Agenda for today

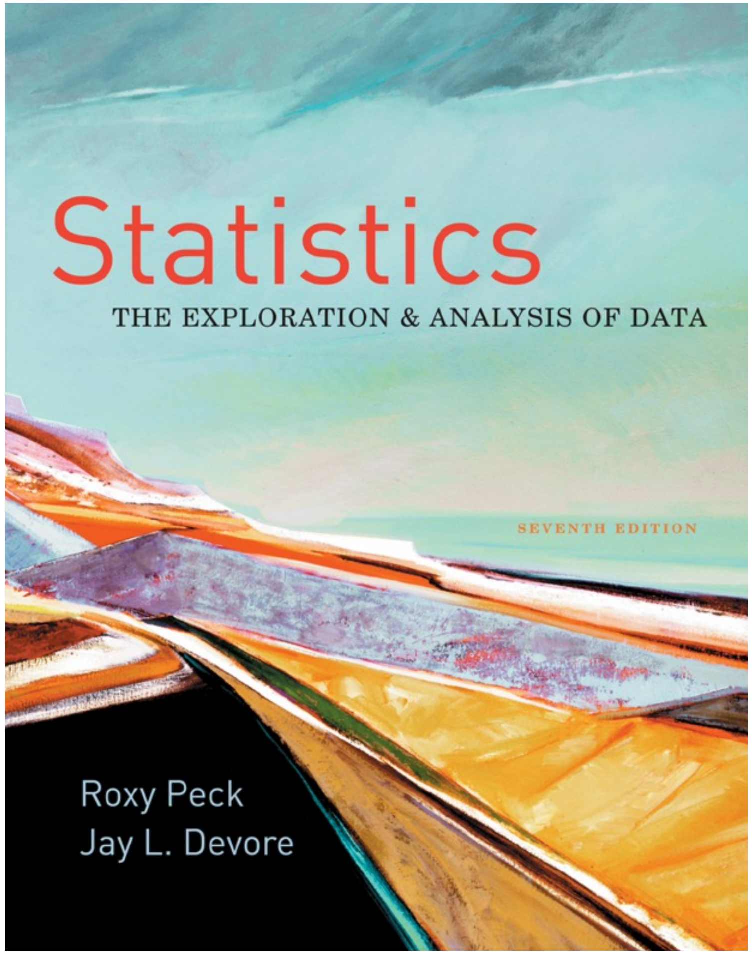
L10 Analysis of Variance

~~L11: Goodness of Fit Test (Not Required)~~

Project 01. Group Project description

More Python tutorials starting from this/next week!

Quiz 5 (today): 9:10-9:40 pm
(Simple Linear regression)



Brief Contents

Sections and/or chapter numbers in color can be found at http://www.cengage.com/statistics/peck

Quiz 6 (Week 11, he last quiz) will be related to L10

L10: Analysis of Variance

ANOVA Models

In previous lectures, we are restricted in testing for the means of ONE or TWO random variables.

Many investigations involve a comparison of more than two random variables.

Here we would add one or more normal random variable, and then **test the equality of their population means.**

An ANOVA, short for "Analysis of Variance", is used to determine whether or not there is a statistically significant difference between the means of three or more independent groups.

Example

Suppose a professor wants to know if **three different studying techniques** lead to **different exam scores**. To test this, he recruits 27 students to participate in a study and randomly assigns each one to use one of the three techniques to prepare for an exam. At the end of one month, all of the students take the same test.

The test scores for each groups are shown below:

How do we compare the scores from these three groups, we have the following approaches?

```
# Multiple sets of example data
data1 = [20, 21, 22, 23, 24, 25, 26, 27, 28]
data2 = [25, 26, 27, 28, 29, 30, 31, 32, 33]
data3 = [30, 31, 32, 33, 34, 35, 36, 37, 38]
```

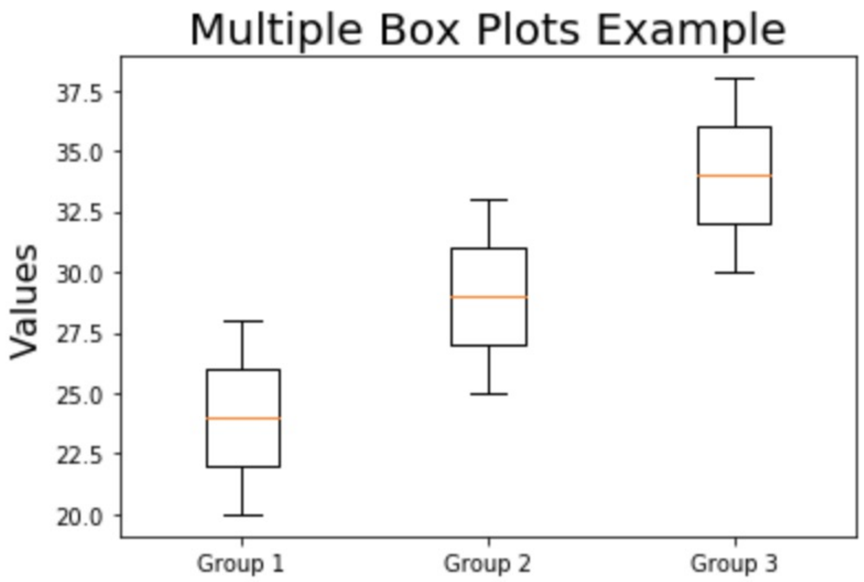
Graphical comparison

To compare the means of $k \geq 2$ groups, we have the following approaches:

1. Graphical comparison:

This allows us to visually compare the distributions of each group.

A ***side-by-side boxplot*** is one of the useful graphical tools for one-way ANOVA analysis. We can create such a graph by using python.

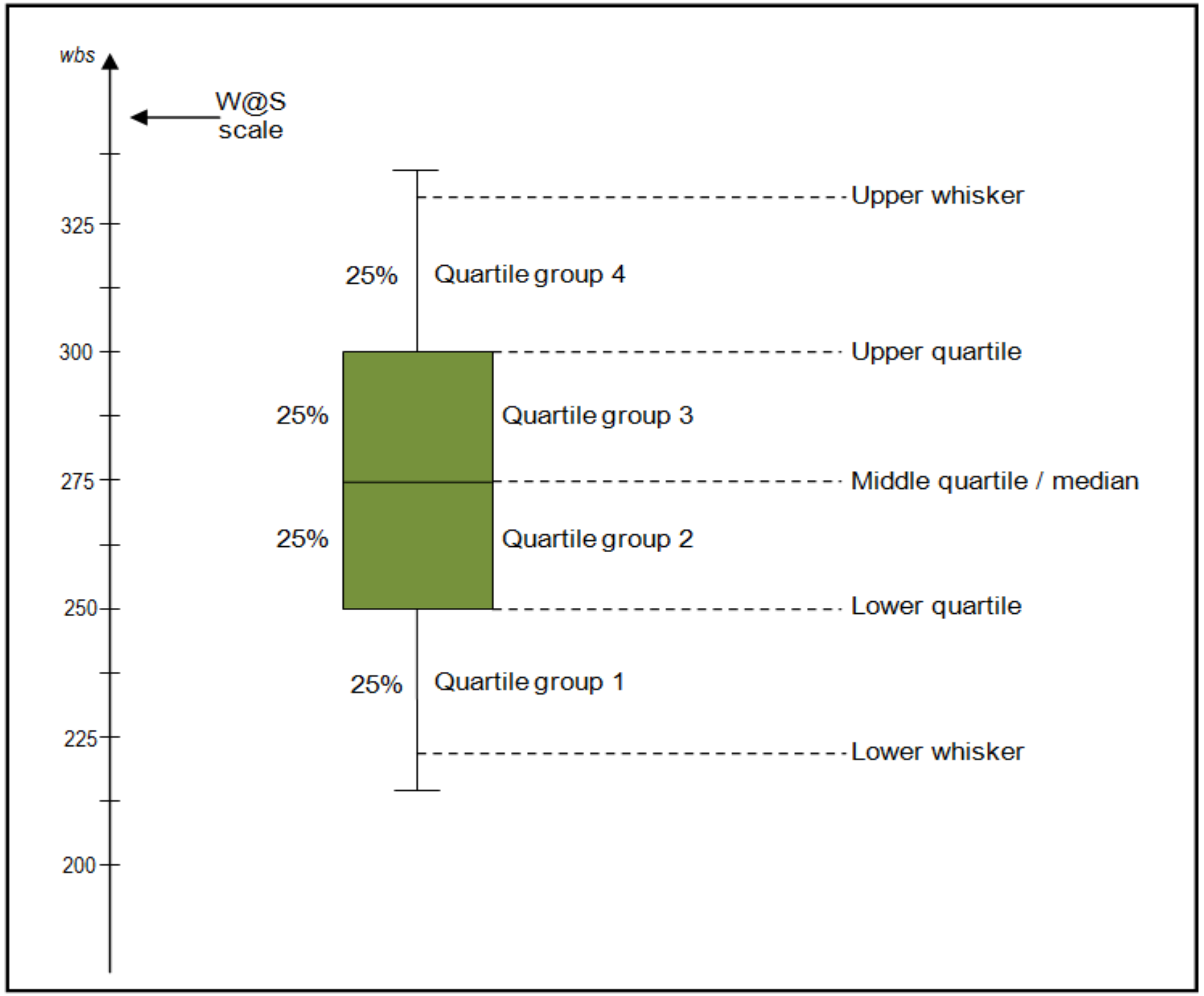



```python
# Multiple sets of example data
data1 = [20, 21, 22, 23, 24, 25, 26, 27, 28]
data2 = [25, 26, 27, 28, 29, 30, 31, 32, 33]
data3 = [30, 31, 32, 33, 34, 35, 36, 37, 38]

# Create side-by-side box plots
plt.boxplot([data1, data2, data3])

# Set the title and labels
plt.title('Multiple Box Plots Example', fontsize=20)
plt.ylabel('Values', fontsize=16)
plt.xticks([1, 2, 3], ['Group 1', 'Group 2', 'Group 3'])
plt.show()
```

Boxplot-Python



Median
The median (middle quartile) marks the mid-point of the data and is shown by the line that divides the box into two parts. Half the scores are greater than or equal to this value and half are less.

Inter-quartile range
The middle "box" represents the middle 50% of scores for the group. The range of scores from lower to upper quartile is referred to as the inter-quartile range. The middle 50% of scores fall within the inter-quartile range.

Upper quartile
75% of the scores fall below the upper quartile.

Lower quartile
25% of scores fall below the lower quartile.

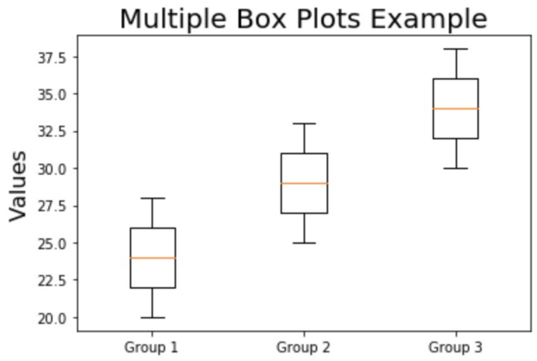
Whiskers
The upper and lower whiskers represent scores outside the middle 50%. Whiskers often (but not always) stretch over a wider range of scores than the middle quartile groups.

One-way ANOVA Analysis

To compare the means of $k \geq 2$ groups, we have the following approaches:

1. Graphical comparison:

This allows us to visually compare the distributions of each group.
A ***side-by-side boxplot*** is one of the useful graphical tools for one-way ANOVA analysis. We can create such a graph by using python.



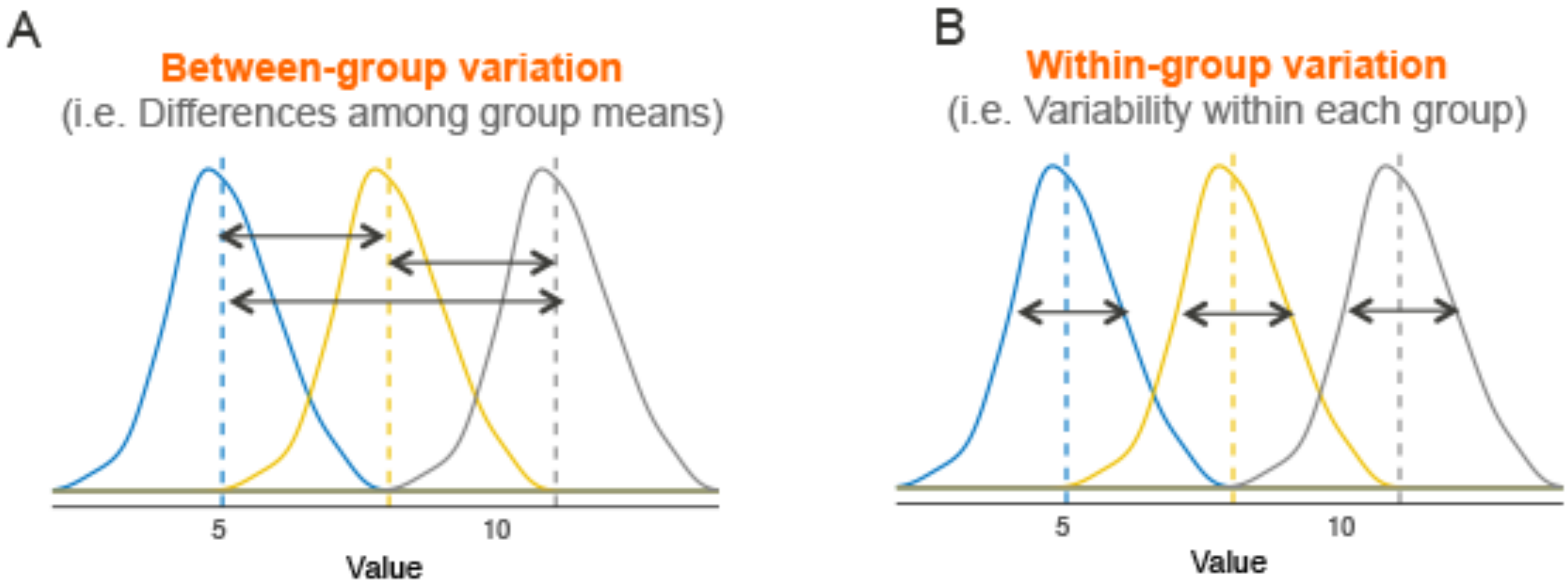
2. Numerical comparison:

The numerical method used for comparisons of two or more groups is the ***ANOVA test***. That is, we can test whether there are any significant differences among the (means of) groups at a certain significance level. ***The null hypothesis tested is the assumption that all groups have the same population mean*.**

Briefly, the mathematical procedure behind the ANOVA test is as follow:

1. Compute the **variance between group means** (see figure, panel A).
2. Compute the **within-group variance**, also known as **residual variance**. This tells us, how different each participant is from their own group mean (see figure, panel B).

3.Produce the F-statistic as the ratio of variance.between.groups/variance.within.groups



Data Structure and Assumptions:

Normal distributions

Equal Unknown Variances

Consider the data from the ***k*** **independent** groups.

