A. Logarithms

1. $a^b = c \to b = \log_a c$ ⟵ Definition
2. $\log_b b = 1,\ \log_b 1 = 0$ ⟵ Special cases
3. $\log_b(\frac{x}{y}) = \log_b x - \log_b y$
4. $\log_b(x \times y) = \log_b x + \log_b y$
5. $\log_b x^y = y \times \log_b x$ ⟵ Follows directly from the previous rule.
6. $\log_b x = \dfrac{\log_c x}{\log_c b}$ ⟵ Changing bases
7. $x^{\log_b y} = y^{\log_b x}$
8. $b^{\log_b x} = x$ ⟵ Follows directly from the previous rule.
9. $\lg(n!) = \sim n \lg n$ ⟵ Stirling's Approximation

Notation:

- $\log n$ (no base) → used with orders of growth to indicate that the base is not important. Logarithms with different bases differ by a constant factor as shown in the listed identities.
- $\lg n$ → base 2.
- $\ln n$ → natural logarithm (base is e)

B. Summations

1. $\sum_{i=1}^{n} i = 1 + 2 + \ldots + n$ ⟵ Definition
2. $\sum_{i=1}^{n} c = c + c + \ldots + c = c \times n$ ⟵ If c does not depend on i.
3. $\sum_{i=1}^{n} c \times f_i = c \times \sum_{i=1}^{n} f_i$
4. $\sum_{i=1}^{n} f_i + g_i = \sum_{i=1}^{n} f_i + \sum_{i=1}^{n} g_i$
5. $\sum_{i=1}^{n} i = 1 + 2 + 3 + \ldots + n = \dfrac{n(n+1)}{2}$
6. $\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$
7. $\sum_{i=0}^{n} r^i = r^0 + r^1 + r^2 + \ldots + r^n = \dfrac{r^{n+1} - 1}{r - 1}, \quad r \neq 1$ ⟵ Geometric Sum.
8. $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1\ , \quad \sum_{i=0}^{n} (\frac{1}{2})^i = 1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n} = \sim 2$ ⟵ Special cases of a geometric sum ($r = 2$ and $r = 0.5$).
9. $\sum_{i=1}^{n} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} \sim \int_1^n \frac{1}{i}\,\mathrm{d}i = \ln n$ ⟵ Harmonic Number H_n.

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