Inventory Management Using Receding Horizon Method

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***Abstract*—This simulation is designed to use receding horizon method for inventory management. There will be 4 tasks in this simulation, and each part will be discussed separately.**

I. Introduction

Receding horizon control (RHC), also known as model predictive control (MPC), is a nonlinear control strategy that deals with input constraints, output constraints, and various control objectives. At present, RHC has been successfully applied to industrial process control, supply chain management and random control in the financial field [1].

The inventory management system minimizes total operating costs (including warehouse costs, short of stock penalties and return costs) by using RHC to reasonably calculate the order number y under the premise of using the optimal re-order level r to avoid investment risks.

The demand for this product is a random variable with a determined probability distribution.

This simulation tries to find the optimal combination of the control and prediction horizon that control the variable order number y to minimize the operational cost.

II. Task 1

Without simulation, selecting an order number at optimal fixed level subjectively.

The optimal fixed level r obtained from assignment 1 is 1. Therefore, the reasonable range of corresponding order number y is 3 or 4. Because the probability of demand is greater than 4, the probability is only 0.04. The short of The mathematical expectation of the stock penalty is only 0.8 coins per week.

So subjectively speaking, the optimal combination is (y,r): (3,1), (4,1). So I choose order number 3 and re-order stock level 1 as my simulation data.

III. Task 2

A. The theoretical knowledge

Receding horizon control (RHC) can be described as a constraint optimization problem solved repeatedly online based on current and past measurement outputs. It is actually a state feedback control.

At each discrete sampling moment, the RHC uses the current state of the system as the initial condition to solve an open loop optimal control problem in finite time domain online, and the optimal control sequence is obtained. And at this time, only the first control signal in the process of the optimal control sequence is actually applied to the system. At the next sampling time, the above process is repeated over time. RHC is an effective control method for systems with control and output horizon.

The RHC procedure works as follows. At time t_0, considering a time interval extending N steps into the future.

Step 1: Predicton

At time t_0 , use the state x_{t0} and the system model to predict the system output x_t as a function of u_t, where $t_0 \leq t < t+N$:

$$\overrightarrow{x_{t_0}} = G\overrightarrow{u_{t_0}} + Hx_{t_0} \tag{1}$$

Step 2: Optimisation

To find u_{t0} to minimise the cost function with constraints. This is actually a quadratic programming (QP) problem.

Step 3: Implement the control input.

Step 4: Repeat the above steps.

These are the basic steps for the RHC.

B. Simulation

In this task, the RHC method is applied. This simulation plan uses control horizon N_u and prediction horizon N_y for constraints. The goal is to obtain an optimal value in the weekly order number y decision, so as to minimize the total cost of 52 weeks. In this simulation, 100 simulations were set up to observe the distribution of the total cost average, and 200 episodes were performed in each simulation to reduce the accidental error of a single simulation.

Assuming control horizon N_u = 3 and prediction horizon N_y = 4, optiaml fixed level r = 1 (obtained in Assignment 1). The simulation results using the RHC method are shown in the left figure of Figure 1, and the right figure is the results obtained using Monte Carlo simulation in Assignment 1.

The mean of total cost of RHC method is 410.74.

The variance of total cost of RHC method is 27.73.

It can be seen that, compared with Monte Carlo simulation, the mean value of the total cost predicted by the RHC method is basically the same, but the variance is much smaller, which proves that the predicted value obtained by the method is more reliable and stable.

The reason for this difference may be that the RHC method uses current and past measurement outputs to solve the optimization problem repeatedly each week, thus obtaining the optimal order number y for the current week to minimize the cost function.