	Group 1 $N(\mu_1, \sigma^2)$	Group 2 $N(\mu_2, \sigma^2)$	…	Group k $N(\mu_k, \sigma^2)$
Data	y_{11} y_{12} $\vdots$ y_{1n_1}	y_{21} y_{22} $\vdots$ y_{2n_2}	… … $\vdots$ …	y_{k1} y_{k2} $\vdots$ y_{kn_k}
Number of data at each group	n_1	n_2	…	n_k
treatment/cell mean at each group	$\bar{y}_{1\cdot}$	$\bar{y}_{2\cdot}$	…	$\bar{y}_{k\cdot}$
Total/grand mean	$\bar{y}_{\cdot\cdot} = \frac{1}{n}\sum_{i=1}^{k}\sum_{j=1}^{n_i} y_{ij} = \frac{n_1\bar{y}_{1\cdot} + \cdots + n_k\bar{y}_{k\cdot}}{n}$			

Note that y_{ij} denote the jth observation from the ith **treatment/group**, and the total number of observations is $n = n_1 + n_2 + \cdots + n_k$.

One-way ANOVA Tests:

We can summarize the ANOVA test as follows:

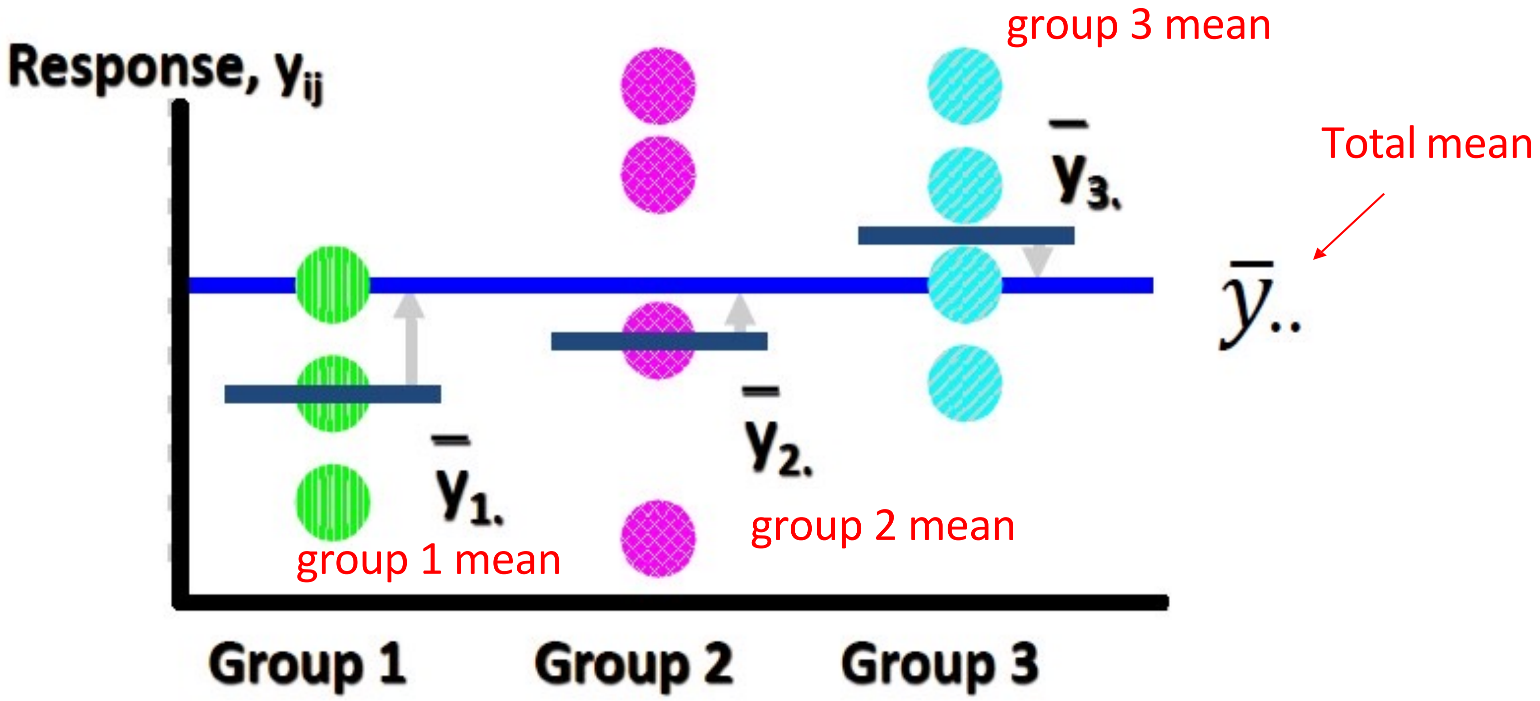
H0: $\mu_1 = \mu_2 = \cdots = \mu_k$, $(i.e., \ no\ group\ effect)$ vs

H1: at least two of the means are not equal.

MAIN idea of ANOVA

The MAIN idea of doing ANOVA test for Problem (A) is to compare the:

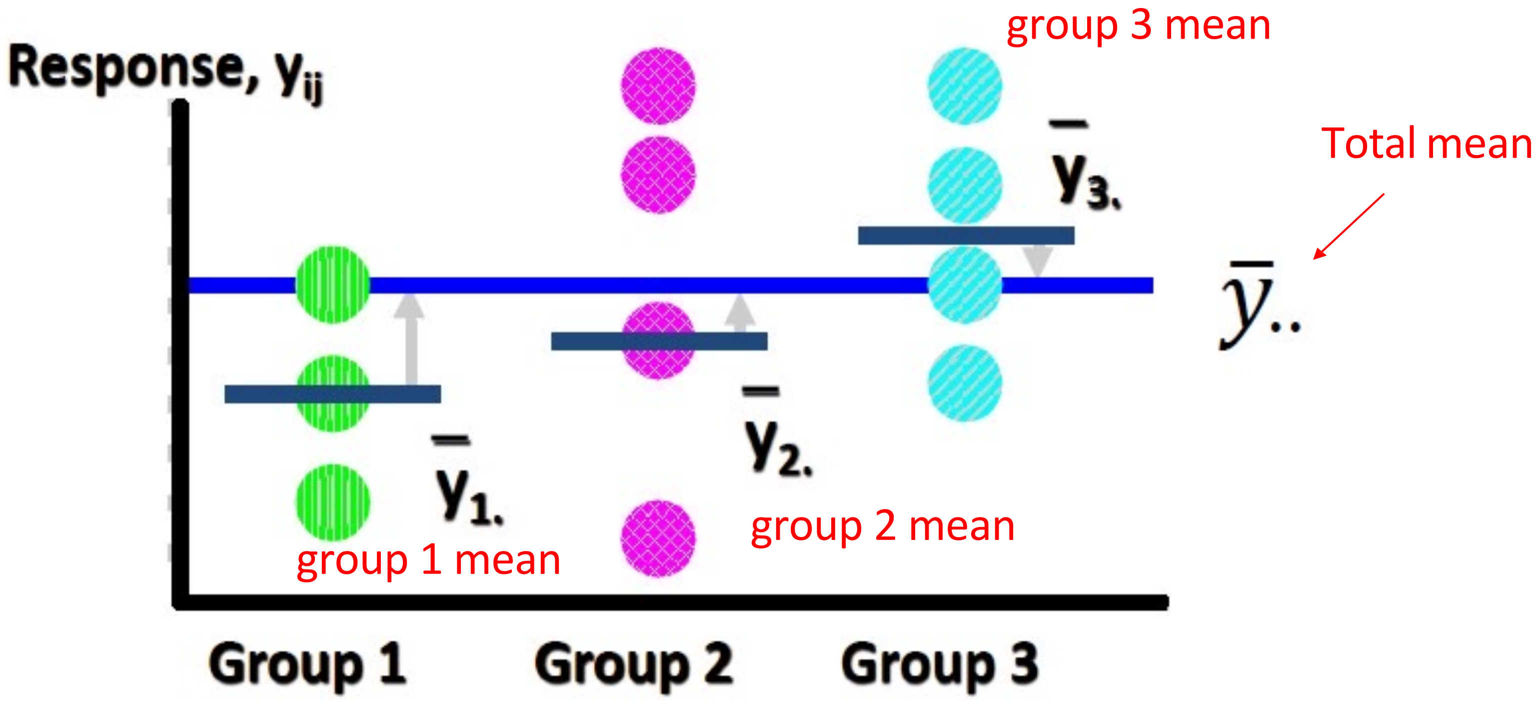
- **mean square treatment (**Based on **Between group variation)**
- **mean square error**. (Based on **Within group variation)**
- **Total Variation**



MAIN idea of ANOVA

- Between group variation (or the treatment sum of squares),

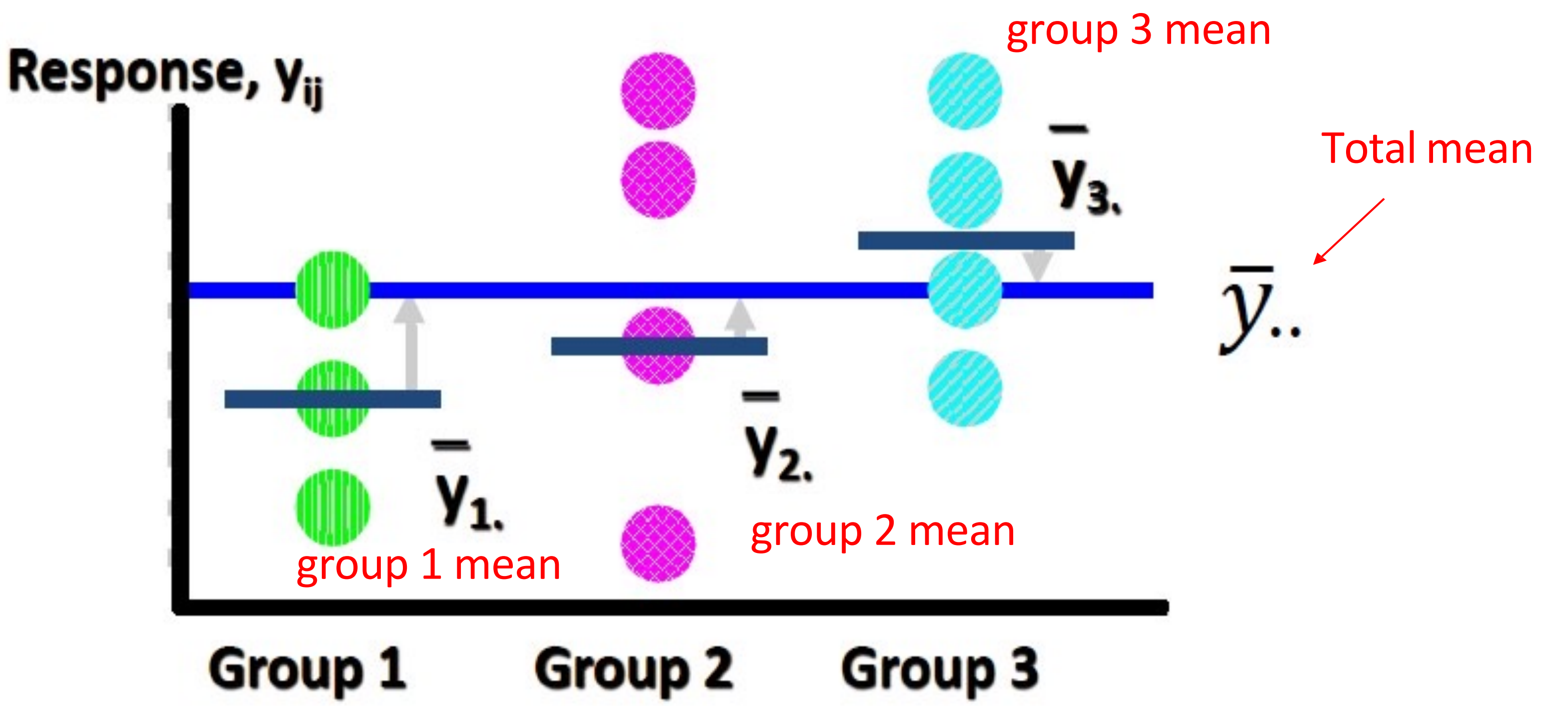
$$\boxed{SS_{\text{Treat}}} = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2 = \sum_{i=1}^{k} n_i(\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2$$



MAIN idea of ANOVA

- Within group variation (or the error sum of squares),

$$\boxed{SSE} = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(y_{ij}-\bar{y}_{i\cdot})^2.$$



MAIN idea of ANOVA

- Between group variation (or the treatment sum of squares),

$$\boxed{SS_{\text{Treat}}} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2 = \sum_{i=1}^{k} n_i (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2, \text{ and}$$

- Within group variation (or the error sum of squares),

$$\boxed{SSE} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i\cdot})^2.$$

In fact, these two variations can be obtained by partitioning the total variation of our data, i.e., the total sum of squares

$$\boxed{SST = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{\cdot\cdot})^2}$$

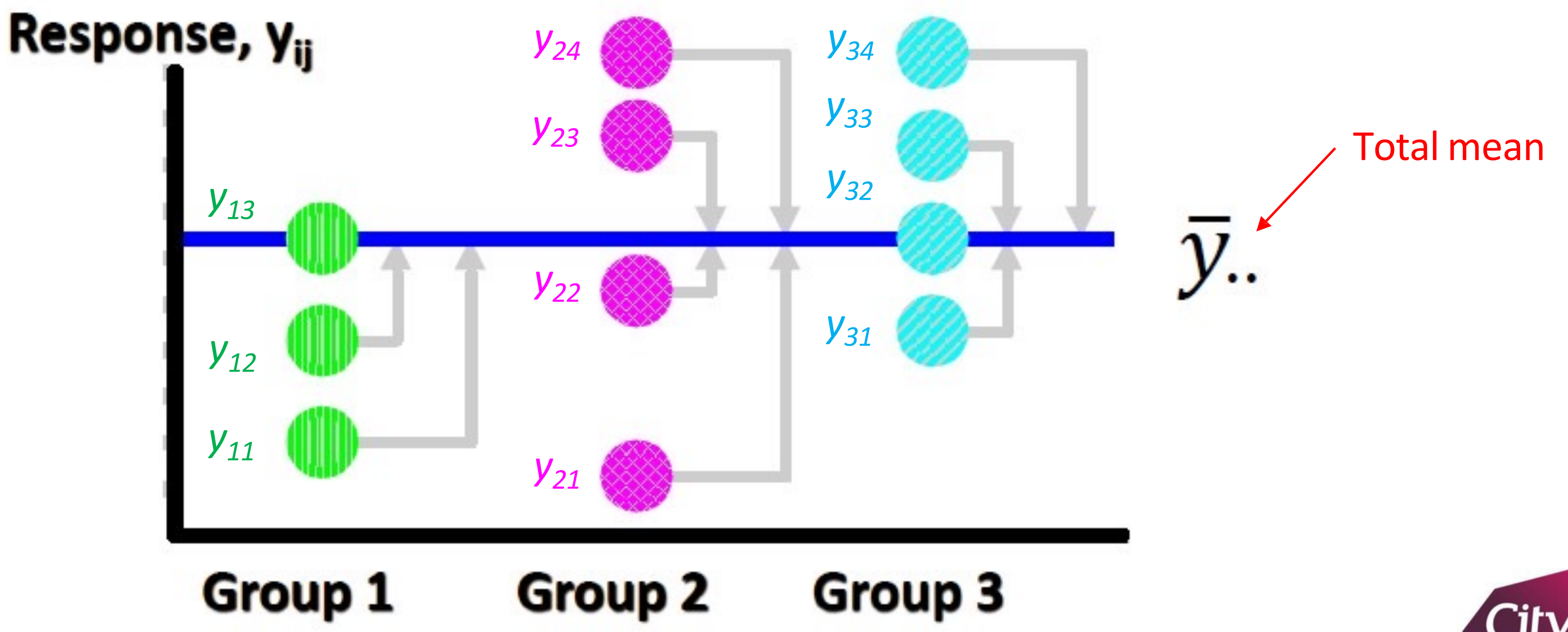
Total Variation

Total Sum of Squares

$$SST = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_{..})^2$$

For k = 3, n_1= 3, n_2= 4 and n_3= 4.

