# A two-agent model for production and outbound distribution scheduling

Sonja Rohmer [1] Jean-Charles Billaut [1]

sukrohmer@outlook.com jean-charles.billaut@univ-tours.fr

[1] Université François-Rabelais Tours

Laboratoire d'Informatique

64 avenue Jean Portalis, 37200 Tours, France

July 2, 2015

**Abstract**: In this paper we consider a two-level supply chain problem composed of a manufacturer and a 3PL provider at the operational level. In the environment that we consider, the number of vehicles provided by the 3PL provider and their departure dates, as well as the maximum delay of delivery for the 3PL provider are given. An economic model is proposed where the manufacturer pays for inventory, tardiness penalties (to the customers) and for the use of vehicles (to the 3PL provider), whereas the 3PL provider pays for its routing and for tardy deliveries (to the manufacturer). A Mixed Integer Linear Programming (MILP) model is proposed for the two-step integrated problem. A number of different scenarios are investigated, changing the environment of the model and new MILP formulations of the models are given. Examples illustrate all the proposed models. Finally, some extensions of the models and a relevant random data generation are presented.

**Keywords**: supply chain, scheduling, vehicle routing, multi-agent, cooperation.

# Contents

# 1 Introduction

In today's business environment, defined by globalisation, fierce competition as well as increasing customer demands, companies are forced to find innovative ways in order to keep costs down and stay competitive while providing high quality of service for their customers. The complexity and interconnectedness of this globalised environment makes planning and logistics important tasks that can cut costs and ensure smooth operations for a company, considering that inventory and transportation/distribution costs make up a substantial amount of a companys cost function. The high competitiveness also demands companies to pay more attention to timeliness and reliability of deliveries. Supply chain management is concerned with the coordination and planning of the supply chain of a business and considers all activities that are involved in the production of a product/service from the supplier up to the delivery to the customer. Supply chains are generally very complex networks consisting of several stages, links and players, however, the main two areas of focus in supply chain management are production and distribution planning. Production planning is concerned with decisions regarding production quantity, i.e. how much to produce, production schedule, i.e. the order of jobs on a machine or when to produce which job, the location of production sites, etc. Distribution planning on the other hand deals with issues such as the delivery of a product for example routing, comprising of decisions regarding delivery times, the order in which costumers are served and delivery quantities depending on the available vehicles. While these two areas are traditionally dealt with individually, they are highly interconnected and more recent considerations have shown that integrated supply chain approaches, that consider both production and distribution decisions at the same time, hold various benefits for organisations both in terms of cost as well as time management. Technological progress and developments such as enterprise resource planning (ERP) systems facilitate this integration and coordination. The level of coordination depends on the strategic supply chain model. An integrated supply chain, in which one cooperation is in control of all or several steps/ links in the supply chain aims to globally optimise the performance whereas in a non-integrated supply chain, players in the supply chain act independently and in their own interest so that coordination only happens on the basis of negotiation between the individual players. The later can be difficult as objectives between the players can be conflicting. The relationship and role of the players and the setup of the supply chain therefore becomes important for these considerations as the coordination decisions might vary depending on which agent is dominating the negotiation or whether the agents stand in competition or collaboration with each other.

The objective of these considerations is to find an optimal global solution to the supply chain scheduling problem for all agents by integrating the two individual problems of production and distribution scheduling into one model. Over recent years such approaches have been more and more covered in the literature. However, one of the difficulties associated with this is the complexity of the two individual models often resulting in simplified models. This research aims to propose an integrated model for the supply chain scheduling that also includes the notion of inventory costs that occur during the production phase in order to obtain a more realistic model. Inventory costs even though often neglected in production scheduling models play a major role in production planning. There are two kinds of inventory to be considered: work in progress (WIP) inventory and finished products inventory.

The problem considered here is an integrated production and distribution scheduling problem. This multi-agent problem consists of two sub problems a permutation flow-shop scheduling problem for the production problem and a vehicle routing problem concerned with the minimisation of total tardiness in terms of the distribution problem.

The paper is organised as follows. In Section 2 we present the literature on integrated production and outbound distribution scheduling. In Section 3 we formally describe the problem and introduce some notations.

## 2 State-of-the-art

While there is a vast amount of literature available on the individual problems of production and distribution scheduling, the integrated approach of supply chain scheduling has only been covered to a greater extend in the literature more recently. Graves (1981) [13] for example gives a classification and early review of the different approaches to production scheduling problems. The classification is mainly done according to the machine environment. Chen (2004) [5] provides a classification for integrated models based on different criteria such as for example the structure of the integration. They differentiate between 3 different kinds of integration:

- Production and outbound transportation
- Inbound transportation and production
- Inbound transportation, production and outbound transportation

The latter includes 3 stages in the supply chain: supplier, manufacturer and customers. Such problems are covered for instance by Sawik (2009) as well as by Hall and Potts (2003). Inbound transportation and production scheduling focuses only on the two problems of supplier and production scheduling; see Agnetis et al. (2006). The focus of this paper however lies on production and outbound transportation models, i.e. models that comprise of a manufacturer and the delivery of the products to one or several customers. These problems have been covered in the literature by for example Chen and Vairaktarakis (2005), Zhong et al. (2010), Ullrich (2013), Wang and Lee (2005), Li et al. (2008), Chang and Lee (2004), Dawande et al. (2006), Li et al. (2005) and Chen et al. (2009). The individual approaches further differ depending on some criteria and constraints such as the machine environment and the objectives of the optimisation. Chen (2010) provides a survey of existing integrated models, classifying them into several classes and giving an overview over the different approaches for each class, as well as proposes a unified notation and representation for such models while also identifying areas for future research. In addition to this, while some literature focuses more on the production side other papers pay more attention to the distribution side of the problem.

Li et al. (2005) study a single machine scheduling problem with an integrated routing to a variety of different customers with the objective function based on the minimisation of the sum of the arrival times. They consider a number of special cases for which they propose good solution algorithms.

Stecke and Zhao (2007) consider a problem for a make-to-order manufacturer and a 3PL provider, where the manufacturer operates in a single machine environment with capacity constraints and the goal to minimise total shipping costs. They consider a range of 8 different distribution states and propose different solution models including linear programming and mixed integer programming models as well as a heuristic model. Low et al. (2013) deal with an integrated production and distribution scheduling model in a single machine environment and delivery to the customer within time windows aiming to minimise the makespan. They develop a nonlinear mathematical solution model as well as two heuristics based on genetic algorithms for solving larger scale problems. Zhong et al. (2010) study an integrated production distribution problem with a single machine environment and committed delivery dates for the manufacturer and a 3PL provider with a linear shipping cost function and fixed pick up times. In order to design a schedule and choose the delivery mode with the goal to minimise the total shipping costs as well as comply with the delivery dates they design a polynomial time heuristic. Chen et al. (2009) consider the special case of an integrated model for perishable food products with decay rates taking into account the demand for the products, time windows for the delivery and capacity constraints of the vehicles. The proposed model is an integer nonlinear programming model determining the optimal quantities, production times and vehicle routing with the objective to maximise the expected total profit of the manufacturer.

Seyedhosseini and Ghoreyshi (2014) also deal with perishable products with one production facility with limited production capacity and multiple customer locations taking into account customer demands and allowing for limited inventory. The cost for the routing is based on the number of trips rather than the distances travelled. The model is aimed at minimising the total costs by determining production quantities, product deliveries and stock management. A two-phase heuristic algorithm is developed in order to solve the problem.

Li et al. (2008) cover a parallel machine assembly and 3PL provider problem with multiple customer destinations. The problem is composed into the two sub-problems and heuristics are developed for the assembly scheduling problem. Chen and Vairaktaratis (2005) deal with integrated production distribution problems optimising the total distribution cost as well as the delivery times considering the case of average delivery times and maximum delivery times. In the paper, they provide both exact algorithms and heuristics for the problems and two machine configurations: single and parallel machine production facilities.

Chang and Lee (2004) study a problem with different job sizes for delivery they cover three problem scenarios with different machine configurations, single and parallel machines and either one or two customer areas that are served by one vehicle with the aim of minimising the makespan. In their paper they design heuristics for the problems and show the performance of these.

Agnetis et al. (2014) analyse an interstage delivery problem where the jobs are processed on machine 1 and then delivered to a different manufacturing site where they are processed on a second machine with two transportation options. The research examines the differences between a scenario in which the manufacturer dominates and a scenario in which the 3PL provider dominates.

Chandra and Fisher (1994) consider a single plant problem with multiple products that have to be delivered to several customer locations. The problem includes customer demands over certain periods of time that have to be met with a possibility of holding inventory. The goal is to minimise the overall costs comprising of production, distribution and inventory. They show through testing that the integrated problem can play a major role in reducing operational cost significantly.

Lei et al. (2006) study a production, inventory and distribution routing problem. The problem consists of several plants with different capacity constraints and cost functions, producing a single product, and a number of customers with variations in demand and inventory capacities. The objective is to find an optimal schedule that minimises the operational costs and complies with the set of constraints associated with the facilities. The problem is solved according to a two phase methodology making use of a mixed integer programming model as well as a heuristic.

Fumero and Vercellis (1999) consider an integrated approach with a production unit producing different kinds of products that have to be delivered to customer locations according to demand requirements. Vehicles with a limited capacity are available at the production site at all times and distribution incurs a cost based on a fixed fee, the distance and the load transported. The notion of inventory is possible in order to fulfil demands. A Lagrangean relaxation is proposed in order to solve the problem. Wang (2012) investigates two kinds of integrated production distribution problems with two different inventory considerations, one that only considers customer inventory and one that also considers intermediate inventory at the manufacturer. The production problem in both cases is a single machine problem with setup times; the jobs are then shipped in batches to the customers in the case a job is finished early or arrives early an inventory cost occurs. The objective is to minimise the total cost function of the problem.

Even though a variety of different problems have been studied in the literature over the last two decades most of the problems in the research outlined above cover single or parallel machine configurations for the production problem and do not take inventory into consideration for the decision making process. Problems that do include inventory generally only focus on one kind of inventory for example inventory of finished goods. However inventory occurs at various stages of the production process and the holding costs are an important cost factor that can have an impact on the optimality of the scheduling decision. As real world applications are generally more complex there is a need for more comprehensive integrated approaches, incorporating various types of inventory, such as for example intermediate and finished product inventory, into the model. In addition to this one can differentiate between two types of objectives that are optimised in the integrated models covered in the literature, time and cost.

### Conflict and Cooperation in Supply Chains

In addition to different modelling approaches for integrated production distribution scheduling or supply chain scheduling there has also been some research been carried out in terms of the structure and relationship between the different agents involved in the problem. This is an important area as it determines the dynamic between the agents and through this the dynamic of the model. Dawande et al. (2006) study the issues of supply chain scheduling in terms of conflict and cooperation. They look at different production and distribution problems and determine a cost of conflict that arises between the two agents in case the others optimal schedule is implemented and further investigate the impact and consequences in the cases of dominance of one of the two agents and the benefits of corporation. Agnetis et al. (2006) also deal with the problem of how the agents optimise their schedule given the optimal schedule of the other agent. They solve their problems by optimising the total interchange cost and from these results determine the conditions under which the two agents benefit from cooperation.

