Production and outbound distribution scheduling: a two-agent approach

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***Abstract*—The problem considered is an integrated production and distribution scheduling problem with two agents: a manufacturer and a third-party-logistics provider. The manufacturer workshop is a flow-shop and the problem is to minimize a cost function composed by inventory, vehicle and tardiness penalty costs. The 3PL provider solves a vehicle routing problem for each vehicle, to minimize a total cost composed by routing costs and tardiness penalties paid to the manufacturer. A general framework is presented and a particular scenario is considered, where the manufacturer dominates and the 3PL provider adjusts. MILP models are proposed and illustrated by a numerical example.**

I. Introduction

In todays business environment, defined by globalisation, fierce competition and increasing customer demands, companies are forced to find innovative ways in order to keep costs down and stay competitive while providing high quality of service for their customers. The complexity and interconnectedness of this globalised environment makes planning and logistics important tasks that can cut costs and ensure smooth operations for a business, considering that inventory and transportation/distribution costs make up a substantial amount of a company's cost function.

Supply chain management is concerned with the coordination and planning of the supply chain of a business and considers all activities that are involved in the production of a product/service from the supplier up to the delivery to the customer. Supply chains are generally very complex networks consisting of several stages, links and players, however, the main two areas of focus in supply chain management are production and distribution planning.

The goal is to optimise the planning decisions for both production and distribution, such as scheduling and routing decisions, according to a set of objectives. While these two areas are traditionally dealt with individually, they are highly interconnected and more recent considerations have shown that integrated supply chain approaches, that consider both production and distribution decisions at the same time, hold various benefits for organisations both in terms of cost as well as time management. The objective of these considerations is to find an optimal global solution to the supply chain scheduling problem for all agents by integrating the two individual problems of production and distribution scheduling into one model. Over recent years, such approaches have been more and more covered in the literature. However, one of the difficulties associated with this is the complexity of the two individual models, often resulting in too simplified models.

The problem considered here is an integrated production and distribution scheduling problem at an operational level. The problem is considered as a multi-agent decision problem, consisting of two sub-problems: a permutation flow-shop scheduling problem for the production problem and a vehicle routing problem concerned with the minimisation of total tardiness in terms of the distribution problem. The model also includes the notion of inventory costs that occur during the production phase, in order to obtain a more realistic model. Inventory costs even though often neglected in production scheduling models play a major role in production planning. Two kinds of inventory are considered here: work in progress inventory and finished products inventory.

The paper is organised as follows. In Section II, we present a state-of-the-art survey on integrated production and outbound distribution scheduling. Section III formally describes the problem that we consider and introduces some notations. In Section IV we propose resolution methods: a mixed integer linear programming model and some (basic) heuristic algorithms. These methods are illustrated with an example. In Section V we present a coordination scenario where the manufacturer dominates and the 3PL provider adjusts. Section VI contains concluding remarks as well as some directions for future research.

II. Literature review

While there is a vast amount of literature available on the individual problems of production and distribution scheduling, the integrated approach of supply chain scheduling has only been covered to a greater extend in the literature more recently. Graves (1981) [11] for example gives a classification and early review of the different approaches to production scheduling problems. The classification is mainly done according to the machine environment. Chen (2004) [5] provides a classification for integrated models based on different criteria such as for example the structure of the integration. They differentiate between three different kinds of integration:

- Production and outbound transportation,
- Inbound transportation and production,
- Inbound transportation, production and outbound transportation.

The latter includes three stages in the supply chain: supplier, manufacturer and customers. Such problems are covered for instance by Sawik (2009) [17] as well as by Hall and Potts (2003) [12]. Inbound transportation and production scheduling focuses only on the two problems of supplier and production scheduling (see [1]). The focus of this paper however lies on production and outbound transportation models, i.e. models that comprise of a manufacturer and the delivery of the products to one or several customers. These problems have been covered in the literature by for example Chen and Vairaktarakis (2005) [6], Zhong et al. (2010) [23], Ullrich (2013) [20], Wang and Lee (2005) [21], Li et al. (2008) [15], Chang and Lee (2004) [4], Dawande et al. (2006) [9], Li et al. (2005) [14] and Chen et al. (2009) [7]. The individual approaches further differ depending on some criteria and constraints such as the machine environment and the objectives of the optimisation. Chen (2010) [8] provides a survey of existing integrated models, classifying them into several classes and giving an overview over the different approaches for each class, as well as proposes a unified notation and representation for such models while also identifying areas for future research. In addition to this, while some literature focuses more on the production side other papers pay more attention to the distribution side of the problem.

Li et al. (2005) [14] study a single machine scheduling problem with an integrated routing to a variety of different customers with the objective function based on the minimisation of the sum of the arrival times. They consider a number of special cases for which they propose good solution algorithms.

