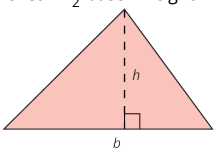
= *only if shooting for 165+*

Squares 1–20	Cubes 1–10	Powers of 2	Other Powers
$1^2 = 1$ $2^2 = 4$ $3^2 = 9$ $4^2 = 16$ $5^2 = 25$ $6^2 = 36$ $7^2 = 49$ $8^2 = 64$ $9^2 = 81$ $10^2 = 100$ $11^2 = 121$ $12^2 = 144$ $13^2 = 169$ $14^2 = 196$ $15^2 = 225$ $16^2 = 256$ $17^2 = 289$ $18^2 = 324$ $19^2 = 361$ $20^2 = 400$	$1^3 = 1$ $2^3 = 8$ $3^3 = 27$ $4^3 = 64$ $5^3 = 125$ $6^3 = 216$ $7^3 = 343$ $8^3 = 512$ $9^3 = 729$ $10^3 = 1000$	$2^0 = 1$ $2^1 = 2$ $2^2 = 4$ $2^3 = 8$ $2^4 = 16$ $2^5 = 32$ $2^6 = 64$ $2^7 = 128$ $2^8 = 256$ $2^9 = 512$ $2^{10} = 1024$	$3^4 = 9^2 = 81$ $3^5 = 243$ $4^4 = 16^2 = 256$ $5^4 = 25^2 = 625$

Fraction/Decimal Conversions		Square Root/Decimal Conversions
$\frac{1}{2} = 0.5$ $\frac{1}{3} = 0.\overline{333}$ $\frac{1}{4} = 0.25$ $\frac{1}{5} = 0.2$ $\frac{1}{6} = 0.1\overline{666}$	$\frac{1}{7} = 0.\overline{142857}$ $\frac{1}{8} = 0.125$ $\frac{1}{9} = 0.\overline{111}$ $\frac{1}{10} = 0.1$ $\frac{1}{11} = 0.\overline{9090}$ $\frac{1}{12} = 0.8\overline{333}$	$\sqrt{2} \approx 1.4$ (think: Valentine's day 2/14) $\sqrt{3} \approx 1.7$ (think: St. Patrick's day 3/17)

Area of a Triangle

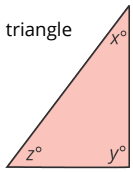
area = $\frac{1}{2}$ *base* × *height*



Angles in a Triangle

All angles in a triangle sum to 180°.

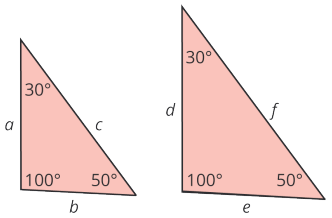
$x + y + z = 180$



Similar Triangles

Similar triangles have all the same angles, and sides that are proportionate.

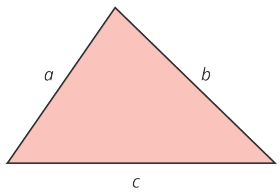
For example, *a*:*c* equals *d*:*f*



Third Side Rule

The third side of a triangle is greater than the difference of the other two sides and less than their sum.

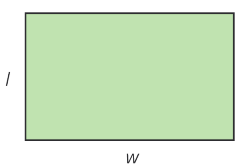
$(a + b) > c > (b - a)$



Area of a Rectangle

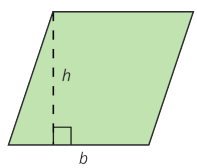
area of a rectangle = *length* × *width*