# 3 Problem description

This research aims to propose an integrated model for the supply chain scheduling problem that also includes several costs (inventory costs, transportation costs, penalty costs, ...) that occur during the production phase and the transportation phase in order to obtain a more realistic model. Inventory costs even though often neglected in production scheduling models play a major role in production planning. There are two kinds of inventory to be considered: work in progress (WIP) inventory and finished products inventory.

The problem considered here is an integrated production and distribution scheduling problem. This multi-agent problem consists of two sub problems: a permutation flow-shop scheduling problem for the production problem and a vehicle routing problem. According to the five-field notation $\alpha|\beta|\pi|\delta|\gamma$ proposed in [8], the notation of the problem that we consider is $F2||V(\infty,\infty), fdep, routing|n|\gamma$ where the field $\pi$ contains $V(\infty,\infty)$ meaning that a sufficient number of vehicles are available and the capacity of each vehicle is unbounded, $fdep$ for *shipping with fixed delivery departure dates*, and *routing* meaning that orders going to different customers can be transported in the same shipment. In the field $\delta$ we have $n$ to indicate that each job belongs to one customer. $\gamma$ is the objective function, detailed later in this section.

## 3.1 Production Problem

The production problem considered here is an $m$-machine permutation flow-shop with work in progress (WIP) and finished product inventory. We suppose that we have a set $\{J_1, ..., J_n\}$ of $n$

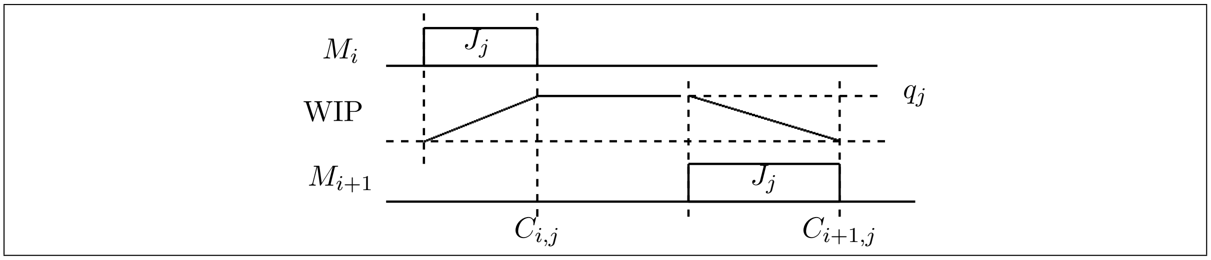


Figure 1: Work-in-process inventory

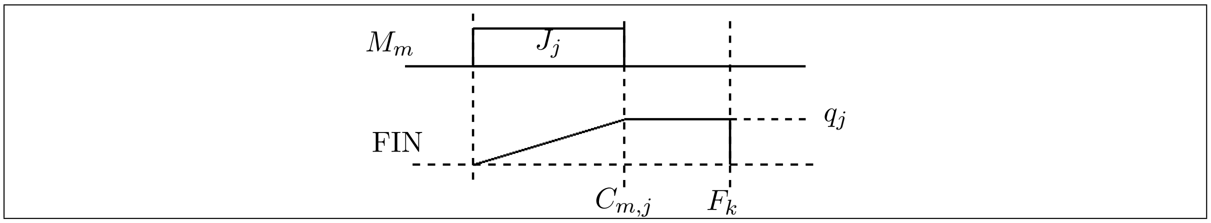


Figure 2: Inventory of final products

jobs to schedule. A manufacturer is looking to design a production schedule for all jobs on the $m$ machines. Each job $J_j$ has a given processing time $p_{i,j}$ on machine $M_i$. With the execution of a job on a machine $M_i$ there is a build-up of WIP inventory. The WIP inventory has to be then stored till the job is processed on machine $M_{i+1}$ and the stock of WIP decreases between $M_i$ and $M_{i+1}$. The holding cost for this work in progress inventory depends on the job and is denoted by $h_j^{WIP}$ for job $J_j$. As the job is processed on machine $M_m$ the inventory of finished products increases. After the completion of the job on machine $M_m$ the finished product inventory is kept until the $k^{th}$ departure date at which the products are collected for distribution, denoted by $F_k$. The holding cost for the finished product inventory again depends on the product, and is denoted by $h_j^{FIN}$ for job $J_j$. Each job $J_j$ has to be delivered to customer $j$ for a given due date denoted by $d_j$. A delay in the delivery of the product generally results in a loss for the manufacturer, this loss can be both financial as well as in terms of the reputation hence the manufacturer incurs a penalty cost $\pi_j^M$ for late delivery of $J_j$ that is paid to the customer.

We denote by $C_{i,j}$ the completion time of job $J_j$ on machine $M_i$, $1 \leq i \leq m$. $D_j$ denotes the delivery completion time. The tardiness of job $J_j$ for the manufacturer is $T_j^M = \max(0, D_j - d_j)$.

The objective of the manufacturer is to minimise his total cost. This cost is composed by an inventory cost, a transportation cost (related to the number of vehicles available), and penalty costs related to tardiness of delivery. At this step, the manufacturer does not know the delivery dates.

We denote by $IC$ the total inventory cost.The work-in-process inventory is represented in Fig. 1 and the final inventory is represented in Fig. 2.

The expression of $IC$ is the following.

$$IC = \sum_{j=1}^{n}\left(\sum_{i=1}^{m-1}\frac{1}{2}(p_{i,j}+p_{i+1,j})q_j + (C_{i+1,j}-p_{i+1,j}-C_{i,j})q_j\right)h_j^{WIP} + \sum_{j=1}^{n}\left((F_k(j)-C_{m,j})q_j + \frac{1}{2}p_{m,j}q_j\right)h_j^{FIN}$$

where $F_k(j)$ denotes the departure time of $J_j$ and $q_j$ is the quantity of items of job $J_j$.

Minimizing this expression is equivalent to minimize the following expression:

$$IC = \sum_{j=1}^{n}(C_{m,j}-C_{1,j})q_j h_j^{WIP} + \sum_{j=1}^{n}(F_k(j)-C_{m,j})q_j h_j^{FIN} \tag{1}$$

We denote by $PC^M$ the penalty cost for tardiness. The expression of $PC^M$ is:

$$PC^M = \sum_{j=1}^{n}\pi_j^M T_j^M$$

Remember that $T_j^M$ depends on the delivery dates $D_j$ that will be given by the 3PL provider, and that are not known at the moment. For this reason, the manufacturer considers a pseudo penalty cost for tardiness $PPC^M$ involving an estimation of the tardiness $PT_j^M$ defined by: $PT_j^M = \max(0, C_{m,j}+T-d_j)$, where $T$ is an estimation of the delivery time that is supposed to be given.

$$PPC^M = \sum_{j=1}^{n}\pi_j^M PT_j^M \tag{2}$$

We denote by $VC$ the cost of vehicles. This cost is defined as follows.

$$VC = c^V V \tag{3}$$

where $c^V$ is the cost of one vehicle and $V$ is the number of vehicles.

The pseudo total cost for the manufacturer is given by:

$$PTC^M = IC + PPC^M + VC \tag{4}$$

### 3.2 Distribution Problem

The distribution problem in this study is a vehicle routing problem in which a third-party-logistics (3PL) provider aims to find an optimal route for the delivery of products from the manufacturer to multiple customer locations. Two hypotheses are considered:

1. Vehicles station at the manufacturing site and depart at fixed dates and the number of vehicles $V$ that are used during the planning horizon $H$ is known (either negociated or imposed).

2. A vehicle takes all the jobs that are available (completed) and does not wait.

Assuming that the planning horizon is $H$, the departure date of vehicle $k$ is set to $kH/V$.

We denote by $t_{i,j}$ the travel time between site $i$ and site $j$ ($i, j \in [0, s+1]$), where site $s$ corresponds to the manufacturer site, site $s+1$ corresponds to the depot of the 3PL provider and site $j$ for $j \in [1, s-1]$ corresponds to the site of customer associated to the job $j$ in the vehicle.

For each trip, the 3PL provider bears the costs for the routing, denoted by $RC_k$ for route of vehicle number $k$, which depends on the travel times and a penalty cost to the manufacturer in the case the delivery time $(D_j - t_0)$ is greater than $T$.The total routing cost is equal to:

$$RC = \sum_{k=1}^{V} RC_k \tag{5}$$

The routing cost is the sum of the costs for all the routes, leaving from the manufacturer location and returning to the depot of the 3PL provider. The penalty cost is denoted by $PC^{3PL}$. We assume that this function is related to the total tardiness, i.e. we denote by $T_j^{3PL}$ the tardiness of delivery of $J_j$ from the point of view of the 3PL provider: $T_j^{3PL} = \max(0, D_j - (t_0 + T))$, where $t_0$ denotes the departure time of the vehicle. $PC^{3PL}$ is defined by:

$$PC^{3PL} = \sum_{j=1}^{n} \pi_j^{3PL} T_j^{3PL} \tag{6}$$

The total cost for the 3PL provider is given by:

$$TC^{3PL} = RC + PC^{3PL} - VC \tag{7}$$

### 3.3 Integrated Problem

The two problems outlined above are interconnected and dependent on each other. We assume that:

- the maximum delay of delivery $T$,
- the number of vehicles $V$,
- the dates of departure of the vehicles ($F_k$, $1 \leq k \leq V$),

have been defined. Several scenarios will be considered for fixing these parameters.

Once these quantities have been negotiated, the manufacturer defines his schedule (setting the completion times of jobs), minimizing $PTC^M$. Notice that $VC$ (vehicle cost) is constant when $V$ is known. Vehicles of the 3PL provider take the jobs and distribute them to the customers at minimum cost. The 3PL provider minimizes $TC^{3PL}$ costs.

The 3PL provider then gives to the manufacturer the values of the delivery dates $D_j$ with implications of tardiness, so that the real penalty cost $PC^M$ can be computed and replace the pseudo penalty cost $PPC^M$ for the manufacturer.

The real cost for the manufacturer and for the 3PL provider are:

$$TC^M = IC + PC^M + VC - PC^{3PL} \tag{8}$$

$$TC^{3PL} = RC + PC^{3PL} - VC \tag{9}$$

The whole process is described more formally in Alg. 1.

**Algorithm 1** General framework

Definition of $V$ and $T$ (scenario dependent).

*// From this values we deduce the fixed delivery departure dates*

The manufacturer optimizes its production schedule: MIN $PTC^M$.

*// From the schedule we deduce the jobs completion times and the batches of delivery*

**for** $k$ **in** 1 **to** $V$ **do**

  The 3PL provider delivers the jobs optimally to minimize his costs $RC_k + PC_k^{3PL}$.

