Monte Carlo Simulation in Inventory Management

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***Abstract*—This simulation is designed to use Monte Carlo methods for inventory management. This simulation is divided into five parts, and each part will be discussed separately.**

I. Introduction

The Monte Carlo method is a numerical calculation method guided by the theory of probability and statistics, and usually uses random numbers to approximate numerical solutions. As a commonly used simulation technology, it has been widely used in the process of quantitative risk analysis in the field of risk management knowledge. This method can produce good results for quantitative analysis in inventory management through scenario generation and what-if analysis. [1]

This inventory management system minimizes the total operating cost, which is composed of warehouse cost, short of stock penalty and return cost, by calculating the inventory level and ordernumber to a reasonable degree to avoid investment risks.

The demand for this product is a random variable with a determined probability distribution.

This simulation tries to find the optimal combination of the order number y and re-order stock level r to minimize the operational cost.

II. Taks 1

Without simulation, judge a reasonable order number and re-order stock level from the subjective judgment.

The reasonable range of order number y is 3 or 4. The reasonable range of corresponding re-order stock level r is 0 or 1. Because the probability of demand is greater than 4, the probability is only 0.04. The short of The mathematical expectation of the stock penalty is only 0.8 coins per week.

So subjectively speaking, the optimal combination is (y,r): (3,0), (3,1), (4,0). So i choose order number 3 and re-order stock level 1 as my simulation data.

III. Task 2

According to the choice of task 1, the order number and re-order stock level of this simulation are 3 and 1. The Monte Carlo simulation uses random numbers, so for Monte Carlo simulation, we use ndom number generator MATLAB: rand () to simulate the sub-model of weekly demand. The simulation process is shown in Table 1.

Because the random number generated by this function obeys a uniform random distribution between 0 and 1, it can simulate a probability space of 0 to 1. Therefore, we divide it into 6 parts according to different probabilities of weekly demand, and the interval of each part represents the given probability. Then use this method to generate different demands every week, and based on this, calculate the operating cost under the given rules.

TABLE I
Generate demand probability using random numbers

Random Number	Demand	Prob	Random Number	Demand	Prob
0-0.04	0	0.04	0.80-0.96	4	0.16
0.04-0.12	1	0.08	0.96-0.98	5	0.02
0.12-0.40	2	0.28	0.98-1.00	6	0.02
0.40-0.80	3	0.4			

After 500 simulations, the distribution of 500 operations is shown in Figure 1.

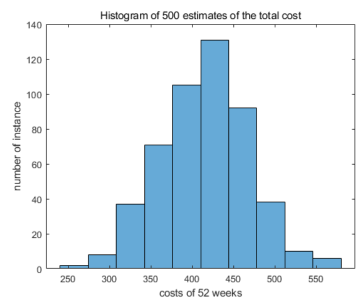

Fig. 1. Histogram of 500 estimates of the total cost

A. Simulation Result

The average of 500 estimated operational cost is 414.50.
The variance of 500 estimated operational cost is 53.45.

IV. Task 3

To use Monte Carlo method to find the optimal order number y and re-order stock level r to minimize the total cost.

In this simulation, we need to iterate through all (y, r) combinations (a total of 49 cases). The method we use here is to simulate a complete 52-week process under each combination. Find the minimum total cost in each simulation, and record the value of order number y and re-order stock level r corresponding to the cost. After N simulations, an optimized combination set is generated, and then the combination set with the most occurrences is found in the set, and the value of this set is selected as the best order number y and re-order stock level r.

Here we choose N as 50 and repeat the simulation 1000 times to increase the accuracy of the simulation. The simulation results are shown in Figure 2. After 1000 simulations, the distribution of 1000 operations is shown in Figure 1.

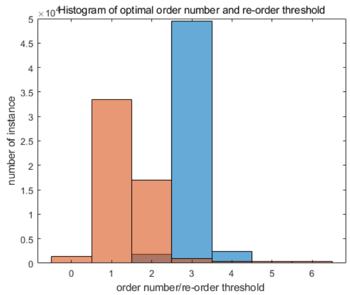

Fig. 2. Histogram of 500 estimates of the total cost

The blue histogram represents the distribution of the optimized order number y, and the orange histogram represents the distribution of the optimized re-order stock level r.