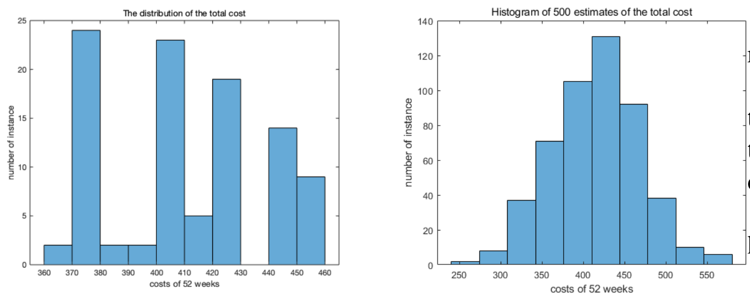

Fig. 1. The distribution of total cost

Using the variable order number y can obtain the optimal solution at the current stage, so the total cost of the entire system is controlled near the limit.

It should be emphasized that the optimization in RHC method is usually a kind of moving optimization with limited time. Therefore, predictive control is not a global optimization, but a relative optimization at each moment, which is different from the monte carlo simulation. Moreover, optimization is not done offline at once, but repeatedly online. In other words, RHC is bootstrapped, while Monte Carlo simulation is not bootstrapped.

IV. Task 3

In order to explore the influence of different control horizon and prediction horizon on the total cost, different combinations of control horizon and prediction horizon were applied to this model in this task, and each combination ran 200 episodes. The results are shown in figure 2.

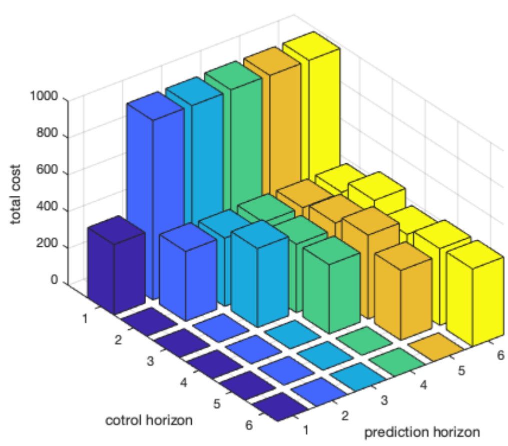

Fig. 2. Total cost under different horizon

As can be seen from the Figure 2, in this simulation, the lowest total cost can be obtained when the control horizon N_u = 3 and the prediction horizon N_y = 4 are used as constraints.

V. Task 4

This simulation basically reached the objectives.

In task 1, order number y was chosen subjectively and the reason was explained.

In task 2, the receding horizon method was used to obtain the current number of optimized order number y per week to reduce the total cost. The receding horizon method is compared with the Monte Carlo method.

In task 3, the effect of different combinations of control horizon and prediction horizon on the total cost is discussed.

But in the whole work, there are some problems for RHC method:

1. Obviously, the RHC method depends on the process model. Therefore, the cumulative effects of inaccurate models and unmeasured interference may lead to inaccurate predictions. This can also be seen from the figure 1.

2. In actual operation, the optimization problem must be solved at each time step. Using traditional numerical optimization techniques, the time required to solve this problem is usually much longer than the time required to calculate control.

3. The RHC method is very sensitive to the selection of control horizon and prediction horizon.

Overall, RHC can obtain a more stable cost forecast, but its limitations are also very obvious. It turns out that the RHC is not the best way to get the lowest cost. In the RHC, the estimated value of the finite horizon is used to replace the average cost of the infinite horizon. Moreover, the RHC does not solve the problem of controller design, which is a thorny non-convex problem with infinite dimensions. However, RHC is a complex heuristic that can be applied to many technologies.

In future research, bias correction can be used to reduce the inaccurate prediction caused by the accumulation of inaccurate models and unmeasured interference. And, in fact, the RHC is a convex optimization problem [2], which means convex optimization tools can be used to solve local optimal solutions to non-convex objectives and constrained problems.

References

[1] J. Mattingley, Y. Wang, and S. Boyd, "Receding horizon control," *IEEE Control Systems Magazine*, vol. 31, no. 3, pp. 52–65, 2011.

[2] S. Boyd and L. Vandenberghe, *Convex Optimization*. USA: Cambridge University Press, 2004.