  *// From the routing of vehicle k we deduce the delivery dates of the jobs in this batch*

**end for**

Compute the total cost of the 3PL provider: $TC^{3PL} = \sum_{k=1}^{V} RC_k + PC_k^{3PL} - VC$

Compute the total cost of the manufacturer: $TC^M = IC + PC^M - PC^{3PL}$+VC

For given values $V$ and $T$ we have a total cost for both the manufacturer and for the 3PL provider. The objective of the study is to provide help to these two agents for setting $V$ and $T$ in order to reduce their costs as much as possible.

### 3.4 Mathematical programming formulations for the integrated problem

This section will provide a detailed description of the mixed integer linear programming (MILP) models and the notations and parameters used.

| | |
|---|---|
| $H$ | planning horizon |
| $V$ | number of vehicles |
| $T$ | estimated delivery time |
| $VC$ | total cost of vehicles |

### 3.4.1 Manufacturer

The data required by the manufacturer is the following.

| | |
|---|---|
| $n$ | number of jobs |
| $m$ | number of machines |
| $p_{i,j}$ | processing time for job $J_j$ on machine $M_i$ |
| $q_j$ | quantity of items of job $J_j$ |
| $d_j$ | delivery due date for job $J_j$ |

The costs that have to be taken into account are the following.

| | |
|---|---|
| $h_j^{WIP}$ | holding cost for WIP inventory of job $J_j$ |
| $h_j^{FIN}$ | holding cost for finished product inventory of job $J_j$ |
| $\pi_j^M$ | penalty cost of the manufacturer for late delivery of job $J_j$ |
| $c^V$ | cost per vehicle |

The variables to determine are the following.

| | |
|---|---|
| $y_{j1,j2}$ | $= 1$ if job $J_{j1}$ is scheduled before job $J_{j2}$, 0 otherwise |
| $z_{j,k}$ | $= 1$ if job $J_j$ departs on tour (or batch) $k$, 0 otherwise |
| $C_{i,j}$ | Completion time of job $J_j$ on machine $M_i$ |
| $F_k$ | departure time of vehicle $k$ |
| $T_j^M$ | tardiness of job $J_j$ in terms of the manufacturer |
| $IC$ | total inventory costs |
| $PPC^M$ | pseudo penalty cost of the manufacturer |
| $PT_j^M \geq 0$ | estimation of the tardiness |

The following variables will be known after the 3PL provider gives the delivery dates to the manufacturer.

| | |
|---|---|
| $D_j$ | delivery completion time of job $J_j$ |
| $T_j^M$ | tardiness of job $J_j$ |

The scheduling problem of the manufacturer is determined by solving the following Mixed Integer

Linear Programming model.

$$\text{Minimize } PTC^M = IC + PPC^M + Vc^V \tag{10}$$

(notice that $Vc^V$ is a constant when $V$ is known)

The relative order between two jobs $J_{j1}$ and $J_{j2}$ ($\forall j1, j2 \in \{1, ..., n\}, j1 \leq j2$) is given by the following constraints:

$$y_{j1,j2} + y_{j2,j1} = 1 \tag{11}$$

The completion time of a job $J_j$ on the first machine is given by ($\forall j \in \{1, ..., n\}$)

$$C_{1,j} \geq p_{1,j} \tag{12}$$

The resource constraints allow to define the completion time of a job on any machine $M_i$ ($\forall i \in \{1, ..., m\}$, $\forall j1, j2 \in \{1, ..., n\}$, $j1 \neq j2$):

$$C_{i,j2} \geq C_{i,j1} + p_{i,j2} - y_{j2,j1}HV \tag{13}$$

The routing constraints are given on any machine $M_i$ ($i \in \{2, ..., m\}$) and for any job $J_j$ ($j \in \{1, ..., n\}$):

$$C_{i,j} \geq C_{i-1,j} + p_{i,j} \tag{14}$$

Each job $J_j$ is transported in a vehicle ($\forall j \in \{1, ..., n\}$), therefore:

$$\sum_{k=1}^{V} z_{j,k} = 1 \tag{15}$$

A job $J_j$ ($\forall j \in \{1, ..., n\}$) is transported by the vehicle leaving at the next departure time. The expression of this constraint is the following:

$$\sum_{k=1}^{V} z_{j,k} F_{k-1} + 1 \leq C_{m,j} \leq \sum_{k=1}^{V} z_{j,k} F_k \tag{16}$$

The costs are given in the following expressions:

$$PT_j^M \geq \sum_{k=1}^{V} z_{j,k} F_k + T - d_j, \quad \forall j \in \{1, ..., n\} \tag{17}$$

$$PPC^M = \sum_{j=1}^{n} \pi_j^M PT_j^M \tag{18}$$

$$IC^{WIP} = \sum_{j=1}^{n} (C_{m,j} - C_{1,j} - \sum_{i=2}^{m} p_{i,j}) q_j h_j^{WIP} \tag{19}$$

$$IC^{FIN} = \sum_{j=1}^{n} (\sum_{k=1}^{V} z_{j,k} F_k - C_{m,j}) q_j) h_j^{FIN} \tag{20}$$

$$IC = IC^{WIP} + IC^{FIN} \tag{21}$$

#### 3.4.2 3PL provider

We assume that the optimization of the routing of the vehicles are independent and that they can be optimized separately. Therefore, for each trip, the only 'due date' to respect is $T$, and we assume that the vehicle leaves the manufacturer site at time $t_0 = 0$.

The data required by the 3PL provider are the following:

| | |
|---|---|
| $T$ | estimated delivery time |
| $s$ | number of sites to visit (sites $0..s-1$ are the customer sites, site $s$ is the manufacturer site and site $s+1$ is the 3PL provider site) |
| $t_{j1,j2}$ | travel time between site $j1$ and site $j2$ ($0 \leq j1, j2 \leq s+1$) |
| $HV$ | an arbitrary high value (can be set to $2 \times \sum_{j1} \sum_{j2} t_{j1,j2}$) |

The costs to be taken into account are the following.

| | |
|---|---|
| $c_{j1,j2}$ | cost of travel time between site $j1$ and site $j2$ ($0 \leq j1, j2 \leq s+1$) |
| $\pi_j^{3PL}$ | penalty cost of 3PL provider for tardiness of job $J_j$ ($0 \leq j \leq s-1$) |

The variables that have to be determined are the following.

| | |
|---|---|
| $x_{j1,j2}$ | $= 1$ if site $j1$ is visited before site $j2$, 0 otherwise ($0 \leq j1, j2 \leq s+1$) |
| $sx_{j1,j2}$ | $= 1$ if site $j1$ is visited **just before** site $j2$, 0 otherwise ($j1 \in [0, s]$, $j2 \in [0, s-1] \cup \{s+1\}$) |
| $D_j$ | date at which site $j$ is visited, i.e. delivery date of $J_j$ ($0 \leq j \leq s+1$) |
| $T_j^{3PL} \geq 0$ | tardiness of delivery of $J_j$ ($0 \leq j \leq s-1$) |
| $RC$ | routing cost |
| $PC^{3PL}$ | total penalty cost of 3PL provider |

The routing of the 3PL provider is determined by solving the following Mixed Integer Linear Programming model.

$$\text{Minimize } TC^{3PL} = RC + PC^{3PL} - Vc^{V} \tag{22}$$

To ensure that there is no subtour in the routing, and each customer is served exactly once, we have, $\forall j1, j2, j3 \in \{0, .., s+1\}$, $j1 \neq j2$, $j1 \neq j3$, $j2 \neq j3$:

$$x_{j1,j2} + x_{j2,j1} = 1 \tag{23}$$

$$x_{j1,j2} + x_{j2,j3} \leq x_{j1,j3} + 1 \tag{24}$$

The connection between variables $x_{j1,j2}$ and variables $sx_{j1,j2}$ is given by ($\forall j1 \in [0, s]$, $\forall j2 \in [0, s-1] \cup \{s+1\}$, $j1 \neq j2$):

$$sx_{j1,j2} \leq x_{j1,j2} \tag{25}$$

To impose that the manufacturer and the 3PL provider have exactly one predecessor and one successor, we have ($\forall j1 \in \{0, .., s\}$)

$$\sum_{j2 \in [0,s-1] \cup \{s+1\}, j1 \neq j2} sx_{j1,j2} = 1 \tag{26}$$

and ($\forall j2 \in [0, s-1] \cup \{s+1\}$):

$$\sum_{j1=0}^{s} sx_{j1,j2} = 1 \tag{27}$$

The arrival date of each job at the customer site is given by the following constraints ($\forall j1 \in [0, s]$, $\forall j2 \in [0, s-1] \cup \{s+1\}$, $j1 \neq j2$):

$$D_{j2}^{3PL} \geq D_{j1}^{3PL} + t_{j1,j2} - HV(1 - sx_{j1,j2}) \tag{28}$$

The costs are given in the following expressions:

$$T_j^{3PL} \geq D_j^{3PL} - T, \quad \forall j \in 0, .., s+1 \tag{29}$$

$$PC^{3PL} = \sum_{j=0}^{s-1} \pi_j^{3PL} T_j^{3PL} \tag{30}$$

$$RC = \sum_{j1=0}^{s} \sum_{j2 \in [0,s-1] \cup \{s+1\}} c_{j1,j2} sx_{j1,j2} \tag{31}$$

### 3.4.3 Example

Let consider the following example. We have $n = 5$ jobs to schedule in a flow shop composed by $m = 2$ machines. The processing times on machine $M_1$ are $(61, 98, 26, 2, 28)$ and $(50, 49, 60, 61, 22)$ on machine $M_2$. The delivery due dates are $d = (212, 257, 277, 258, 249)$ and the tardiness penalties $\pi^M = (60, 70, 80, 90, 100)$. The holding costs are equal to $h_j^{WIP} = h_j^{FIN} = 1$ and $q_j = 10$ for any job $J_j$. The traveling durations are given by the following matrix and we define $c_{j1,j2} = t_{j1,j2}$. The timeout penalties are equal to $\pi^{3PL} = 100$ for any job and the price of one vehicle is fixed to $c^V = 1500$.

$$(t_{i,j}) = \left(\begin{array}{ccccc|c|c} 0 & 44 & 23 & 35 & 18 & 21 & 41 \\ 44 & 0 & 64 & 29 & 52 & 38 & 77 \\ 23 & 64 & 0 & 58 & 14 & 43 & 44 \\ 35 & 29 & 58 & 0 & 52 & 17 & 54 \\ 18 & 52 & 14 & 52 & 0 & 39 & 53 \\ \hline 21 & 38 & 43 & 17 & 39 & 0 & 39 \\ \hline 41 & 77 & 44 & 54 & 53 & 39 & 0 \end{array}\right)$$

Suppose that the number of vehicles is equal to $V = 2$ and the delivery time is equal to $T = 20$.