Therefore, the optimal (y, r) combination can be obtained by Monte Carlo method as (3,1), which is basically consistent with subjective judgment. However, using the Monte Carlo method, the better (y, r) combination is (3,1), (3,2), and there is no previous subjective prediction (4,0).

V. Task 4

In the method of task three, we solve the optimization problem by establishing an optimal combination set and selecting most of them.

In this simulation, when N is 50, the combination (y, r) of order number y and re-order stock level r is (3, 1), and the probability of majority combination is about 98%. Repeat this process 1000 times to get the probability distribution of most combinations in the best combination. Its distribution is shown in Figure 3.

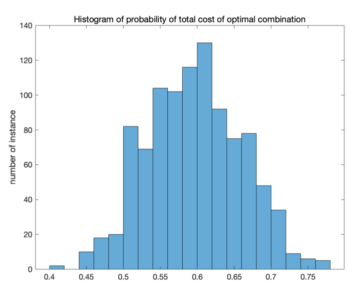

Fig. 3. Distribution

Here, we construct it into a standard normal distribution according to the central limit theorem, as shown in equation (1).

$$\frac{\bar{x} - X}{\delta/\sqrt{n}} \to N(0,1) \tag{1}$$

$$(L(x), U(x)) = (\bar{x} - Z_{\frac{1-\alpha}{2}} \frac{\delta}{\sqrt{n}}, \bar{x} + Z_{\frac{1-\alpha}{2}} \frac{\delta}{\sqrt{n}}) \tag{2}$$

Equation (2) represents the confidence interval of the distribution.

When we choose a confidence level of 99%, the confidence interval for this distribution is shown in Figure 4.

3×5 table

	MEANS	STANDART_DERIVATIONS	CONFIDENCE_LEVELS	LOWER_BOUNDS	UPPER_BOUNDS
ORDER NUMBER	3.0043	0.28487	90%	3	3
STOCK NUMBER	1.3743	0.77715	90%	0.92643	1.0736
MINIMUM COST	415.48	7.6496	90%	414.92	416.04

Fig. 4. Confidence intervals for y, r and cost

From the figure we can see that under the condition of N = 50 and 90% confidence level, the confidence interval of order number y is (3, 3.0043) and the confidence interval of re-order stock level is (0.9264, 1.3743). The confidence interval for the total cost is (414.92,415.48). So we can say that our conclusion in task three is very credible.

In order to increase the confidence level, we can expand the confidence interval or reduce the variance of the distribution, but because the Monte Carlo method uses random numbers for simulation, here we choose to expand the confidence interval method to improve our confidence level. The results are shown in Figure 5.

3×5 table

	MEANS	STANDART_DERIVATIONS	CONFIDENCE_LEVELS	LOWER_BOUNDS	UPPER_BOUNDS
ORDER NUMBER	3.0043	0.29339	99%	3	3
STOCK NUMBER	1.3832	0.79126	99%	0.88462	1.1154
MINIMUM COST	415.62	7.7669	99%	414.72	416.51

Fig. 5. Confidence intervals for y, r and cost

We can see that with a small amount of confidence intervals, our confidence level has increased to 99%, which shows that this method of increasing the confidence level is completely available in this simulation.

VI. Task 5

In this project, we used Monte Carlo methods to simulate inventory management and operating costs, and evaluated our results based on confidence levels and confidence intervals.

However, because the rand () command in MATLAB actually generates a pseudo-random number, the simulation results still have some limitations. Moreover, in this simulation, we only used one method, and did not use more methods to compare and test, so we were not sure whether this method was the optimal solution.

To improve the accuracy of the model, we need a test set to validate the model. But because the weekly demand is random during the simulation, there is no test set to validate the model. Therefore, if there is a standard test set, the accuracy of the model will be improved to a certain extent.

In practice, the situation will be more complicated, and we need to consider more parameters and make stricter assumptions. For example, the product's self-loss situation and time cost.

References

[1] G. E. Evans and B. Jones, "The application of monte carlo simulation in finance, economics and operations management," in *2009 WRI World Congress on Computer Science and Information Engineering*, vol. 4, March 2009, pp. 379–383.