$$SST = (y_{11} - \bar{y}_{..})^2 + (y_{12} - \bar{y}_{..})^2 + (y_{13} - \bar{y}_{..})^2 + (y_{21} - \bar{y}_{..})^2 + \cdots + (y_{33} - \bar{y}_{..})^2 + (y_{34} - \bar{y}_{..})^2$$



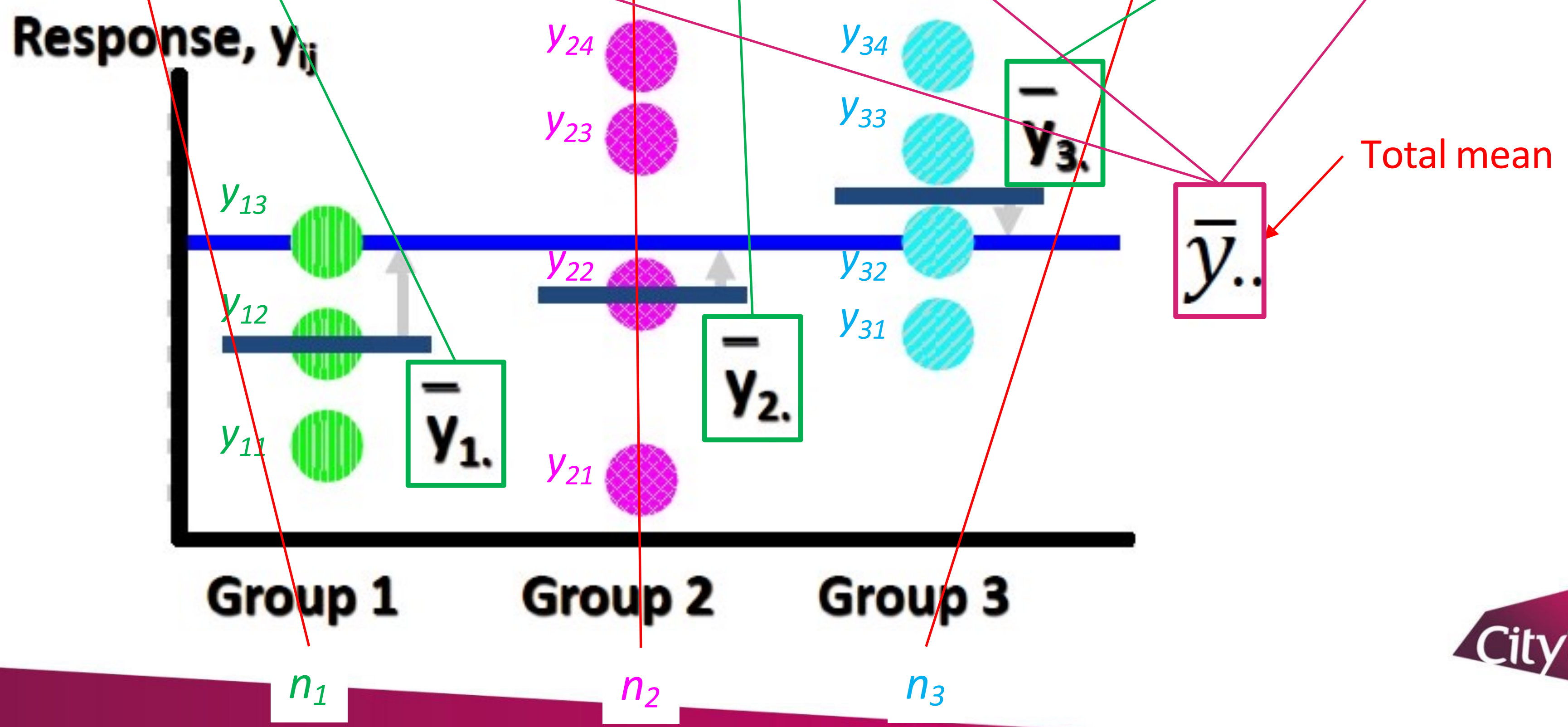
Between-group Variation

Treatment Sum of Squares

$$SS_{\text{Treat}} = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(\bar{y}_{i.} - \bar{y}_{..})^2 = \sum_{i=1}^{k} n_i(\bar{y}_{i.} - \bar{y}_{..})^2$$

For k = 3, n_1= 3, n_2= 4 and n_3= 4.

$$SS_{Treat} = \boxed{n_1}(\bar{y}_{1.} - \bar{y}_{..})^2 + \boxed{n_2}(\bar{y}_{2.} - \bar{y}_{..})^2 + \boxed{n_3}(\bar{y}_{3.} - \bar{y}_{..})^2$$



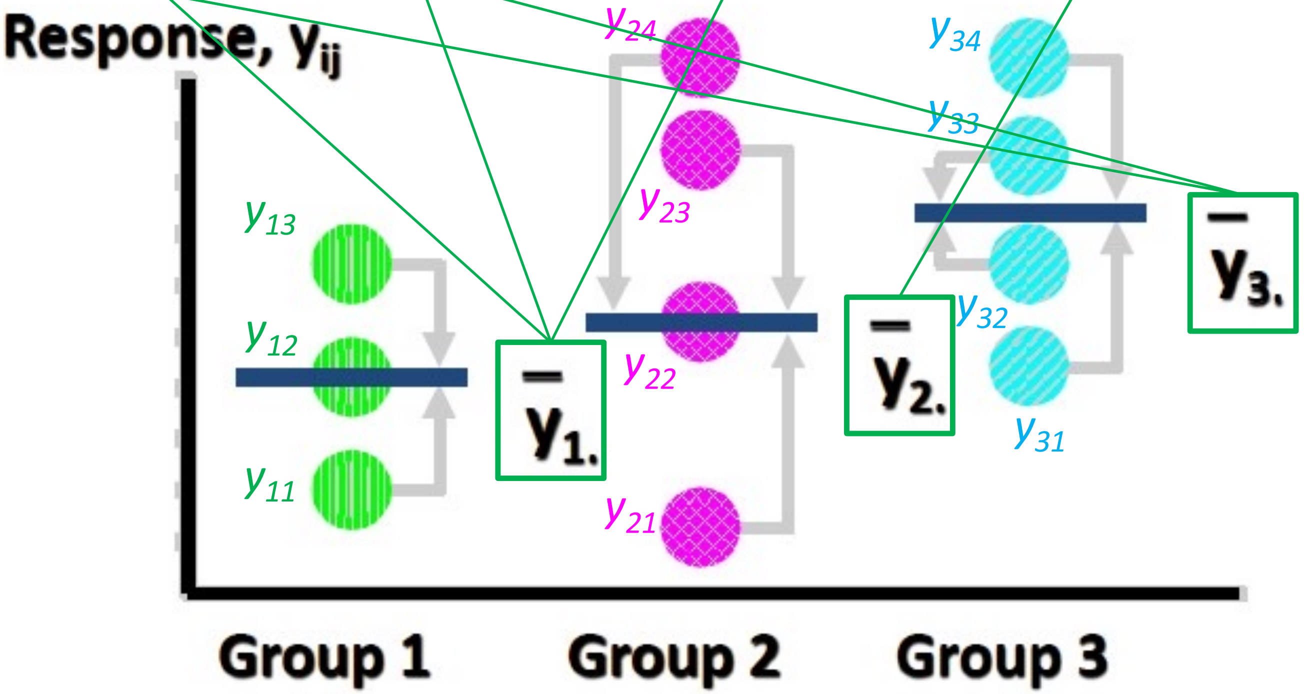
Within-group Variation

Error Sum of Squares

$$SSE = (n_1-1)s_1^2+(n_2-1)s_2^2+(n_3-1)s_3^2+\ldots+(n_k-1)s_k^2$$

For $k = 3$, $n_1 = 3$, $n_2 = 4$ and $n_3 = 4$.

$$SSE = (y_{11}-\bar{y}_{1\cdot})^2+(y_{12}-\bar{y}_{1\cdot})^2+(y_{13}-\bar{y}_{1\cdot})^2+(y_{21}-\bar{y}_{2\cdot})^2+\cdots+(y_{33}-\bar{y}_{3\cdot})^2+(y_{34}-\bar{y}_{3\cdot})^2$$



MAIN idea of ANOVA

Other form of abbreviation:
SST=SSTo
$SS_{Treat} = SST_r$

The relationship between SST, SS_{Treat} , and SSE

$$\boxed{SST = SS_{Treat} + SSE}$$

SST total variation

SS_{Treat} Between group variation

SSE Within group variation

which is called the *fundamental identity for single-factor ANOVA.*

total variation = explained variation + unexplained variation

(residual variance)

SS_{Treat} represents variation that can be explained by any differences between means

SSE results from measuring variability separately within each sample and then combining as indicated in the formula for SSE. Such within-sample variability is present regardless of whether or not *H0* is true

MAIN idea of ANOVA

Other form of abbreviation:
SST=SSTo
SS_{Treat} = SST_r

The relationship between SST, SS_{Treat} , and SSE

$$\boxed{SST = SS_{Treat} + SSE}$$

Then, we can construct the following ANOVA table:

Source	d.f.	Sum of Squares (SS)	Mean sum of squares (MS)	F-value
Treatment/Between/Model	k-1	SS_{Treat}	$MS_{Treat} = SS_{Treat}/(k-1)$	$F = MS_{Treat}/MSE$
Error/Within	n-k	SSE	$MSE = SSE/(n-k)$	
Total	n-1	SST		

where

$$\boxed{MS_{Treat} = \frac{SS_{Treat}}{k-1}}$$ and $$\boxed{MSE = \frac{SSE}{n-k}.}$$

mean square treatment   mean square error

are two estimates of the common population variance σ^2

H0: $\mu_1 = \mu_2 = \cdots = \mu_k$, ($i.e.,$ $no\ group\ effect$) vs
H1: at least two of the means are not equal.

Then the random variable $F_{n,m} = \frac{X/n}{Y/m}$

Is said to have *F-Distribution* *with* *n and m degrees of freedom*

That is, if H_0 is true, we expect that

$$\boxed{F = \frac{MS_{Treat}}{MSE} \text{ is small}}$$

Thus, if this ratio is large, then we would tend to believe that H_0 is false and **then reject H_0**.

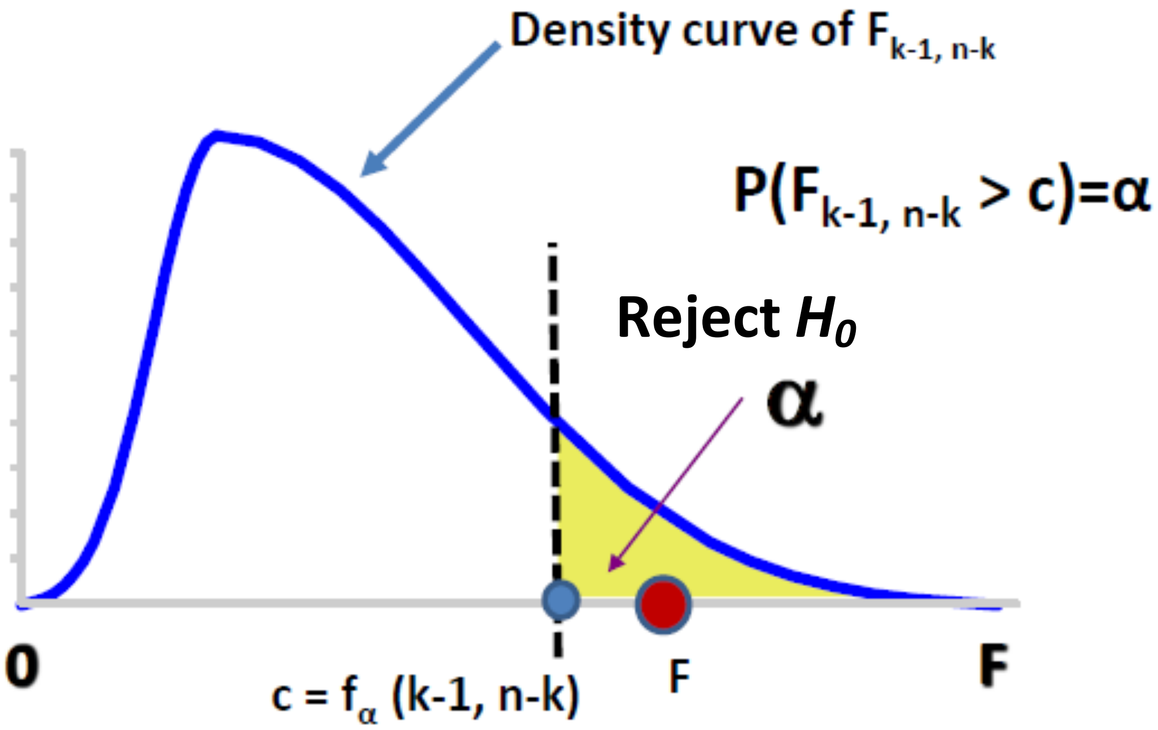
How large is large?

Using a concept of hypothesis testing, we can answer how large *F* is for the rejection of H_0.

One-Way ANOVA Test, Critical Value

Under H_0 (i.e. all means are equal),

Reject H_0 at a significance level α if we get a large F, say $F > c$.