area of a square = $side^2$



Area of a Parallelogram

area of a parallelogram = *base* × *height*



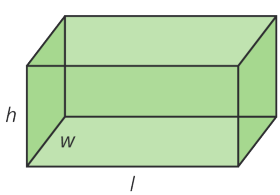
Sum of Angles in a Polygon

sum of angles in a polygon = $180(n - 2)$, where n = number of sides

Volume and Surface Area of a Box

volume = $l \times w \times h$

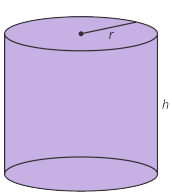
surface area = $2lw + 2wh + 2lh$



Volume and Surface Area of a Cylinder

volume = $\pi r^2 h$

surface area = $2\pi r^2 + 2\pi rh$

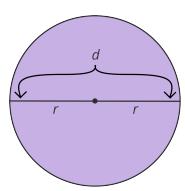


Circles Facts

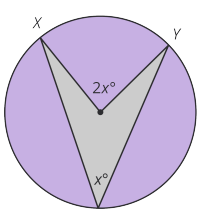
diameter = $2r$

circumference = πd or $2\pi r$

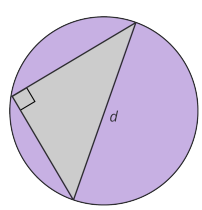
area = πr^2



A central angle has a vertex that lies at the center point of the circle. An inscribed angle has its vertex on the circle itself rather than on the center of the circle. An inscribed angle is equal to half of the arc it intercepts.

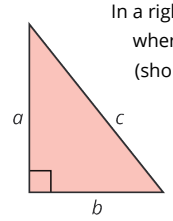


If one of the sides of an inscribed triangle is a diameter of the circle, then the triangle must be a right triangle.



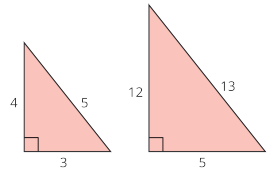
Pythagorean Theorem

In a right triangle, $a^2 + b^2 = c^2$, where a and b are the legs (shorter sides) and c is the hypotenuse.



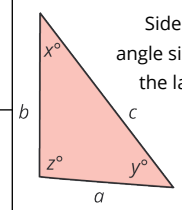
Pythagorean Triples

The most common triples are 3–4–5, 5–12–13, 7–24–25, and 8–15–17.

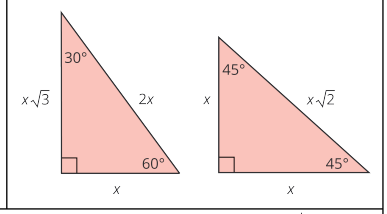


Angle/side Relationship

Side length corresponds with angle size. Angle z and side c are the largest. Angle x and side a are the smallest.



Special Right Triangles

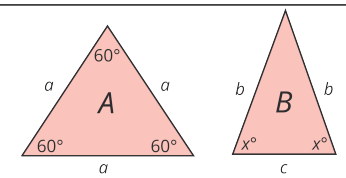


Isosceles and Equilateral

equilateral = all sides are the same length

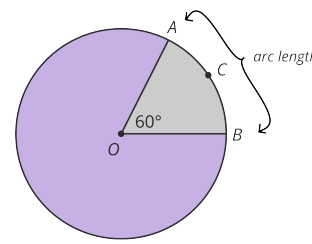
isosceles = two sides are the same length

In an equilateral triangle, all angles are 60°.



Sector

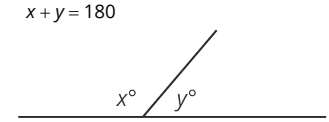
Sector area and arc length are proportional to angle size. Here, the angle is 60°. Since there are 360° in a circle, the sector (grey area) takes up $\frac{1}{6}$ of the circle. Thus, its area is $\frac{1}{6}$ of the total area, and its arc length is $\frac{1}{6}$ of the circumference.



Line and Angles

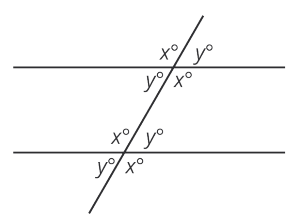
All angles in a line add up to 180°.

$x + y = 180$



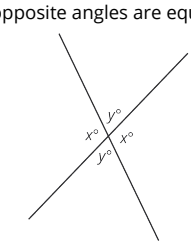
Parallel Lines

When a line intersects with two parallel angles, the resulting intersections are identical.



Intersecting Lines

When two lines intersect, opposite angles are equal.