**Solving the manufacturer problem**

The schedule obtained by the manufacturer leads to sequence $(4, 1, 3, 2, 5)$. The jobs' completion times are given in the following table (the schedule is not active – left shifted – because of the consideration of inventory costs):

| $j$ | 4 | 1 | 3 | 2 | 5 |
|---|---|---|---|---|---|
| $C_{1,j}$ | 2 | 63 | 89 | 187 | 236 |
| $C_{2,j}$ | 77 | 127 | 187 | 236 | 258 |

Two batches are composed, one per vehicle. The first batch contains jobs $J_1$ and $J_4$, the second batch contains jobs $J_2$, $J_3$ and $J_5$. The first batch leaves at time 129, the second batch at time 258. The schedule is represented in Fig. 3

The inventory costs $IC$ are given by the work in process inventory plus the cost of inventory of final products. The costs per job are given below:

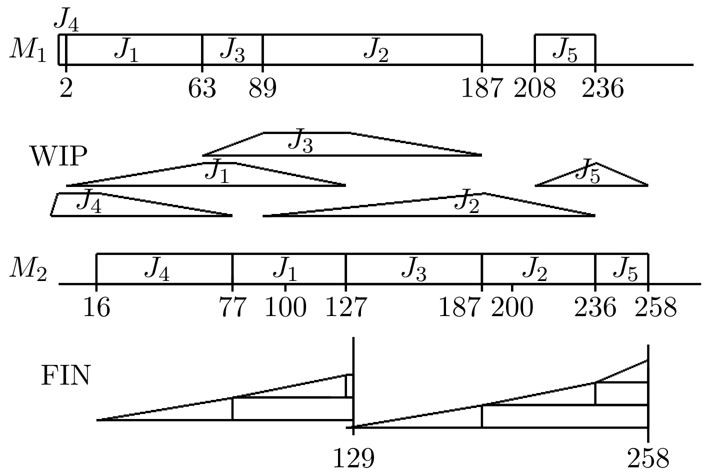


Figure 3:

| $j$ | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| WIP | 140 | 0 | 380 | 140 | 0 | 660 |
| FIN | 20 | 220 | 710 | 520 | 0 | 1470 |

which makes a total inventory cost of $IC = 660 + 1470 = 2130$.

The expected total tardiness $PPC^M$ is given by:

| $j$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $C_{2,j}$ | 127 | 236 | 187 | 77 | 258 |
| $F_1 + T$ | 149 | 278 | 278 | 149 | 278 |
| $d_j$ | 212 | 257 | 277 | 258 | 249 |
| $PT_j^M$ | 0 | 21 | 1 | 0 | 29 |
| $\pi_j^M PT_j^M$ | 0 | 1470 | 80 | 0 | 2900 |

The expected penalty cost for tardiness is equal to $PPC^M = 4450$. Because $VC = 3000$, the pseudo total cost is then equal to $PTC^M = 4450 + 2130 + 3000 = 9580$.

**Solving the 3PL provider problem**

The 3PL provider has two independent vehicle routing problems to solve. The solution is illustrated in Fig. 4.

For the first trip, the 3PL provider delivers job $J_4$ and then job $J_1$. For the second trip, the 3PL provider delivers job $J_3$, $J_5$ and $J_2$ in this order. The delivery times and tardiness of jobs for the 3PL provider are given in the following table:

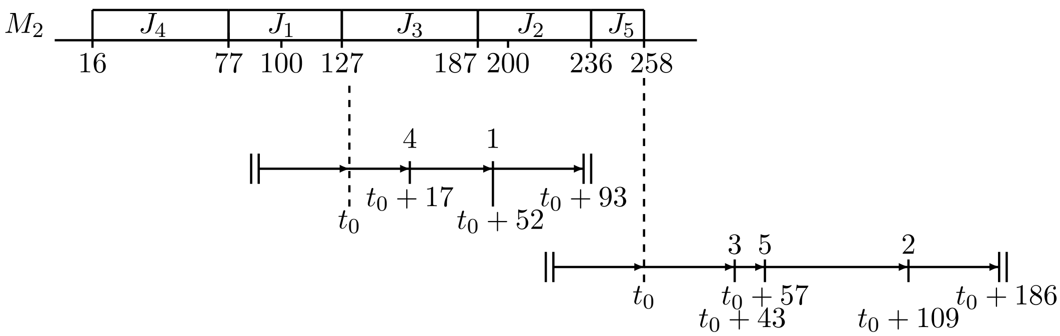


Figure 4:

| $j$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $D_j$ | $t_0 + 52$ | $t_0 + 109$ | $t_0 + 43$ | $t_0 + 17$ | $t_0 + 57$ |
| $T_j^{3PL}$ | 32 | 89 | 23 | 0 | 37 |
| $\pi_j^{3PL} T_j^{3PL}$ | 3200 | 8900 | 2300 | 0 | 3700 |

The penalty cost for the 3PL provider is equal to $PC^{3PL} = PC_1^{3PL} + PC_2^{3PL} - VC$. We have $PC_1^{3PL} = 3200$ and $PC_2^{3PL} = 14900$, i.e. $PC^{3PL} = 18100$. The routing cost is equal to $RC_1 + RC_2$ with $RC_1 = 132 (= 93 + 39)$ and $RC_2 = 225 (= 186 + 39)$, therefore we have $RC = 357$. His total cost is equal to $TC^{3PL} = 18100 + 357 - 3000 = 15457$.

The manufacturer knows the delivery dates and can now compute his total cost. The real delivery times and the tardiness of jobs for the manufacturer are given in the following table:

| $j$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| | $129 + 52$ | $258 + 109$ | $258 + 43$ | $128 + 17$ | $258 + 57$ |
| $D_j$ | $= 181$ | $= 367$ | $= 301$ | $= 146$ | $= 315$ |
| $d_j$ | 212 | 257 | 277 | 258 | 249 |
| $T_j^M$ | 0 | 110 | 24 | 0 | 66 |
| $\pi_j^M T_j^{3PL}$ | 0 | 7700 | 1920 | 0 | 6600 |

The total penalty cost is equal to $PC^M = 16220$. The total cost for the manufacturer is equal to $TC^M = 2130 + 16220 + 3000 - 18100 = 3250$.

Therefore, if $V = 2$ and $T = 20$, the costs $(TC^M, TC^{3PL})$ are equal to $(3250, 15457)$.

### Using different parameters

We can easily understand that if $T$ decreases, the 3PL provider will have more difficulties to reach the deadline, so his total cost function will increase to the profit of the Manufacturer. If $T$ increases,

If $V = 2$ and $T = 10$ we obtain $(TC^M, TC^{3PL}) = (-1450, 20157)$.

However, if $V$ increases, there is a reward for the 3PL provider and a cost for the manufacturer. But the manufacturer may decrease his inventory costs. If $V = 3$ and $T = 20$, we obtain $(TC^M, TC^{3PL}) = (7850, 10936)$.

# 4 Scenarios

In this section we consider several scenarios of the problem outlined above. In the first scenario, the manufacturer and the 3PL provider are supposed to belong to the same company. Then, we considertwo scenarios, one in which the manufacturer dominates and the 3PL provider adjusts and one the other way around in which the 3PL provider has the dominant position and the manufacturer adjusts to the decision of the 3PL provider. Finally we study the case where the manufacturer and the 3PL provider cooperate.

## 4.1 Global model for the Manufacturer and 3PL provider in the same company

In this scenario, we assume that the manufacturer and the 3PL provider belong to the same company. Therefore, the company has to solve the integrated problem. The cost that is considered is a global cost. We consider that the cost is a linear combination of the costs of the manufacturer and the 3PL provider.

In this case, let $TC$ denote the global total cost. We have:

$$TC = \alpha TC^M + (1-\alpha)TC^{3PL} \tag{32}$$

$$= \alpha IC + \alpha PC^M + (1-\alpha)RC + (2\alpha - 1)VC + (1-2\alpha)PC^{3PL} \tag{33}$$

If $\alpha = 0.5$, $RC$ and $PC^{3PL}$ are not considered. If $\alpha < 0.5$, then $2\alpha - 1 < 0$ and $VC$ is maximized (maximum number of vehicles will be used). And if $\alpha > 0.5$, then $1 - 2\alpha < 0$, $PC^{3PL}$ will be maximized. We can see that in this case, $VC$ and $PC^{3PL}$ generate particular behaviors. These costs - in this scenario - can be considered as 'internal costs'. We decide to not consider $PC^{3PL}$ in the integrated model, and therefore $T$ is no more a decision variable.

The objective function becomes

$$TC = \alpha TC^M + (1-\alpha)TC^{3PL} + VC \tag{34}$$

We can consider in this scenario that there are two agents plus a global agent.

The new data required in this model is the following:

| | |
|---|---|
| $\alpha_M$ | coefficient of $TC^M$ in the linear combination |
| $\alpha_{3PL}$ | $= 1-\alpha_M$ coefficient of $TC^{3PL}$ |
| $UB_t$ | upper bound for t |

The variables to determine are $y_{j1,j2}$, $z_{j,k}$ and $C_{i,j}$ as defined in the model of the manufacturer (see Section 3.4.1). Variables $T_j^M$, $IC^{WIP}$, $IC^{FIN}$ remain also unchanged. The variables $x_{j1,j2}$ and $RC$ defined in the model of the 3PL provider (see Section 3.4.2) become $x_{j1,j2,k}$ and $RC_k$, introducing index $k$ for the tour $k$. Variables $D_j$ remain unchanged.

The new variables are the following:

| | |
|---|---|
| $V \geq 0$ | number of vehicles |
| $PC^M \geq 0$ | penalty cost of the manufacturer |
| $x_{i,j,k}$ | $= 1$ if arc $i, j$ is in tour $k$, 0 otherwise |
| $az_k$ | $= 1$ if tour $k$ is 'active', 0 otherwise |
| $F_k \geq 0$ | start of tour $k$ |
| $S_j \geq 0$ | departure date of job $J_j$ |
| $TC^M \geq 0$ | total cost of the manufacturer |
| $RC_k \geq 0$ | routing cost for trip $k$ |
| $RC^{3PL} \geq 0$ | routing cost for 3PL provider |
| $PC^M$ | total penalty cost of the manufacturer |

The global problem for an integrated version, where manufacturer and 3PL provider are part of the same company is solved by the following Mixed Integer Linear Programming model.

$$\text{Minimize } TC = \alpha TC^M + (1-\alpha)TC^{3PL} + VC \tag{35}$$

The constraints are listed in the following:

(11), (13), (12), (14), (15)