Stecke and Zhao (2007) [19] consider a problem for a make-to-order manufacturer and a 3PL provider, where the manufacturer operates in a single machine environment with capacity constraints and the goal to minimise total shipping costs. They consider a range of eight different distribution states and propose different solution models including linear programming and mixed integer programming models as well as a heuristic model. Low et al. (2013) [16] deal with an integrated production and distribution scheduling model in a single machine environment and delivery to the customer within time windows aiming to minimise the makespan. They develop a nonlinear mathematical solution model as well as two heuristics based on genetic algorithms for solving larger scale problems. Zhong et al. (2010) [23] study an integrated production distribution problem with a single machine environment and committed delivery dates for the manufacturer and a 3PL provider with a linear shipping cost function and fixed pick up times. In order to design a schedule and choose the delivery mode with the goal to minimise the total shipping costs as well as comply with the delivery dates they design a polynomial time heuristic. Chen et al. (2009) [7] consider the special case of an integrated model for perishable food products with decay rates taking into account the demand for the products, time windows for the delivery and capacity constraints of the vehicles. The proposed model is an integer nonlinear programming model determining the optimal quantities, production times and vehicle routing with the objective to maximise the expected total profit of the manufacturer.

Seyedhosseini and Ghoreyshi (2014) [18] also deal with perishable products with one production facility with limited production capacity and multiple customer locations taking into account customer demands and allowing for limited inventory. The cost for the routing is based on the number of trips rather than the distances travelled. The model is aimed at minimising the total costs by determining production quantities, product deliveries and stock management. A two-phase heuristic algorithm is developed in order to solve the problem.

Li et al. (2008) [15] cover a parallel machine assembly and 3PL provider problem with multiple customer destinations. The problem is composed of the two sub-problems and heuristics are developed for the assembly scheduling problem. Chen and Vairaktaratis (2005) [6] deal with integrated production distribution problems optimising the total distribution cost as well as the delivery times considering the case of average delivery times and maximum delivery times. In the paper, they provide both exact algorithms and heuristics for the problems and two machine configurations: single and parallel machine production facilities.

Chang and Lee (2004) [4] study a problem with different job sizes for delivery, covering three problem scenarios with different machine configurations, single and parallel machines and either one or two customer areas that are served by one vehicle with the aim of minimising the makespan. In their paper they design heuristics for the problems and show the performance of these.

Agnetis et al. (2014) [2] analyse an interstage delivery problem where the jobs are processed on a first machine and then delivered to a different manufacturing site where they are processed on a second machine with two transportation options. The research examines the differences between a scenario in which the manufacturer dominates and a scenario in which the 3PL provider dominates.

Chandra and Fisher (1994) [3] consider a single plant problem with multiple products that have to be delivered to several customer locations. The problem includes customer demands over certain periods of time that have to be met with a possibility of holding inventory. The goal is to minimise the overall costs comprising of production, distribution and inventory. They show through testing that the integrated problem can play a major role in reducing operational cost significantly.

Lei et al. (2006) [13] study a production, inventory and distribution routing problem. The problem consists of several plants with different capacity constraints and cost functions, producing a single product, and a number of customers with variations in demand and inventory capacities. The objective is to find an optimal schedule that minimises the operational costs and complies with the set of constraints associated with the facilities. The problem is solved according to a two phase methodology making use of a mixed integer programming model as well as a heuristic.

Fumero and Vercellis (1999) [10] consider an integrated approach with a production unit producing different kinds of products that have to be delivered to customer locations according to demand requirements. Vehicles with a limited capacity are available at the production site at all times and distribution incurs a cost based on a fixed fee, the distance and the load transported. The notion of inventory is possible in order to fulfil demands. A Lagrangean relaxation is proposed in order to solve the problem.

Wang (2012) [22] investigates two kinds of integrated production distribution problems with two different inventory considerations, one that only considers customer inventory and one that also considers intermediate inventory at the manufacturer. The production problem in both cases is a single machine problem with setup times; the jobs are then shipped in batches to the customers in the case a job is finished early or arrives early an inventory cost occurs. The objective is to minimise the total cost function of the problem.

Even though a variety of different problems have been studied in the literature over the last two decades most of the problems in the research outlined above cover single or parallel machine configurations for the production problem and do not take inventory into consideration for the decision making process. Problems that do include inventory generally only focus on one kind of inventory for example inventory of finished goods. However inventory occurs at various stages of the production process and the holding costs are an important cost factor that can have an impact on the optimality of the scheduling decision. As real world applications are generally more complex there is a need for more comprehensive integrated approaches, incorporating various types of inventory, such as for example intermediate and finished product inventory, into the model. In addition to this, one can differentiate between two types of objectives that are optimised in the integrated models covered in the literature, time and cost.

III. A TWO-AGENT MODEL

This research aims to propose an integrated model for a supply chain scheduling problem that includes several costs (inventory costs, transportation costs, penalty costs, ...) and which occurs at the operational level of the production and transportation phases. Inventory costs, even though often neglected in production scheduling models, play a major role in production planning. Here, two kinds of inventory are considered: work in progress inventory and finished products inventory.

The integrated production and distribution scheduling problem, considered here, is a multi-agent problem consisting of two sub-problems: a permutation flow-shop scheduling problem and a vehicle routing problem.

According to the five-field notation $\alpha|\beta|\pi|\delta|\gamma$ proposed in [8], the notation of the problem that we consider is $Fm||V(\infty,\infty), fdep, routing|n|\gamma$, where the field $\alpha = Fm$ means that we consider an m-machine flow-shop scheduling problem, β is empty, the field π contains $V(\infty,\infty)$ meaning that a sufficient number of vehicles are available and the capacity of each vehicle is unbounded, $fdep$ for *shipping with fixed delivery departure dates*, and $routing$ meaning that orders going to different customers can be transported in the same shipment. In the field δ we have n to indicate that each job belongs to one customer. γ is the objective function, detailed later in this section.

