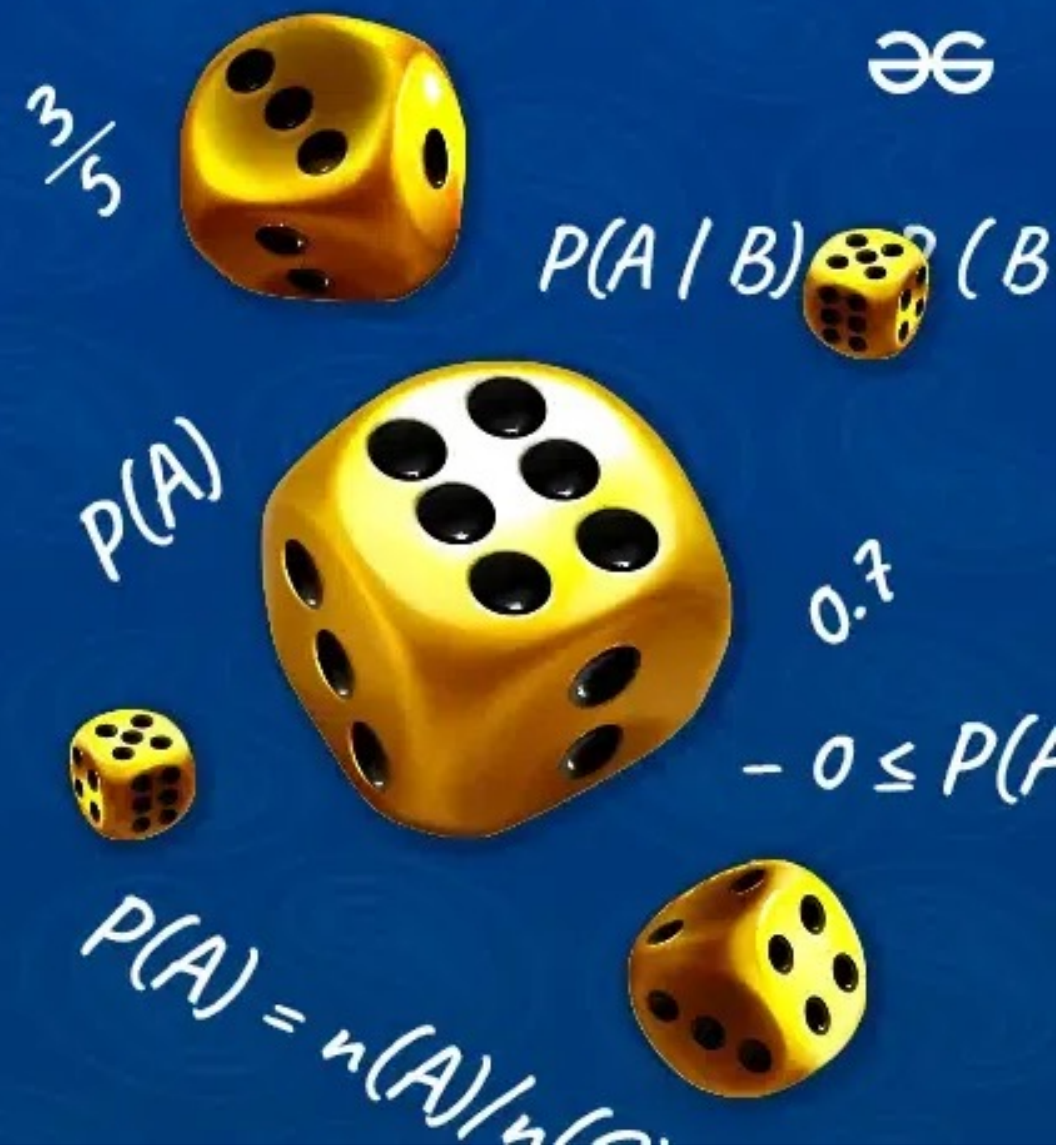
L03:

PROBABILITY



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Introduction

What is probability?

- Probability is the study of randomness. A probability of an uncertain outcome/event is the chance (or likelihood) that it will occur.

Probability ≡ the **relative frequency** of **an event** *in the* ***sample space*** … alternatively… the proportion of times an event is *expected* to occur **in the long run**

Example of Relative frequency interpretation of probability

In a given year: 100 traffic fatalities (events) in a sample of
N = 1,000,000 (1M)

Probability of event

= relative freq in samples

= 100 / 1,000,000

= .0001

Example of Relative frequency interpretation of probability

A package delivery service promises 2-day delivery between Shenzhen and Hong Kong but is often able to deliver packages in just 1 day.

Suppose that the company reports that that the probability of next-day delivery is .5, implying that in the long run, 50% of all packages arrive in 1 day.

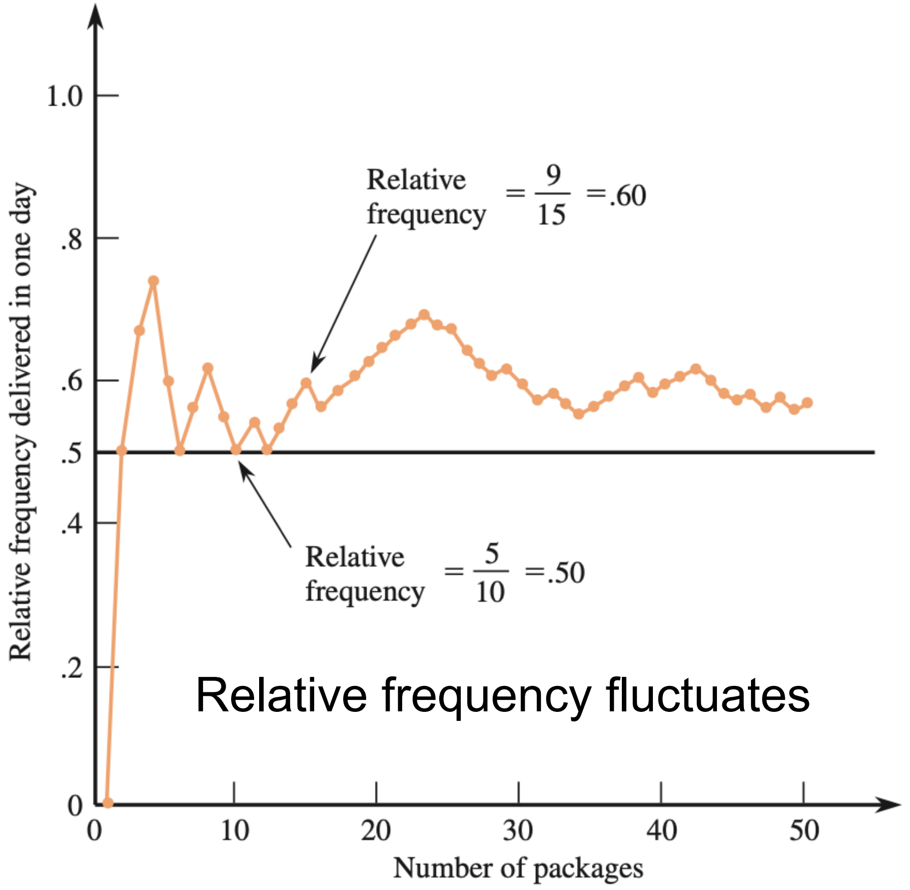
We track the delivery of packages shipped with this company. With each new package shipped, we could compute the relative frequency of packages shipped so far that arrived in 1 day:

$$\frac{\text{number of packages that arrived in 1 day}}{\text{total number of packages shipped}}$$

The results for the first 10 packages might be as follows:

