One Small Assignment on Centipede Game

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Consider some extension on centipede game's payoffs. In three-rounds-form game, we can design a game satisfying that backward induction outcome (BIO) is both P1 and P2 choose the strategy 'continue' while there exist one Nash Equilibrium (NE) that P1 chooses 'stop' at first period. At the same time, it is proved that it is impossible to design a game that BIO contains P1 chooses 'stop' while there exist a NE that both players choose 'continue'.

I. INTRODUCTION

Centipede game, first introduced by Robert Rosenthal in 1981, is an extensive form game in which two players take turns choosing either to take a slightly larger share of an increasing pot, or to pass the pot to the other player. When it is passed to her opponent who could take the pot, as a result, the first player could only get lower payoff than last round. However, if her opponent chooses pass, the first player's payoff will increase if she choose to take the pot. Generally, although at each round one player has incentives to leave, they are better to wait. In this assignment, we change the payoff settings in order to get the expected result.

II. SOLUTIONS

A. Norm-Form Centipede Game

Taking the norm-form centipede game as an example, the game tree is shown in Figure 1.

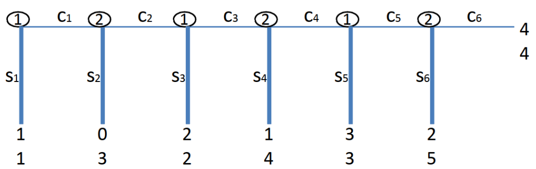

FIG. 1: Norm Form Centipede Game

When using the backward-induction we can get the results: $s_6 \to s_5 \to s_4 \to s_3 \to s_2 \to s_1$

Obviously, both players choose to defect on their turns. There are a large number of NEs in a centipede game, but in each, the first player defects on the first round. However, defection by the first player is the unique sub-game perfect equilibrium and required by any NE. Note that being in a NE does not require the strategies be rational at every point in the game as in the sub-game perfect equilibrium, i.e. those strategies that are cooperative in the never-reached later rounds of the game could still be in a NE.

B. Three Rounds Centipede Game

Figure 2 gives one toy example of three rounds centipede game. Player1 chooses to play either 'stop' or 'continue' in the first round, Player2 has the same action space in the second round, then back to Player1. The game finishes after third round.

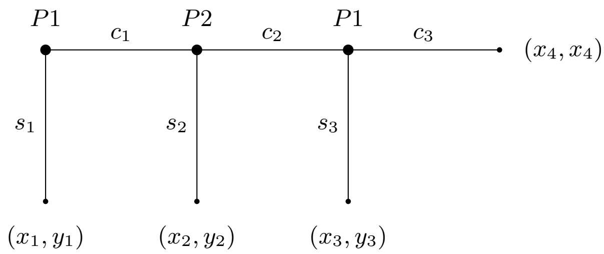

FIG. 2: Instance of Centipede Game with Uncertainty

Now we explore the first problem: constructing players' payoff such that the BIO is $\{c_1, c_2, c_3\}$ and there exist one NE contains s_1. Using the backward-induction, we can construct the game's payoff as Figure 3:

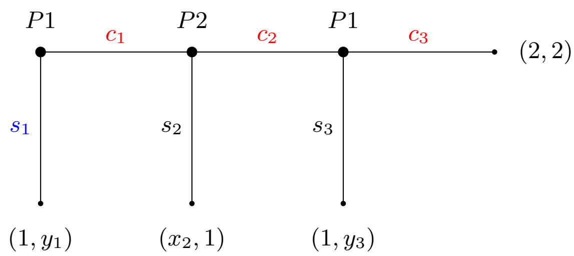

FIG. 3: Payoff Satisfying BIO

Here actions marked with red consist of the BIO and action s_1 is marked with blue represent we are going to find one NE contains it. Fixed s_1, it is obvious that there are four different types of NE's strategies form the possible NE set $\mathcal{T}$: $\mathcal{T} = \{\{s_1, c_2, c_3\}, \{s_1, c_2, s_3\}, \{s_1, s_2, c_3\}, \{s_1, s_2, s_3\}\}$. They are denoted as $\tau_1, \tau_2, \tau_3, \tau_4$. Next we will analyse them respectively. First consider $\tau_1 = \{s_1, c_2, c_3\}$. Fixed Player1's actions s_1, c_3, it is not hard to verify that Player2 has no motivation to deviate from the current status. Since Player1 chose s_1 in first round, no matter what action Player2 takes will not increase her utility. On the other side, fixed Player2's action c_2, we can find Player1 has incentive to deviate from s_1 because choosing c_1 and c_3 will get 2 bigger than 1. Thus, τ_1 does not satisfy our setting. Similarly we can verify the other three cases in turn. Result is shown in Table I.

Possible NE strategies	Whether exist or not?
$\tau_1 = \{s_1, c_2, c_3\}$	No, P1: $s_1 \to c_1$
$\tau_2 = \{s_1, c_2, s_3\}$	No, P1: $s_1, s_3 \to c_1, c_3$
$\tau_3 = \{s_1, s_2, c_3\}$	Yes, When $x_2 < 1$
$\tau_4 = \{s_1, s_2, s_3\}$	Yes, when $x_2 < 1$

TABLE I: Table of Possible NE strategies

We can see that when $x_2 < 1$, τ_3 and τ_4 could be NE with s_1 under the BIO $\{c_1, c_2, c_3\}$.

Next task we will consider the reverse one, BIO contains s_1 and $\{c_1, c_2, c_3\}$ is a NE. The possible BIO set is defined as $\mathcal{O} = \{\{s_1, c_2, c_3\}, \{s_1, c_2, s_3\}, \{s_1, s_2, c_3\}, \{s_1, s_2, s_3\}\}$. They are denoted as $\alpha_1, \alpha_2, \alpha_3, \alpha_4$. Table II shows the possible BIO and corresponding answer about whether $\{c_1, c_2, c_3\}$ is a NE or not.

Possible BIO strategies	$\{c_1, c_2, c_3\}$ is NE or not?
$\alpha_1 = \{s_1, c_2, c_3\}$	No, P1: $c_1 \to s_1$
$\alpha_2 = \{s_1, c_2, s_3\}$	No, P1: $c_1 \to s_1$
$\alpha_3 = \{s_1, s_2, c_3\}$	No, P2: $c_2 \to s_2$
$\alpha_4 = \{s_1, s_2, s_3\}$	No, P1: $c_3 \to s_3$

TABLE II: Whether $\{c_1, c_2, c_3\}$ is a NE

It is shown that it is impossible that $\{c_1, c_2, c_3\}$ can be a NE under the BIO contains s_1. Details for different payoff in all different possible BIOs can be induced by the means like that in Figure 3. Here we will not illustrate them one by one.

C. Four Rounds Centipede Game

Now we consider the centipede game's payoff setting under four rounds as shown in Figure 4.

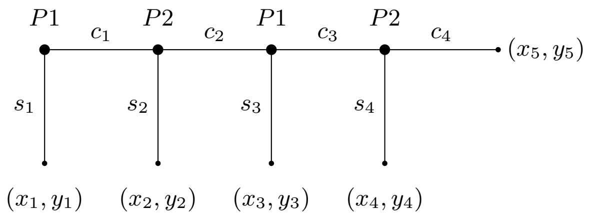

FIG. 4: Four Rounds Centipede Game

This Time the problem changes, on one hand, we first consider whether there exist a NE where Player 2 chooses s_2 while the BIO is $\{c_1, c_2, c_3, c_4\}$; also, the reverse one on the other hand.

The former case is similar with that in three rounds game. Under $\{c_1, c_2, c_3, c_4\}$ BIO, there exist NE that satisfies Player 2 chooses s_2.

There are eight cases of BIO contains s_2 and we can verify them respectively, finally you will find that it is also impossible for the existence of NE($\{c_1, c_2, c_3, c_4\}$) under the BIO contains s_2.

III. CONCLUSIONS

This small task is given by Professor Qiao in my game theory course. It is interesting to induce the restrictions under the centipede game and construct the payoff leading to expected result.