

# Binary search tree rotations

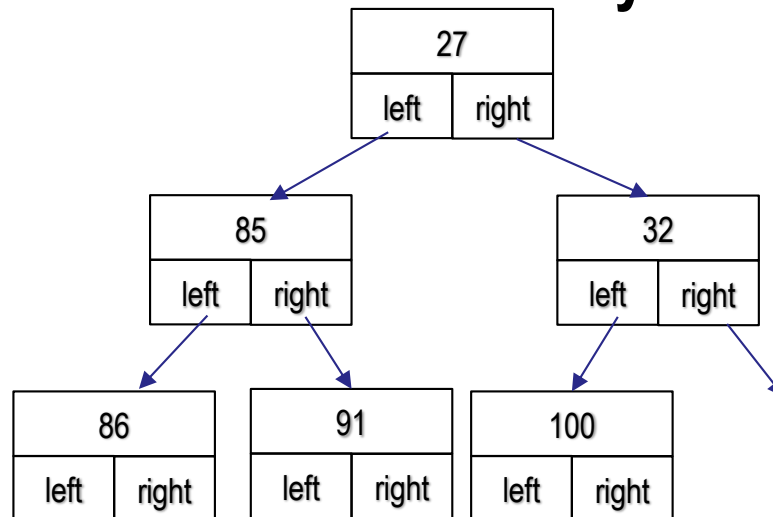
- **Constantly doing binary tree rotations can be**
  - ▶ **Costly**
  - ▶ **More error prone, just for the complexity of the rotations**
  - ▶ **Limiting in multi-threaded trees**
- **Rotations can lead to**
  - ▶ **A balanced tree, which makes searches faster**
  - ▶ **Balance robustness relative to the order in which data is added to the tree**

# Min-Heap

- **A heap is a data structure that allows you to remove the smallest element efficiently.**
  - ▶ Not intended for you to search for elements
  - ▶ Not intended for you to remove arbitrary elements (although you can get the code to do it)
- **There is a variant called a Max-Heap to let you remove the largest element efficiently.**

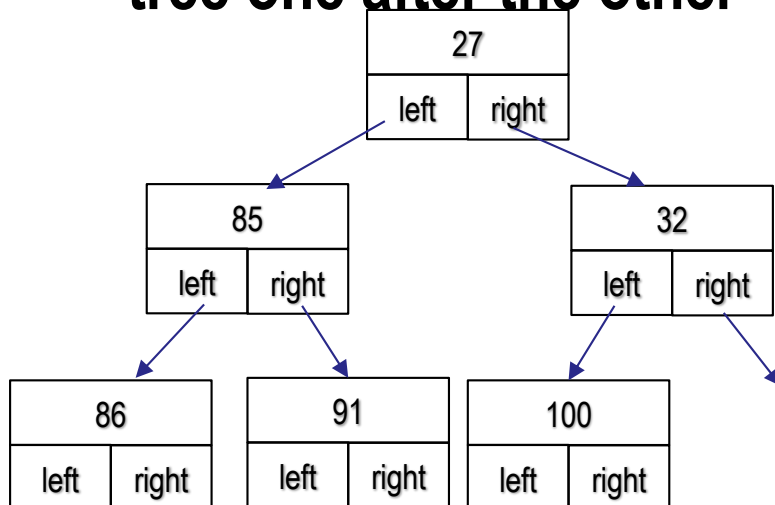
# Min-Heap

- Most often described / modeled as a binary tree:
  - ▶ The parent is smaller than both children
  - ▶ The binary tree remains balanced
  - ▶ The children are not necessarily stored in any particular order



# Min-Heap

- The binary tree thought, as a complete binary tree, is often implemented through an array:
  - ▶ Store the root at array index 1
  - ▶ The children of node at index  $x$  are found at indices  $2x$  and  $2x+1$
  - ▶ The array looks like you have stored the levels of the binary tree one after the other