The departure time $S_j$ of a job $J_j$ is defined by the following constraints ($\forall j \in \{1, ..., n\}$, $\forall k \in \{1, ..., n\}$):

$$S_j \geq F_k - HV(1 - z_{j,k}) \tag{36}$$

$$S_j \leq F_k + HV(1 - z_{j,k}) \tag{37}$$

The tardiness is given in the following constraints ($\forall j \in \{1, ..., n\}$):

$$T_j^M \geq D_j - d_j \tag{38}$$

The number of vehicles is determined by the following expression ($\forall k \in \{1, ..., n\}$):

$$V = \sum_{k=1} az_k \tag{39}$$

To ensure that, a job $J_j$ can only be transported on an 'active' tour, we have, ($\forall j \in \{1, ..., n\}$, $\forall k \in \{1, ..., n\}$)

$$z_{j,k} \leq az_k \tag{40}$$

and $\forall k \in \{2, ..., n\}$

$$az_{k-1} \geq az_k \tag{41}$$

The departure time of a tour $k$ is given by the following expression ($forallj \in \{1, ..., n\}$, $\forall k \in \{1, ..., n\}$)

$$F_k \geq C_{m,j} - HV(1 - z_{j,k}) \tag{42}$$

We introduce constraints for breaking symmetries in the model ($\forall k \in \{1, ..., n\}$):

$$F_k \leq F_{k+1} \tag{43}$$

The delivery time of a job $J_j$ is defined by the following constraints ($\forall i \in \{0, ..., n\}$, $\forall j \in \{1, ..., n\}$, $\forall k \in \{1, ..., n\}$, $i \neq j$)

$$D_0 = 0 \tag{44}$$

$$D_j \geq D_i + t_{i,j} - HV(1 - x_{i,j,k}) \tag{45}$$

$$D_j \geq F_k + t_{0,j} - HV(1 - z_{j,k}) \tag{46}$$

To ensure that each customer location has exactly one arriving arc and one leaving arc we introduce the following constraints, $\forall j \in \{1, ..., n\}$

$$\sum_{k \in [1,..,n] i \in 0,..,n]} x_{i,j,k} = 1 \tag{47}$$

and $\forall k \in \{1, ..., n\}$, $\forall j \in \{1, ..., n+1\}$

$$\sum_{k \in [1,..,n] i \in 1,..,n+1]} x_{i,j,k} = 1 \tag{48}$$

Departure and arrival loactions of a tour $k$ are defined, $\forall k \in \{1, ..., n\}$, $\forall j \in \{1, ..., n\}$, by:

$$\sum_{i \in [0..n]} x_{i,j,k} \geq z_{j,k}, \tag{49}$$

$$\sum_{i \in [1..n+1]} x_{j,i,k} \geq z_{j,k}, \tag{50}$$

and $\forall i \in \{0, ..., n\}$, $\forall k \in \{1, ..., n\}$

$$x_{i,i,k} = 0 \tag{51}$$

To prevent loops in the routing, we introduce the following constraints ($\forall i \in \{0, ..., n\}$, $\forall k \in \{1, ..., n\}$):

$$x_{i,i,k} = 0 \tag{52}$$

The costs are given in the following expressions:

$$RC_k = \sum_{i \in [0,..,n] j \in 1,..,n+1]} c_{i,j} x_{i,j,k}, \quad \forall k \in \{1, ..., n\} \tag{53}$$

$$IC^{WIP} = \sum_{j=1}^{n} \sum_{i=1}^{m} (C_{i+1,j} - p_{i+1,j} - C_{i,j}) q_j h_{i,j} \tag{54}$$

$$IC^{FIN} = \sum_{j=1}^{n} (S_j - C_{m,j}) q_j h_{m,j} \tag{55}$$

$$TC^M = IC^{WIP} + IC^{FIN} + PC^M \tag{56}$$

$$PC^M = \sum_{j=1}^{n} \pi_j^M T_j^M \tag{57}$$

$$RC^{3PL} = \sum_{k=1}^{n} RC_k \tag{58}$$

$$VC = c^V V \tag{59}$$

$$TC^{3PL} = RC^{3PL} \tag{60}$$

**Example**

Lets consider a similar example as used before for the model in which the manufacturer and the 3PL provider are independent agents. We consider the same data for the manufacturer as before, except for higher holding costs of $h_j^{WIP} = h_j^{FIN} = 10$ and a vehicle cost $c^V = 5000$.The coefficient $\alpha$ is set to 0.5. The travel distances and the travel costs of the 3PL provider are given in the following matrices:

$$(t_{i,j}) = \left(\begin{array}{ccccc|c|c} 0 & 21 & 38 & 43 & 17 & 39 & 39 \\ 21 & 0 & 44 & 23 & 35 & 18 & 41 \\ 38 & 44 & 0 & 64 & 29 & 52 & 77 \\ 43 & 23 & 64 & 0 & 58 & 14 & 44 \\ 17 & 35 & 29 & 58 & 0 & 52 & 54 \\ \hline 39 & 18 & 52 & 14 & 52 & 0 & 39 \\ \hline 39 & 41 & 77 & 44 & 54 & 39 & 0 \end{array}\right)$$

$$(c_{i,j}) = \left(\begin{array}{ccccc|c|c} 0 & 210 & 380 & 430 & 170 & 390 & 390 \\ 210 & 0 & 440 & 230 & 350 & 180 & 410 \\ 380 & 440 & 0 & 640 & 290 & 520 & 770 \\ 430 & 230 & 64 & 0 & 580 & 140 & 440 \\ 170 & 350 & 29 & 580 & 0 & 52 & 54 \\ \hline 390 & 180 & 52 & 140 & 520 & 0 & 390 \\ \hline 390 & 410 & 77 & 440 & 540 & 390 & 0 \end{array}\right)$$

**Solving the problem**

The schedule obtained by the manufacturer leads to sequence $(4, 5, 3, 1, 2)$. The jobs' completion times are given in the following table:

| $j$ | 4 | 5 | 3 | 1 | 2 |
|---|---|---|---|---|---|
| $C_{1,j}$ | 2 | 58 | 84 | 145 | 243 |
| $C_{2,j}$ | 63 | 85 | 145 | 195 | 292 |

Three batches are composed, one per vehicle. The first batch contains jobs $J_4$ and $J_5$, the second batch contains job $J_3$, and $J_1$ and the fourth batch contains job $J_2$. The first batch leaves at time 85, the second batch at time 195 and the third batch at time 292.

The inventory costs $IC$ are given by the work in process inventory plus the cost of inventory of final products. The costs per job are given below:

| $j$ | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| WIP | 0 | 0 | 100 | 500 | 0 | 600 |
| FIN | 2200 | 0 | 5000 | 0 | 0 | 7200 |

which makes a total inventory cost of $IC = 600 + 7200 = 7800$.

The penalty cost for tardiness $PC^M$ is given by:

| $j$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $C_{2,j}$ | 195 | 292 | 145 | 63 | 85 |
| $D$ | 216 | 330 | 188 | 102 | 164 |
| $d_j$ | 212 | 257 | 277 | 258 | 249 |
| $PT_j^M$ | 4 | 73 | 0 | 0 | 0 |
| $\pi_j^M PT_j^M$ | 240 | 5110 | 0 | 0 | 0 |

The total penalty cost for tardiness is equal to $PC^M = 5350$. The total cost for the manufacturer is then equal to $TC^M = 7800 + 5350 = 13130$.

The routing cost is equal to $RC_1 + RC_2 + RC_3$ with $RC_1 = 1080(= 170 + 520 + 390)$, $RC_2 = 880(= 210 + 230 + 440)$ and $RC_3 = 1150(= 380 + 770)$, therefore we have $RC = 3110$.The total cost of the 3PL provider are equal to the routing cost $TC^{3PL} = 3110$.

The global total cost is equal to $TC = 0.5 * 13130 + 0.5 * 3110 + 15000 = 23130$, as vehicle costs are equal to $VC = 15000$.

## 4.2 Manufacturer dominates

In the case of dominance, the manufacturer will determine an optimal schedule, minimising his costs, regardless of the 3PL provider. It is therefore assumed that the manufacturer will choose the number of vehicles $V$ and impose a maximum delivery time $T$ on the 3PL provider in such a way, that the manufacturer's schedule is optimised, hence $V$ and $T$ are treated as variables in this model. Generally, though depending on the penalty costs, the manufacturer aims to keep the value of $T$ as low as possible shifting the cost of delay onto the 3PL provider. Therefore, there is no reason to fix $T$ to another value than 0. For this reason, in order to avoid a non-realistic definition of $T$ we impose a lower bound, so that $T \geq t_{0,j}$

The manufacturer computes $C_{i,j}$, $F_k$ and $T$ and estimates the costs resulting from the optimal schedule. Given $F_k$ and $T$ from the optimal schedule of the manufacturer, the 3PL provider then makes a routing decision optimising its costs, after which the manufacturer can update its estimation and calculate the real costs. The adapted model is outlined below.

The data required by the manufacturer is the same as before (see Section 3.4.1) plus the delivery times $t_{0,j}$. The variables are also the same except for the new variables to determine, which are the following.

$V$ number of vehicles

$T$ estimated delivery time

$S_j$ the departure time of $J_j$

The scheduling problem of the manufacturer is determined by solving the following Mixed Integer Linear Programming model.

$$\text{Minimize } PTC^M = IC + PPC^M + Vc^V \tag{61}$$

Notice that $VC$ is a variable in this model. The constraints (11), (13), (14), (15), (18), (19), (21), (86), (87) are still valid.

To determine the number of vehicles, we introduce the following constraints ($\forall j \in \{1, ..., n\}$):

$$\sum_{j=1}^{n} z_{k,j} \leq V \tag{62}$$

We introduce constraints for breaking symmetries in the model ($\forall k \in \{1, .., n\}$):

$$F_k + 1 \leq F_{k+1} \tag{63}$$

A lower bound for the maximum delivery time $T$ is introduced ($\forall j \in \{1, ..., n\}$):

$$T \geq t_{0,j} \tag{64}$$

The following constraints replace constraints (16) ($\forall j \in \{1, ..., n\}, \forall k \in \{2, ..., n\}$):

$$F_{k-1} + 1 - HV(1 - z_{k,j}) \leq C_{m,j} \leq F_k + HV(1 - z_{k,j}) \tag{65}$$

The following constraints replace constraints (17) ($\forall j \in \{1, ..., n\}$):

$$PT_j^M \geq D_j + T - d_j \tag{66}$$

Finally, the following constraints replace constraints (20) ($\forall j \in \{1, ..., n\}$):

$$IC^{FIN} = \sum_{j=1}^{n} (S_j - C_{m,j}) q_j h_j^{FIN} \tag{67}$$

Once the manufacturer has optimised his schedule the 3PL provider makes his routing decision as in the general model for the 3PL provider using the parameters for $T$, $V$ and $S_j$ that have been determined in the dominance model outlined above.

### Example

Given the same instance for the manufacturer as in the example where manufacturer and 3PL provider are independent agents and solving the model with manufacturer dominance for 2 different vehicle cost considerations ($c^V = 100$ and $c^V = 5000$), one obtains the following results:

In the first instance with $c^V = 100$, the optimal solution for the manufacturer is obtained by the sequence (4,1,5,3,2). The completion times of the jobs are given in the following table:

| $j$ | 4 | 1 | 5 | 3 | 2 |
|---|---|---|---|---|---|
| $C_{1,j}$ | 2 | 63 | 113 | 139 | 237 |
| $C_{2,j}$ | 63 | 113 | 135 | 199 | 286 |

The five jobs are each treated as an individual batch so that five batches and five vehicles are used to distribute the jobs to the customers. Each batch leaves immediately after the completion time of the job associated with the batch. Resulting in zero finished product inventory cost so that total inventory cost IC are only based on the work in progress inventory, which is in this case also equal to zero for all jobs. We have therefore no inventory costs.