One-Way ANOVA Test, Critical Value

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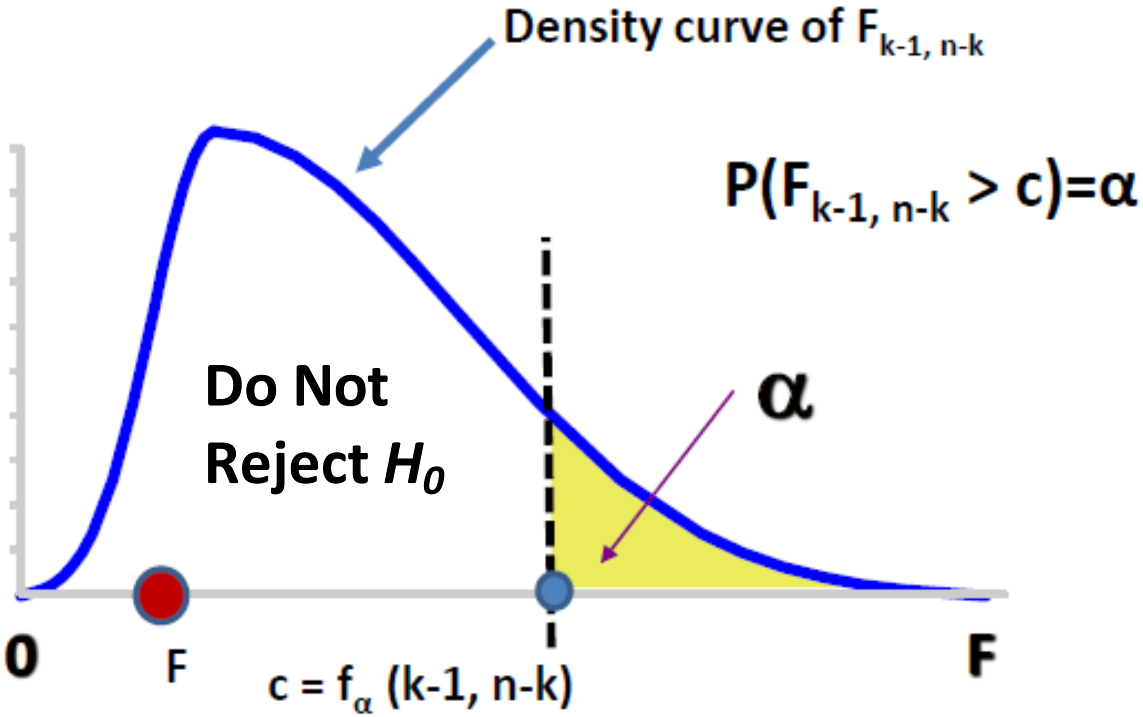


One-Way ANOVA Test, Critical Value

Test

H_0: $\mu_1 = \mu_2 = \cdots = \mu_k$ **VS** H_1: **Otherwise**

at a significance level α.

$\boxed{\text{We would reject } H_0 \text{ at a significance level } \alpha \text{ if } F > f_\alpha(k\text{-}1, n\text{-}k)}$

The F-Distribution

$$F = \frac{MS_{Treat}}{MSE} \text{ is small}$$

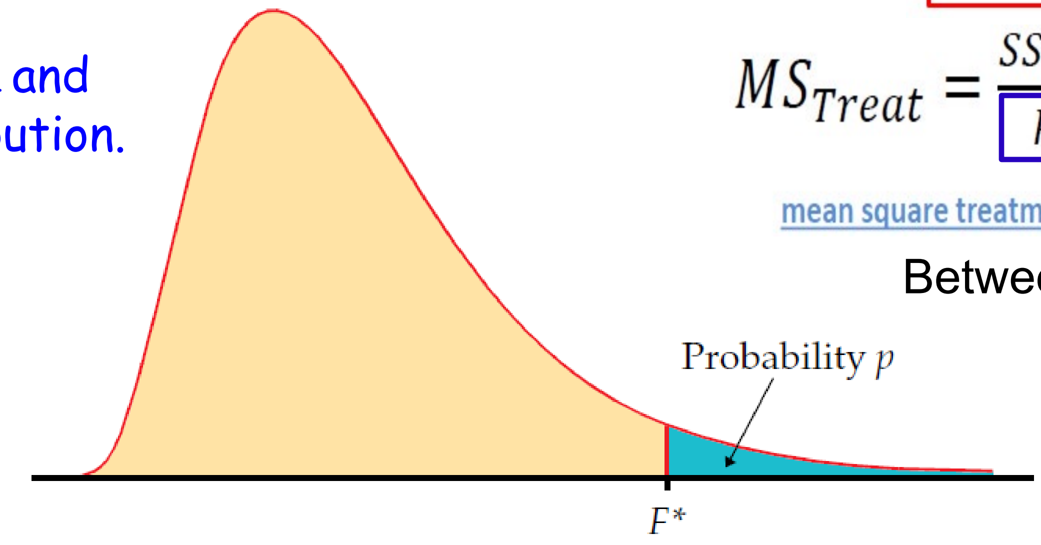
$$MS_{Treat} = \frac{SS_{Treat}}{\boxed{k-1}}$$ and $$MSE = \frac{SSE}{\boxed{n-k}}$$

mean square treatment — df1 — Between group

mean square error — df2 — Within group

Each different combination of df1 and df2 produces a different F distribution.

Table entry for p is the critical value F^* with probability p lying to its right.



K: number of groups
n: total number of observations is $n = n_1 + n_2 + \cdots + n_k$.

TABLE E

F critical values

		df1 Degrees of freedom in the numerator								
df2	p	1	2	3	4	5	6	7	8	9
1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
	.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
	.025	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
	.010	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
	.001	405284	500000	540379	562500	576405	585937	592873	598144	602284

Table entry for p is the critical value F^* with probability p lying to its right.

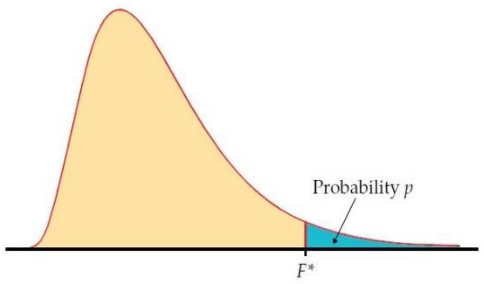


TABLE E

F critical values

Degrees of freedom in the denominator	p	1	2	3	4	5	6	7	8	9
		Degrees of freedom in the numerator								
1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
	.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
	.025	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
	.010	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
	.001	405284	500000	540379	562500	576405	585937	592873	598144	602284
2	.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
	.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
	.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
	.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39
3	.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
	.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
	.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86
4	.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
	.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
	.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47
5	.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
	.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
	.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24
6	.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
	.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
	.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69
7	.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
	.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
	.001	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33

Table entry for p is the critical value F^* with probability p lying to its right.

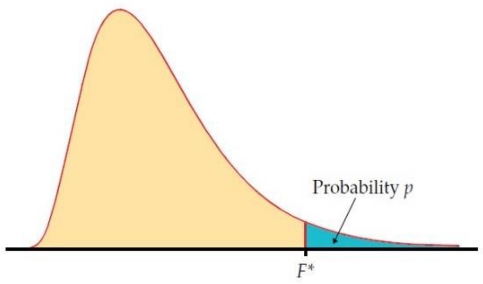


TABLE E

F critical values (continued)

Degrees of freedom in the numerator										
10	12	15	20	25	30	40	50	60	120	1000
60.19	60.71	61.22	61.74	62.05	62.26	62.53	62.69	62.79	63.06	63.30
241.88	243.91	245.95	248.01	249.26	250.10	251.14	251.77	252.20	253.25	254.19
968.63	976.71	984.87	993.10	998.08	1001.4	1005.6	1008.1	1009.8	1014.0	1017.7
6055.8	6106.3	6157.3	6208.7	6239.8	6260.6	6286.8	6302.5	6313.0	6339.4	6362.7
605621	610668	615764	620908	624017	626099	628712	630285	631337	633972	636301
9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.49
19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48	19.48	19.49	19.49
39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.48	39.49	39.50
99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.48	99.49	99.50
999.40	999.42	999.43	999.45	999.46	999.47	999.47	999.48	999.48	999.49	999.50
5.23	5.22	5.20	5.18	5.17	5.17	5.16	5.15	5.15	5.14	5.13
8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.58	8.57	8.55	8.53
14.42	14.34	14.25	14.17	14.12	14.08	14.04	14.01	13.99	13.95	13.91
27.23	27.05	26.87	26.69	26.58	26.50	26.41	26.35	26.32	26.22	26.14
129.25	128.32	127.37	126.42	125.84	125.45	124.96	124.66	124.47	123.97	123.53
3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.80	3.79	3.78	3.76
5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.63
8.84	8.75	8.66	8.56	8.50	8.46	8.41	8.38	8.36	8.31	8.26
14.55	14.37	14.20	14.02	13.91	13.84	13.75	13.69	13.65	13.56	13.47
48.05	47.41	46.76	46.10	45.70	45.43	45.09	44.88	44.75	44.40	44.09
3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.12	3.11
4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.44	4.43	4.40	4.37
6.62	6.52	6.43	6.33	6.27	6.23	6.18	6.14	6.12	6.07	6.02
10.05	9.89	9.72	9.55	9.45	9.38	9.29	9.24	9.20	9.11	9.03
26.92	26.42	25.91	25.39	25.08	24.87	24.60	24.44	24.33	24.06	23.82
2.94	2.90	2.87	2.84	2.81	2.80	2.78	2.77	2.76	2.74	2.72
4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.75	3.74	3.70	3.67
5.46	5.37	5.27	5.17	5.11	5.07	5.01	4.98	4.96	4.90	4.86
7.87	7.72	7.56	7.40	7.30	7.23	7.14	7.09	7.06	6.97	6.89
18.41	17.99	17.56	17.12	16.85	16.67	16.44	16.31	16.21	15.98	15.77
2.70	2.67	2.63	2.59	2.57	2.56	2.54	2.52	2.51	2.49	2.47
3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.32	3.30	3.27	3.23
4.76	4.67	4.57	4.47	4.40	4.36	4.31	4.28	4.25	4.20	4.15
6.62	6.47	6.31	6.16	6.06	5.99	5.91	5.86	5.82	5.74	5.66
14.08	13.71	13.32	12.93	12.69	12.53	12.33	12.20	12.12	11.91	11.72

(Continued)

TABLE E

F critical values (continued)

Degrees of freedom in the denominator	*p*	Degrees of freedom in the numerator: 1	2	3	4	5	6	7	8	9
8	.100	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	.050	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
	.025	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
	.010	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
	.001	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77
9	.100	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
	.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
	.010	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
	.001	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11
10	.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
	.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
	.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
	.001	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96
11	.100	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
	.050	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
	.025	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59
	.010	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
	.001	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12
12	.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
	.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
	.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
	.001	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48
13	.100	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16
	.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
	.025	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31
	.010	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
	.001	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98
14	.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12
	.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
	.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21
	.010	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
	.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58
15	.100	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
	.025	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
	.010	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
	.001	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26
16	.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06
	.050	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
	.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05
	.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
	.001	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98
17	.100	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03
	.050	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
	.025	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98
	.010	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68
	.001	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75

TABLE E

F critical values (continued)

Degrees of freedom in the denominator	*p*	Degrees of freedom in the numerator: 10	12	15	20	25	30	40	50	60	120	1000
8	.100	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.30
	.050	3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.02	3.01	2.97	2.93
	.025	4.30	4.20	4.10	4.00	3.94	3.89	3.84	3.81	3.78	3.73	3.68
	.010	5.81	5.67	5.52	5.36	5.26	5.20	5.12	5.07	5.03	4.95	4.87
	.001	11.54	11.19	10.84	10.48	10.26	10.11	9.92	9.80	9.73	9.53	9.36
9	.100	2.42	2.38	2.34	2.30	2.27	2.25	2.23	2.22	2.21	2.18	2.16
	.050	3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.80	2.79	2.75	2.71
	.025	3.96	3.87	3.77	3.67	3.60	3.56	3.51	3.47	3.45	3.39	3.34
	.010	5.26	5.11	4.96	4.81	4.71	4.65	4.57	4.52	4.48	4.40	4.32
	.001	9.89	9.57	9.24	8.90	8.69	8.55	8.37	8.26	8.19	8.00	7.84
10	.100	2.32	2.28	2.24	2.20	2.17	2.16	2.13	2.12	2.11	2.08	2.06
	.050	2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.64	2.62	2.58	2.54
	.025	3.72	3.62	3.52	3.42	3.35	3.31	3.26	3.22	3.20	3.14	3.09
	.010	4.85	4.71	4.56	4.41	4.31	4.25	4.17	4.12	4.08	4.00	3.92
	.001	8.75	8.45	8.13	7.80	7.60	7.47	7.30	7.19	7.12	6.94	6.78
11	.100	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	1.98
	.050	2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.51	2.49	2.45	2.41
	.025	3.53	3.43	3.33	3.23	3.16	3.12	3.06	3.03	3.00	2.94	2.89
	.010	4.54	4.40	4.25	4.10	4.01	3.94	3.86	3.81	3.78	3.69	3.61
	.001	7.92	7.63	7.32	7.01	6.81	6.68	6.52	6.42	6.35	6.18	6.02
12	.100	2.19	2.15	2.10	2.06	2.03	2.01	1.99	1.97	1.96	1.93	1.91
	.050	2.75	2.69	2.62	2.54	2.50	2.47	2.43	2.40	2.38	2.34	2.30
	.025	3.37	3.28	3.18	3.07	3.01	2.96	2.91	2.87	2.85	2.79	2.73
	.010	4.30	4.16	4.01	3.86	3.76	3.70	3.62	3.57	3.54	3.45	3.37
	.001	7.29	7.00	6.71	6.40	6.22	6.09	5.93	5.83	5.76	5.59	5.44
13	.100	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88	1.85
	.050	2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.31	2.30	2.25	2.21
	.025	3.25	3.15	3.05	2.95	2.88	2.84	2.78	2.74	2.72	2.66	2.60
	.010	4.10	3.96	3.82	3.66	3.57	3.51	3.43	3.38	3.34	3.25	3.18
	.001	6.80	6.52	6.23	5.93	5.75	5.63	5.47	5.37	5.30	5.14	4.99
14	.100	2.10	2.05	2.01	1.96	1.93	1.91	1.89	1.87	1.86	1.83	1.80
	.050	2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.24	2.22	2.18	2.14
	.025	3.15	3.05	2.95	2.84	2.78	2.73	2.67	2.64	2.61	2.55	2.50
	.010	3.94	3.80	3.66	3.51	3.41	3.35	3.27	3.22	3.18	3.09	3.02
	.001	6.40	6.13	5.85	5.56	5.38	5.25	5.10	5.00	4.94	4.77	4.62
15	.100	2.06	2.02	1.97	1.92	1.89	1.87	1.85	1.83	1.82	1.79	1.76
	.050	2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.18	2.16	2.11	2.07
	.025	3.06	2.96	2.86	2.76	2.69	2.64	2.59	2.55	2.52	2.46	2.40
	.010	3.80	3.67	3.52	3.37	3.28	3.21	3.13	3.08	3.05	2.96	2.88
	.001	6.08	5.81	5.54	5.25	5.07	4.95	4.80	4.70	4.64	4.47	4.33
16	.100	2.03	1.99	1.94	1.89	1.86	1.84	1.81	1.79	1.78	1.75	1.72
	.050	2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.12	2.11	2.06	2.02
	.025	2.99	2.89	2.79	2.68	2.61	2.57	2.51	2.47	2.45	2.38	2.32
	.010	3.69	3.55	3.41	3.26	3.16	3.10	3.02	2.97	2.93	2.84	2.76
	.001	5.81	5.55	5.27	4.99	4.82	4.70	4.54	4.45	4.39	4.23	4.08
17	.100	2.00	1.96	1.91	1.86	1.83	1.81	1.78	1.76	1.75	1.72	1.69
	.050	2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.08	2.06	2.01	1.97
	.025	2.92	2.82	2.72	2.62	2.55	2.50	2.44	2.41	2.38	2.32	2.26
	.010	3.59	3.46	3.31	3.16	3.07	3.00	2.92	2.87	2.83	2.75	2.66
	.001	5.58	5.32	5.05	4.78	4.60	4.48	4.33	4.24	4.18	4.02	3.87