A. Production Problem

The production problem considered here is an m-machine permutation flow-shop with work in progress (WIP) and finished product inventory. We suppose that we have a set $\{J_1, ..., J_n\}$ of n jobs to schedule. A manufacturer is looking to design a production schedule for all jobs on the m machines, where each job J_j has a given processing time $p_{i,j}$ on machine M_i. With the execution of a job on a machine M_i there is a build-up of WIP inventory. The WIP inventory has to be then stored till the job is processed on machine M_{i+1} and the stock of WIP decreases between M_i and M_{i+1}. The holding cost for this work in progress inventory depends on the job and is denoted by h_j^{WIP} for job J_j. As the job is processed on machine M_m the inventory of finished products increases. After the completion of the job on machine M_m the finished product inventory is kept until the k^{th} departure date at which the products are collected for distribution, denoted by F_k. The holding cost for the finished product inventory again depends on the product, and is denoted by h_j^{FIN} for job J_j. Each job J_j has to be delivered to customer j for a given due date denoted by d_j. A delay in the delivery of the product generally results in a loss for the manufacturer, this loss can be both financial as well as in terms of reputation, hence the manufacturer incurs a penalty cost π_j^M for late delivery of J_j that is paid to the customer.

We denote by $C_{i,j}$ the completion time of job J_j on machine M_i, $1 \leq i \leq m$. D_j denotes the delivery completion time. The tardiness of job J_j for the manufacturer is $T_j^M = \max(0, D_j - d_j)$.

The objective of the manufacturer is to minimise his total cost. This cost is composed of an inventory cost, a transportation cost (related to the number of vehicles available), and penalty costs related to tardiness of delivery. At this step, the manufacturer does not know the delivery dates.

We denote by IC the total inventory cost. The work-in-process inventory is represented in Fig. 1 and the final inventory is represented in Fig. 2.

The expression of IC is the following.

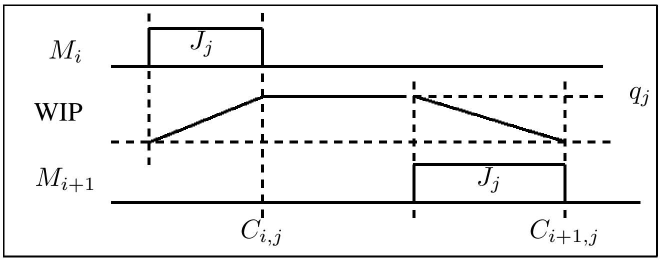

Fig. 1. Work-in-process inventory

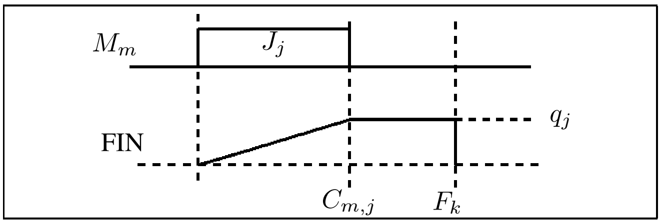

Fig. 2. Inventory of final products

$$IC = \sum_{j=1}^{n} \left(\sum_{i=1}^{m-1} \frac{1}{2}(p_{i,j} + p_{i+1,j})q_j + (C_{i+1,j} - p_{i+1,j} - C_{i,j}) q_j \right) h_j^{WIP} + \sum_{j=1}^{n} \left((F_k(j) - C_{m,j})q_j + \frac{1}{2}p_{m,j}q_j \right) h_j^{FIN}$$

where $F_k(j)$ denotes the departure time of J_j and q_j is the quantity of items of job J_j. Minimizing this expression is equivalent to minimize the following expression:

$$\begin{aligned} IC = &\sum_{j=1}^{n}(C_{m,j} - C_{1,j} - \sum_{i=2}^{m} p_{i,j})q_j h_j^{WIP} \\ &+ \sum_{j=1}^{n}(F_k(j) - C_{m,j})q_j h_j^{FIN} \end{aligned} \tag{1}$$

We denote by PC^M the penalty cost for tardiness. The expression of PC^M is:

$$PC^M = \sum_{j=1}^{n} \pi_j^M T_j^M$$

Remember that T_j^M depends on the delivery dates D_j that will be given by the 3PL provider, and remain unknown for the moment. For this reason, the manufacturer considers a *pseudo penalty cost* for tardiness PPC^M involving an estimation of the tardiness PT_j^M defined by: $PT_j^M = \max(0, C_{m,j} + T - d_j)$, where T is considered as an estimation of the delivery time (supposed to be given). We have:

$$PPC^M = \sum_{j=1}^{n} \pi_j^M PT_j^M$$

We denote by VC the cost of vehicles. This cost is defined as follows.

$$VC = c^V V \tag{2}$$

where c^V is the cost of one vehicle and V is the number of vehicles.

The pseudo total cost for the manufacturer is given by:

$$PTC^M = IC + PPC^M + VC \tag{3}$$

B. Distribution Problem

The distribution problem in this study is a vehicle routing problem in which a third-party-logistics (3PL) provider aims to find an optimal route for the delivery of products from the manufacturer to multiple customer locations. In the problem considered here, the vehicles are stationed at the manufacturing site.