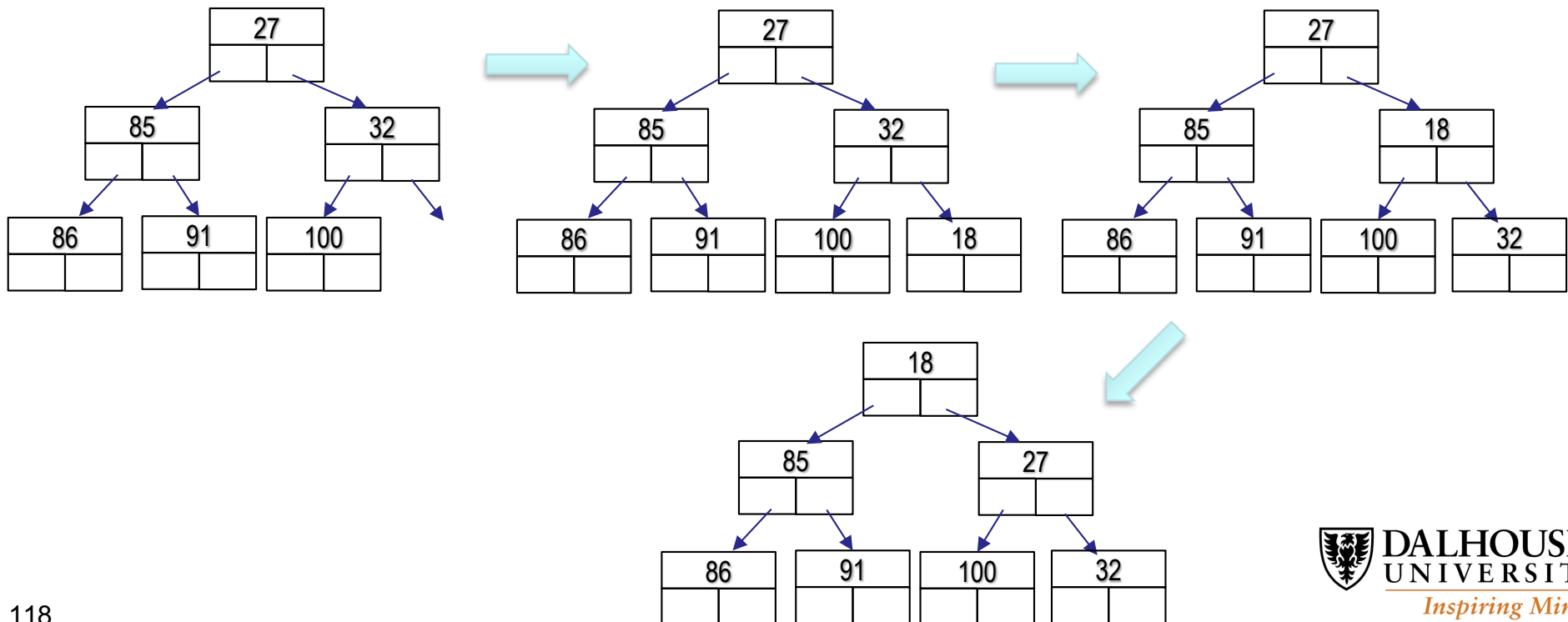


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# Min-Heap

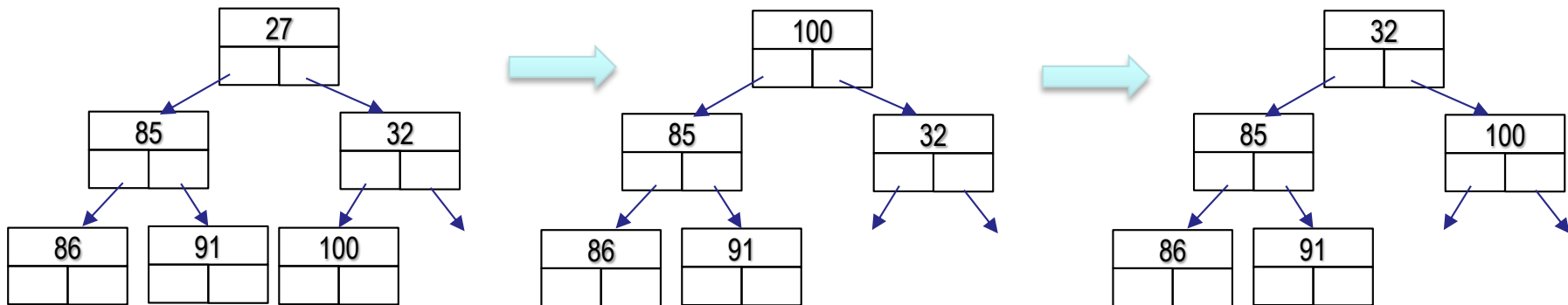
- To add an item:

- ▶ Store it in the next spot in the bottom level
- ▶ Continually swap it with its parent if it is smaller than the parent



# Min-Heap

- To remove an item:
  - ▶ Remove the top-most item.
  - ▶ Move the last item in the lowest level to the top
  - ▶ Continually compare this moved item to its two children and swap it with its smallest child



# Min-Heap

- **A min-heap is a common implementation of a priority queue.**

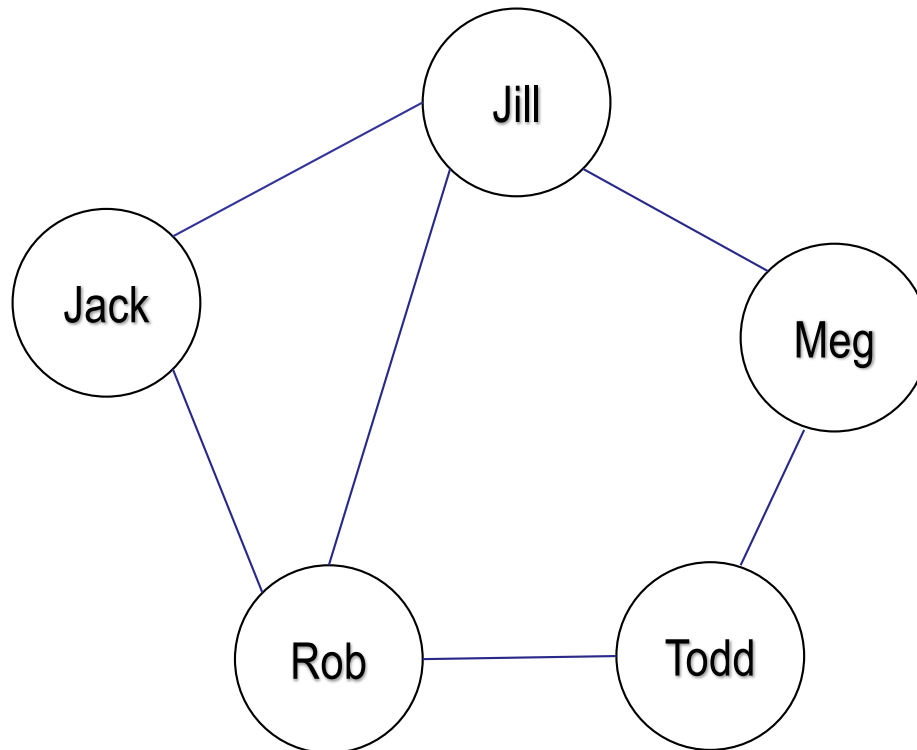
# Graph

- An ADT for a relations between elements.
- Defined by a set of vertices  $V$  and a set of edges  $E$  where  $E$  captures the relations (subset of  $V \times V$ )
  - ▶ Operators include add/remove vertex or edge, access adjacent vertex, test if an edge exists, and traverse
- Example:
  - ▶ Graph of people who know one another
    - $V$  = set of people,  $E$  = edges between people who know each other



# Graph

- Graph of people who know each other



# Sample uses of graphs

- Finite state machine
- Transition model representation
- Process flow
- Computer network topology
- Neural network
- Language structure description
- ...