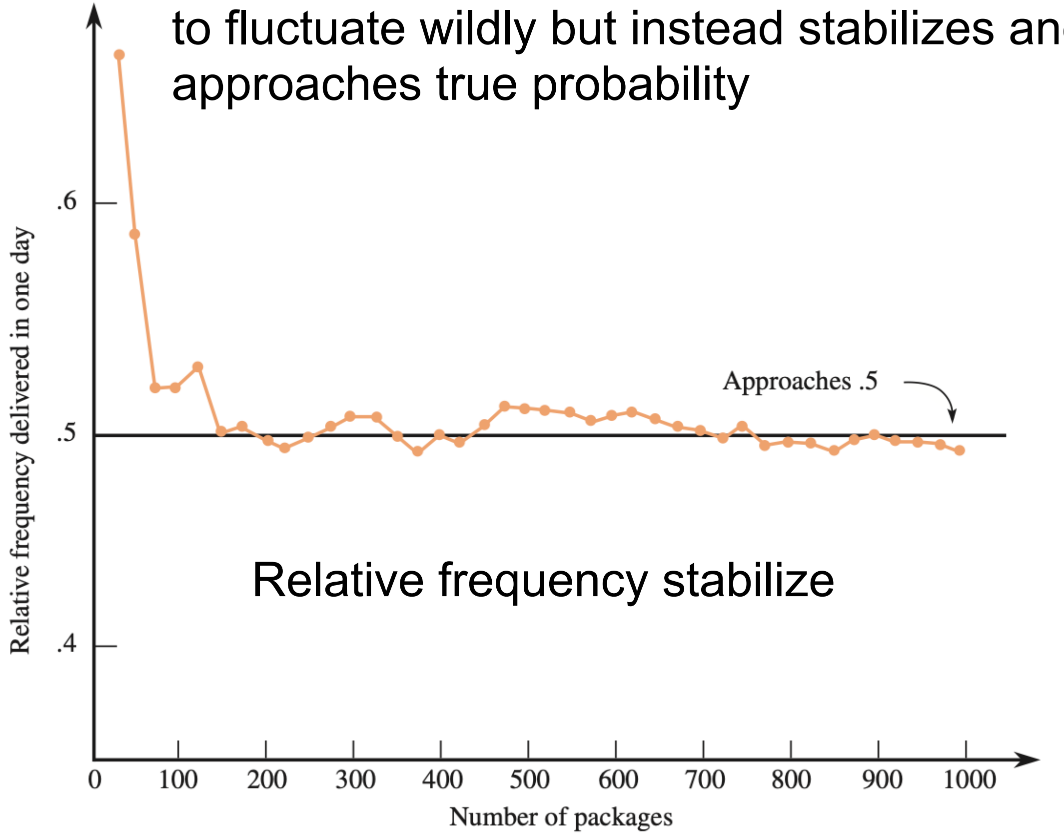
Package number	1	2	3	4	5	6	7	8	9	10
Arrived next day	N	Y	Y	Y	N	N	Y	Y	N	N
Relative frequency delivered next day	0	.5	.667	.75	.6	.5	.571	.625	.556	.5

Example of Relative frequency interpretation of probability



Relative frequency fluctuates

As the number of packages in the sequence increases, the relative frequency does not continue to fluctuate wildly but instead stabilizes and approaches true probability



Relative frequency stabilize

Quantifying Uncertainty

Probability is used to quantify levels of belief:

Probability	Expression
0.00	Never
0.05	Seldom
0.20	Infrequent
0.50	As often as not
0.80	Very frequent
0.95	Highly likely
1.00	Always

Properties of Probability

- The probability of any event A is between 0 and 1.

$$0 \leq P(A) \leq 1$$

- If outcomes cannot occur simultaneously, then the probability that any one of them will occur is the sum of the outcome probabilities. The sum of the probabilities of all outcomes in the sample space is 1.

$$\sum_{all\ outcomes} P(A) = 1$$

Properties of Probability

Compound Events are formed by combining several simple events.

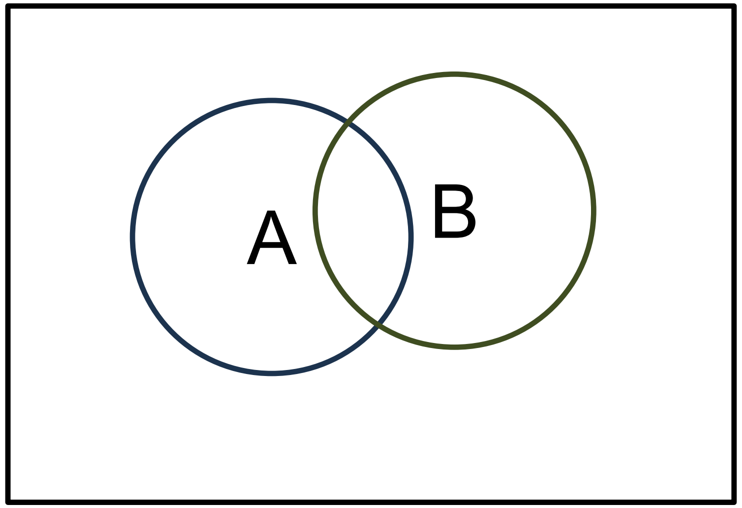
- The probability that either event A or event B (union) will occur (called union probability):

$$P(A \text{ or } B) = P(A \cup B)$$

- The probability that both events A and B (intersection) will occur (called Joint probability):

$$P(A \text{ and } B) = P(A \cap B)$$

Venn diagram



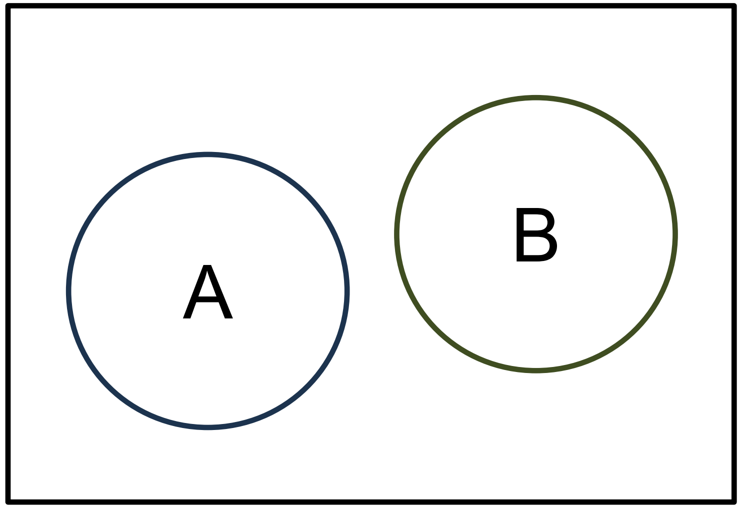
Mutually Exclusive Events

❖ ***Definition***:

Events defined in such a way that the occurrence of one event precludes the occurrence of any of the other events.

(In short, if one of them happens, the others cannot happen.)

A and B are two disjoint events $A \cap B = \emptyset$

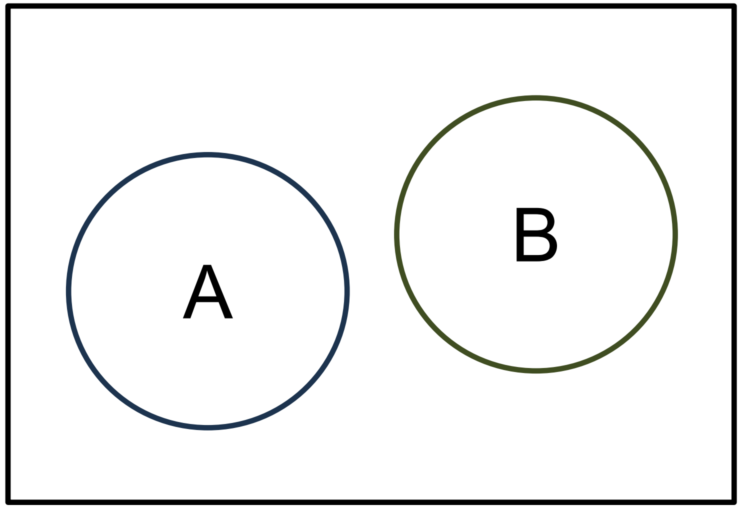


Mutually Exclusive Event Examples:

- **Flipping a Coin once:**
  1. Event A: Landing on heads.
  2. Event B: Landing on tails.

- **Drawing a Card once:**
  1. Event A: Drawing a queen.
  2. Event B: Drawing a king.

- **Answering a True/False Question:**
  1. Event A: Answering true.
  2. Event B: Answering false.

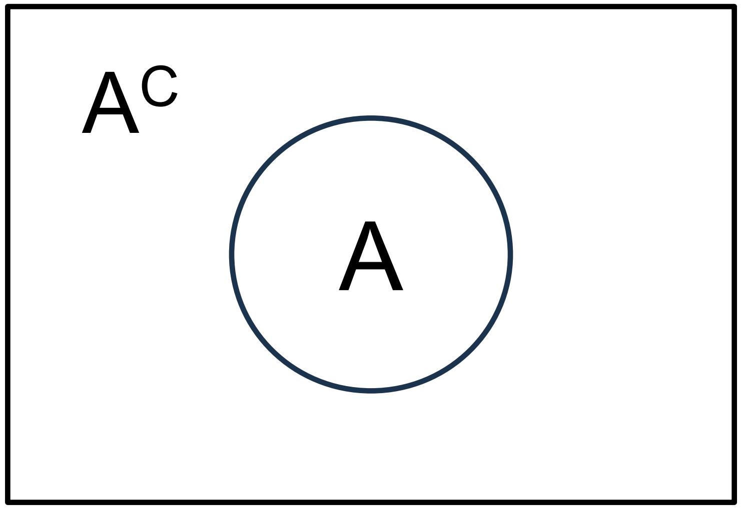


Properties of Probability

Complementary Events

❖ ***Definition*:**

Complementary events are two events that exist such that one event will occur if and only if the other does not take place.