(*Continued*)

TABLE E

F critical values (continued)

Degrees of freedom in the denominator	*p*	Degrees of freedom in the numerator								
		1	2	3	4	5	6	7	8	9
18	.100	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00
	.050	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
	.025	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93
	.010	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
	.001	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56
19	.100	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98
	.050	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
	.025	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88
	.010	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
	.001	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39
20	.100	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
	.050	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
	.025	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
	.010	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
	.001	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24
21	.100	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95
	.050	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
	.025	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80
	.010	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
	.001	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11
22	.100	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93
	.050	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
	.025	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76
	.010	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35
	.001	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99
23	.100	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92
	.050	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
	.025	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73
	.010	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30
	.001	14.20	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89
24	.100	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
	.050	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
	.025	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
	.010	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
	.001	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80
25	.100	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89
	.050	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
	.025	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68
	.010	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22
	.001	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71
26	.100	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88
	.050	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
	.025	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65
	.010	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18
	.001	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64
27	.100	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87
	.050	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
	.025	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63
	.010	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15
	.001	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57

TABLE E

F critical values (continued)

Degrees of freedom in the denominator	*p*	Degrees of freedom in the numerator										
		10	12	15	20	25	30	40	50	60	120	1000
18	.100	1.98	1.93	1.89	1.84	1.80	1.78	1.75	1.74	1.72	1.69	1.66
	.050	2.41	2.34	2.27	2.19	2.14	2.11	2.06	2.04	2.02	1.97	1.92
	.025	2.87	2.77	2.67	2.56	2.49	2.44	2.38	2.35	2.32	2.26	2.20
	.010	3.51	3.37	3.23	3.08	2.98	2.92	2.84	2.78	2.75	2.66	2.58
	.001	5.39	5.13	4.87	4.59	4.42	4.30	4.15	4.06	4.00	3.84	3.69
19	.100	1.96	1.91	1.86	1.81	1.78	1.76	1.73	1.71	1.70	1.67	1.64
	.050	2.38	2.31	2.23	2.16	2.11	2.07	2.03	2.00	1.98	1.93	1.88
	.025	2.82	2.72	2.62	2.51	2.44	2.39	2.33	2.30	2.27	2.20	2.14
	.010	3.43	3.30	3.15	3.00	2.91	2.84	2.76	2.71	2.67	2.58	2.50
	.001	5.22	4.97	4.70	4.43	4.26	4.14	3.99	3.90	3.84	3.68	3.53
20	.100	1.94	1.89	1.84	1.79	1.76	1.74	1.71	1.69	1.68	1.64	1.61
	.050	2.35	2.28	2.20	2.12	2.07	2.04	1.99	1.97	1.95	1.90	1.85
	.025	2.77	2.68	2.57	2.46	2.40	2.35	2.29	2.25	2.22	2.16	2.09
	.010	3.37	3.23	3.09	2.94	2.84	2.78	2.69	2.64	2.61	2.52	2.43
	.001	5.08	4.82	4.56	4.29	4.12	4.00	3.86	3.77	3.70	3.54	3.40
21	.100	1.92	1.87	1.83	1.78	1.74	1.72	1.69	1.67	1.66	1.62	1.59
	.050	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.94	1.92	1.87	1.82
	.025	2.73	2.64	2.53	2.42	2.36	2.31	2.25	2.21	2.18	2.11	2.05
	.010	3.31	3.17	3.03	2.88	2.79	2.72	2.64	2.58	2.55	2.46	2.37
	.001	4.95	4.70	4.44	4.17	4.00	3.88	3.74	3.64	3.58	3.42	3.28
22	.100	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.60	1.57
	.050	2.30	2.23	2.15	2.07	2.02	1.98	1.94	1.91	1.89	1.84	1.79
	.025	2.70	2.60	2.50	2.39	2.32	2.27	2.21	2.17	2.14	2.08	2.01
	.010	3.26	3.12	2.98	2.83	2.73	2.67	2.58	2.53	2.50	2.40	2.32
	.001	4.83	4.58	4.33	4.06	3.89	3.78	3.63	3.54	3.48	3.32	3.17
23	.100	1.89	1.84	1.80	1.74	1.71	1.69	1.66	1.64	1.62	1.59	1.55
	.050	2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.88	1.86	1.81	1.76
	.025	2.67	2.57	2.47	2.36	2.29	2.24	2.18	2.14	2.11	2.04	1.98
	.010	3.21	3.07	2.93	2.78	2.69	2.62	2.54	2.48	2.45	2.35	2.27
	.001	4.73	4.48	4.23	3.96	3.79	3.68	3.53	3.44	3.38	3.22	3.08
24	.100	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.57	1.54
	.050	2.25	2.18	2.11	2.03	1.97	1.94	1.89	1.86	1.84	1.79	1.74
	.025	2.64	2.54	2.44	2.33	2.26	2.21	2.15	2.11	2.08	2.01	1.94
	.010	3.17	3.03	2.89	2.74	2.64	2.58	2.49	2.44	2.40	2.31	2.22
	.001	4.64	4.39	4.14	3.87	3.71	3.59	3.45	3.36	3.29	3.14	2.99
25	.100	1.87	1.82	1.77	1.72	1.68	1.66	1.63	1.61	1.59	1.56	1.52
	.050	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.84	1.82	1.77	1.72
	.025	2.61	2.51	2.41	2.30	2.23	2.18	2.12	2.08	2.05	1.98	1.91
	.010	3.13	2.99	2.85	2.70	2.60	2.54	2.45	2.40	2.36	2.27	2.18
	.001	4.56	4.31	4.06	3.79	3.63	3.52	3.37	3.28	3.22	3.06	2.91
26	.100	1.86	1.81	1.76	1.71	1.67	1.65	1.61	1.59	1.58	1.54	1.51
	.050	2.22	2.15	2.07	1.99	1.94	1.90	1.85	1.82	1.80	1.75	1.70
	.025	2.59	2.49	2.39	2.28	2.21	2.16	2.09	2.05	2.03	1.95	1.89
	.010	3.09	2.96	2.81	2.66	2.57	2.50	2.42	2.36	2.33	2.23	2.14
	.001	4.48	4.24	3.99	3.72	3.56	3.44	3.30	3.21	3.15	2.99	2.84
27	.100	1.85	1.80	1.75	1.70	1.66	1.64	1.60	1.58	1.57	1.53	1.50
	.050	2.20	2.13	2.06	1.97	1.92	1.88	1.84	1.81	1.79	1.73	1.68
	.025	2.57	2.47	2.36	2.25	2.18	2.13	2.07	2.03	2.00	1.93	1.86
	.010	3.06	2.93	2.78	2.63	2.54	2.47	2.38	2.33	2.29	2.20	2.11
	.001	4.41	4.17	3.92	3.66	3.49	3.38	3.23	3.14	3.08	2.92	2.78

(*Continued*)

TABLE E

F critical values (continued)

Degrees of freedom in the numerator

Degrees of freedom in the denominator	*p*	1	2	3	4	5	6	7	8	9
28	.100	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87
	.050	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
	.025	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61
	.010	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
	.001	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50
29	.100	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86
	.050	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
	.025	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59
	.010	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
	.001	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45
30	.100	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
	.050	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
	.025	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
	.010	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
	.001	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
40	.100	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79
	.050	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
	.025	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45
	.010	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
	.001	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02
50	.100	2.81	2.41	2.20	2.06	1.97	1.90	1.84	1.80	1.76
	.050	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07
	.025	5.34	3.97	3.39	3.05	2.83	2.67	2.55	2.46	2.38
	.010	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78
	.001	12.22	7.96	6.34	5.46	4.90	4.51	4.22	4.00	3.82
60	.100	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
	.050	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
	.025	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
	.010	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
	.001	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
100	.100	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69
	.050	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97
	.025	5.18	3.83	3.25	2.92	2.70	2.54	2.42	2.32	2.24
	.010	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59
	.001	11.50	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.44
200	.100	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66
	.050	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93
	.025	5.10	3.76	3.18	2.85	2.63	2.47	2.35	2.26	2.18
	.010	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50
	.001	11.15	7.15	5.63	4.81	4.29	3.92	3.65	3.43	3.26
1000	.100	2.71	2.31	2.09	1.95	1.85	1.78	1.72	1.68	1.64
	.050	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89
	.025	5.04	3.70	3.13	2.80	2.58	2.42	2.30	2.20	2.13
	.010	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43
	.001	10.89	6.96	5.46	4.65	4.14	3.78	3.51	3.30	3.13

TABLE E

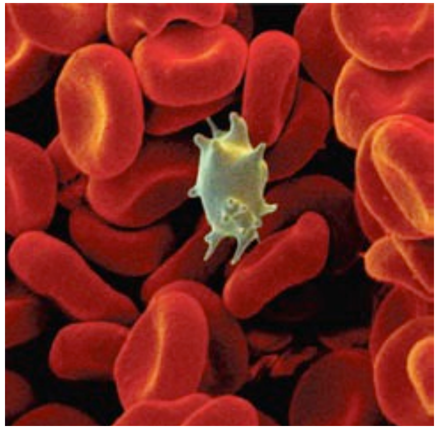
F critical values (continued)

Degrees of freedom in the numerator

Degrees of freedom in the denominator	*p*	10	12	15	20	25	30	40	50	60	120	1000
28	.100	1.84	1.79	1.74	1.69	1.65	1.63	1.59	1.57	1.56	1.52	1.48
	.050	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.79	1.77	1.71	1.66
	.025	2.55	2.45	2.34	2.23	2.16	2.11	2.05	2.01	1.98	1.91	1.84
	.010	3.03	2.90	2.75	2.60	2.51	2.44	2.35	2.30	2.26	2.17	2.08
	.001	4.35	4.11	3.86	3.60	3.43	3.32	3.18	3.09	3.02	2.86	2.72
29	.100	1.83	1.78	1.73	1.68	1.64	1.62	1.58	1.56	1.55	1.51	1.47
	.050	2.18	2.10	2.03	1.94	1.89	1.85	1.81	1.77	1.75	1.70	1.65
	.025	2.53	2.43	2.32	2.21	2.14	2.09	2.03	1.99	1.96	1.89	1.82
	.010	3.00	2.87	2.73	2.57	2.48	2.41	2.33	2.27	2.23	2.14	2.05
	.001	4.29	4.05	3.80	3.54	3.38	3.27	3.12	3.03	2.97	2.81	2.66
30	.100	1.82	1.77	1.72	1.67	1.63	1.61	1.57	1.55	1.54	1.50	1.46
	.050	2.16	2.09	2.01	1.93	1.88	1.84	1.79	1.76	1.74	1.68	1.63
	.025	2.51	2.41	2.31	2.20	2.12	2.07	2.01	1.97	1.94	1.87	1.80
	.010	2.98	2.84	2.70	2.55	2.45	2.39	2.30	2.25	2.21	2.11	2.02
	.001	4.24	4.00	3.75	3.49	3.33	3.22	3.07	2.98	2.92	2.76	2.61
40	.100	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.48	1.47	1.42	1.38
	.050	2.08	2.00	1.92	1.84	1.78	1.74	1.69	1.66	1.64	1.58	1.52
	.025	2.39	2.29	2.18	2.07	1.99	1.94	1.88	1.83	1.80	1.72	1.65
	.010	2.80	2.66	2.52	2.37	2.27	2.20	2.11	2.06	2.02	1.92	1.82
	.001	3.87	3.64	3.40	3.14	2.98	2.87	2.73	2.64	2.57	2.41	2.25
50	.100	1.73	1.68	1.63	1.57	1.53	1.50	1.46	1.44	1.42	1.38	1.33
	.050	2.03	1.95	1.87	1.78	1.73	1.69	1.63	1.60	1.58	1.51	1.45
	.025	2.32	2.22	2.11	1.99	1.92	1.87	1.80	1.75	1.72	1.64	1.56
	.010	2.70	2.56	2.42	2.27	2.17	2.10	2.01	1.95	1.91	1.80	1.70
	.001	3.67	3.44	3.20	2.95	2.79	2.68	2.53	2.44	2.38	2.21	2.05
60	.100	1.71	1.66	1.60	1.54	1.50	1.48	1.44	1.41	1.40	1.35	1.30
	.050	1.99	1.92	1.84	1.75	1.69	1.65	1.59	1.56	1.53	1.47	1.40
	.025	2.27	2.17	2.06	1.94	1.87	1.82	1.74	1.70	1.67	1.58	1.49
	.010	2.63	2.50	2.35	2.20	2.10	2.03	1.94	1.88	1.84	1.73	1.62
	.001	3.54	3.32	3.08	2.83	2.67	2.55	2.41	2.32	2.25	2.08	1.92
100	.100	1.66	1.61	1.56	1.49	1.45	1.42	1.38	1.35	1.34	1.28	1.22
	.050	1.93	1.85	1.77	1.68	1.62	1.57	1.52	1.48	1.45	1.38	1.30
	.025	2.18	2.08	1.97	1.85	1.77	1.71	1.64	1.59	1.56	1.46	1.36
	.010	2.50	2.37	2.22	2.07	1.97	1.89	1.80	1.74	1.69	1.57	1.45
	.001	3.30	3.07	2.84	2.59	2.43	2.32	2.17	2.08	2.01	1.83	1.64
200	.100	1.63	1.58	1.52	1.46	1.41	1.38	1.34	1.31	1.29	1.23	1.16
	.050	1.88	1.80	1.72	1.62	1.56	1.52	1.46	1.41	1.39	1.30	1.21
	.025	2.11	2.01	1.90	1.78	1.70	1.64	1.56	1.51	1.47	1.37	1.25
	.010	2.41	2.27	2.13	1.97	1.87	1.79	1.69	1.63	1.58	1.45	1.30
	.001	3.12	2.90	2.67	2.42	2.26	2.15	2.00	1.90	1.83	1.64	1.43
1000	.100	1.61	1.55	1.49	1.43	1.38	1.35	1.30	1.27	1.25	1.18	1.08
	.050	1.84	1.76	1.68	1.58	1.52	1.47	1.41	1.36	1.33	1.24	1.11
	.025	2.06	1.96	1.85	1.72	1.64	1.58	1.50	1.45	1.41	1.29	1.13
	.010	2.34	2.20	2.06	1.90	1.79	1.72	1.61	1.54	1.50	1.35	1.16
	.001	2.99	2.77	2.54	2.30	2.14	2.02	1.87	1.77	1.69	1.49	1.22

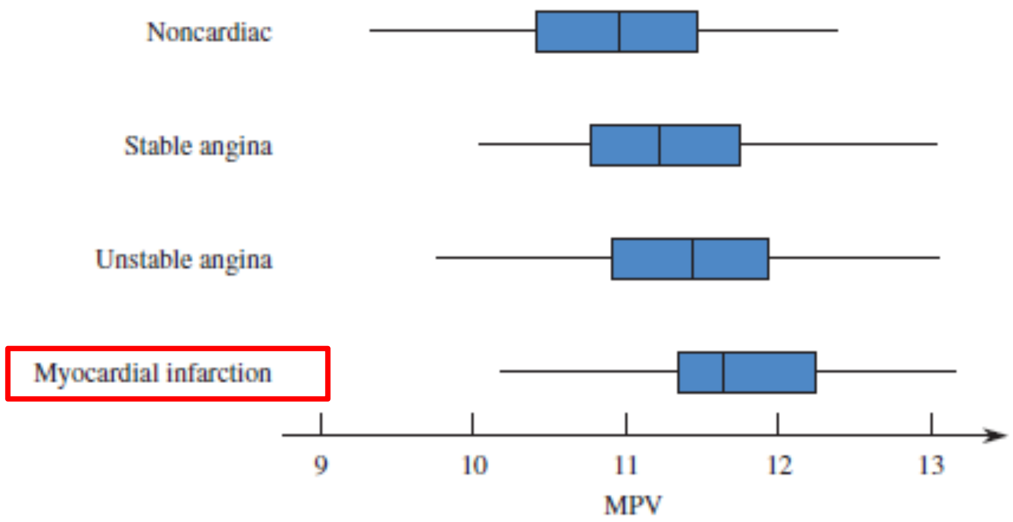
Example

The article "**Could Mean Platelet Volume Be a Predictive Market for Acute Myocardial Infarction (heart attack**?" described an experiment in which four groups of **patients seeking treatment for chest pain were compared** with respect to mean platelet volume (MPV, measured in fL). The four groups considered were based on the clinical diagnosis and were (1) noncardiac chest pain, (2) stable angina pectoris, (3) unstable angina pectoris, and (4) myocardial infarction (heat attack).



