

CS648 : Randomized Algorithms

CSE, IIT Kanpur

Practice sheet 3: **Duration of a randomized experiment**

1. **Coupon Collector Problem**

Recall the coupon collector problem with n different coupons. What is the expected number of coupons to be collected to have $0.99n$ different coupons ?

2. **A biased random walk**

Recall the random walk on a line discussed in the class. Suppose, probability of taking a step in the right direction is $3/4$ and the probability of taking a step in the left direction is $1/4$. What is the expected number of steps till the particle reaches n th milestone ?

3. **Random walk on a complete graph**

We can extend the notion of random walk on a line to a graph in a very natural manner. Suppose we are at a vertex v . If u_1, \dots, u_t are the neighbors of v . The particle selects a vertex randomly uniformly from $\{u_1, \dots, u_t\}$ and moves to that vertex in the next step.

We are given a complete graph G on n vertices (there is an edge between each pair of vertices). Consider a random walk in G starting from a vertex u .

(a) Let v be a vertex other than u . What is the expected number of steps of the random walk to visit v ?

(b) What is the expected number of steps to visit each vertex of the graph at least once ?

(c) What is the expected number of steps to visit half of the vertices ?

4. **A variant of client server problem**

Consider a parallel computer consisting of n processors and n memory modules. In the first step, each processor sends a memory request to a memory module selected randomly uniformly. If more than one processor sends request to the same memory module, the memory module discards all of them. If a memory module receives only one request, that request gets satisfied. After the first step, all those processors whose memory request was satisfied, leave the system. The remaining processors follow the same protocol in the following round. What is the expected number of rounds when all the memory request have been satisfied ?