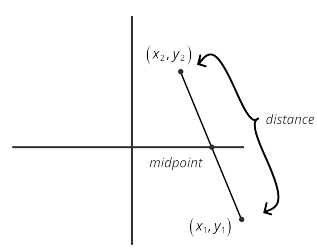


Distance and Midpoint on a Coordinate Plane

If you have endpoints (x_1, y_1) and (x_2, y_2) of a line on a coordinate plane:

$$\text{midpoint} = \left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)$$

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



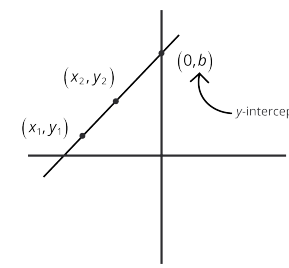
Slope, *x*-, *y*-intercept

$y = mx + b$, where m = slope, b = y-intercept

$$\text{slope} = \frac{\text{rise}}{\text{run}} \text{ or } \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

The y-intercept is the y-coordinate on a line at which $x = 0$.

The x-intercept is the x-coordinate on a line at which $y = 0$.



ALGEBRA

Linear Equations

• Solve/simplify/manipulate **one-variable** questions or **multivariable** systems of equations.
• Need 2 unique equations to solve for 2 variables
• When are 2 equations *not enough*? (they're **copies** of each other, or at least one of them has multiplied variables, creating a **quadratic**)
• When is 1 equation enough? (solving for **COMBO**, or variables = **pos integers**)

Exponents & Roots

• **Can't distribute** across addition or subtraction
• If bases are same, combine exponents
• If exponents are same, combine bases (never combine both)
• If bases are different *and* exponents are different: can't use rules!
$12^5 \times 12^3 = 12^{5+3}$ $(12^5)^3 = 12^{15}$ $12^5 / 12^3 = 12^{5-3}$
$12^5 \times 3^5 = 36^5$ $4^3 \times 4^3 = 16^3$ or $4^3 \times 4^3 = 4^6$
• **Adding or subtracting**: factor out and calculate what's left
e.g. $12^5 + 12^7 = 12^5(1 + 12^2) = 12^5(145)$
• **Multiplying or dividing**: break down bases until they match
e.g. $12^5 \times 10^4 = 2^x \times 3^y \times 5^z \rightarrow (2^5 \times 2^5 \times 3^5) \times (2^4 \times 5^4) = 2^{14} \times 3^5 \times 5^4$
• **Approximate square root**
$\sqrt{40}$ is between 6 and 7, because $\sqrt{36} < \sqrt{40} < \sqrt{49}$
• **Simplify square root** $\sqrt{40} = \sqrt{4 \times 10} = \sqrt{4} \times \sqrt{10} = 2\sqrt{10}$

Quadratics

• Even power? Beware: multiple solutions are possible!
• Set equal to 0 and find possible values by factoring
• 3 Special Quadratics $x^2 - y^2 = (x + y)(x - y)$
$(x + y)^2 = x^2 + 2xy + y^2$
$(x - y)^2 = x^2 - 2xy + y^2$

Formulas & Functions

• Don't panic. They're just inventing some *translation process*. Each time you see this function symbol they've invented and defined, you have to perform that translation process.

Sequences

• Immediately draw out 5 or 6 blanks and number them underneath
• Input the givens. $a_4 = 12 \rightarrow$ "the 4th term is 12".
• Read the rule as a process. $a_n = 3(a_{n-1} + a_{n-2}) \rightarrow$ "to find the next term, add up the two previous terms and multiply by 3".
• If not given numbers, add variables to the blanks.

Inequalities

• Think about: **negatives** and fractions! Don't multiply or divide by a variable if its sign is unknown. If they tell you variables are positive, they want you to manipulate the algebra.
• Test Weird/Extreme cases
• System of two inequalities: stack & add them (align the signs)
• " > 0" or " < 0" is really testing **positive/negative** rules
• If given $|x - 5| < 8$, consider both the positive and negative possibilities: $(x - 5) < 8$ or $-(x - 5) < 8$

NUMBER PROPERTIES

Divisibility & Primes

• "x is div by y" "x is a mult of y" "y is a factor of x" "$\frac{x}{y}$ is an integer" all mean: "x has **at least** the primes that are in y."