We denote by $t_{i,j}$ the travel time between site i and site j ($i, j \in [0, s+1]$), where site s corresponds to the manufacturer site, site $s+1$ corresponds to the depot of the 3PL provider and site j for $j \in [0, s-1]$ corresponds to the site of the customer associated to the job j in the vehicle.

For each trip, the 3PL provider bears the costs for the routing, denoted by RC_k for route of vehicle number k, which depends on the travel times and a penalty cost to the manufacturer in the case the delivery time $(D_j - t_0)$ is greater than T, where t_0 is the starting time of a tour at the manufacturer site. The total routing cost is equal to:

$$RC = \sum_{k=1}^{V} RC_k \tag{4}$$

The routing cost is the sum of the costs for all the routes, leaving from the manufacturer location and returning to the depot of the 3PL provider. The penalty cost is denoted by PC^{3PL}. We assume that this function is related to the total tardiness, i.e. we denote by T_j^{3PL} the tardiness of delivery of J_j from the point of view of the 3PL provider: $T_j^{3PL} = \max(0, D_j - (t_0 + T))$ and PC^{3PL} is defined by:

$$PC^{3PL} = \sum_{j=1}^{n} \pi_j^{3PL} T_j^{3PL} \tag{5}$$

The total cost for the 3PL provider is given by:

$$TC^{3PL} = RC + PC^{3PL} - VC \tag{6}$$

C. Integrated Problem

The two problems outlined above are interconnected and dependent on each other. We assume that:

- the maximum delay of delivery T,
- the number of vehicles V,
- the dates of departure of the vehicles (F_k, $1 \leq k \leq V$),

have been defined (negotiated or imposed). Once these parameters are known, the manufacturer constructs his schedule (setting of jobs completion times), minimizing PTC^M. Notice that VC (vehicle cost) is constant when V is known. Vehicles of the 3PL provider take the jobs and distribute them to the customers at minimum cost. The 3PL provider minimizes his total costs, TC^{3PL}.

The 3PL provider then gives the values of the delivery dates D_j with implications of tardiness to the manufacturer, so that the real penalty cost PC^M can be computed and replace the pseudo penalty cost PPC^M for the manufacturer.

The real cost for the manufacturer and for the 3PL provider are:

$$TC^M = IC + PC^M + VC - PC^{3PL} \quad (7)$$

$$TC^{3PL} = RC + PC^{3PL} - VC \quad (8)$$

The whole process is described more formally in Alg. 1.

Algorithm 1 General framework

Definition of V, T and F_k dates (scenario dependent).
The manufacturer optimizes its production schedule:
MIN PTC^M
// The schedule gives the jobs completion times and batches of delivery
for k **in** 1 **to** V **do**
The 3PL provider delivers the jobs optimally to minimize his costs: $RC_k + PC_k^{3PL}$.
// The routing of vehicle k gives the delivery dates of the jobs in this batch
end for
Compute the total cost of the 3PL provider:
$TC^{3PL} = RC + PC^{3PL} - VC$
Compute the total cost of the manufacturer:
$TC^M = IC + PC^M + VC - PC^{3PL}$

For given values of V, T and F_k, we have a total cost for both the manufacturer and for the 3PL provider.

IV. Resolution approaches

This section outlines two different approaches for solving the production scheduling and outbound distribution problem described in this paper. The first one is a two-step MILP model that solves the problem for the two agents to optimality and the second approach proposed is based on two heuristics aimed at solving the problem. An example terminates the section in order to illustrate the whole model and resolution approaches.

A. Mixed Integer Linear Programming models

This section will provide a detailed description of the mixed integer linear programming (MILP) models for both agents and the notations and parameters used. The general data is the following:

- V number of vehicles
- T estimated delivery time
- F_k departure date of vehicle number k

1) Manufacturer MILP model: The data required by the manufacturer is the following.

- n number of jobs
- m number of machines
- $p_{i,j}$ processing time for job J_j on machine M_i
- q_j quantity of items of job J_j
- d_j delivery due date for job J_j

The costs that have to be taken into account are the following.

- h_j^{WIP} holding cost for WIP inventory of job J_j
- h_j^{FIN} holding cost for finished product inventory of job J_j
- π_j^M penalty cost of the manufacturer for late delivery of job J_j
- c^V cost per vehicle

The variables to determine are defined as follows.

- $y_{j1,j2}$ $= 1$ if job J_{j1} is scheduled before job J_{j2}, 0 otherwise
- $z_{j,k}$ $= 1$ if job J_j departs on tour (or batch or vehicle) k, 0 otherwise
- $C_{i,j}$ Completion time of job J_j on machine M_i
- T_j^M tardiness of job J_j in terms of the manufacturer
- IC total inventory costs
- PPC^M pseudo penalty cost of the manufacturer
- PT_j^M ≥ 0 estimation of the tardiness

The following variables will be known after the 3PL provider gives the delivery dates to the manufacturer.