Note:

- Complementary events are also mutually exclusive.
- Mutually exclusive events are **not** necessarily complementary.

$$P(A) + P(A^c) = 1$$

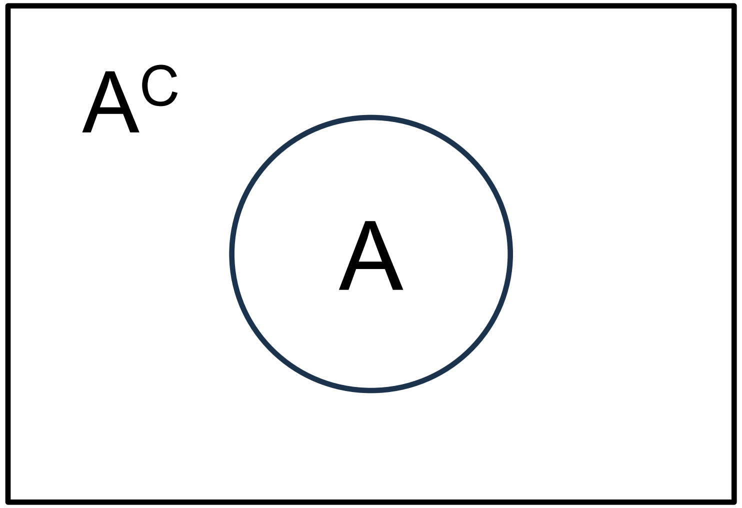
Sometime also $\overline{A}$ to denote complementary of A

Complementary Event Examples:

- **Flipping a Coin once:**
  1. Event A: Getting a heads when flipping a coin.
  2. Complement of A : Getting a tails (T).

- **Drawing a Card once:**
  1. Event A: Drawing a heart from a standard deck of playing cards.
  2. Complement of A: Drawing any card that is not a heart (could be clubs, spades, or diamonds).

- **Passing a Test:**
  1. Event A: Passing a test.
  2. Complement of A: Not passing the test.



Example

The following table summarizes a group of diabetic patients visited a local medical clinic last month, classified by gender and by type of diabetes (Type-1 and Type-2).

Gender	Type of Diabetes		Total
	Type-1	Type-2	
Male (M)	110	80	190
Female (F)	160	100	260
Total	270	180	450

One patient from this group is selected at random.

Define

event A as “the patient selected has Type-1 diabetes”;

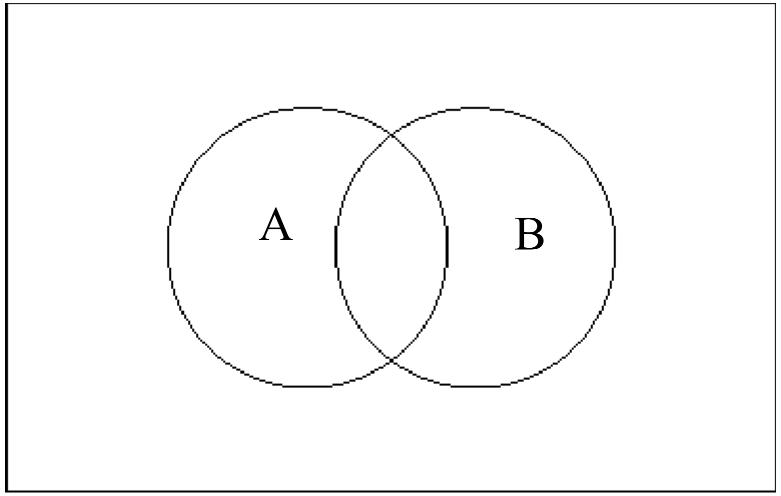
event B as “the patient selected has Type-2 diabetes”; and

event C as “the patient selected is female”.

1. Are events A and B mutually exclusive?
2. Are events A and C mutually exclusive?

Properties of Probability

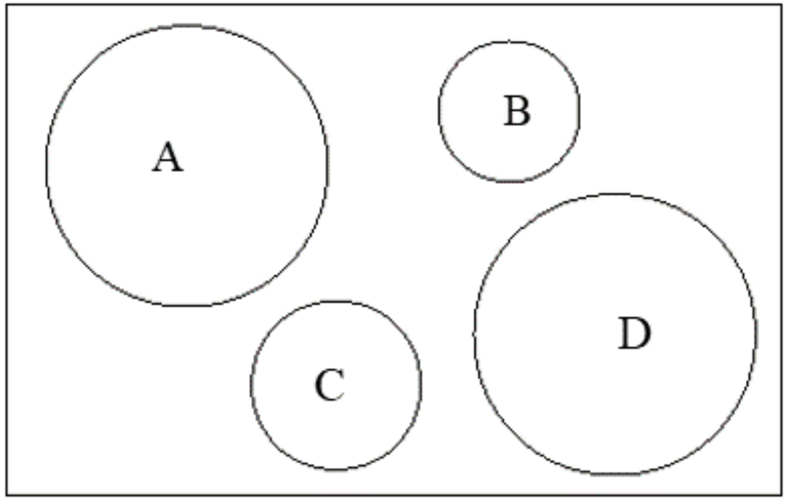
Illustration:



General Addition Rule: Let A and B be two events defined in a sample space *S*.

$$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$$

Note: If two events A and B are mutually exclusive: P(A and B) = 0



Special Addition Rule: If A and B are mutually exclusive events, then

$$P(\text{A or B}) = P(\text{A}) + P(\text{B})$$

This can be expended to consider more than two mutually exclusive events:

$$P(\text{A or B or C or D}\ldots) = P(\text{A})+ P(\text{B})+ P(\text{C})+ P(\text{C})$$

Example

Consumer is selected at random. The probability the consumer has tried a snack food (F) is 0.5, tried a new soft drink (D) is 0.6, and tried both the snack food and the soft drink is 0.2.

The probability of the consumer

1) Tried the snack food or the soft drink?

2) Not tried the snack food?

3) Tried neither the snack food nor the soft drink?

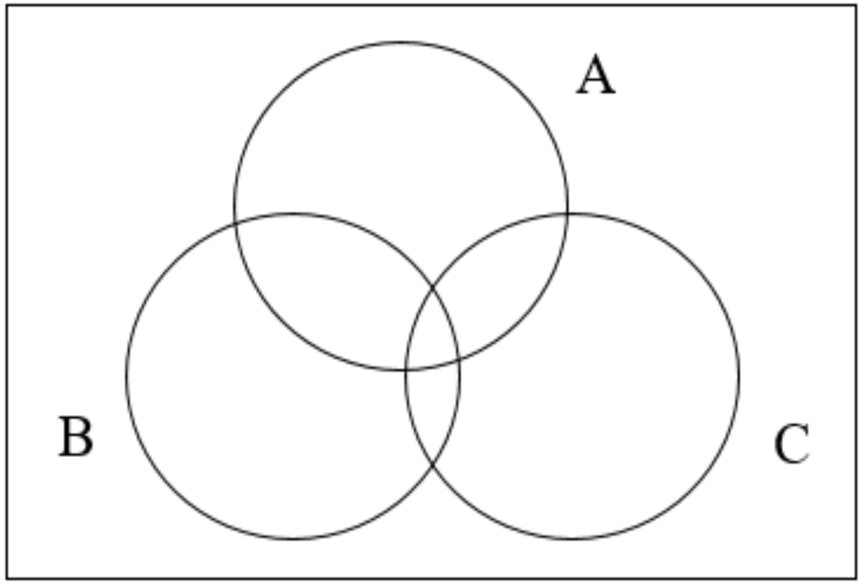
4) Tried only the soft drink?

Properties of Probability

General Addition Rule **for 3 events** :

Let A, B and C be three events defined in a sample space S

$P(\text{A or B or C}) = P(\text{A}) + P(\text{B}) + \text{P(C)} - P(\text{A and B}) - \text{P(A and C)} - \text{P(B and C)} + \text{P(A and B and C)}$



Properties of Probability

Multiplication Rule for Independent Events

If two events are independent, the probability that *both* events occur is the product of the individual outcome probabilities.

$$P(\text{A and B}) = P(\text{A})P(\text{B})$$

Independent events: Two events are said to be **independent** if the probability that one event occurs is not affected by knowledge of whether the other event has occurred.