Group Number	Group Description	Sample Size	Sample Mean	Sample Standard Deviation
1	Noncardiac chest pain	35	10.89	0.69
2	Stable angina pectoris	35	11.25	0.74
3	Unstable angina pectoris	35	11.37	0.91
4	Myocardial infarction (heart attack)	35	11.75	1.07

comparative boxplot
for the four samples

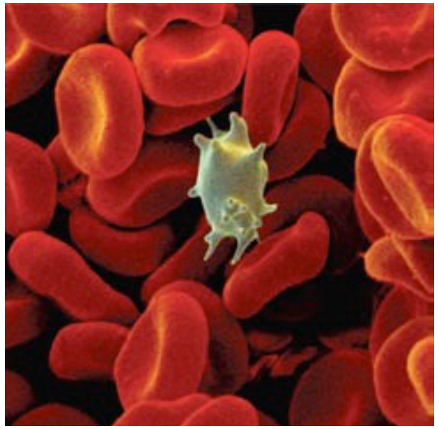


With μi denoting the true mean MPV for group *i* (*i* 1, 2, 3, 4), let's consider the *H0:* $\mu_1=\mu_2=\mu_3=\mu_4$

it is not obvious whether H_0 is true or false.

Example

The article "**Could Mean Platelet Volume Be a Predictive Market for Acute Myocardial Infarction (heart attack**?" described an experiment in which four groups of **patients seeking treatment for chest pain were compared** with respect to mean platelet volume (MPV, measured in fL). The four groups considered were based on the clinical diagnosis and were (1) noncardiac chest pain, (2) stable angina pectoris, (3) unstable angina pectoris, and (4) myocardial infarction (heat attack).



null hypothesis H_0: m_1 m_2 m_3 m_4

Group Number	Group Description	Sample Size	Sample Mean	Sample Standard Deviation
1	Noncardiac chest pain	35	10.89	0.69
2	Stable angina pectoris	35	11.25	0.74
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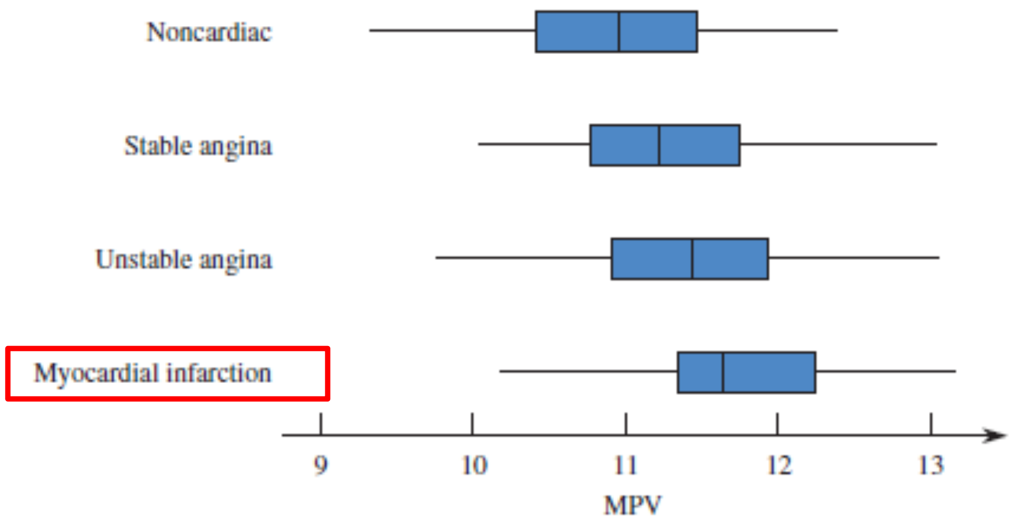
comparative boxplot
for the four samples



The mean MPV for the **heart attack sample** is **larger than for the other three samples.**
However, because the four boxplots show substantial overlap.

Let's compute the variance between groups and variance within groups

Compute SS_{Treat}, SSE, MS_{Treat} and MSE

Please compute SS_{Treat}, SSE, MS_{Treat} and MSE

Group Number	Group Description	Sample Size	Sample Mean	Sample Standard Deviation
1	Noncardiac chest pain	35	10.89	0.69
2	Stable angina pectoris	35	11.25	0.74
3	Unstable angina pectoris	35	11.37	0.91
4	Myocardial infarction (heart attack)	35	11.75	1.07

Solution

The grand mean $\overline{\overline{x}}$ was computed to be 11.315 with $\overline{x_1}$=10.89, $\overline{x_2}$=11.25, $\overline{x_3}$=11.37, $\overline{x_4}$=11.75, and n_1= n_2= n_3= n_4=35.

Compute SS_{Treat}, SSE, MS_{Treat} and MSE

Group Number	Group Description	Sample Size	Sample Mean	Sample Standard Deviation
1	Noncardiac chest pain	35	10.89	0.69
2	Stable angina pectoris	35	11.25	0.74
3	Unstable angina pectoris	35	11.37	0.91
4	Myocardial infarction (heart attack)	35	11.75	1.07

$$SS_{Treat} = n_1(\bar{y}_{1.} - \bar{y}_{..})^2 + n_2(\bar{y}_{2.} - \bar{y}_{..})^2 + n_3(\bar{y}_{3.} - \bar{y}_{..})^2$$

$$\text{SSE} = (n_1-1)s_1^2+(n_2-1)s_2^2+(n_3-1)s_3^2+\ldots+(n_k-1)s_k^2$$

$$MS_{Treat} = \frac{SS_{Treat}}{k-1} \quad \text{and} \quad MSE = \frac{SSE}{n-k}.$$

mean square treatment

mean square error

Compute SS_{Treat}, SSE, MS_{Treat} and MSE

Group Number	Group Description	Sample Size	Sample Mean	Sample Standard Deviation
1	Noncardiac chest pain	35	10.89	0.69
2	Stable angina pectoris	35	11.25	0.74
3	Unstable angina pectoris	35	11.37	0.91
4	Myocardial infarction (heart attack)	35	11.75	1.07

Solution

The grand mean $\bar{\bar{x}}$ was computed to be 11.315 with $\overline{x_1}$=10.89, $\overline{x_2}$=11.25, $\overline{x_3}$=11.37, $\overline{x_4}$=11.75, and n_1= n_2= n_3= n_4=35.

$$\boxed{SS_{Treat} = n_1(\bar{y}_{1.} - \bar{y}_{..})^2 + n_2(\bar{y}_{2.} - \bar{y}_{..})^2 + n_3(\bar{y}_{3.} - \bar{y}_{..})^2}$$

$$SS_{treat} = n_1(\overline{x_1} - \bar{\bar{x}})^2 + n_2(\overline{x_2} - \bar{\bar{x}})^2 + n_3(\overline{x_3} - \bar{\bar{x}})^2 + \ldots$$
$$+ n_k(\overline{x_k} - \bar{\bar{x}})^2$$

$$SS_{treat} =$$
$$35(10.89 - 11.315)^2 + 35(11.25 - 11.315)^2 +$$
$$35(11.37 - 11.315)^2 + 35(11.75 - 11.315)^2$$

$$SS_{treat} = 13.199$$

Compute SS_{Treat}, SSE, MS_{Treat} and MSE

Solution

The grand mean $\bar{\bar{x}}$ was computed to be 11.315 with $\overline{x_1}$=10.89, $\overline{x_2}$=11.25, $\overline{x_3}$=11.37, $\overline{x_4}$=11.75, and n_1= n_2= n_3= n_4=35.

Group Number	Group Description	Sample Size	Sample Mean	Sample Standard Deviation
1	Noncardiac chest pain	35	10.89	0.69
2	Stable angina pectoris	35	11.25	0.74
3	Unstable angina pectoris	35	11.37	0.91
4	Myocardial infarction (heart attack)	35	11.75	1.07

$$\boxed{SS_{Treat} = n_1(\bar{y}_{1.} - \bar{y}_{..})^2 + n_2(\bar{y}_{2.} - \bar{y}_{..})^2 + n_3(\bar{y}_{3.} - \bar{y}_{..})^2}$$

Compute SS_{Treat}, SSE, MS_{Treat} and MSE

Solution

The grand mean $\bar{\bar{x}}$ was computed to be 11.315 with x_1=10.89, x_2=11.25, x_3=11.37, x_4=11.75, and n_1= n_2= n_3= n_4=35.

$$SS_{treat} = n_1(\overline{x_1} - \bar{\bar{x}})^2 + n_2(\overline{x_2} - \bar{\bar{x}})^2 + n_3(\overline{x_3} - \bar{\bar{x}})^2 + \ldots + n_k(\overline{x_k} - \bar{\bar{x}})^2$$

$$SS_{treat} = 35(10.89 - 11.315)^2 + 35(11.25 - 11.315)^2 + 35(11.37 - 11.315)^2 + 35(11.75 - 11.315)^2$$

SS_{treat} = 13.199

The number of degrees of freedom are treatment df = k – 1 = 4 – 1 = 3

Error df = N – k = 35 + 35 + 35 + 35 - 4 = 136

S_1=0.69, S_2=0.74, S_3=0.91, S_4=1.07

$$\text{SSE} = (n_1-1)S_1^2 + (n_2-1)S_2^2 + (n_3-1)S_3^2 + \ldots + (n_k-1)S^2{}_k$$

$$\text{SSE} = (35 - 1)0.4761 + (35-1)0.5476 + (35 - 1)0.8281 + (35 -$$

From which

$$MS_{treat} = \frac{SS_{treat}}{k-1} = \frac{13.199}{3} = 4.4$$

$$MSE = \frac{SSE}{N-k} = \frac{101.888}{136} = 0.749$$

Compute SS_{Treat}, SSE, MS_{Treat} and MSE

Group Number	Group Description	Sample Size	Sample Mean	Sample Standard Deviation
1	Noncardiac chest pain	35	10.89	0.69
2	Stable angina pectoris	35	11.25	0.74
3	Unstable angina pectoris	35	11.37	0.91
4	Myocardial infarction (heart attack)	35	11.75	1.07

Solution

The grand mean $\bar{\bar{x}}$ was computed to be 11.315 with $\overline{x_1}$=10.89, $\overline{x_2}$=11.25, $\overline{x_3}$=11.37, $\overline{x_4}$=11.75, and n_1= n_2= n_3= n_4=35.

$$\boxed{SS_{Treat} = n_1(\bar{y}_{1.} - \bar{y}_{..})^2 + n_2(\bar{y}_{2.} - \bar{y}_{..})^2 + n_3(\bar{y}_{3.} - \bar{y}_{..})^2}$$

$$SS_{treat} = n_1(\overline{x_1} - \bar{\bar{x}})^2 + n_2(\overline{x_2} - \bar{\bar{x}})^2 + n_3(\overline{x_3} - \bar{\bar{x}})^2 + \ldots + n_k(\overline{x_k} - \bar{\bar{x}})^2$$

Compute SS_{Treat}, SSE, MS_{Treat} and MSE

Solution

The grand mean $\bar{\bar{x}}$ was computed to be 11.315 with x_1=10.89, x_2=11.25, x_3=11.37, x_4=11.75, and n_1= n_2= n_3= n_4=35.

$$SS_{treat} = 35(10.89 - 11.315)^2 + 35(11.25 - 11.315)^2 + 35(11.37 - 11.315)^2 + 35(11.75 - 11.315)^2$$

SS_{treat} = 13.199

The number of degrees of freedom are treatment df = k – 1 = 4 – 1 = 3

Error df = N – k = 35 + 35 + 35 + 35 - 4 = 136

S_1=0.69, S_2=0.74, S_3=0.91, S_4=1.07

$$SSE = (n_1 - 1)S_1^2 + (n_2 - 1)S_2^2 + (n_3 - 1)S_3^2 + \ldots + (n_k - 1)S_k^2$$

$$SSE = (35 - 1)0.4761 + (35 - 1)0.5476 + (35 - 1)0.8281 + (35 -$$

From which

$$MS_{treat} = \frac{SS_{treat}}{k-1} = \frac{13.199}{3} = 4.4$$

$$MSE = \frac{SSE}{N-k} = \frac{101.888}{136} = 0.749$$

Is the ratio MS_{Treat}/MSE large enough to suggest H_o is false at a significance level of 0.001?

so MS_{treat} is about 6 times as large as MSE.

Is the ratio MS_{Treat}/MSE large enough to suggest H_o is false at a significance level of 0.001?

Group Number	Group Description	Sample Size	Sample Mean	Sample Standard Deviation
1	Noncardiac chest pain	35	10.89	0.69
2	Stable angina pectoris	35	11.25	0.74
3	Unstable angina pectoris	35	11.37	0.91
4	Myocardial infarction (heart attack)	35	11.75	1.07

Solution

MS_{treat} = 4.4 and MSE = 0.749

The value of the *F* statistic is then

$$F = \frac{MS_{treat}}{MSE} = \frac{4.4}{0.749} = 5.87$$

Is the ratio MS_{Treat}/MSE large enough to suggest H_o is false at a significance level of 0.001?