- D_j delivery completion time of job J_j
- T_j^M tardiness of job J_j

The objective function of the scheduling problem of the manufacturer is:

$$\text{Minimize } PTC^M = IC + PPC^M + VC \quad (9)$$

(notice that $VC = Vc^V$ is a constant when V is known)

The relative order between two jobs J_{j1} and J_{j2} ($\forall j1, j2 \in \{1, ..., n\}, j1 \leq j2$) is given by the following constraints:

$$y_{j1,j2} + y_{j2,j1} = 1 \quad (10)$$

The resource constraints allow to define the completion time of a job on any machine M_i ($\forall i \in \{1, ..., m\}$, $\forall j1, j2 \in \{1, ..., n\}$, $j1 \neq j2$):

$$C_{i,j2} \geq C_{i,j1} + p_{i,j2} - y_{j2,j1} HV \quad (11)$$

The routing constraints are given on any machine M_i ($i \in \{2, ..., m\}$) and for any job J_j ($j \in \{1, ..., n\}$):

$$C_{i,j} \geq C_{i-1,j} + p_{i,j} \quad (12)$$

Each job J_j is transported in a vehicle ($\forall j \in \{1, ..., n\}$), therefore:

$$\sum_{k=1}^{V} z_{j,k} = 1 \quad (13)$$

A job J_j ($\forall j \in \{1, ..., n\}$) is transported by the vehicle leaving at the next departure time. The expression of this constraint is the following:

$$\sum_{k=1}^{V} z_{j,k} F_{k-1} + 1 \leq C_{m,j} \leq \sum_{k=1}^{V} z_{j,k} F_k \quad (14)$$

The costs are given in the following expressions:

$$PT_j^M \geq \sum_{k=1}^{V} z_{j,k} F_k + T - d_j, \quad \forall j \in \{1, ..., n\} \quad (15)$$

$$PPC^M = \sum_{j=1}^{n} \pi_j^M PT_j^M \quad (16)$$

$$IC^{WIP} = \sum_{j=1}^{n} (C_{m,j} - C_{1,j} - \sum_{i=2}^{m} p_{i,j}) q_j h_j^{WIP} \quad (17)$$

$$IC^{FIN} = \sum_{j=1}^{n} (\sum_{k=1}^{V} z_{j,k} F_k - C_{m,j}) q_j) h_j^{FIN} \quad (18)$$

$$IC = IC^{WIP} + IC^{FIN} \quad (19)$$

2) 3PL provider MILP model: We assume that the optimization of the routing of the individual vehicles is independent and therefore can be done separately. Hence, for each trip, the only 'due date' to respect is T, and we assume that the vehicle leaves the manufacturer site at time $t_0 = 0$. Furthermore, in the following model, site j does not correspond to job J_j, but it corresponds to the j^{th} job in the vehicle. That is the reason why their index varies between 0 and $s-1$, the number of sites to visit.

The data required by the 3PL provider is the following:

- T estimated delivery time
- s number of sites to visit (sites $0..s-1$ are the customer sites, site s is the manufacturer site and site $s+1$ is the 3PL provider site)
- $t_{j1,j2}$ travel time between site $j1$ and site $j2$ ($0 \leq j1, j2 \leq s+1$)
- HV an arbitrary high value (can be set to $2 \times \sum_{j1} \sum_{j2} t_{j1,j2}$)

The costs to be taken into account are the following.

- $c_{j1,j2}$ cost of travel time between site $j1$ and site $j2$ ($0 \leq j1, j2 \leq s+1$)
- π_j^{3PL} penalty cost of 3PL provider for tardiness of job J_j ($0 \leq j \leq s-1$)

The variables that have to be determined are the following.

- $x_{j1,j2}$ = 1 if site $j1$ is visited before site $j2$, 0 otherwise ($0 \leq j1, j2 \leq s+1$)
- $sx_{j1,j2}$ = 1 if site $j1$ is visited **just before** site $j2$, 0 otherwise ($j1 \in [0, s]$, $j2 \in [0, s-1] \cup \{s+1\}$)
- D_j^{3PL} date at which site j is visited, i.e. delivery date of J_j for the 3PL provider ($0 \leq j \leq s+1$)
- T_j^{3PL} ≥ 0, tardiness of delivery of J_j ($0 \leq j \leq s-1$)
- RC routing cost
- PC^{3PL} total penalty cost of 3PL provider

The objective function of the vehicle routing problem of the 3PL provider is the following:

$$\text{Minimize } TC^{3PL} = RC + PC^{3PL} - VC \quad (20)$$

To ensure that there is no subtour in the routing, and each customer is served exactly once, we have ($\forall j1, j2, j3 \in \{0, .., s+1\}$, $j1 \neq j2$, $j1 \neq j3$, $j2 \neq j3$):

$$x_{j1,j2} + x_{j2,j1} = 1 \quad (21)$$

$$x_{j1,j2} + x_{j2,j3} \leq x_{j1,j3} + 1 \quad (22)$$

The connection between variables $x_{j1,j2}$ and variables $sx_{j1,j2}$ is given by ($\forall j1 \in [0, s]$, $\forall j2 \in [0, s-1] \cup \{s+1\}$, $j1 \neq j2$):

$$sx_{j1,j2} \leq x_{j1,j2} \quad (23)$$

To impose that the manufacturer and the 3PL provider have exactly one predecessor and one successor, we have ($\forall j1 \in \{0, .., s\}$)

$$\sum_{j2 \in [0, s-1] \cup \{s+1\}, j1 \neq j2} sx_{j1,j2} = 1 \quad (24)$$

and ($\forall j2 \in [0, s-1] \cup \{s+1\}$):

