Differential Equations
(MATH 2051)

EXERCISE 7

1. Solve the given initial value problem.

(a) $9y'' - 12y' + 4y = 0, \quad y(0) = 2, \; y(0) = 2, y'(0) = -1$

(b) $9y'' + 6y' + 82y = 0, \quad y(0) = -1, \; y(0) = 2, y'(0) = 2$

Solution:

2. Consider the initial value problem :

$$9y'' + 12y' + 4y = 0, y(0) = a > 0, y'(0) = -1.$$

(a) Solve the initial value problem.

(b) Find the critical value of a that separates solutions that become negative from those that are always positive.

Solution:

3. **(a)** Consider the equation $y'' + 2ay' + a^2y = 0$. Show that the roots of the characteristic equation are $r_1 = r_2 = -a$, so that one solution of the equation is e^{-at}.

(b) Use Abel's formula to show that the Wronskian of any two solutions of the given equation is $W(t) = y_1(t)y_2{}'(t) - y_1{}'(t)y_2(t) = c_1e^{-2at}$ at, where c_1 is a constant.

(c) Let $y_1(t) = e^{-at}$ and use the result of part (b) to obtain a differential equation satisfied by a second solution $y_2(t)$. By solving this equation, show that $y_2(t) = te^{-at}$