Group Number	Group Description	Sample Size	Sample Mean	Sample Standard Deviation
1	Noncardiac chest pain	35	10.89	0.69
2	Stable angina pectoris	35	11.25	0.74
3	Unstable angina pectoris	35	11.37	0.91
4	Myocardial infarction (heart attack)	35	11.75	1.07

Solution

MS_{treat} = 4.4 and MSE = 0.749

The value of the F statistic is then

$$F = \frac{MS_{treat}}{MSE} = \frac{4.4}{0.749} = 5.87$$

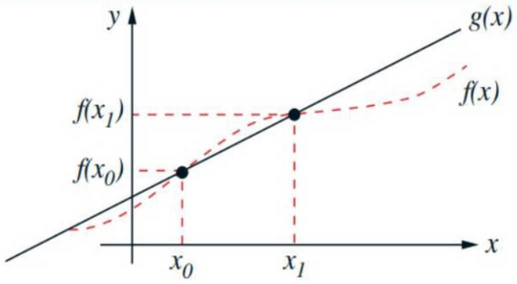
a significance level of 0.001

$$df_1 = k - 1 = 3$$
$$df_2 = N - k = 140 - 4 = 136$$

TABLE E

F critical values (continued)

Degrees of freedom in the denominator	p	Degrees of freedom in the numerator: 1	2	3	4	5	6	7	8	9
	.100	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87
	.050	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
28	.025	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61
	.010	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
	.001	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50
	.100	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86
	.050	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
29	.025	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59
	.010	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
	.001	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45
	.100	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
	.050	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
30	.025	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
	.010	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
	.001	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
	.100	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79
	.050	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
40	.025	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45
	.010	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
	.001	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02
	.100	2.81	2.41	2.20	2.06	1.97	1.90	1.84	1.80	1.76
	.050	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07
50	.025	5.34	3.97	3.39	3.05	2.83	2.67	2.55	2.46	2.38
	.010	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78
	.001	12.22	7.96	6.34	5.46	4.90	4.51	4.22	4.00	3.82
	.100	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
	.050	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
60	.025	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
	.010	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
	.001	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
	.100	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69
	.050	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97
100	.025	5.18	3.83	3.25	2.92	2.70	2.54	2.42	2.32	2.24
	.010	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59
	.001	11.50	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.44
	.100	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66
	.050	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93
200	.025	5.10	3.76	3.18	2.85	2.63	2.47	2.35	2.26	2.18
	.010	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50
	.001	11.15	7.15	5.63	4.81	4.29	3.92	3.65	3.43	3.26
	.100	2.71	2.31	2.09	1.95	1.85	1.78	1.72	1.68	1.64
	.050	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89
1000	.025	5.04	3.70	3.13	2.80	2.58	2.42	2.30	2.20	2.13
	.010	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43
	.001	10.89	6.96	5.46	4.65	4.14	3.78	3.51	3.30	3.13



f(x) = the exact function for which values are known only at a discrete set of data points

g(x) = the interpolated approximation to *f(x)*

x_0, x_1 = the data points (also referred to as interpolation points or nodes)

Substituting A and B into equation $g(x) = Ax + B$

$$g(x) = f(x_0)\frac{(x_1 - x)}{(x_1 - x_0)} + f(x_1)\frac{(x - x_0)}{(x_1 - x_0)}$$

Linear Interpolation

136?

Is the ratio MS_{Treat}/MSE large enough to suggest H_o is false at a significance level of 0.001?

Group Number	Group Description	Sample Size	Sample Mean	Sample Standard Deviation
1	Noncardiac chest pain	35	10.89	0.69
2	Stable angina pectoris	35	11.25	0.74
3	Unstable angina pectoris	35	11.37	0.91
4	Myocardial infarction (heart attack)	35	11.75	1.07

Solution

MS_{treat} = 4.4 and MSE = 0.749

The value of the F statistic is then

$$F = \frac{MS_{treat}}{MSE} = \frac{4.4}{0.749} = 5.87$$

a significance level of 0.001

$$df_1 = k - 1 = 3$$
$$df_2 = N - k = 140 - 4 = 136$$

TABLE E

F critical values (continued)

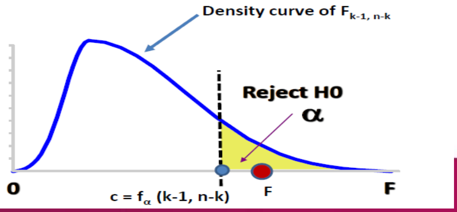
Degrees of freedom in the denominator	*p*	1	2	3	4	5	6	7	8	9
28	.100	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87
	.050	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
	.025	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61
	.010	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
	.001	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50
29	.100	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86
	.050	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
	.025	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59
	.010	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
	.001	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45
30	.100	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
	.050	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
	.025	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
	.010	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
	.001	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
40	.100	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79
	.050	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
	.025	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45
	.010	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
	.001	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02
50	.100	2.81	2.41	2.20	2.06	1.97	1.90	1.84	1.80	1.76
	.050	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07
	.025	5.34	3.97	3.39	3.05	2.83	2.67	2.55	2.46	2.38
	.010	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78
	.001	12.22	7.96	6.34	5.46	4.90	4.51	4.22	4.00	3.82
60	.100	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
	.050	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
	.025	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
	.010	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
	.001	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
100	.100	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69
	.050	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97
	.025	5.18	3.83	3.25	2.92	2.70	2.54	2.42	2.32	2.24
	.010	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59
	.001	11.50	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.44
200	.100	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66
	.050	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93
	.025	5.10	3.76	3.18	2.85	2.63	2.47	2.35	2.26	2.18
	.010	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50
	.001	11.15	7.15	5.63	4.81	4.29	3.92	3.65	3.43	3.26
1000	.100	2.71	2.31	2.09	1.95	1.85	1.78	1.72	1.68	1.64
	.050	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89
	.025	5.04	3.70	3.13	2.80	2.58	2.42	2.30	2.20	2.13
	.010	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43
	.001	10.89	6.96	5.46	4.65	4.14	3.78	3.51	3.30	3.13

Linear Interpolation

Using df_1=3 and df_2=136, F-distribution table shows that 5.78 captures tail area 0.001

Since 5.87> 5.78 (lies in the critical region)

So there is compelling evidence for rejecting H_o.



Example

Leaf surface area is an important variable in plant gas-exchange rates. Dry matter per unit surface area (mg/cm^3) was measured for different types of trees. Is there evidence that the mean surface area is not the same for all four groups? Test the relevant hypotheses using $\alpha = 0.01$.

	Oak	Maple	Pond Apple	Custard Apple
Sample Size	106	255	314	36
$\bar{x}$	2.00	3.40	3.07	2.84
s	1.56	1.68	1.66	1.89

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Leaf surface area is an important variable in plant gas-exchange rates. Dry matter per unit surface area (mg/cm3) was measured for different types of trees. Is there evidence that the mean surface area is not the same for all four groups? Test the relevant hypotheses using $\alpha = 0.01$.

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Sample Size	106	255	314	36
$\bar{x}$	2.00	3.40	3.07	2.84
s	1.56	1.68	1.66	1.89

Solution

The grand mean $\bar{\bar{x}}$ was computed to be 3.02 with $\overline{x_1}$=2, $\overline{x_2}$=3.4, $\overline{x_3}$=3.07, $\overline{x_4}$=2.84, and n_1= 106, n_2=255, n_3= 314, n_4=36.

$$SS_{treat} = n_1(\overline{x_1} - \bar{\bar{x}})^2 + n_2(\overline{x_2} - \bar{\bar{x}})^2 + n_3(\overline{x_3} - \bar{\bar{x}})^2 + n_4(\overline{x_4} - \bar{\bar{x}})^2$$
$$SS_{treat} = 106(2 - 3.02)^2 + 255(3.4 - 3.02)^2 + 314(3.07 - 3.02)^2 + 36(2.84 - 3.02)^2$$
$$SS_{treat} = 149.0558$$

S_1=1.56, S_2=1.68, S_3=1.66, S_4=1.89

$$\text{SSE} = (n_1 - 1)S_1^2 + (n_2 - 1)S_2^2 + (n_3 - 1)S_3^2 + (n_4 - 1)S_4^2$$
$$\text{SSE} = (106 - 1)2.4336 + (255 - 1)2.8224 + (314 - 1)2.7556 + (36 - 1)3.5721$$
SSE = 1959.9439

The number of degrees of freedom are treatment

df = k – 1 = 4 – 1 = 3

Error df = N – k = 106 + 255 + 314 + 36 - 4 = 707

From which

$$MS_{treat} = \frac{SS_{treat}}{k-1} = \frac{149.0558}{3} = 49.69$$
$$MSE = \frac{SSE}{N-k} = \frac{1959.9439}{707} = 2.772$$

CityU

Example

Leaf surface area is an important variable in plant gas-exchange rates. Dry matter per unit surface area (mg/cm^3) was measured for different types of trees. Is there evidence that the mean surface area is not the same for all four groups? Test the relevant hypotheses using $\alpha = 0.01$.

	Oak	Maple	Pond Apple	Custard Apple
Sample Size	106	255	314	36
$\bar{x}$	2.00	3.40	3.07	2.84
s	1.56	1.68	1.66	1.89

Solution

The grand mean $\bar{\bar{x}}$ was computed to be 3.02 with $\overline{x_1}$=2, $\overline{x_2}$=3.4, $\overline{x_3}$=3.07, $\overline{x_4}$=2.84, and n_1= 106, n_2=255, n_3= 314, n_4=36.

$$SS_{treat} = n_1(\overline{x_1} - \bar{\bar{x}})^2 + n_2(\overline{x_2} - \bar{\bar{x}})^2 + n_3(\overline{x_3} - \bar{\bar{x}})^2 + n_4(\overline{x_4} - \bar{\bar{x}})^2$$
$$SS_{treat} = 106(2 - 3.02)^2 + 255(3.4 - 3.02)^2 + 314(3.07 - 3.02)^2 + 36(2.84 - 3.02)^2$$
$$SS_{treat} = 149.0558$$

S_1=1.56, S_2=1.68, S_3=1.66, S_4=1.89

$$\text{SSE} = (n_1 - 1)S_1^2 + (n_2 - 1)S_2^2 + (n_3 - 1)S_3^2 + (n_4 - 1)S_4^2$$
$$\text{SSE} = (106 - 1)2.4336 + (255 - 1)2.8224 + (314 - 1)2.7556 + (36 - 1)3.5721$$

SSE = 1959.9439

The number of degrees of freedom are treatment

df = k – 1 = 4 – 1 = 3

Error df = N – k = 106 + 255 + 314 + 36 - 4 = 707

From which

$$MS_{treat} = \frac{SS_{treat}}{k-1} = \frac{149.0558}{3} = 49.69$$

$$MSE = \frac{SSE}{N-k} = \frac{1959.9439}{707} = 2.772$$

The value of the F statistic is then

$$F = \frac{MS_{treat}}{MSE} = \frac{49.69}{2.772} = 17.92$$

CityU

Example

Leaf surface area is an important variable in plant gas-exchange rates. Dry matter per unit surface area (mg/cm3) was measured for different types of trees. Is there evidence that the mean surface area is not the same for all four groups? Test the relevant hypotheses using $\alpha = 0.01$.

	Oak	Maple	Pond Apple	Custard Apple
Sample Size	106	255	314	36
$\bar{x}$	2.00	3.40	3.07	2.84
s	1.56	1.68	1.66	1.89

Solution

The grand mean $\bar{\bar{x}}$ was computed to be 3.02 with $\overline{x_1}=2$, $\overline{x_2}=3.4$, $\overline{x_3}=3.07$, $\overline{x_4}=2.84$, and $n_1=106$, $n_2=255$, $n_3=314$, $n_4=36$.

$$SS_{treat} = n_1(\overline{x_1}-\bar{\bar{x}})^2+n_2(\overline{x_2}-\bar{\bar{x}})^2+n_3(\overline{x_3}-\bar{\bar{x}})^2+n_4(\overline{x_4}-\bar{\bar{x}})^2$$
$$SS_{treat} = 106(2-3.02)^2+255(3.4-3.02)^2+314(3.07-3.02)^2+36(2.84-3.02)^2$$
$$SS_{treat} = 149.0558$$

$S_1=1.56$, $S_2=1.68$, $S_3=1.66$, $S_4=1.89$

$$\text{SSE} = (n_1-1)S_1^2+(n_2-1)S_2^2+(n_3-1)S_3^2+(n_4-1)S_4^2$$
$$\text{SSE} = (106-1)2.4336+(255-1)2.8224+(314-1)2.7556+(36-1)3.5721$$
$$\text{SSE} = 1959.9439$$

The number of degrees of freedom are treatment df = k – 1 = 4 – 1 = 3

Error df = N – k = 106 + 255 + 314 + 36 - 4 = 707

From which

$$MS_{treat} = \frac{SS_{treat}}{k-1} = \frac{149.0558}{3} = 49.69$$
$$MSE = \frac{SSE}{N-k} = \frac{1959.9439}{707} = 2.772$$

The value of the F statistic is then

$$F = \frac{MS_{treat}}{MSE} = \frac{49.69}{2.772} = 17.92$$

Using $df_1=3$ and $df_2=707$, F-distribution table shows that 3.809 captures tail area 0.01

TABLE E

F critical values (continued)

Linear Interpolation:

	p	1	2	3
200	.100	2.73	2.33	2.11
	.050	3.89	3.04	2.65
	.025	5.10	3.76	3.18
	.010	6.76	4.71	3.88
	.001	11.15	7.15	5.63
1000	.100	2.71	2.31	2.09
	.050	3.85	3.00	2.61
	.025	5.04	3.70	3.13
	.010	6.66	4.63	3.80
	.001	10.89	6.96	5.46

Example

Leaf surface area is an important variable in plant gas-exchange rates. Dry matter per unit surface area (mg/cm3) was measured for different types of trees. Is there evidence that the mean surface area is not the same for all four groups? Test the relevant hypotheses using $\alpha = 0.01$.

	Oak	Maple	Pond Apple	Custard Apple
Sample Size	106	255	314	36
$\bar{x}$	2.00	3.40	3.07	2.84
s	1.56	1.68	1.66	1.89

Solution

The grand mean $\bar{\bar{x}}$ was computed to be 3.02 with $\overline{x_1}$=2, $\overline{x_2}$=3.4, $\overline{x_3}$=3.07, $\overline{x_4}$=2.84, and n_1= 106, n_2=255, n_3= 314, n_4=36.

$$SS_{treat} = n_1(\overline{x_1} - \bar{\bar{x}})^2 + n_2(\overline{x_2} - \bar{\bar{x}})^2 + n_3(\overline{x_3} - \bar{\bar{x}})^2 + n_4(\overline{x_4} - \bar{\bar{x}})^2$$
$$SS_{treat} = 106(2 - 3.02)^2 + 255(3.4 - 3.02)^2 + 314(3.07 - 3.02)^2 + 36(2.84 - 3.02)^2$$
$$SS_{treat} = 149.0558$$

S_1=1.56, S_2=1.68, S_3=1.66, S_4=1.89

$$\text{SSE} = (n_1 - 1)S_1^2 + (n_2 - 1)S_2^2 + (n_3 - 1)S_3^2 + (n_4 - 1)S_4^2$$
$$\text{SSE} = (106 - 1)2.4336 + (255 - 1)2.8224 + (314 - 1)2.7556 + (36 - 1)3.5721$$

SSE = 1959.9439

The number of degrees of freedom are treatment df = k – 1 = 4 – 1 = 3

Error df = N – k = 106 + 255 + 314 + 36 - 4 = 707

From which

$$MS_{treat} = \frac{SS_{treat}}{k-1} = \frac{149.0558}{3} = 49.69$$
$$MSE = \frac{SSE}{N-k} = \frac{1959.9439}{707} = 2.772$$

The value of the F statistic is then

$$F = \frac{MS_{treat}}{MSE} = \frac{49.69}{2.772} = 17.92$$

Using df_1 =3 and df_2 =707, F-distribution table shows that 3.809 captures tail area 0.01

Since 17.925.87 > 3.809 (lies in the critical region)

The P-value (3.809) is smaller than any reasonable α, so there is compelling evidence for rejecting H_o.