$$\sum_{j1=0}^{s} sx_{j1,j2} = 1 \quad (25)$$

The arrival date of each job at the customer site is given by the following constraints ($\forall j1 \in [0, s]$, $\forall j2 \in [0, s-1] \cup \{s+1\}$, $j1 \neq j2$):

$$D_{j2}^{3PL} \geq D_{j1}^{3PL} + t_{j1,j2} - HV(1 - sx_{j1,j2}) \quad (26)$$

The costs are given in the following expressions:

$$T_j^{3PL} \geq D_j^{3PL} - T, \quad \forall j \in 0, .., s+1 \quad (27)$$

$$PC^{3PL} = \sum_{j=0}^{s-1} \pi_j^{3PL} T_j^{3PL} \quad (28)$$

$$RC = \sum_{j1=0}^{s} \sum_{j2 \in [0, s-1] \cup \{s+1\}} c_{j1,j2} sx_{j1,j2} \quad (29)$$

B. Heuristic algorithms

Two heuristics are also proposed in order to find a solution to the integrated problem. The production scheduling problem is solved using the earliest due date first (EDD) sorting rule, that is widely used for scheduling problems. In a second step, the outbound distribution scheduling problem, is solved by using the Nearest Neighbour (NN) heuristic. These two (basic) heuristic algorithms correspond to a logical decision process for both agents. For the manufacturer, the idea is to ensure the delivery as soon as possible in order to reduce the tardiness penalty costs. For the 3PL provider, the idea is to minimize (intuitively) his total routing costs.

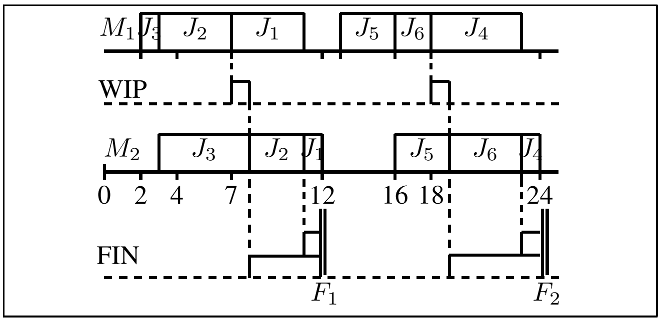

Fig. 3. Solution for the manufacturer

C. Example

The following example allows to illustrate the proposed model. We consider a set of $n = 6$ jobs to schedule on $m = 2$ machines. Processing times are $p_{1,j} = (4, 4, 1, 5, 3, 2)$ and $p_{2,j} = (1, 3, 5, 1, 3, 4)$. Delivery due dates are equal to $d_j = (18, 20, 23, 27, 30, 31)$. The matrix of travel times is equal to:

$$(t_{j1,j2}) = \left(\begin{array}{cccccc|c|c} 0 & 2 & 2 & 10 & 11 & 11 & 5 & 8 \\ 2 & 0 & 3 & 12 & 14 & 13 & 7 & 9 \\ 2 & 3 & 0 & 9 & 11 & 10 & 5 & 5 \\ 10 & 12 & 9 & 0 & 2 & 3 & 6 & 7 \\ 11 & 14 & 11 & 2 & 0 & 3 & 7 & 8 \\ 11 & 13 & 10 & 3 & 3 & 0 & 8 & 6 \\ \hline 5 & 7 & 5 & 6 & 7 & 8 & 0 & 8 \\ \hline 8 & 9 & 5 & 7 & 8 & 6 & 8 & 0 \end{array}\right)$$

Remember that site 7 corresponds to the manufacturer site and site 8 corresponds to the 3PL provider site. The routing costs are equal to the travel times.

The price of one vehicle is equal to $c^v = 150$. Finally, for any job J_j, the remaining data is:

- holding costs $h_j^{WIP} = 1$ and $h_j^{FIN} = 2$,
- quantities are equal to $q_j = 10$,
- penalty costs are equal to $\pi_j^M = 60$ and $\pi_j^{3PL} = 100$.

The parameters V, T and F_k are set to the following values:

- $V = 2$
- $T = 7$
- $F_k = (12, 24)$

Solution given by the MILP models: For the manufacturer, the sequence of production is $(J_3, J_2, J_1, J_5, J_6, J_4)$, illustrated in Fig. 3.

The total inventory cost is equal to $IC = IC^{WIP} + IC^{FIN}$ with $IC^{WIP} = 20$ and $IC^{FIN} = 220$. The value of the pseudo penalty cost for tardiness is given in Table I.

Therefore, the value of PPC^M is equal to 360. The pseudo total cost for the manufacturer is equal to $PTC^M = IC + PPC^M + VC = 240 + 360 + 300 = 900$

The 3PL provider has two problems to solve. The first one consists in finding an optimal route for jobs J_1, J_2, J_3, the second problem for jobs J_4, J_5, J_6.