Example

Probability of Compound Events (Snow and Rain)

Let A : probability of **snow** A=0.15;

Let B : probability of **rain** B=0.10;

There are 4 possible events:

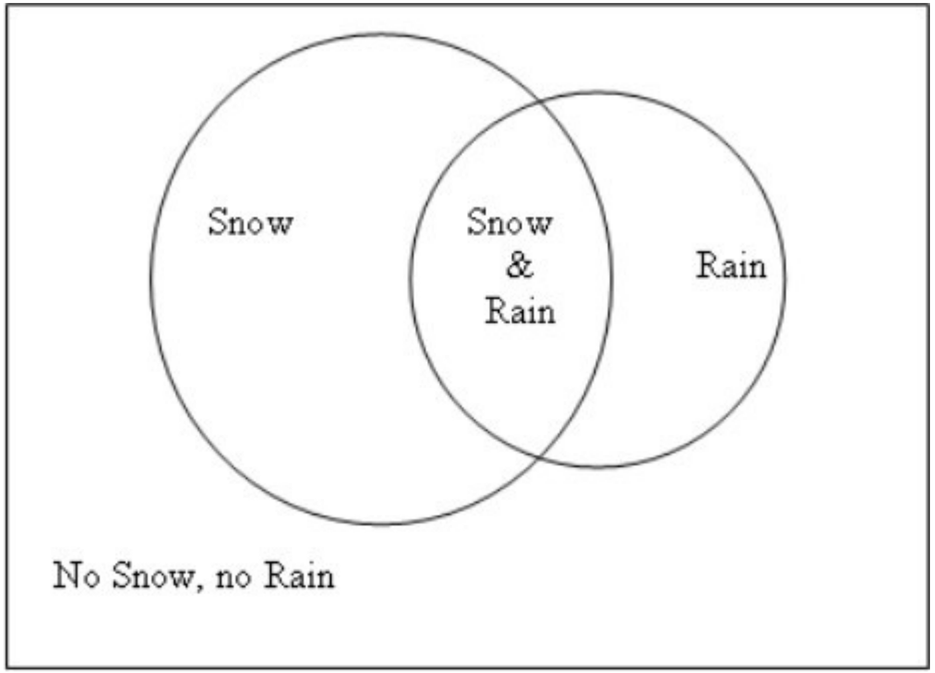
Snow and rain P1:

Snow, but no rain P2:

No snow, but rain P3:

No snow, and no rain P4:

We can consider the snow and rain to be independent events here



Independence of Events

Special multiplication rule for three independent events

$$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$$

Example

Suppose the event A is "Allen gets a cold this winter," B is "Bob gets a cold this winter," and C is "Chris gets a cold this winter." $P(A) = 0.15$, $P(B) = 0.25$, $P(C) = 0.3$, and all three events are independent.

Find the probability that:

1. All three get colds this winter.
2. Allen and Bob get a cold but Chris does not.
3. None of the three gets a cold this winter.

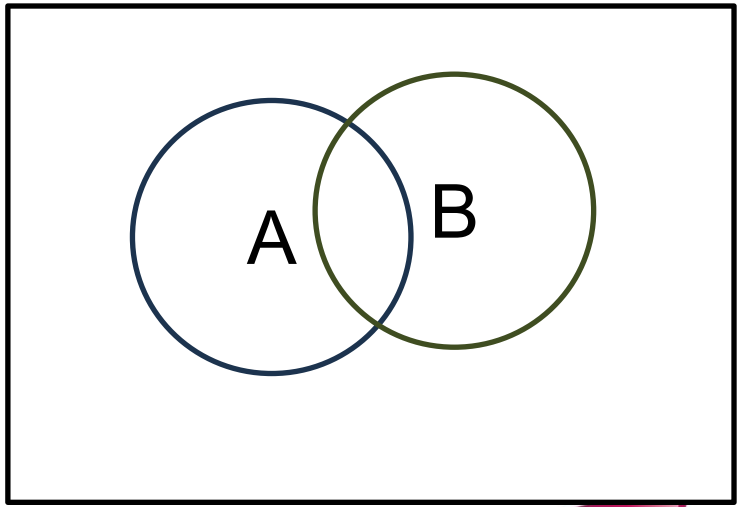
Summary

- **Complement Rule:** The probability that an event **will NOT occur** is equal to 1 minus the probability that the event will **occur**.

$$P(A^c) + P(A) = 1 \text{ or } P(\overline{A}) + P(A) = 1$$

- **General Addition Rule**: Let A and B be two events defined in a sample space *S*.

$$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$$



- **Multiplication Rule for Independent Events:**

$$P(\text{A and B}) = P(\text{A})P(\text{B})$$

Space with Finite Equally Likely Outcomes

- Outcomes are said to be equally likely if they have equal chances to occur. For example, If you toss a fair coin, a Head (H) and a Tail (T) are equally likely to occur.

> **Theorem:**
>
> Consider a sample space $S = \{w_1, w_2, \dots, w_N\}$ with N equally likely outcomes, where N is a finite positive integer. Let E be any event in S. Then
>
> $$P(E) = \frac{number\ of\ outcomes\ in\ E}{N}.$$

Example

1.What is the probability of rolling a four with a fair six-sided die?

$$P\ (E) = \frac{1}{6}$$

2. If you roll a die twice, what is the probability that you get a four on the first roll and a two on the second?

Two independent events $P\ (E) = P(E1) * P(E2) = \frac{1}{6} * \frac{1}{6} = \frac{1}{36}$

Example

A fair die is thrown 5 times. What is the probability of

A = {**at least** one ‘6’},

B = {**at most** three ‘6’}?

Conditional probability

Conditional Probability

Conditional probability is defined as the likelihood of an event or outcome occurring, based on the occurrence of a previous event or outcome.

Definition:

Let B be an event with non-zero probability, A is another event. The probability of A given B is

$$P(A|B) = \frac{P(A\ and\ B)}{P(B)} \text{ or } P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example

Toss a fair die once, getting an odd number given it is greater than 1

Example

Example: Let a fair die be rolled for twice. Find the probability that a sum of 6 is obtained, given that the (absolute) difference of the numbers obtained is 4.

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Example

Let's roll two fair dices, what is the probability that at least one of them is 6 given that the two dice show different numbers?

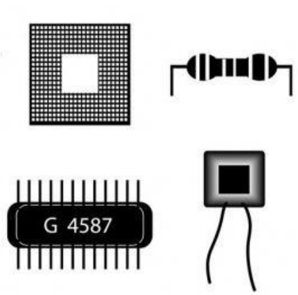
Let

E = {at least one of them is 6}

F = {two dice show different numbers}

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Example



A bin contains

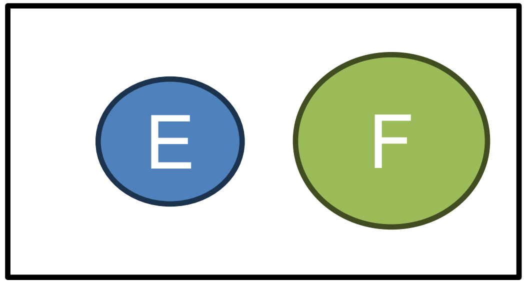
- 5 defective (immediately fail when put in use)
- 10 partially defective (fail after a couple of hours of use)
- 25 acceptable transistors

A transistor is chosen at random from the bin and put into use. If it does not immediately fail, what is the probability it is acceptable?

Conditional Probability

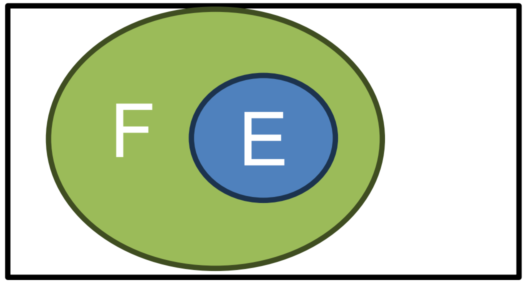
- What can we say about P(E | F) if E and F are mutually exclusive?

$$P(E|F) = \frac{P(EF)}{P(F)} = \frac{0}{P(F)} = 0$$



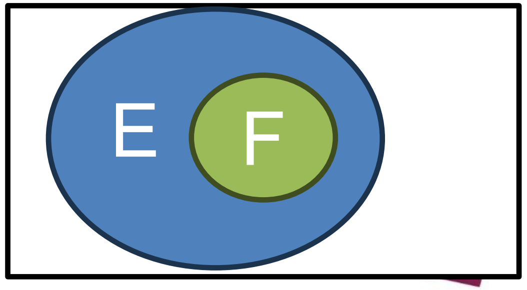
- What can we say about P(E | F) if $E \subset F$

$$P(E|F) = \frac{P(EF)}{P(F)} = \frac{P(E)}{P(F)} \geq P(E)$$



- What can we say about P(E | F) if $F \subset E$

$$P(E|F) = \frac{P(EF)}{P(F)} = \frac{P(F)}{P(F)} = 1$$



General Multiplication Rule

Conditional Probability

$$P(B|A) = \frac{P(AB)}{P(A)}$$

$$\Rightarrow$$

General Multiplication Rule

$$P(AB) = P(B|A)P(A)$$

Conditional Probability

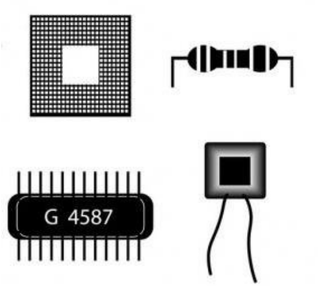
- **Multiplication rule for 3 events:**

$$P(A \cap B \cap C) = P(C|(A \cap B))P(B|A)P(A)$$

This formula can be used to calculate the joint probability of any number of events occurring in sequence.

