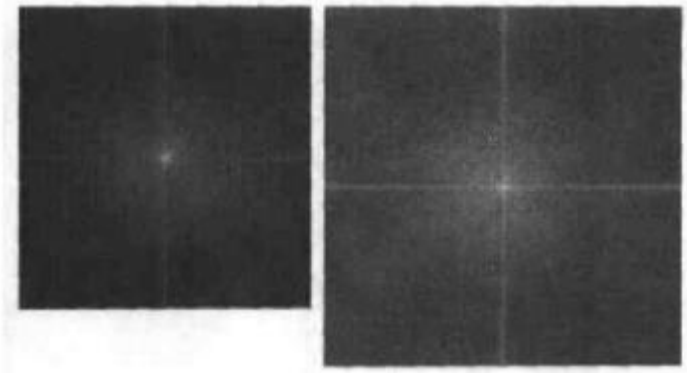

Figure 1

1.3 (10pts)Find the equivalent filter, H(u,v), that implements in the frequency domain the spatial operation performed by Laplacian mask in Fig 2.

0	1	0
1	−4	1
0	1	0

Figure 2

1.4 (10pts)Consider the images shown. The image on the right was obtained by: (a)multiplying the image on the left by $(-1)^{x+y}$; (b)computing the DFT; (c)taking the complex conjugate of the transform; (d)computing the inverse DFT; and (e)multiplying the real part of the result by $(-1)^{x+y}$. Explain (mathematically) why the image on the right appears as it does.

Figure 3

1.5 (10pts)Let f (x, y) be the original image and g(x,y)=f(x,y)+η(x,y) be the image with random noise η(x,y). Given the filter formula of inverse harmonic filter:

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Explain why when parameter Q > 0, pepper noise (isolated black spot) can be removed, but salt noise (isolated white spot) is ineffective (even worse).

1.6 (20pts)1) Brief description of the relationship between frequency domain spectrum image and image spatial features;

2) Figure 4 is a centralized spectrogram, indicating which information of the original spatial image corresponds to the regions indicated by the two arrows.

3) Figure 5 is an intensity image of a filter in the spectrum domain. Please indicate whether it is a low-pass filter or a high-pass filter.

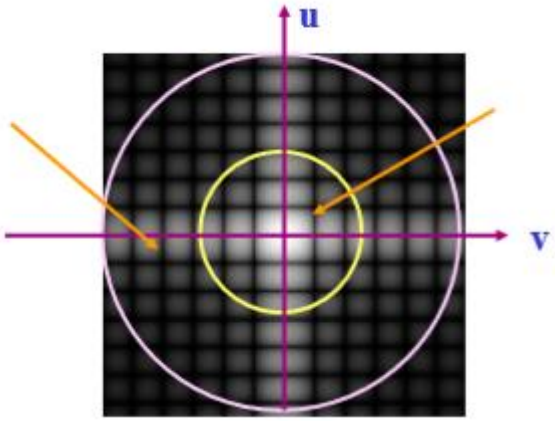

Figure 4

Figure 5

2 (30pts) Programing: Fast Fourier Transform(30pts)
You are required to manually implement the Fast Fourier Transform (FFT). The function prototype is "fft2d(input_img, flags)→ output_img", returning the FFT / IFFT result of the given input. "flags" is a parameter to specify whether FFT or IFFT is required. "fft2d" should produce every similar results in comparison to"dft2d". However, your implementation may be limited to images whose sizes are integer powers of 2. We recommend you handle this problem by simply padding the given input so as to obtain a proper size. For the report, please load your input image and use your program to:

1. **Perform FFT and manually paste the (centered) Fourier spectrum on your report. (10 Points)**
2. **Perform IFFT on the result of the last question, and paste the real part on your report. (10 Points)**
3. **Explain why does FFT hove a lower time complexity than DFT. (5Points)**

4. **Detailed discuss how you implement FFT/IFFT in less than 2 pages. (5Points)**