With $T = 43$, the expected total tardiness $PPC^M$ is given by:

| $j$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $C_{2,j}$ | 113 | 286 | 199 | 63 | 135 |
| $F_1 + T$ | 156 | 329 | 242 | 106 | 177 |
| $d_j$ | 212 | 257 | 277 | 258 | 249 |
| $PT_j^M$ | 0 | 72 | 0 | 0 | 0 |
| $\pi_j^M PT_j^M$ | 0 | 5040 | 0 | 0 | 0 |

The expected penalty cost for tardiness is equal to $PPC^M$=5040. The pseudo total cost is therefore equal to $PTC^M$=5040+0+500=5540 with a vehicle cost equal to VC=500.

Solving the model for the 3PL provider using the same data as in the global model we obtain the following results for the routing:

The delivery times are given in the following table:

| $j$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $D_j$ | 21 | 38 | 43 | 17 | 39 |

The routing costs (referring to each job) are composed of $RC_1 = 620$ (210+410), $RC_2 = 1150$ (380+770), $RC_3 = 870$ (430+440), $RC_4 = 710$ (170+540) and $RC_5 = 780$ (390+390), leading to a total routing cost equal to $RC = 620 + 1150 + 870 + 710 + 780 = 4130$. The 3PL provider does not incure a penalty cost given the maximum delay equal to $T = 43$. Hence, the total cost of the 3PL provider are equal to $TC^{3PL} = 4130 - 500 = 3630$.

In this case, the cost estimation of the manufacturer was accurate and the pseudo total cost $PTC^M$ is equal to the total cost of the manufacturer $TC^M$= 5540

The the second instance with a vehicle cost equal to $c^V$=5000, leads to an optimal schedule with the scheduling sequence (4,1,5,3,2). The completion times for the jobs on the two machines are shown in the following table:

| $j$ | 4 | 1 | 5 | 3 | 2 |
|---|---|---|---|---|---|
| $C_{1,j}$ | 2 | 63 | 113 | 139 | 237 |
| $C_{2,j}$ | 63 | 113 | 135 | 199 | 286 |

The jobs are split into four batches, the first batch comprising of job 4, departing immediately after the completion time of job 4, the second batch comprising of job 1 and 5 departing immediately after the completion of job 5, batch 3 comprising of job 3, departing immediately after the completion time of job 3 and batch 4 comprising of job 2 and leaving immediately after the completion time of job 2. The inventory costs for the individual jobs are shown in the following table:

| $j$ | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| WIP | 0 | 0 | 0 | 0 | 0 | 0 |
| FIN | 2200 | 0 | 0 | 0 | 0 | 2200 |

The total inventory cost is therefore equal to IC=0+2200=2200

The expected total tardiness $PPC^M$ is given by:

| $j$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $C_{2,j}$ | 113 | 286 | 199 | 63 | 135 |
| $F_1 + T$ | 178 | 329 | 242 | 106 | 178 |
| $d_j$ | 212 | 257 | 277 | 258 | 249 |
| $PT_j^M$ | 0 | 72 | 0 | 0 | 0 |
| $\pi_j^M PT_j^M$ | 0 | 5040 | 0 | 0 | 0 |

The expected penalty cost for tardiness is equal to $PPC^M$=5040. The pseudo total cost is therefore equal to $PTC^M$=5040+2200+20000=27240 with a vehicle cost equal to $VC = 20000$ for four vehicles.

Solving the model for the 3PL provider we obtain the following results for the routing:

The delivery times are given in the following table:

| $j$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $D_j$ | 21 | 38 | 43 | 17 | 65 |
| $PC^{3PL}$ | 0 | 0 | 0 | 0 | 2200 |

The routing costs are composed of $RC_1 = 710$ (170+540), $RC_2 = 780$ (210+ 180+ 390), $RC_3 = 870$ (430+440) and $RC_4 = 1150$ (380+770), leading to a total routing cost equal to $RC = 710 + 780 + 870 + 1150 = 3510$. The 3PL provider incures a penalty cost of $PC^{3PL}$= 2200 given the maximum delay equal to $T = 43$. Hence, the total cost of the 3PL provider are equal to $TC^{3PL} = 3510 + 2200 - 20000 = -14290$ and the 3PL provider earns a reward due to the high vehicle costs.

Even with the 3PL provider exceeding the maximum delivery time for job 5 by 22 time units, the job is still delivered in time and the cost estimation of the manufacturer for the penalty cost was accurate, therefore, the pseudo penalty cost $PPC^M$ equals $PC^M$, the real penalty cost of the manufacturer. However, the manufacturer now receives a reward from the 3PL provider for exceeding the maximum delivery time T, so that the total cost of the manufacturer is equal to $TC^M = 27240 - 2200 = 25040$.

## 4.3 3PL provider dominates

In the scenario, in which the 3PL provider takes the role of the dominant agent in the model, the 3PL provider aims to build its routings and minimise its cost by determining the number of vehicles $V$ and setting the maximum delivery time $T$ as high as possible in order to shift the load of the penalty cost onto the manufacturer. $T$ therefore becomes the duration of the longest trip, i.e. $T = \max_{1 \leq j \leq n}(t_{0,j})$. In addition to this, the 3PL provider also imposes the composition of the batches on the manufacturer. Given the routings and batch composition, the manufacturer then adjusts in a second step defining the completion time $C_{i,j}$ of job $J_j$ on machine $M_i$ as well as the departure time $F_k$ of tour $k$.

The data required by the 3PL provider is the same as before (see Section 3.4.2), except that $T$ is no longer treated as a date but as a variable. The variables that have to be determined are the following:

| | |
|---|---|
| $V$ | number of vehicles |
| $T$ | allowed delay for 3PL provider |
| $D_j$ | delivery date |
| $x_{i,j}$ | = 1 if site $j1$ is visted before $j2$, 0 otherwise |
| $z_{k,j}$ | =1 if job $J_j$ in tour $k$ |
| $az_k$ | =1 if tour $k$ is active |
| $RC_k$ | cost for routing of tour $k$ |
| $RC$ | total routing cost |
| $PC^{3PL}$ | total penalty cost of 3PL provider |
| $VC$ | vehicle costs |

The routing of the 3PL provider is determined by solving the following Mixed Integer Linear Programming model.

$$\text{Minimize } TC^{3PL} = RC + PC^{3PL} - VC \tag{68}$$

The constraints are listed in the following:

To ensure that, a job $J_j$ can only be transported on an 'active' tour, we have, ($\forall j \in \{1, ..., n\}$, $\forall k \in \{1, ..., n\}$)

$$z_{j,k} \leq az_k, \tag{69}$$

and $\forall k \in \{2, ..., n\}$

$$az_{k-1} \geq az_k, \tag{70}$$

An active tour has to carry at least one job ($\forall k \in \{1, ..., n\}$), therefore:

$$az_k \leq \sum_{j=1}^{n} z_{k,j} \tag{71}$$

The number of vehicles is defined by the following expression:

$$V = \sum_{k=1}^{n} az_k \tag{72}$$

The delivery time of a job $J_j$ is defined by the following constraints ($\forall i \in \{0, ..., n\}$, $\forall j \in \{1, ..., n\}$, $\forall k \in \{1, ..., n\}$, $i \neq j$)

$$D_0 = 0, \tag{73}$$

$$D_j \geq D_i + t_{i,j} - HV(1 - x_{i,j,k}) \tag{74}$$

To ensure that each customer location has exactly one arriving arc and one leaving arc we introduce the following constraints, $\forall j \in \{1, ..., n\}$

$$\sum_{k \in [1,..,n] i \in 0,..,n]} x_{i,j,k} = 1 \tag{75}$$

and $\forall k \in \{1, ..., n\}$, $\forall j \in \{1, ..., n+1\}$

$$\sum_{k \in [1,..,n] i \in 1,..,n+1]} x_{i,j,k} = 1 \tag{76}$$

Departure and arrival loactions of a tour $k$ are defined, $\forall k \in \{1, ..., n\}$, $\forall j \in \{1, ..., n\}$, by:

$$\sum_{i \in [0..n]} x_{i,j,k} \geq z_{j,k} \tag{77}$$

$$\sum_{i \in [1..n+1]} x_{j,i,k} \geq z_{j,k} \tag{78}$$

and $\forall i \in \{0, ..., n\}$, $\forall k \in \{1, ..., n\}$

$$x_{i,i,k} = 0 \tag{79}$$

To prevent loops in the routing, we introduce the following constraints ($\forall i \in \{0, ..., n\}$, $\forall k \in \{1, ..., n\}$):

$$x_{i,i,k} = 0 \tag{80}$$

An upper bound is introduced for the maximum delay $T$, $\forall j \in \{1, ..., n\}$, by:

$$T \leq t_{0,j} + t_{j,n+1} \tag{81}$$

The costs are given in the following expressions:

$$RC_k = \sum_{i \in [0,..,n] j \in 1,..,n+1]} c_{i,j} x_{i,j,k}, \ \ \forall k \in \{1, ..., n\} \tag{82}$$

$$RC^{3PL} = \sum_{k=1} RC_k \tag{83}$$

$$VC = c^V V \tag{84}$$

Once the 3PL provider has optimised his routing decisions, the manufacturer adjusts given the data from the optimal schedule of the 3PL provider.

The data required by the manufacturer is the same as in the original model for the manufacturer (see Section 3.4.1), with the following additions that were before treated as variables in the model:

| | |
|---|---|
| $l$ | number of batches |
| $z_{k,j}$ | if job $J_j$ is included in batch $k$ or not |
| $D_j$ | the delivery date of job $J_j$ |

The variables to determine are $y_{j1,j2}$ and $C_{i,j}$ as defined in the model of the manufacturer (see Section 3.4.1). Variables $T_j^M$, $IC^{WIP}$, $IC^{FIN}$ remain also unchanged. The variables $PC^M$, $F_k$, $S_j$ and $TC^M$ are the same as in the global model (see Section 4.1).

The schedule of the manufacturer and the pick-up dates are determined by solving the following Mixed Integer Linear Programming model.

$$\text{Minimize } TC^M = IC^{WIP} + IC^{FIN} + PC^M \tag{85}$$

Note, we do not consider vehicle costs as they can be treated as a constant in this model.