# Graph representation

## ● What data structures let you store a graph?

### ▶ Adjacency matrix

- Have a  $|V| \times |V|$  matrix, with each element of  $V$  represented in a row and a column
- Put a value of 1 in the matrix where the element in the row and the column are related

### ▶ Incidence matrix

- Have a  $|V| \times |E|$  matrix, with each element of  $V$  represented in a row and each edge represented by a column
- Put a value of 1 in the matrix where the row is an endpoint of the edge that is given by the column

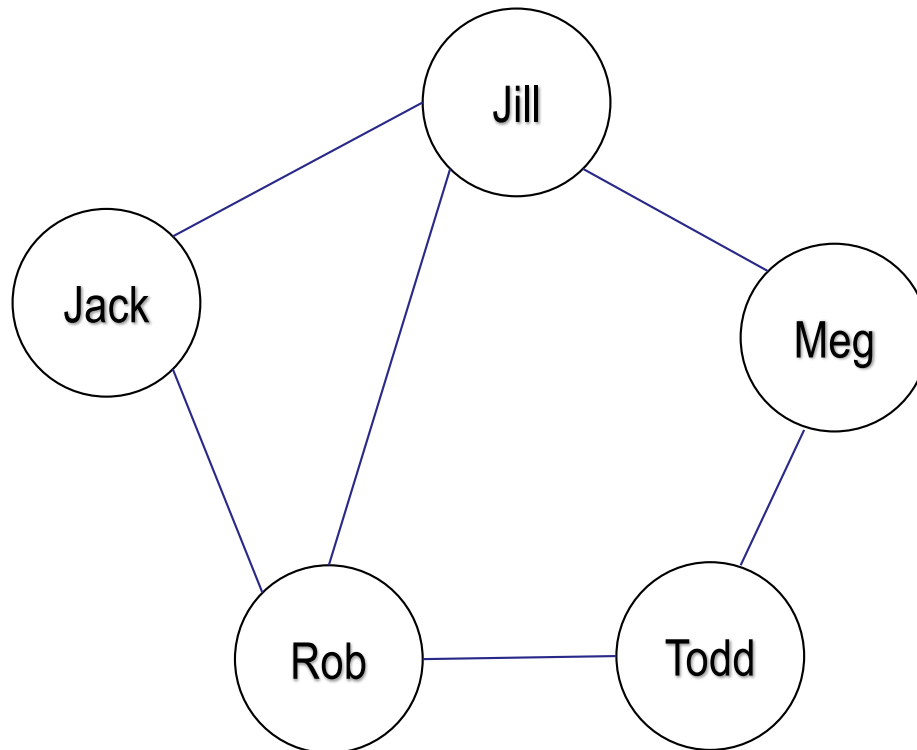
### ▶ Adjacency list

- For each vertex, store the list of vertices to which it is related

## ● Why choose one over another?

# Graph

- Graph of people who know each other



# Adjacency Matrix

	Jack	Jill	Meg	Todd	Rob
Jack	0	1	0	0	1
Jill	1	0	1	0	1
Meg	0	1	0	1	0
Todd	0	0	1	0	1
Rob	1	1	0	1	0

# Incidence Matrix

	E1	E2	E3	E4	E5	E6
Jack	1	1				
Jill	1		1	1		
Meg				1	1	
Todd					1	1
Rob		1	1			1

# Adjacency List

Vertex	Neighbours
Jack	{Jill, Rob}
Jill	{Jack, Meg, Rob}
Meg	{Jill, Todd}
Todd	{Meg, Rob}
Rob	{Jack, Jill, Todd}

# Types of Graphs

## ● Edge influence

- ▶ Undirected: you can traverse an edge in any direction
- ▶ Directed: there is just one way by which you can follow an edge

## ● Connectivity

- ▶ Connected: you can get from one vertex to any other vertex
- ▶ Unconnected: there are some vertices that can't reach one another



# Types of Graphs

## ● Edge multiplicity

- ▶ Simple: there are no edges back to the same vertex (a loop) and at most one edge between pairs of vertices
- ▶ Multi-edge: you can have loops and several edges between the same pair of vertices.

## ● Edge weight

- ▶ Unweighted: no values associated with edges
- ▶ Weighted: edges can have a “weight”, either as a cost to traverse, a number of times that you can use it, a capacity for the edge, ...

# Common graph problems

- **Traversals (depth-first or breadth-first)**
- **Shortest path (Dijkstra's algorithm)**
- **Graph cycle detection (variant of depth-first search)**
- **Minimum spanning tree – get rid of cycles**
  - ▶ Prim's algorithm, Kruskal's algorithm
- **Connectivity**
- **Network flow**
- **Topological sorting**