- P(C | (A ∩ B)) : the probability of event C occurring given that both event A and event B have occurred.
- P(B | A): the probability of event B occurring given that both event A have occurred.
- P(A): the probability of event A occurring

Example



A box of fuses contains 20 fuses, of which 5 are defective.
If three of the fuses are selected randomly and removed from the box in succession without replacement, what is the probability that all three fuses are defective?

Example

Let us suppose the TV forecast shows that

1) the probability of rain tomorrow is 80%,
2) if it rains tomorrow, then the probability of lightning is 25%.

What then is the probability that tomorrow there is both rain and lightning?

Sequential Conditioning Formula

General Multiplication Rule

Two events $$\boxed{P(A_1A_2) = P(A_1)P(A_2|A_1)}$$

Three events $$P(A_1A_2A_3) = P(A_1)P(A_2|A_1)P(A_3|A_1A_2)$$

$$\ldots = \ldots$$

More events $$P(A_1A_2\ldots A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1A_2)\ldots P(A_n|A_1A_2\ldots A_{n-1})$$

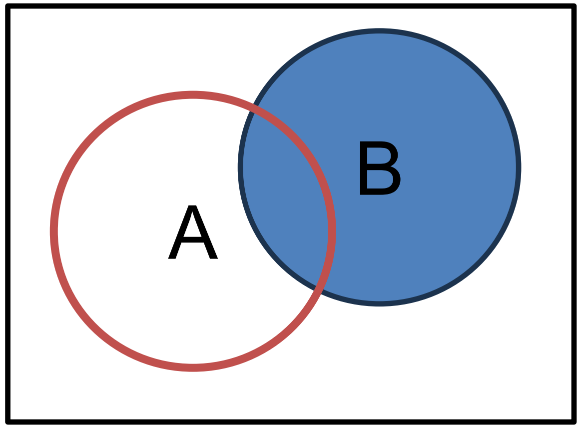
This extended formula assumes that the probability of each event can be conditioned on the occurrence of all previous events in the sequence.

Properties of conditioning probability

1. Complement rule for conditional probability:

$$P(A|B) + P(A^c|B) = 1$$

Recall $P(A) + P(A^C) = 1$



Properties of conditioning probability

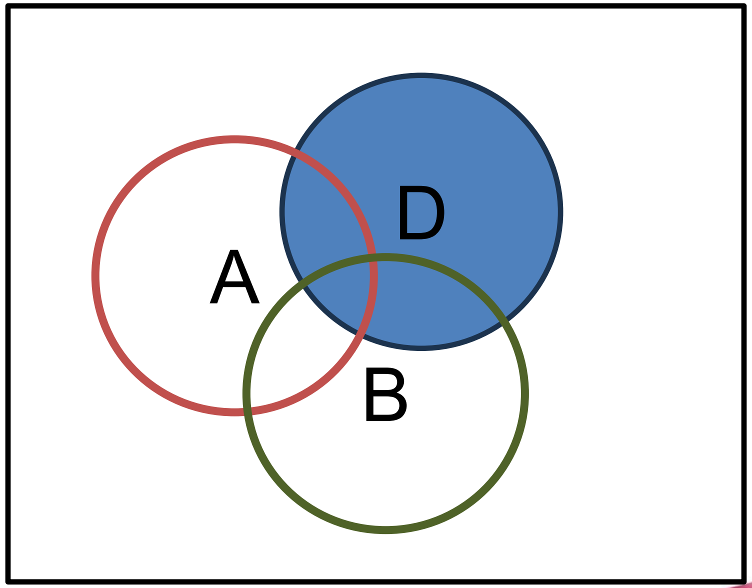
2. General Addition Rule for conditional probabilities

The probability of A or B given event D occurred

$P((A \cup B)|D) = P(A|D) + P(B|D) - P((A \cap B)|D)$

Recall

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



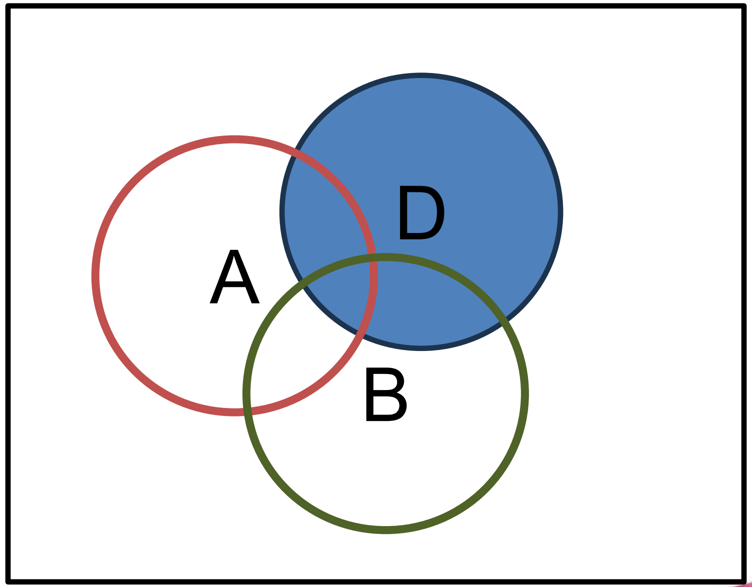
Properties of conditioning probability

3. Specific instance of the general multiplication rule for conditional probabilities

The conditional probability of A and B occurring given D in terms of other conditional probabilities

$$P((A \cap B)|D) = P(A|(B \cap D)) \times P(B|D)$$

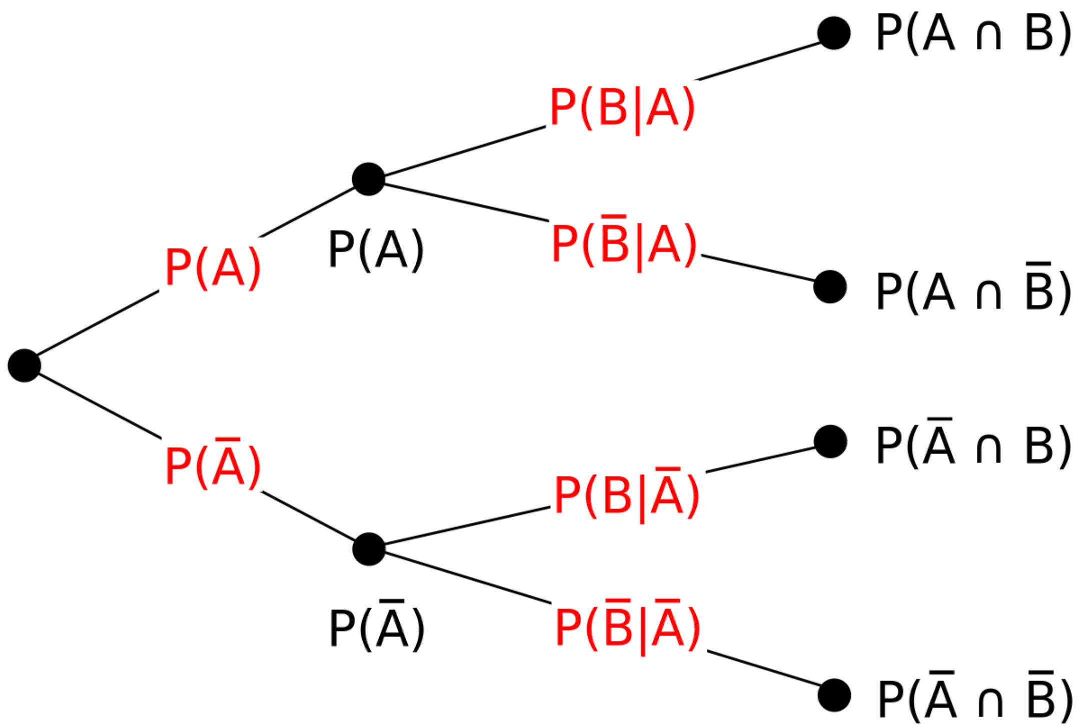
It relates the conditional probability of the intersection of events **A and B given D** to the product of the conditional probabilities of **A given B and D, and B given D**.



Combining the Rules of Probability

- Many probability problems can be represented by tree diagrams.
- Using the tree diagram, the addition and multiplication rules are easy to apply.