The constraints are outlined in the following:

(11), (13), (12), (14)

The departure time $S_j$ of a job $J_j$ is defined by the following constraints ($\forall j \in \{1, ..., n\}$, $\forall k \in \{1, ..., n\}$):

$$S_j \geq F_k - HV(1 - z_{j,k}) \tag{86}$$

$$S_j \leq F_k + HV(1 - z_{j,k}) \tag{87}$$

The departure time of a tour $k$ is given by the following expression ($forall j \in \{1, ..., n\}$, $\forall k \in \{1, ..., n\}$):

$$F_k \geq C_{m,j} - HV(1 - z_{j,k}) \tag{88}$$

We introduce constraints for breaking symmetries in the model ($\forall k \in \{1, ..., n\}$):

$$F_k \leq F_{k+1} \tag{89}$$

The tardiness constraints change to the following expression, taking into account $S_j$ ($\forall j \in \{1, ..., n\}$):

$$T_j^M \geq D_j + S_j - d_j \tag{90}$$

The costs are given in the following expressions:

$$IC^{WIP} = \sum_{j=1}^{n}\sum_{i=1}^{m}(C_{i+1,j} - p_{i+1,j} - C_{i,j})q_j h_{i,j} \tag{91}$$

$$IC^{FIN} = \sum_{j=1}^{n}(S_j - C_{m,j})q_j h_{m,j} \tag{92}$$

$$TC^M = IC^{WIP} + IC^{FIN} + PC^M \tag{93}$$

$$PC^M = \sum_{j=1}^{n}\pi_j^M T_j^M \tag{94}$$

**Example**

Considering the same example as before for the global model and solving it for the case in which the 3PL provider dominates for two different vehicle cost considerations ($c^V = 100$ and $c^V = 5000$) we obtain the following results:

For the first instance with $c^V = 100$, the optimal solution for the 3PL provider is found by using 2 vehicles, delivering the jobs in two batches, with one of the batches containing jobs 1, 3 and 5 with routing (0,1,5,3,6) and the other batch containing job 2 and 4 with routing (0,4,2,6). The delivery times are given in the following table:

| $j$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $D_j$ | 30 | 46 | 62 | 17 | 48 |

The routing cost is equal to $RC_1 + RC_2$ with $RC_1 = 1230(= 170 + 290 + 770)$ and $RC_2 = 970(= 210 + 180 + 140 + 440)$, therefore we have $RC = 2200$.The total cost of the 3PL provider are equal to $TC^{3PL} = RC + PC^{3PL} - VC = 2200 + 0 - 200 = 2000$, as the 3PL provider does not incure a penalty cost due to a high maximum delay $T = 62$ and a vehicle cost of $VC = 200$ as two vehicles are used.

Given the batches of the 3PL provider the manufacturer obtains the following optimal schedule with sequence $(3, 1, 5, 4, 2)$. The jobs' completion times are given in the following table:

| $j$ | 3 | 1 | 5 | 4 | 2 |
|---|---|---|---|---|---|
| $C_{1,j}$ | 26 | 87 | 120 | 122 | 220 |
| $C_{2,j}$ | 86 | 137 | 159 | 220 | 269 |

The inventory costs $IC$ are given by the work in process inventory plus the cost of inventory of final products. The costs per job are given below:

| $j$ | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| WIP | 0 | 0 | 3700 | 1700 | 0 | 5400 |
| FIN | 2200 | 0 | 7300 | 4900 | 0 | 14400 |

which makes a total inventory cost of $IC = 5400 + 14400 = 19800$.

The total penalty $PC^M$ is given by:

| $j$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $C_{2,j}$ | 137 | 269 | 86 | 220 | 137 |
| $D_j$ | 30 | 46 | 62 | 17 | 48 |
| $d_j$ | 212 | 257 | 277 | 258 | 249 |
| $T_j^M$ | 0 | 74 | 0 | 28 | 0 |
| $\pi_j^M PT_j^M$ | 0 | 5180 | 0 | 2520 | 0 |

The total penalty cost for tardiness is equal to $PC^M = 7700$. The total cost for the manufacturer is then equal to $TC^M = 19800 + 7700 = 27500$.

The second instance with $c^V = 5000$, the optimal solution for the 3PL provider is found by using 5 vehicles, meaning one vehicle per job so that each job is delivered individually. The delivery times are given in the following table:

| $j$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $D_j$ | 21 | 62 | 62 | 17 | 62 |

The routing cost is equal to $RC_1 + RC_2 + RC_3 + RC_4 + RC_5$ with $RC_1 = 710(= 170 + 540)$, $RC_2 = 1150(= 380 + 770)$, $RC_3 = 780(= 390 + 390)$, $RC_4 = 870(= 430 + 440)$ and $RC_5 = 620(= 210 + 410)$ therefore we have $RC = 4130$.The total cost of the 3PL provider are equal to $TC^{3PL} = RC + PC^{3PL} - VC = 4130 + 0 - 25000 = -20870$, as the 3PL provider does not incure a penalty cost due to a high maximum delay $T = 62$ and a vehicle cost of $VC = 25000$ as two vehicles are used. In this case the 3PL provider receives a reward rather than incuring a cost.

Given the batches and delivery times of the 3PL provider the manufacturer obtains the following optimal schedule with sequence $(4, 2, 5, 3, 1)$. The jobs' completion times are given in the following table:

| $j$ | 4 | 2 | 5 | 3 | 1 |
|---|---|---|---|---|---|
| $C_{1,j}$ | 2 | 100 | 145 | 171 | 232 |
| $C_{2,j}$ | 63 | 149 | 1171 | 231 | 282 |

The inventory costs $IC$ are given by the work in process inventory plus the cost of inventory of final products. The costs per job are given below:

| $j$ | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| WIP | 0 | 0 | 400 | 0 | 0 | 400 |
| FIN | 0 | 0 | 0 | 0 | 0 | 0 |

which makes a total inventory cost of $IC = 5400 + 14400 = 400$.

The total penalty $PC^M$ is given by:

| $j$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $C_{2,j}$ | 282 | 149 | 231 | 63 | 117 |
| $D_j$ | 21 | 62 | 62 | 17 | 62 |
| $d_j$ | 212 | 257 | 277 | 258 | 249 |
| $T_j^M$ | 91 | 0 | 16 | 0 | 0 |
| $\pi_j^M PT_j^M$ | 5460 | 0 | 1280 | 0 | 0 |

The total penalty cost for tardiness is equal to $PC^M = 6740$. The total cost for the manufacturer is then equal to $TC^M = 400 + 6740 = 7140$.

Note, the vehicle costs are not included in the cost of the manufacturer as they are a constant in the model, the actual cost including vehicle costs would be equal to $TC^M + VC = 7140 + 25000 = 32140$.

# 5 Extensions of the model

## 5.1 Reward of the 3PL provider for an early delivery

We assume in this section that the 3PL provider is aware of the due dates and has an incentive to respect them. Therefore, the $T$ delay for delivery is not usefull. Notice that it is not the same to say that 'the 3PL provider has a reward in case of early delivery, paid by the manufacturer' and 'the 3PL provider pays a cost to the manufacturer if the delivery is late'. The main reason is that the 3PL provider knows now the due dates and has a direct interest in respecting them. The tardiness penalties paid to the manufacturer are not considered any more.

In the case of the integrated model, where 3PL provider and manufacturer act as independent agents, the manufacturer model remains unchanged, except for the computation of the total cost.

Indeed, the pseudo penalty cost takes $T$ into account. The manufacturer has to define a new estimation of the tardiness. For example considering

$$PT_j^M = \max(0, C_{m,j} + t_{ref} - d_j)$$

with $t_{ref} = \min_{1 \leq j \leq n} t_{0,j}$ or $t_{ref} = \max_{1 \leq j \leq n} t_{0,j}$, etc.

The definition of $PPC^M$ and of $PC^M$ are unchanged. However, the total cost $TC^M$ is now defined by:

$$TC^M = IC + VC + PC^M + EC^{3PL}$$

where $EC^{3PL}$ is the earliness cost paid to the 3PL provider equal to $EC^{3PL} = \sum_{j=1}^{n} \rho_j^{3PL} E_j^{3PL}$ with $\rho^{3PL}$ the reward for the 3PL provider and $E_j^{3PL} = \begin{cases} 1 & \text{if } D_j \leq d_j \\ 0 & \text{otherwise} \end{cases}$ (the importance lies in not being late rather than maximising earliness).

For the 3PL provider we now consider due dates, therefore for each tour $k$, we have to add $F_k$ to the delivery dates obtained by the model. We introduce the definition of $EC^{3PL}$ and the total cost for the 3PL provider is now equal to:

$$TC^{3PL} = RC - EC^{3PL} - VC$$

## 5.2 Considering vehicle capacity

While we assumed in previous considerations that the capacity of the vehicles, used for delivery, is unbounded, in real world applications this is highly unlikely as vehicles usually have a maximum load in terms of wheight and spacial restrictions and hence capacity constraints for the vehicles have to be taken into account. In this section, we extend the model therefore, move away from the previous assumptions and now consider vehicle capacity in the model. We denote the capacity of a vehicle $k$ by $K$. As we are dealing with a homogenous fleet all vehicles have the same capacity $K$.

The following constraint is added to the model:

$$K \geq \sum_{j=1}^{n} z_{j,k} q_j \forall k \in 1, ..., n \tag{95}$$

In the case of the two step model, in which the manufacturer and 3PL provider act as independent agents and the number of vehicles $V$ was given as a parameter, resulting from negotiation between the two agents, $V$ now becomes a variable and the capacity constraint replaces the negotiation between the agents.

To determine the number of vehicles, we introduce the following constraints ($\forall j \in \{1, ..., n\}$):

$$\sum_{j=1}^{n} z_{k,j} \leq V \tag{96}$$

### 5.3 Considering perishable products

We consider here that the products are perishable (the case in chemotherapy production, food production, concrete production, etc.). It means that the delivery must be done after a given delay after the production completion time.

If we consider the global model, the following constraint is added:

$$D_j - C_{m,j} \leq \Delta_j, \forall k \in 1, ..., n \tag{97}$$

where $\Delta_j$ is the given delay after which delivery must occur.

The problem becomes more difficult if we are not in the hypothesis of the global model (only one company), as the manufacturer does not know the delivery dates $D_j$ beforehand in the other models/scenarios and hence has to find a way to compel the 3PL provider to adhere to the given delay and through this ensure delivery is done in time.

## 6 Data generation

We refer to [12] for a global view of transportation costs in logistic systems and to [9] for the evaluation of routing costs.

We consider an integrated production and delivery planning problem at an operational level, meaning that the planning horizon is 'short', and hence the distances considered are short. We assume that the maximum distance between the manufacturer and the customer locations does not exceed 150 km. In addition to this we consider the average speed to be 60km/h, so that the travel times are equal to the distances. We randomly generate the coordinates of the manufacturer, the 3PL provider and the $n$ customer locations so that $x_j$ and $y_j$ are in $[1, \frac{150}{\sqrt{2}}]$. The distance between two locations is the euclidean distance.

The processing time of job $J_j$ is generated between 1 and $150\alpha$ with $\alpha$ a coefficient in $\{0.75, 1, 1.25\}$. $\alpha$ corresponds to the ratio between the average processing time and the average transportation time. So, for example, if $\alpha = 0.75$, the average processing time is smaller than the average transportation time. The due dates are generated in $[75, 75n]$.

We assume, that the products, that are produced by the manufacturer, are manufactured in small quantities and transported in small vehicles (less than 3 tonnes). The quantities $q_j$ are generated in $[10, 200]$ (a quantity is given in kilograms) and the capacity of a vehicle is generated in $[1000, 3000]$ kgs.

**Definition of the costs**

The holding cost of product $J_j$ depends on the price of the product and of the cost of deterioration and obsolescence: $h_j^{FIN} = \delta_j t$ with $\delta_j$ the price of one unit of the product and $t = 20\%$. We define the inventory cost of work-in-process as a ratio of $h_j^{FIN}$: $h_j^{WIP} = \beta h_j^{FIN}$ with $\beta = 10\%$.

