CS648A : Randomized Algorithms
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How well did you internalize the proof of Chernoff Bound?

Consider a collection $X_1, \ldots, X_n$ of n independent geometrically distributed random variables with expected value 2. Let $X = \sum_{i=1}^{i=n} X_i$ and $\delta > 0$.

1. Derive a bound on $\mathbf{P}(X \geq (1+\delta)(2n))$ by applying the Chernoff bound to a sequence of $(1+\delta)(2n)$ fair coin tosses.
2. Directly derive a Chernoff like bound on $\mathbf{P}(X \geq (1+\delta)(2n))$ from scratch.
3. Which bound is better?

Estimating all-pairs distances exactly

Consider an undirected unweighted graph G on n vertices. For simplicity, assume that G is connected. We are also given a partial distance matrix M : For a pair of vertices i, j the entry $M[i, j]$ stores exact distance if i and j are separated by distance $\leq n/100$, otherwise M stores a symbol # indicating that distance between vertex i and vertex j is greater than $n/100$. Unfortunately, there are $\Theta(n^2)$ # entries in M, i.e., for $\Theta(n^2)$ pairs of vertices, the distance is not known. Design a Monte Carlo algorithm to compute exact distance matrix for G in $O(n^2 \log n)$ time. All entries of the distance matrix have to be correct with probability exceeding $1 - 1/n^2$.