- Each node represents a particular outcome and the probabilities are written on the branches of the tree.
- The probabilities shown on the edges are **conditional on the previous node**, and the probability of climbing from the base to any of the branches is **obtained by multiplying the probabilities**.



Combining the Rules of Probability

Example:

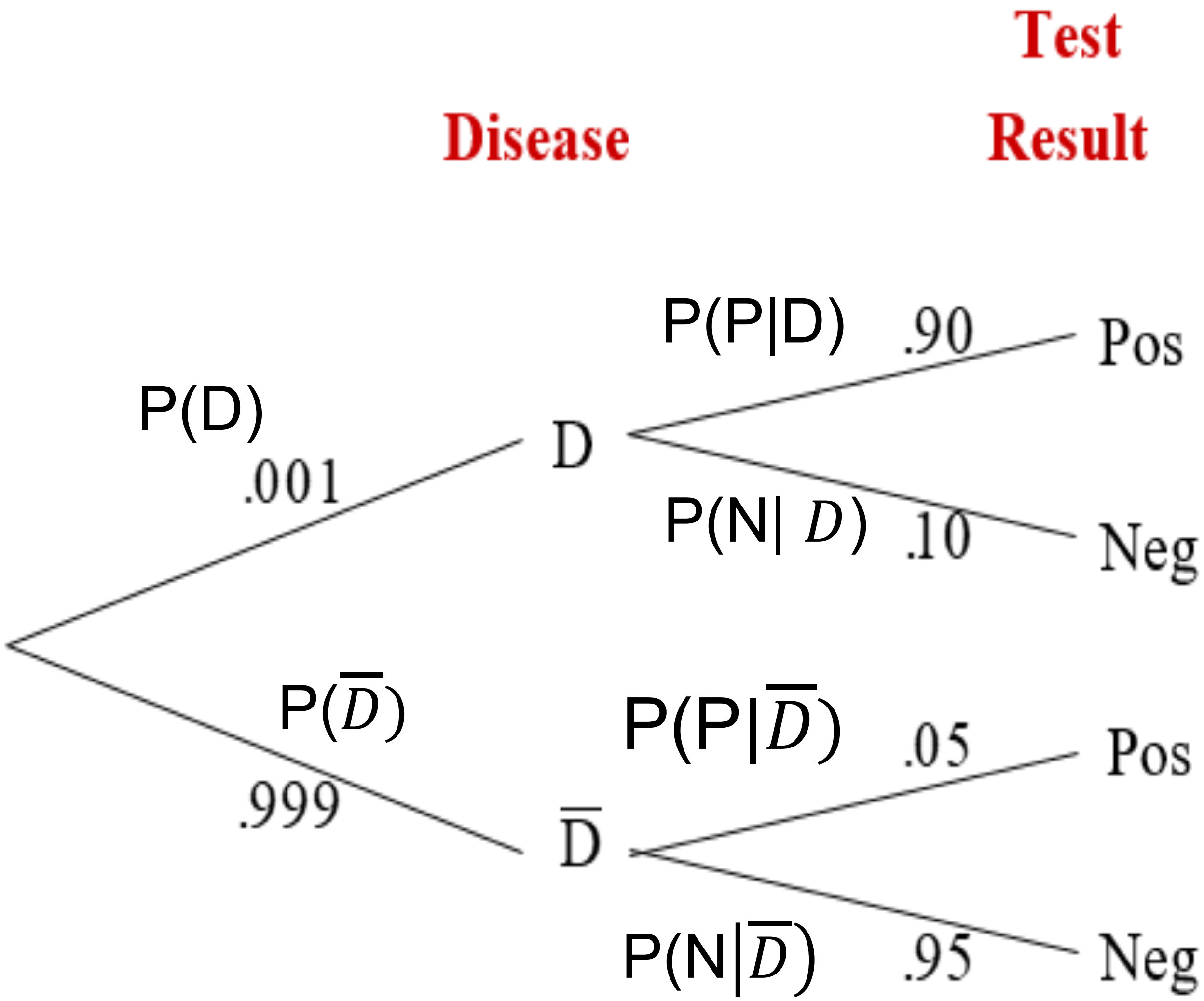
This problem involves testing individuals for the presence of a disease. Suppose the probability of having the disease (D) is 0.001. If a person has the disease, the probability of a positive test result (Pos) is 0.90. If a person does not have the disease, the probability of a negative test result (Neg) is 0.95. For a person selected at random:

1. Find the probability of a negative test result given the person has the disease.
2. Find the probability of having the disease and a positive test result.
3. Find the probability of a positive test result.

Combining the Rules of Probability

Suppose the probability of having the disease (D) is 0.001.

- If a person has the disease, the probability of a positive test result (Pos) is 0.90.
- If a person does not have the disease, the probability of a negative test result (Neg) is 0.95.



Independent event and dependent event

Independence and dependent of events

Note that it is possible that the knowledge/information that an event B has occurred **does not influence** the probability that A will occur.
That is, **P(A|B) = P(A)**

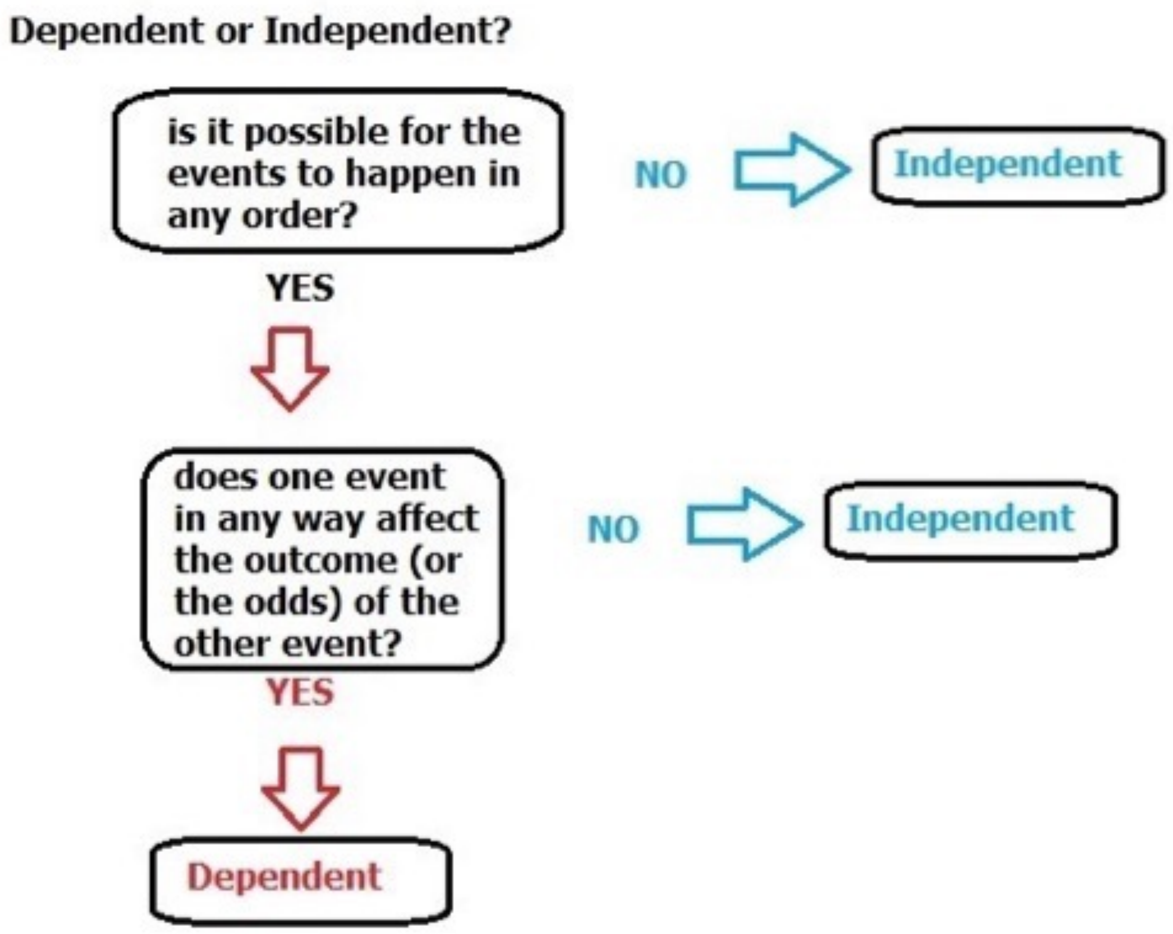
In this case, we would say that the events A and B are independent.

By the previous multiplication rule, we can notice that events A and B are **independent** if and only if:
$\boldsymbol{P(A \cap B) = P(A)P(B)}$.

If the events are **NOT independent**, then they are said to be **dependent**.