TABLE E

F critical values (continued)

Linear Interpolation

	p	1	2	3
200	.100	2.73	2.33	2.11
	.050	3.89	3.04	2.65
	.025	5.10	3.76	3.18
	.010	6.76	4.71	3.88
	.001	11.15	7.15	5.63
1000	.100	2.71	2.31	2.09
	.050	3.85	3.00	2.61
	.025	5.04	3.70	3.13
	.010	6.66	4.63	3.80
	.001	10.89	6.96	5.46

Example

Self-discharge is a phenomenon in batteries in which internal chemical reactions reduce the stored charge of the battery without any connection between the electrodes. Self-discharge decreases the shelf life of batteries and causes them to initially have less than a full charge when actually put to use. Below shows the data of stored charge of battery at different storage time. Is there sufficient evidence to conclude that the mean stored charge is not the same for the four different storage times? Use the value of F from the ANOVA table to test the appropriate hypotheses at significance level 0.05.

Storage Period	Observations					
0 months	58.75	57.94	58.91	56.85	55.21	57.30
1 month	58.87	56.43	56.51	57.67	59.75	58.48
2 months	59.13	60.38	58.01	59.95	59.51	60.34
4 months	62.32	58.76	60.03	59.36	59.61	61.95

Example

Self-discharge is a phenomenon in batteries in which internal chemical reactions reduce the stored charge of the battery without any connection between the electrodes. Self-discharge decreases the shelf life of batteries and causes them to initially have less than a full charge when actually put to use. Below shows the data of stored charge of battery at different storage time. Is there sufficient evidence to conclude that the mean stored charge is not the same for the four different storage times? Use the value of F from the ANOVA table to test the appropriate hypotheses at significance level 0.05.

Storage Period	Observations					
0 months	58.75	57.94	58.91	56.85	55.21	57.30
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2 months	59.13	60.38	58.01	59.95	59.51	60.34
4 months	62.32	58.76	60.03	59.36	59.61	61.95

The grand mean $\bar{\bar{x}}$ was computed to be 58.87 with $\bar{x}_1$=57.49, $\bar{x}_2$=57.95, $\bar{x}_3$=59.56, and , $\bar{x}_4$=60.34, and $n_1 = n_2 = n_3 = n_4 = 6$, and $s_1 = 1.37$, $s_2 = 1.33$, $s_3 = 0.9$, $s_4 = 1.46$.

The number of degrees of freedom are treatment df = $k - 1 = 4 - 1 = 3$; Error df = $N - k = 24 - 4 = 20$

$$SS_{treat} = n_1(\bar{x}_1 - \bar{\bar{x}})^2 + n_2(\bar{x}_2 - \bar{\bar{x}})^2 + n_3(\bar{x}_3 - \bar{\bar{x}})^2 + n_4(\bar{x}_4 - \bar{\bar{x}})^2$$

$$SS_{treat} = 6(57.49 - 58.87)^2 + 6(57.95 - 58.87)^2 + 6(59.56 - 58.87)^2 + 6(60.34 - 58.87)^2$$

$$SS_{treat} = 32.33$$

$$\text{SSE} = (n_1 - 1)S_1^2 + (n_2 - 1)S_2^2 + (n_3 - 1)S_3^2 + (n_4 - 1)S_4^2$$

$$\text{SSE} = (6-1)1.88 + (6-1)1.77 + (6-1)0.81 + (6-1)2.13$$

SSE = 32.95

SST = SS_{treat} + SSE = 65.28

$$MS_{treat} = \frac{SS_{treat}}{k-1} = 10.78; \; MSE = \frac{SSE}{N-k} = 1.65; \; F = \frac{MS_{treat}}{MSE} = 6.53$$

Example

Self-discharge is a phenomenon in batteries in which internal chemical reactions reduce the stored charge of the battery without any connection between the electrodes. Self-discharge decreases the shelf life of batteries and causes them to initially have less than a full charge when actually put to use. Below shows the data of stored charge of battery at different storage time. Is there sufficient evidence to conclude that the mean stored charge is not the same for the four different storage times? Use the value of F from the ANOVA table to test the appropriate hypotheses at significance level 0.05.

Storage Period	Observations					
0 months	58.75	57.94	58.91	56.85	55.21	57.30
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2 months	59.13	60.38	58.01	59.95	59.51	60.34
4 months	62.32	58.76	60.03	59.36	59.61	61.95

The grand mean $\bar{\bar{x}}$ was computed to be 58.87 with $\bar{x}_1$=57.49, $\bar{x}_2$=57.95, $\bar{x}_3$=59.56, and , $\bar{x}_4$=60.34, and $n_1 = n_2 = n_3 = n_4 = 6$, and $s_1 = 1.37$, $s_2 = 1.33$, $s_3 = 0.9$, $s_4 = 1.46$.

The number of degrees of freedom are treatment df = $k - 1 = 4 - 1 = 3$; Error df = $N - k = 24 - 4 = 20$

$$SS_{treat} = n_1(\bar{x}_1 - \bar{\bar{x}})^2 + n_2(\bar{x}_2 - \bar{\bar{x}})^2 + n_3(\bar{x}_3 - \bar{\bar{x}})^2 + n_4(\bar{x}_4 - \bar{\bar{x}})^2$$

$$SS_{treat} = 6(57.49 - 58.87)^2 + 6(57.95 - 58.87)^2 + 6(59.56 - 58.87)^2 + 6(60.34 - 58.87)^2$$

$$SS_{treat} = 32.33$$

$$SSE = (n_1 - 1)S_1^2 + (n_2 - 1)S_2^2 + (n_3 - 1)S_3^2 + (n_4 - 1)S_4^2$$

$$SSE = (6-1)1.88 + (6-1)1.77 + (6-1)0.81 + (6-1)2.13$$

$$SSE = 32.95$$

$$SST = SS_{treat} + SSE = 65.28$$

$$MS_{treat} = \frac{SS_{treat}}{k-1} = 10.78;\ MSE = \frac{SSE}{N-k} = 1.65;\ F = \frac{MS_{treat}}{MSE} = 6.53$$

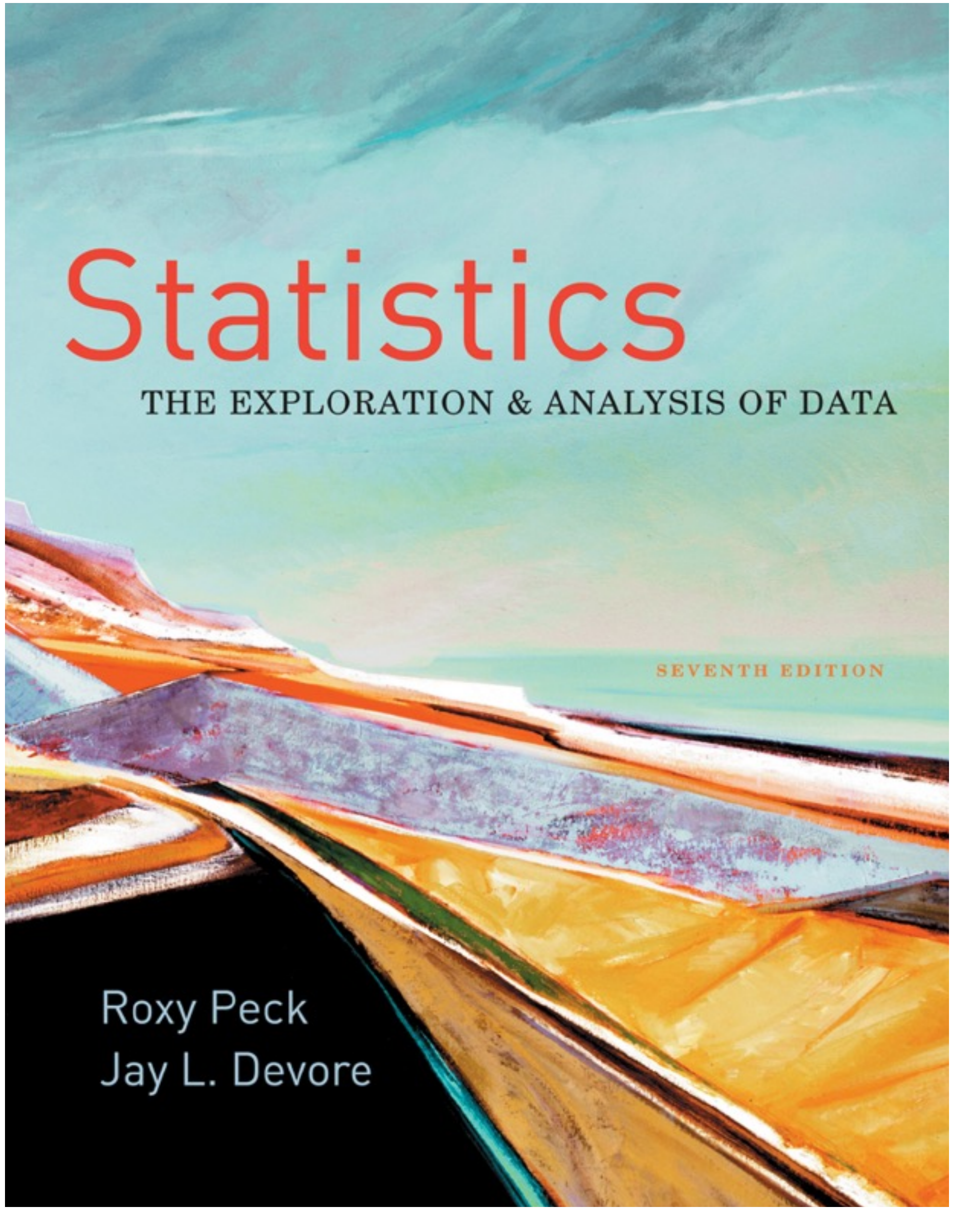
Using df_1=3 and df_2=20, F-distribution table shows that 3.10 captures tail area 0.05

Since 6.53 > 3.1 (lies in the critical region). So there is sufficient evidence for rejecting H_o.

E

al values (continued)

	p	1	2	3
18	.100	3.01	2.62	2.42
	.050	4.41	3.55	3.16
	.025	5.98	4.56	3.95
	.010	8.29	6.01	5.09
	.001	15.38	10.39	8.49
19	.100	2.99	2.61	2.40
	.050	4.38	3.52	3.13
	.025	5.92	4.51	3.90
	.010	8.18	5.93	5.01
	.001	15.08	10.16	8.28
20	.100	2.97	2.59	2.38
	.050	4.35	3.49	3.10
	.025	5.87	4.46	3.86
	.010	8.10	5.85	4.94
	.001	14.82	9.95	8.10



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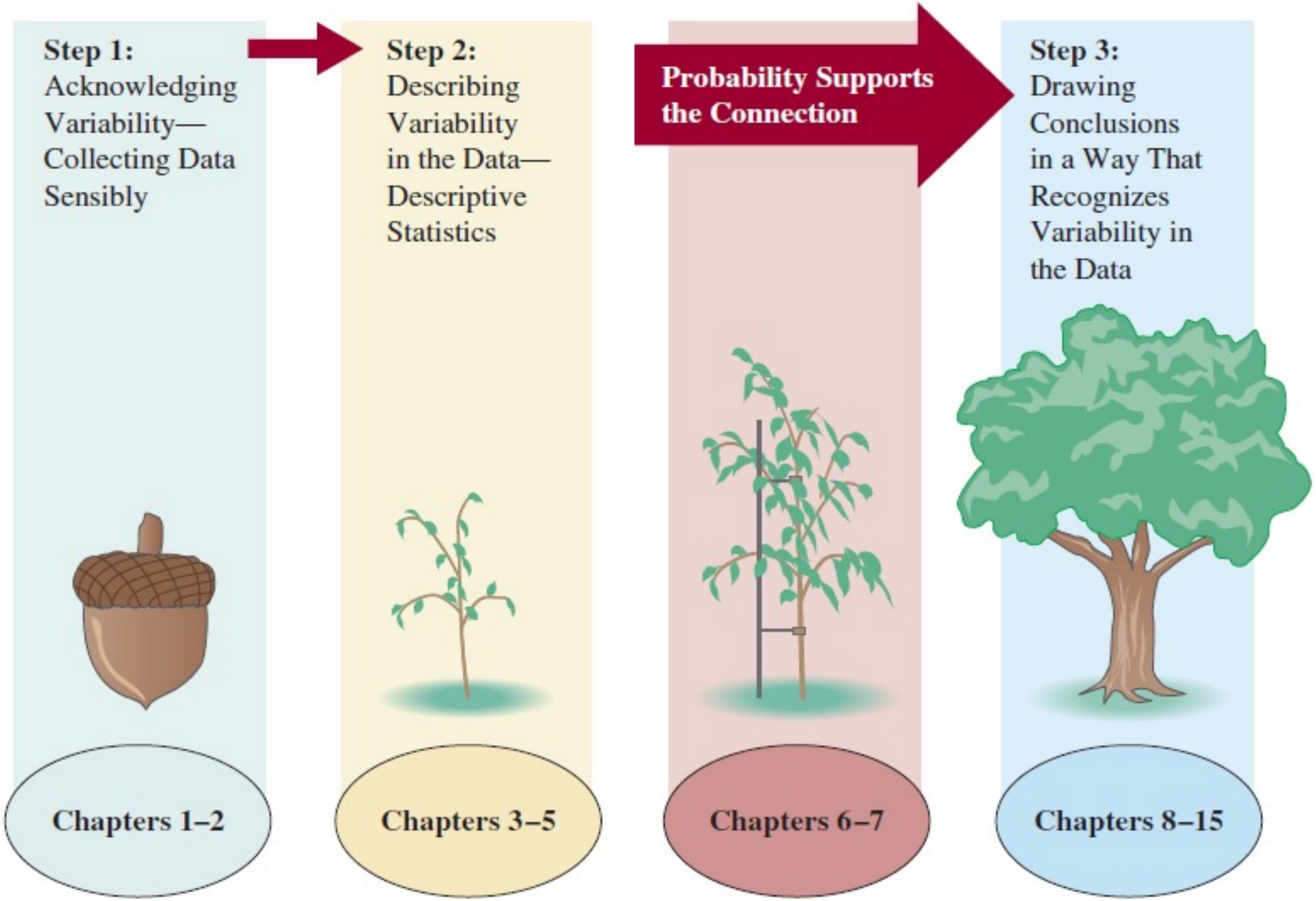
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CityU

Thank you!