Self-Practice Week 1 - Fundamentals (part 1)

Developing Algorithms

The goal of this assignment is to practice designing, implementing and testing basic algorithms.

Exercise 1 – Union-Find

Implement the Union-Find algorithm seen at lectures in Python. Test its functioning.

Exercise 2 – Social Network Connectivity

Given a social network containing N members and a log file containing a sequence of friendships requests, design an algorithm to determine the earliest time at which all members are connected (i.e., every member is a friend of a friend of a friend ... of a friend). Assume that the log file is temporally sorted, and that friendship is an equivalence relation. What is the running time of your algorithm?

Hint

Use Union-Find to model friendships

Exercise 3 – Extended Union-Find

Extend Union-Find with a method find so that find(i) returns the largest element in the connected component containing i. For example, if one of the connected components is $\{1,2,6,9\}$, then the find method should return 9 for each of the four elements in the connected components. The operations union(), connected(), and find() should all take logarithmic time or better.

Hint

Maintain an extra array that stores, for each i, the largest element in the connected component containing i.