• "**prime** factors of 12" (2, 2, 3) vs. "factors of 12" (1, 2, 3, 4, 6, 12)

• LCM of x & y = smallest number that has **minimum** primes of x and y.
• GCF of x & y = product of the primes that x and y have in common.

• For any number: [mult of x] ± [mult of x] = mult of x
e.g. $(26x + 65)$ is a multiple of 13, because $26x$ and 65 are multiples of 13

[RARE: To calculate how many factors a number would have, you add 1 to all the exponents of its prime factorization and multiply them.
$600 = 2^3 \times 3^1 \times 5^2 \rightarrow (3 + 1) \times (1 + 1) \times (2 + 1) = 4 \times 2 \times 3 = 24$ factors]

Odds & Evens

E ± O = O O ± O = E E ± E = E
E × O = E O × O = O E × E = E
(biggest thing: **even × anything = even**)
$3x + 4y$ is odd $\rightarrow 3x + E = O \rightarrow 3x = O \rightarrow x = O$
$12x + 5y$ is even $\rightarrow E + 5y = E \rightarrow 5y = E \rightarrow y = E$

Positives & Negatives

$x+y > 0$ (at least one pos) $x-y > 0$ ($x > y$) $xy > 0$ or $x/y > 0$ (same sign)
$x+y < 0$ (at least one neg) $x-y < 0$ ($x < y$) $xy < 0$ or $x/y < 0$ (opp signs)

"x/y is even" $\rightarrow$ "$x/y = E$" $\rightarrow$ "$x = E \times y$" $\rightarrow$ x is even, y is ??
"x/y is odd" $\rightarrow$ "$x/y = O$" $\rightarrow$ "$x = O \times y$" $\rightarrow$ both even or both odd
raised to an **even** power, sign is hidden
(given: $x^2 > 0$, x could be pos or neg)
raised to an **odd** power, sign is visible
(if $x^3 > 0$, then $x > 0$. if $x^3 < 0$, then $x < 0$)

Combinatorics

From **9**, how many ways to pick 3? $\frac{9!}{3!6!} = \frac{Pool!}{In!\,Out!}$

Anagrams: $\frac{Total\ Letters!}{Repeating\ Letters!}$ Anagrams of BANANA = $\frac{6!}{3!2!}$

• Slot method (For each decision, draw & label a slot. Fill slots with # left in pool & multiply. Divide by factorial if labels match)

Probability

• Solve as fraction: $\frac{\#\ of\ Applicable\ Possibilities}{Total\ \#\ of\ Possibilities}$

• Event-by-event (figure out the probability of each event occurring and **multiply** them together)
• Prob of X **or** Y = Prob X + Prob Y – Prob of both
• Prob of "**at least**" = Calculate the probability that it DOESN'T happen and use 1 – x.

GEOMETRY

Polygons

• Triangles have 180°. Every time you add a side, you add another 180°.
Or: $(n - 2) \times 180$ = total degrees (4 sides: 360°, 5 sides: 540°, 6 sides: 720°…)
• Area of Trapezoid = (Avg. of Bases) × Height
• Area of Parallelogram = Base × Height
• Volume of Rectangular Solid: Vol = $l \times w \times h$
• Space Diagonal of Rectangular Solid: $a^2 + b^2 + c^2 = d^2$
• Volume of Cube: Vol = s^3

Triangles & Diagonals

$\sqrt{2} \approx 1.4$ (Valentine's) $\sqrt{3} \approx 1.7$ (St Paddy's)

• Special Angle Rt Triangles: 45-45-90 ($x : x : x\sqrt{2}$) & 30–60–90 ($x : x\sqrt{3} : 2x$)
• Pythagorean Triple Triangles: 3:4:5 & 5:12:13 (& 8:15:17 & 7:24:25)
• Diagonal of square = hypotenuse of 45–45–90
• Two equal sides ↔ Two equal angles
• Area of equilateral triangle = $(s^2 \times \sqrt{3}) / 4$
(or break into two 30–60–90 and solve that way)
• Third side rule: Difference < 3rd side of a triangle < Sum
• Exterior angle rule: If an angle is supplemental to one of a triangle's three angles, it's equal to the sum of the triangle's other two angles.
• Similar Triangles: Triangles with the same set of three angles will have proportional sides. This usually occurs when there are mini-triangles within bigger triangles.