- For the first set of jobs, the optimal routing is given by the sequence (J_3, J_1, J_2). This sequence has a total cost of $RC_1 = 18$ (including the cost from J_2 to the 3PL provider site). Job J_2 is delivered 9 time units after the departure of the vehicle, so the delay $T = 7$ is exceeded by 2 time units, leading to a cost $PC_1^{3PL} = 200$.
- For the second set of jobs, the optimal routing is given by the sequence (J_4, J_5, J_6). This sequence has a total cost of $RC_2 = 17$ (including the cost from J_6 to the 3PL provider site). Job J_5 is delivered 1 time unit after the departure of the vehicle, and job J_6 is delivered 4 times units late , both generating a tardiness cost. This leads to a cost $PC_2^{3PL} = 500$.

TABLE I
COSTS AND VARIABLES OBTAINED BY THE MILP MODELS

j	1	2	3	4	5	6	Total
$C_{1,j}$	11	7	3	23	16	18	
$C_{2,j}$	12	11	8	24	19	23	
F_k	12	12	12	24	24	24	
$F_k + T$	19	19	19	31	31	31	
d_j	18	20	23	27	30	31	
PT_j^M	1	0	0	4	1	0	6
D_j^{3PL}	7	9	5	6	8	11	
T_j^{3PL}	0	2	0	0	1	4	7
D_j	19	21	17	30	32	35	
T_j^M	1	1	0	3	2	4	10

We can now compute the real tardiness of delivery (see T_j^M in Table I).

These results are summarized in Table I. Because there are two vehicles used, $VC = 300$ and the total cost for the 3PL provider is equal to $TC^{3PL} = 35 + 700 - 300 = 435$. The total cost for the manufacturer is equal to $TC^M = 240 + 660 - 700 + 300 = 500$.

Solution given by the heuristics: For the manufacturer, the sequence of production is now $(J_1, J_2, J_3, J_4, J_5, J_6)$. Because job J_3 completes after $F_1 = 12$, the first vehicle only takes jobs J_1 and J_2, the other jobs are being transported by the second vehicle. The inventory cost is equal to $IC^{WIP} = 80$ plus $IC^{FIN} = 480$, i.e. $IC = 560$. The pseudo penalty cost for tardiness is equal to $PPC^M = 14 \times 60 = 840$.

For the 3PL provider, the route of the first vehicle is (J_1, J_2) and the route for the second vehicle is (J_3, J_4, J_5, J_6). The delivery dates are reported in Table II. The total routing cost is equal to $RC = 16 + 25 = 41$. The total penalty cost is equal to $PC^{3PL} = 28 \times 100 = 2800$.

Finally, the total cost for the 3PL provider is equal to $TC^{3PL} = 41 + 2800 - 300 = 2541$. The total cost for the manufacturer is equal to $TC^M = 560 + 2340 - 2800 + 300 = 400$.

V. ONE COORDINATION SCENARIO: MANUFACTURER DOMINATES

A. Model

In the case of dominance between the two agents, we consider here that the manufacturer dominates. In this case, we assume that he can decide the number of vehicles V he wants to use, the dates F_k at which they depart and the delay

TABLE II
COSTS AND VARIABLES OBTAINED BY THE HEURISTICS

j	1	2	3	4	5	6	Total
$C_{1,j}$	4	8	9	14	17	19	
$C_{2,j}$	7	8	11	16	17	20	24
F_k	12	12	24	24	24	24	
F_k+T	19	19	31	31	31	31	
d_j	18	20	23	27	30	31	
PT_j^M	1	0	8	4	1	0	14
D_j^{3PL}	5	7	5	14	16	19	
T_j^{3PL}	0	0	0	7	9	12	28
D_j	17	19	29	38	40	43	
T_j^M	0	0	6	11	10	12	39

TABLE III
COSTS AND VARIABLES OBTAINED BY THE HEURISTICS

j	1	2	3	4	5	6	Total
$C_{1,j}$	11	4	14	22	17	7	
$C_{2,j}$	12	7	19	23	22	11	
F_k	12	12	23	23	23	12	
F_k+T	12	12	23	23	23	12	
d_j	18	20	23	27	30	31	
PT_j^M	0	0	0	0	0	0	0
D_j^{3PL}	5	7	19	6	8	20	
T_j^{3PL}	5	7	19	6	8	20	65
D_j	17	19	42	29	31	32	
T_j^M	0	0	19	2	1	1	23

T imposed on the 3PL provider. He also determines an optimal schedule, minimising his costs, regardless of the 3PL provider.

Generally, though depending on the penalty costs, the manufacturer aims to keep the value of T as low as possible shifting the cost of delay onto the 3PL provider. Therefore, there is no reason to fix T to another value than 0. Given F_k and T from the optimal schedule of the manufacturer, the 3PL provider then makes a routing decision optimising its costs. The adapted model is outlined below.

The data required by the manufacturer is the same as before. The variables remain the same, except for the new variables to determine, which are the following.

- V number of vehicles
- T estimated delivery time
- F_k the departure time of vehicle k
- S_j the departure time of job J_j (equal to F_k if J_j is in vehicle k)

The scheduling problem of the manufacturer is determined by solving the following MILP model.

$$\text{Minimize } PTC^M \tag{30}$$

Notice that VC is a variable in this model. The constraints (10), (11), (12), (13), (16), (17), (19) are still valid in this model.