Conditional probability: $P(A \cap B) = P(A|B)P(B)$

Dependent or Independent? Steps



Dependent or Independent? Steps

There are four formulas for checking for independence of events:

1. $P(B \mid A) = P(B)$
2. $P(A \mid B) = P(A)$
3. $P(B \mid A) = P(B \mid \text{not } A)$
4. $P(A \text{ and } B) = P(A)\, P(B)$

If *any* of these conditions hold, events are independent.

Example

A bag contains 5 red and 5 blue balls. We remove a random ball from the bag, record its color and put it back into the bag. We then remove another random ball from the bag and record its color.

- What is the probability that the first ball is red?
- What is the probability that the second ball is blue?
- What is the probability that the first ball is red and the second ball is blue?
- Are the first ball being red and the second ball being blue independent events?

Example

Does smoking cause lung cancer?

It is believed that smoking can cause lung cancer. We can do a survey to ascertain whether this is a fact.

We looked through the records of 1,000 cases in a hospital and found the following

Number of smokers = 130,

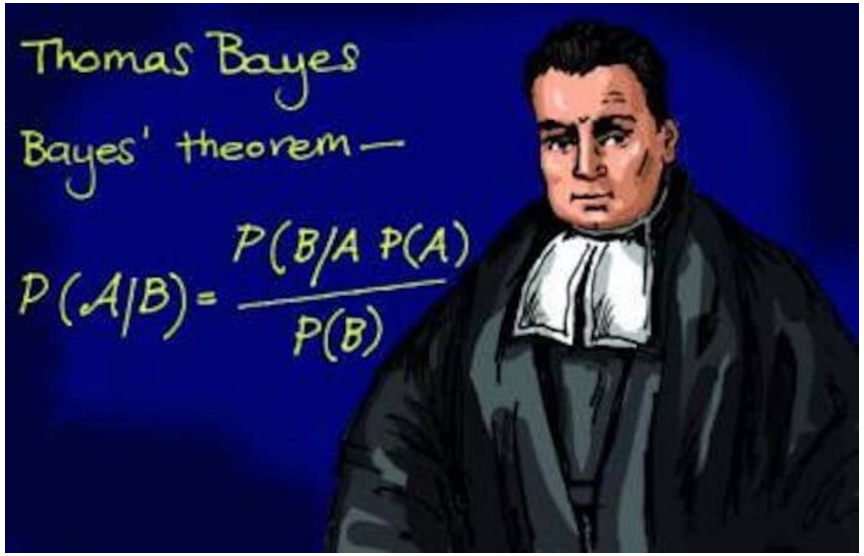
Number of people died of lung cancer = 46,

Number of people who smoked and died of lung cancer = 17.

Bayes' theorem

(Thomas Bayes 1701-1761)

Bayes' theorem describes the probability of occurrence of an event related to any condition. It is also considered for the case of conditional probability.



Bayes' Formula

Recall： For two events A and B, we define the conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$\Rightarrow$ Express P(A|B) in terms of P(B|A)

Bayes' Formula

$\Rightarrow$ Express $P(A|B)$ in terms of $P(B|A)$

The derivation is very simple, we just use $A \cap B = B \cap A \Rightarrow P(A \cap B) = P(B \cap A)$

Bayes' Formula

$\Rightarrow$ Express $P(A|B)$ in terms of $P(B|A)$

The derivation is very simple, we just use $A \cap B = B \cap A \Rightarrow P(A \cap B) = P(B \cap A)$

and the definition of conditional probability

Bayes' Formula

$\Rightarrow$ Express $P(A|B)$ in terms of $P(B|A)$

The derivation is very simple, we just use $A \cap B = B \cap A \Rightarrow P(A \cap B) = P(B \cap A)$

and the definition of conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \Rightarrow \quad P(A \cap B) = \mathrm{P}\,(A|B)P(B)$$

Bayes' Formula

$\Rightarrow$ Express P(A|B) in terms of P(B|A)

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Bayes' formula (Bayes' theorem)

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}}$$

CityU

Example

Detecting aircraft

A radar is designed to detect aircraft. If an aircraft is present, it is detected with probability 0.99. When no aircraft is present, the radar generates an alarm probability 0.02 (false alarm). We assume that an aircraft is present with probability 0.05. If the radar generates an alarm, what is the probability than an aircraft is present?

Importance of Bayes' theorem

- There are many reasons why this theorem is important (for example in the interpretation of probability, in games theory, etc.)

In science, very often we have access to P(A|B) (for example by some experiments) but what we really want to know is P(B|A)

We can then use Bayes' theorem, provided we also know $P(A)$ and $P(B)$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Importance of Bayes' theorem

When developing the test in a lab, we take certain persons who are known to have the disease and run the test.

⇒ Therefore the scientists are measuring P(positive|disease), P(negative|disease).

⇒ We also perform the test on certain persons who do not have the disease, we then measure P(positive|no disease), P(negative|no disease).

Now we use the test in practise. If the results are positive, we want to know the probability that the patient really has the disease, so we want to know **P(disease|positive).**

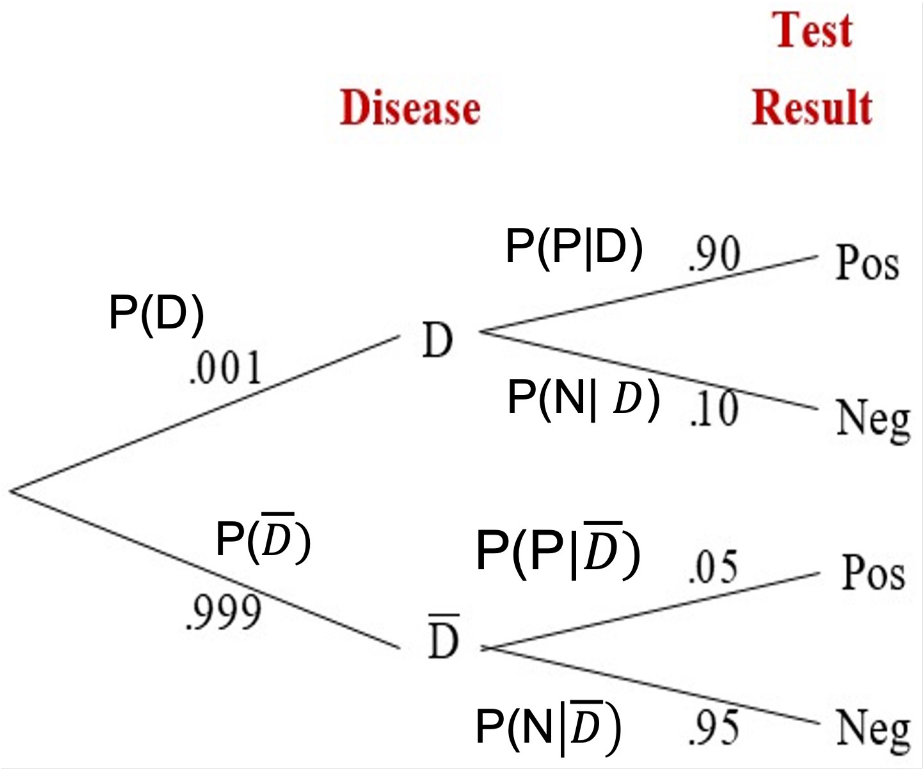
We want to find $P(disease \mid positive)$ knowing $P(positive \mid disease)$ ⇒ Bayes' theorem

Example

Previous example continued:

If a person has a positive test result, find the probability they actually have the disease using Bayes's Formula

Suppose the probability of having the disease (D) is 0.001.
If a person has the disease, the probability of a positive test result (Pos) is 0.90.
If a person does not have the disease, the probability of a negative test result (Neg) is 0.95.



Importance of Bayes' theorem

Solving the Puzzle

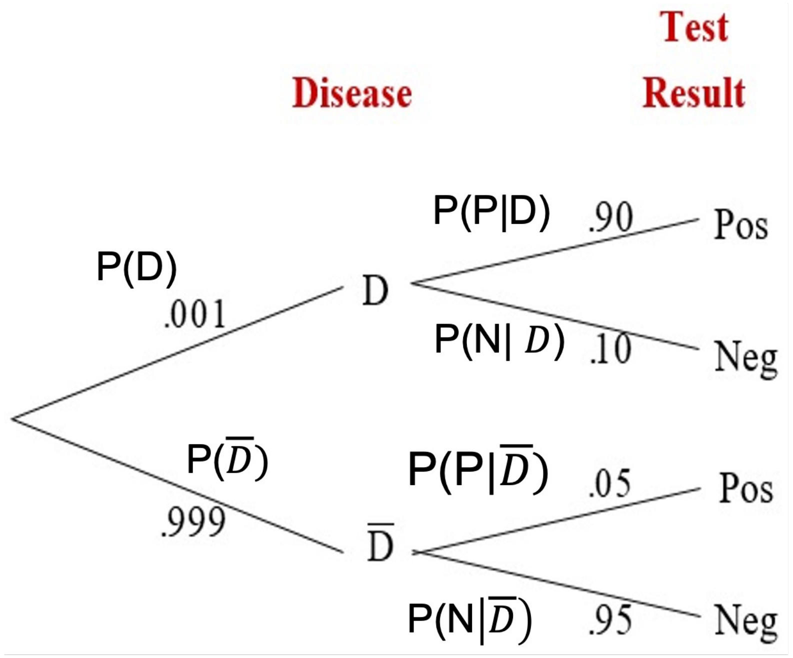
The disease is rare: the probability of *no disease* is a very close to one (0.999).