Circles & Cylinders

• $d = 2r$ Circumf = πd Area = $\pi \times r^2$
• Arc length = (central angle° / 360°) × Circumference
• Area of sector = (central angle° / 360°) × Area
• Capacity of cylinder: Vol = $\pi r^2 \times h$
• Volume of liquid: Vol = $\pi r^2 \times h$ (of liquid)
• Rectangle inscribed in circle? Rectangle Diagonal = Circle Diameter
• Circle inscribed in square? Circle Diameter = Square Side
• Triangle inscribed in semi-circle? Right triangle
• Two of a triangle's sides are radii? Isosceles

Lines & Angles

• Angles on a line add up to 180°
• When two lines are parallel, all the small angles are the same, and all the big angles are the same
• If told lines are parallel, **find the clone angles**

Coordinate Plane

• Slope: (change in y / change in x) = (rise / run)
• Slope-intercept: $y = mx + b$
• "Line contains point" means you could plug a coordinate pair, e.g. Find $(5, d)$ on the line $y = 3x + 4$ by replacing $x = 5$ and $y = d$ ... $d = 3(5) + 4$
• Parallel lines (do not intersect ... same slope)
• Perpendicular lines (slope is opposite reciprocal ... the product of their slopes is –1)
• Finding distance between two points (make a right triangle where the distance is the hypotenuse)

FRACTIONS / DECIMALS / PERCENTS

Digits & Decimals

• Clean up decimals as (Integer × Power of 10) e.g., $0.0034 = 34 \times 10^{-4}$
• Units digit patterns:
  • Ends in 0, 1, 5, or 6? Units digit stays the same forever.
  • Ends in 4 or 9? Powers of 4 end in 4 or 6 / Powers of 9 end in 9 or 1, based on whether exponent is odd or even.
  • Ends in 2, 3, 7 or 8? Goes through a cycle of four numbers. Powers of 2 (2, 4, 8, 6) ... Powers of 3 (3, 9, 7, 1) ... Powers of 7 (7, 9, 3, 1) ... Powers of 8 (8, 4, 2, 6). Figure out how far exponent is from a mult of 4.

Fractions

• Add / Subtract / Multiply / Divide mechanics
• When picking #s: multiply denominators together
• "Split Up" addition or subtraction in *numerator*

$$\frac{x+y}{y} = \frac{x}{y} + \frac{y}{y} = \frac{x}{y} + 1$$

Percents

• Translate into simplified fractions
x/2 = multiples of 50 %
x/3 = multiples of 33.3 %
x/4 = multiples of 25 %
x/5 = multiples of 20 %
x/8 = multiples of 12.5 %
x/10 = multiples of 10 %
x/20 = multiples of 5 %

• Percent of = $\frac{Part}{Whole} \times 100$

• Percent change = $\frac{Difference}{Original} \times 100$

• When a question asks "What **percent of** *x* is *y* ?", just ask yourself, "What **fraction** is *x* of *y* ?". Then: **Fraction × 100 = Percent**
• When picking #s: use 100 for straightforward math

Ratios

• Use "Multiple of ____ " thinking a lot, or you might be fooled. Most ratio problems have an implied integer constraint, so if you have a ratio of blues to reds that is 5 to 11, think "Blues are a multiple of 5. Reds are a multiple of 11. The total is a multiple of 16."

• Scale It Up (Write ratio horizontally and add a "Total" column). A real # for any of the pieces, or for the sum or difference between any of the pieces, allows us to solve for the multiplier and figure out all the real values.

• Combine Multiple Ratios: When you have two separate ratios, focus on the piece that's common to both ratios. Scale both ratios up/down until the common piece is the same.