To determine the number of vehicles, we introduce the following constraints ($\forall j \in \{1, ..., n\}$):

$$\sum_{j=1}^{n} z_{k,j} \leq V \tag{31}$$

We introduce constraints for breaking symmetries in the model ($\forall k \in \{1, .., n\}$):

$$F_k + 1 \leq F_{k+1} \tag{32}$$

We use the following constraints to set the value of S_j variables ($\forall j \in \{1, ..., n\}, \forall k \in \{1, .., n\}$):

$$S_j \geq F_k - HV(1 - z_{j,k}) \tag{33}$$

$$S_j \leq F_k + HV(1 - z_{j,k}) \tag{34}$$

The following constraints replace constraints (14) ($\forall j \in \{1, ..., n\}, \forall k \in \{2, ..., n\}$):

$$F_{k-1} + 1 - HV(1 - z_{k,j}) \leq C_{m,j} \leq F_k + HV(1 - z_{k,j}) \tag{35}$$

The following constraints replace constraints (15) ($\forall j \in \{1, ..., n\}$):

$$PT_j^M \geq D_j + T - d_j \tag{36}$$

Finally, the following constraints replace constraints (18) ($\forall j \in \{1, ..., n\}$):

$$IC^{FIN} = \sum_{j=1}^{n} (S_j - C_{m,j}) q_j h_j^{FIN} \tag{37}$$

Once the manufacturer has optimised his schedule the 3PL provider makes his routing decision as in the general model for the 3PL provider, using the parameters for T, V and F_k that have been determined in the dominance model outlined above.

B. Example

With the data presented before, we obtain the following solution for the manufacturer. The number of vehicles is equal to $V = 2$, we obtain $T = 0$ and $F_1 = 12$ and $F_2 = 23$.

The inventory cost is equal to $IC^{WIP} = 20$ plus $IC^{FIN} = 220$, i.e. $IC = 240$. We have $PPC^M = 0$. At the end, the optimal pseudo cost if the manufacturer dominates is equal to $PTC^M = 240 + 300 = 540$.

Of course, this optimization does not take the decisions of the 3PL provider into account. When the 3PL provider solves his problem to optimality, he obtains the following results. The route of the first vehicle is (J_1, J_2, J_6) and the route for the second vehicle is (J_4, J_5, J_3). The delivery dates are reported in Table III. The total routing cost is equal to $RC = 26 + 24 = 50$. The total penalty cost is equal to $PC^{3PL} = 65 \times 100 = 6500$.

Finally, the total cost for the 3PL provider is equal to $TC^{3PL} = 50 + 6500 - 300 = 6250$. The total cost for the manufacturer is equal to $TC^M = 1380 + 240 - 6500 + 300 = -4580$. So, there is a reward for the manufacturer and a cost for the 3PL provider.

VI. CONCLUSION AND FUTURE RESEARCH DIRECTIONS

In this paper, we consider a two-level supply chain problem. A manufacturer has to produce some jobs, and a 3PL provider delivers the jobs to customers. The manufacturer and the 3PL provider are considered as two independent agents, which interact as part of a business relationship. A contract links their activities at an operational level. The manufacturer pays the 3PL provider to have vehicles available for the delivery, and he also pays tardiness penalty costs to the customers in case of late delivery. It is assumed that the dates at which the vehicles are available are demanded by the manufacturer. On the other hand, the 3PL provider has to deliver all the jobs within a given fixed deadline, that is negotiated. If the deadline is not respected, the 3PL provider has to pay the manufacturer a penalty cost. The workshop of the manufacturer has a flowshop organization and the cost function contains inventory costs, vehicle costs plus customer tardiness penalty costs while the cost function of the 3PL provider contains routing costs plus tardiness penalty costs to the manufacturer.

Mixed Integer Linear Programming models are proposed for the optimization of both, the manufacturer problem and the 3PL provider problem. A heuristic method, representing an intuitive behavior for each agent is proposed. These models are integrated in a global algorithm (the customer penalty costs are only known after the delivery), and illustrated by a numerical example. Finally a scenario where the manufacturer dominates the negotiation is proposed, and the MILP model associated to this scenario is presented.

There are a lot of possible research directions associated to this model. From a resolution point of view, due to the complexity of these problems, some heuristic methods can be proposed to replace the MILP formulations. From a modeling point of view, several directions can be investigated. First, the setting of T can be considered with a higher accuracy. For instance, it is not practical/acceptable to fix this parameter to a value smaller than the maximum travel time between the manufacturer site and the customers (given that there is no big deviation in the data). On the other hand, it makes no sense to set this parameter to a high value. This remark highlights the difficulty to design 'interesting' data sets. Indeed, if the travel times are a lot longer than the processing times, the scheduling problem does not play an important role, and reciprocally, if the processing times are a lot longer than the travel times, the routing problem is not really important. Furthermore, the problem of setting T, V and F_k such that the costs of the manufacturer and the 3PL provider are minimised is of great interest. Finding (T, V, F_k) can also lead to compromise solutions between the two agents.

Some other scenarios can be considered. Of course, the case where the 3PL provider dominates can be coverd, using the same reasoning. Similarly, the case where both agents belong to the same company is also interesting. In this case, the vehicle cost and the penalty cost of the 3PL provider can be considered as internal costs, making the problem a multiobjective problem, or a *multiagent problem with a global agent*. Another model can be defined where the 3PL provider has an interest (a reward from the manufacturer) if the jobs are delivered on time. In this case, the 3PL provider knows the due dates contracted between the manufacturer and the customers and both agents may increase their profit (or decrease their cost).

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