The penalty costs are defined as follows. The manufacturer tardiness penalty cost is a ratio of the price of one unit of the product per time unit of tardiness: $\pi_j^M = \varepsilon \delta_j q_j$. In the following we take $\varepsilon \in \{3\%, 5\%, 7\%\}$.

We consider in the following two different kinds of production:

- production of some very expensive products (production of chemotherapy drugs, production in the semi-conductor sector or of expensive electronic components). In this case, we have $\delta_j \in \{50, 200\}$ euros per unit of product.
- production of some very cheap products (milk or water production). In this case, we have $\delta_j \in \{0.05, 0.20\}$ euros per unit of product.

The cost of one vehicle for the manufacturer depends on the quantity transported and on the distance. For a given quantity, the cost is a linear function of the distance. Therefore, $c^v$ is a variable, depending on the tour (quantities transported, distances). The expression of the cost of one vehicle is given by the following expression:

$$c^v(tt, q) = (c_1 \times tt + c_2) \times c_3 + q^{c_4}$$

We take the following values for the constants: $c_1 = 0.06$, $c_2 = 10$, $c_3 = 3$ and $c_4 = -0.05$.

The cost of one vehicle for the 3PL provider is a ratio of this cost:

$$c_{j1,j2}(tt, q) = \lambda c^v(tt, q)$$

with $\lambda = 20\%$.

# 7 Conclusion and future research directions

In this paper, we consider a two-level supply chain problem., in which a manufacturer has to produce some jobs, and a 3PL provider delivers these jobs to the customers. The manufacturer and the 3PL provider are considered as two independent agents, which interact as part of a business relationship. A contract links their activities at an operational level. The manufacturer has to consider inventory costs, pays the 3PL provider to have vehicles available for the delivery of the jobs, while in addition to this paying tardiness penalty costs to the customers in case of late delivery. It is assumed that the dates at which the vehicles are available are demanded by the manufacturer. On the other hand, the 3PL provider has to deliver all the jobs within a given fixed deadline, that is negotiated. If the deadline is not respected, the 3PL provider has to pay the manufacturer a penalty cost. The workshop of the manufacturer has a flow shop organization and the cost function contains inventory costs, vehicle costs plus customer tardiness penalty costs while the cost function of the 3PL provider contains routing costs plus tardiness penalty costs to the manufacturer.

Mixed Integer Linear Programming models are proposed for the optimization of both, the manufacturer problem and the 3PL provider problem. These models are integrated in a global algorithm (the customer penalty costs are only known after the delivery), and illustrated by a numerical example. Then, several scenarios are proposed: we first propose a global model where the manufacturer and the 3PL provider belong to the same company and a global decision is made; then, we propose a model in which the manufacturer dominates the negotiation and a model in which the 3PL provider dominates. All scenarios considered are illustrated by numerical examples. Finally, we consider various extensions of the model. For each case, the mathematical programming formulations are presented. The document terminates with a Section about possible random data generation, that corresponds more or less to a realistic application of the models.

There are a lot of possible future research directions associated to this research project, that could, due to time limitations, unfortunately not be covered in this project. While, from a general point of view, more extensive computational experiments should be carried out in the case of future research, we also consider the following three main directions of research: different resolution approaches, different modelling approaches and other scenarios.

### Heuristics and better models

From a resolution point of view, due to the complexity of these problems, some heuristic methods can be proposed to replace the MILP formulations. This allows to solve larger instances and make more intensive computational tests, especially as real life problems are generally large instances for which it is sufficient to find a good solution fast, rather than using a time consuming exact method. Another possibilty is to design better exact models that are able to deal with bigger instances within reasonable computation time. However, our feeling is that exact methods will quickly reach their limits and therefore not be sufficient, due to the complexity of the problems.

### Higher accuracy of $T$, delivery date and cost estimation

From a modeling point of view, the setting of $T$ and the cost estimation of the manufacturer can be considered with a higher accuracy. For instance, it is not practical/acceptable to fix this parameter to a value smaller than the maximum travel time (given that there is no big deviation in the data) between the manufacturer site and the customers. On the other hand, it makes no sense to set this parameter to a high value. This remark highlights the difficulty to design 'interesting' data sets. Indeed, if the travel times are a lot longer than the processing times, the scheduling problem does not play an important role, and reciprocally, if the processing times are a lot longer than the travel times, the routing problem is not really important. Furthermore, the problem of setting $T$, $V$ and $F_k$ such that the costs of the manufacturer and the 3PL provider are minimised is of great interest. Finding $(T, V, F_k)$ can also lead to compromise solutions between the two agents.

In terms of the cost estimation for the pseudo cost of the manufacturer, the following points should be taken into consideration:

- The manufacturer has to take into account tardiness independent of $T$, and based on an estimation of the delivery time.
- $T$ should be used in the context of the tardiness associated to the 3PL provider.
- The manufacturer should make an estimation of the 3PL provider's penalty cost and consider this in his cost function, therefore taking into account the difference between the penalty cost received by the 3PL provider and the one paid to the customer.

Resulting in a cost function for the pseudo penalty cost similar to the following:

$$PPC^M = (\pi_j^M - \pi_j^{3PL})D_j + \pi_j^{3PL}(F_k + T) - \pi_j^M d_j \tag{98}$$

As the manufacturer does not know the exact delivery date $D_j$, it is necessary to make an estimation of the delivery date, hence to introduce a pseudo delivery date $PD_j$. One suggestion for estimating $PD_j$ is to take the departure date of the job plus the maximum distance from the manufacturer site to a job in the same batch, plus the average distance between jobs in the same batch. Another consideration could be to also take the number of jobs within a batch into account, as the more jobs in a batch the longer it will take to deliver the last job in the batch and the more likely it is that a job will be delayed.

### Taking realistic vehicle costs into account

Another aspect that can be improved in terms of accuracy from a modelling point of view is the vehicle cost consideration, based on the assumptions made in the chapter about data generation, so that vehicle costs are no longer a fixed cost but become a variable dependent on the distance and quantity transported. This makes the model more complex, especially if we consider both transport costs of the 3PL provider and the vehicle costs paid by the manufacturer as a variable in the model.

### Perishable products, time-windows and just-in-time delivery

The extension in terms of perishable products can be applied to the other scenarios rather than just the global model. In addition to this, we can consider the cases of delivery within time windows and the special case fo just-in-time delivery, especially as lean manufacturing and just-in-time deliveries become increasingly important in the industry in order to increase customer value and reduce costs.

### Manufacturer and 3PL provider cooperate

Other scenarios can be considered, such as the scenario in which the manufacturer and the 3PL provider cooperate. A possible idea for a solution approach for such a scenario could be to create a bi-criteria model based on the global model, minimising $TC^{3PL}$ subject to $TC^{M} \leq Q$ or the other way round and then for example enumerate a Pareto front, providing the optimal solutions for manufacturer and 3PL provider given the cost of the other agent.

## References


[1] Agnetis, A., Hall, N. G., Pacciarelli, D. (2006). Supply chain scheduling: Sequence coordination. Discrete Applied Mathematics, 154(15), 2044-2063.

[2] Agnetis, A., Aloulou, M. A., Fu, L. L. (2014). Coordination of production and interstage batch delivery with outsourced distribution. European Journal of Operational Research, 238(1), 130-142.

[3] Chandra, P., Fisher, M. L. (1994). Coordination of production and distribution planning. European Journal of Operational Research, 72(3), 503-517.

[4] Chang, Y. C., Lee, C. Y. (2004). Machine scheduling with job delivery coordination. European Journal of Operational Research, 158(2), 470-487.

[5] Chen, Z. L. (2004). Integrated Production and Distribution Operations. InHandbook of quantitative supply chain analysis (pp. 711-745). Springer US.

[6] Chen, Z. L., Vairaktarakis, G. L. (2005). Integrated scheduling of production and distribution operations. Management Science, 51(4), 614-628.

[7] Chen, H. K., Hsueh, C. F., Chang, M. S. (2009). Production scheduling and vehicle routing with time windows for perishable food products. Computers & Operations research, 36(7), 2311-2319.

[8] Chen Z-L. (2010). Integrated Production and Outbound Distribution Scheduling: Review and Extensions Operations Research, 58(1), 130148.

[9] Comité National Routier. *http://www.cnr.fr/Outils-simulation2/Simulateurs*. Last access July 2015.

[10] Dawande, M., Geismar, H. N., Hall, N. G., Sriskandarajah, C. (2006). Supply chain scheduling: distribution systems. Production and Operations Management, 15(2), 243-261.

[11] Fumero, F., Vercellis, C. (1999). Synchronized development of production, inventory, and distribution schedules. Transportation science, 33(3), 330-340.

[12] Gacogne, V. (2003). Impact des couts de transport sur les systèmes logistiques par une modélisation en dynamique des systèmes. PhD thesis, Ecole Nationale des Ponts et Chaussées, 2003.

[13] Graves, S. C. (1981). A review of production scheduling. Operations Research,29(4), 646-675.

[14] Hall, N. G., Potts, C. N. (2003). Supply chain scheduling: Batching and delivery. Operations Research, 51(4), 566-584.

[15] Lei, L., Liu, S., Ruszczynski, A., Park, S. (2006). On the integrated production, inventory, and distribution routing problem. IIE Transactions,38(11), 955-970.

[16] Li, C. L., Vairaktarakis, G., Lee, C. Y. (2005). Machine scheduling with deliveries to multiple customer locations. European Journal of Operational Research, 164(1), 39-51.

[17] Li, K., Sivakumar, A. I., Ganesan, V. K. (2008). Analysis and algorithms for coordinated scheduling of parallel machine manufacturing and 3PL transportation. International Journal of Production Economics, 115(2), 482-491.

[18] Low, C., Li, R. K., Chang, C. M. (2013). Integrated scheduling of production and delivery with time windows. International Journal of Production Research,51(3), 897-909.

[19] Sawik, T. (2009). Coordinated supply chain scheduling. International Journal of Production Economics, 120(2), 437-451.

[20] Seyedhosseini, S. M., Ghoreyshi, S. M. (2014). An Integrated Model for Production and Distribution Planning of Perishable Products with Inventory and Routing Considerations. Mathematical Problems in Engineering, 2014.

[21] Stecke, K. E., Zhao, X. (2007). Production and transportation integration for a make-to-order manufacturing company with a commit-to-delivery business mode. Manufacturing & Service Operations Management, 9(2), 206-224.

[22] Ullrich, C. A. (2013). Integrated machine scheduling and vehicle routing with time windows. European Journal of Operational Research, 227(1), 152-165.

[23] Wang, D. (2012). Problmes d'ordonnancement intgr entre la production et le transport avec stocks intermdiares et prise en compte de dates dues(Doctoral dissertation, Universit de Technologie de Belfort-Montbeliard).

[24] Wang, H., Lee, C. Y. (2005). Production and transport logistics scheduling with two transport mode choices. Naval Research Logistics (NRL), 52(8), 796-809.

[25] Zhong, W., Chen, Z. L., Chen, M. (2010). Integrated production and distribution scheduling with committed delivery dates. Operations Research Letters, 38(2), 133-138.