• Old Ratio → New Ratio
1. write Old and New as **two** fraction equations, then solve system
2. use multiplier on Old, then set up one fraction equation for the New
3. or Backsolve

WORD PROBLEMS

Algebraic Translations

• **Backsolve** or translate into a 2 equation system and solve algebraically
• Solve systems with **substitution** (isolate a variable) or by **elimination** (stack the equations and then add or subtract) ?
"of" = multiply "percent" = 1/100 "ratio of *x* to *y*" = x/y
"verb" = equal sign "total" = sum "8 less than *x*" = *x* – 8
"there are __ as many" = __ goes on 2nd thing (there are 2/3 as many apples as pears → $a = \frac{2}{3}p$)

Rates / Work / Distance

• Average speed = $\frac{Total\ Distance}{Total\ Time}$
• Opposite Directions = add rates
• Same Direction = subtract rates
• When things **work together**, add rates.
• When things **work against each other**, subtract rates.
• Time together = $\frac{Product\ of\ Two\ Separate\ Times}{Sum\ of\ Two\ Separate\ Times}$
• 1/2 (smaller time) < Time Together < 1/2 (larger time)
• Pick smart #s if Work or Distance not given

Statistics

• See "**avg**"? Calculate **sum**
• **Range**: difference between highest & lowest data points
• **Median**: arrange in order. If odd # of things, median is the middle data point. If even # of things, median is the avg of middle two data points.
• **Standard deviation**: need distance of each data point from average. *the farther the data points are from avg, the bigger the standard deviation*
• If **avg** is 40 and **standard deviation** is 3, then the interval within one standard deviation of the mean is 37 to 43.
• **Normal Distribution**: draw bell curve and write 2, 14, 34, 34, 14, 2
• **Weighted averages:** the distance the middle number is from each endpoint *is* the ratio, in reverse. *20% of undergrads and 50% of grads at my school drink tea. Overall, 33% of students drink tea.* (33 is **13** away from 20. 33 is **17** away from 50. So ratio of U:G is 17:13)

Evenly Spaced Sets

• Median = Average (midpoint of the set)
Adding up evenly spaced sets: Sum = (Avg) × (#)
• Avg = Avg of the 1st and Last term
• # of terms = $\frac{Range}{Spacing} + 1$

Overlapping Sets

• Default: double set matrix
• Group Equation: Total = $Group_1$ + $Group_2$ – Both + Neither
• "Everyone was A, B, or both" = No Neither (put a 0 in Neither)
• Subgroup facts: "30% of **students who like jazz** also like rap" ... put a variable in total for jazz, and then put 0.3x into the spot for 'both jazz and rap', and 0.7x into the spot for 'jazz but not rap'.
• 3 group formula **Total = G1 + G2 + G3 – B's – 2A + Neither**
(B's: people in 2 out of 3 groups; A: people in 3 out of 3 groups)

OTHER

Remainder

• Know how to quickly generate 3 or 4 possible numbers when given a constraint like
"when *n* is divided by 8 the remainder is 3"
Think: *multiples of 8 plus 3* (first multiple is 0, so first *n* is remainder itself)
n = 3, 11, 19, 27, 35 ... (list increases by 8)

• Decimal Remainder = $\frac{Remainder}{Divisor}$

$\frac{13}{4}$ = 3.**25**, or 3 R*1* 0.**25** = $\frac{1}{4}$

Terminating Decimal

If the simplified denominator of a fraction has any primes besides 2 and 5, it will **not** terminate.

Any simplified denominator whose prime factorization only consists of 2's and / or 5's **will** terminate.

$\frac{7}{160}$ will be a terminating decimal, because the denominator's prime factorization only has 2's and 5's.

$\frac{7}{12}$ will not terminate, because the denominator's prime factorization has something besides 2's and 5's (it has a 3).

Unit Conversion

• Set up clear, vertical fractions to cancel units

Go from 120mph to ____ feet per second

$$\frac{120\ mi}{1\ hr} \times \frac{5280\ feet}{1\ mi} \times \frac{1\ hr}{60\ min} \times \frac{1\ min}{60\ sec}$$

Optimization (greatest/least possible)

• If trying to maximize a function, make some nonnegative quantity zero.

e.g. To get the maximum value of $f(x) = 87 - (x + 5)^2$, make $x = -5$.
The maximum value of $f(x)$ is 87, when $x = -5$.

• In statistics problems, when trying to maximize one data point, aim to minimize all the other data points. You normally know the sum of the entire set, so you're working from a budget.

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