Therefore the probability of being positive and having the disease

$$P(positive \cap disease) = 0.90 \times 0.001 = 0.0009 \text{ is}$$

small compared to the probability of being a "false positive"

$$P(positive \cap no\ disease) = 0.05 \times 0.999 = 0.04995$$



The probability of being positive

$$P(positive) = P(positive \cap disease) + P(positive \cap no\ disease)$$
$$= 0.0009 + 0.04995 = 0.05085$$

is largely dominated by $P(positive \cap no\ disease)$

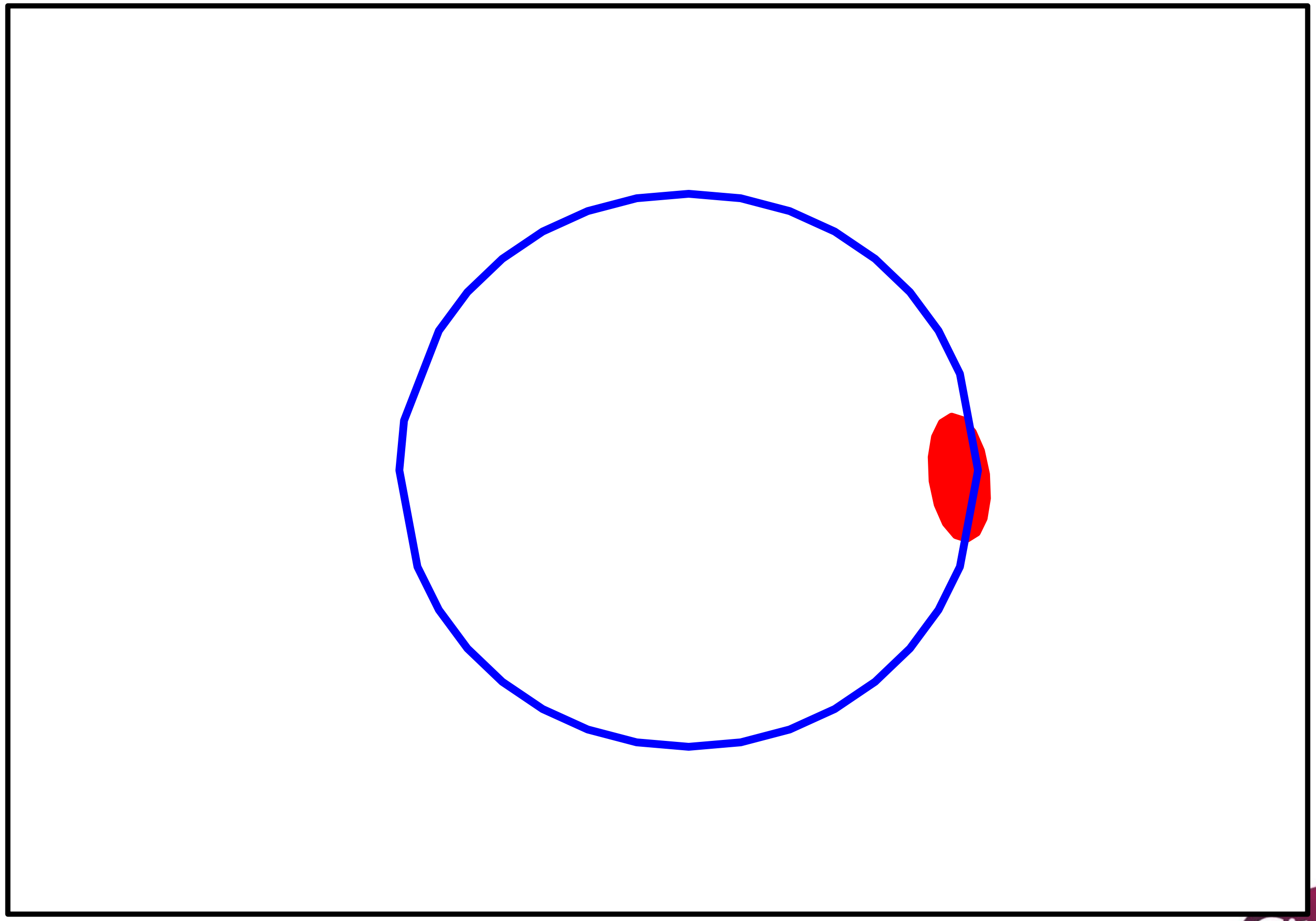
In other words: if somebody is tested positive, it is very likely that he is *false positive*

$$P(no\ disease \mid positive) = \frac{0.04995}{0.05085} \sim 0.98 \qquad P(disease \mid positive) = \frac{0.0009}{0.05085} \sim 0.0177$$

Importance of Bayes' theorem

Solving the Puzzle

If we pick up a random person detected *positive*, most likely it is a *false positive*



Blue *positive*
Red *infected*

Importance of Bayes' theorem

Let us check by changing the numbers

Suppose the probability of having the disease (D) is 0.001.
If a person has the disease, the probability of a positive test result (Pos) is 0.90.
If a person does not have the disease, the probability of a negative test result (Neg) is 0.95.

$$P(disease\,|positive)$$
$$= \frac{P(positive|disease)P(disease)}{P(positive|disease)P(disease) + P(positive|no\ disease)P(\ nodisease)}$$

$$= \frac{0.90\ * 0.001}{0.90\ * 0.001 + 0.05 * 0.999} \qquad \sim 0.0177$$

Importance of Bayes' theorem

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$$= \frac{0.95\ * 0.001}{0.95\ * 0.001 + 0.05 * 0.999} \quad \sim 0.01866$$

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$$= \frac{0.99\ * 0.001}{0.99\ * 0.001 + 0.05 * 0.999} \quad \sim 0.01943$$

Importance of Bayes' theorem

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$$P(disease \mid positive)$$
$$= \frac{P(positive|disease)P(disease)}{P(positive|disease)P(disease) + P(positive|no\ disease)P(\ nodisease)}$$

$$= \frac{0.999 \ * 0.001}{0.999 \ * 0.001 + 0.05 * 0.999} \quad \sim 0.01961$$

Importance of Bayes' theorem

Let us check by changing the numbers

Suppose the probability of having the disease (D) is 0.001.
If a person has the disease, the probability of a positive test result (Pos) is 1.
If a person does not have the disease, the probability of a negative test result (Neg) is 0.95.

$$P(disease\ |positive)$$
$$= \frac{P(positive|disease)P(disease)}{P(positive|disease)P(disease) + P(positive|no\ disease)P(\ nodisease)}$$

$$= \frac{1\ * 0.001}{1\ * 0.001 + 0.05 * 0.999} \qquad \sim 0.01963$$

Importance of Bayes' theorem

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~0.4734

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$$P(disease\ |positive)$$
$$= \frac{P(positive|disease)P(disease)}{P(positive|disease)P(disease) + P(positive|no\ disease)P(\ nodisease)}$$

$$= \frac{0.90\ * 0.001}{0.90\ * 0.001 + (1-0.9999)\ * 0.999} \qquad \sim 0.9001$$

Importance of Bayes' theorem

Bayes' theorem plays a crucial part in probability and statistics.
Has a lot of important applications.

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Has a lot of important applications.

Example

Suppose a carbon monoxide (CO) measuring badge has been developed that is imperfect, with random error that has been studied and is well understood. Originally, the design called for a monitor that would give the user a positive reading whenever the integrated CO concentration exceeded some public health criterion (e.g., 73 ppm-hr in 8 hours). Experiments conducted on the badge show that the monitor correctly indicated a violation of the health criterion in 95% of the cases in which a true violation occurred. The monitor also occasionally gives false positive reading 10% if the time it reports a CO problem when none really exists. Suppose it is known the CO problems occur in approximately 10% of the high-rise building in a given city. If this badge is deployed in a field study, and a positive reading is obtained for a given building, what is the probability that a real CO problem exists there?

A = event “a real CO problem exists”

B = event “positive reading on the monitor”

A positive reading (B) is obtained for a given building, what is the probability that a real CO problem (A) exists there?