(2) Find [(3,5)].

$[(3,5)] = \{(3,5), (1,3), (2,4)\}$

(3) Compute A/R ~~[(5,1)] = {(5,1)}~~

$A/R = \{\{(5,1)\}, \{(4,1),(5,2)\}, \{(3,1),(4,2),(5,3)\}, \{(2,1),(3,2),(4,3),(5,4)\}, \{(1,1),(2,2),(3,3),(4,4),(5,5)\},$
$\{(1,5)\}, \{(1,4),(2,5)\}, \{(1,3),(2,4),(3,5)\}, \{(1,2),(2,3),(3,4),(4,5)\}\}$

9

4. [9 points] In the questions below find the matrix that represents the given relation. Use elements in the order given to determine rows and columns of the matrix.

a) R on $\{-2,-1,0,1,2,\}$ where aRb means $a^2 = b^2$.

$$M_R = \begin{bmatrix} 1&0&0&0&1 \\ 0&1&0&1&0 \\ 0&0&1&0&0 \\ 0&1&0&1&0 \\ 1&0&0&0&1 \end{bmatrix}$$

b) R^2, where R is the relation on on $\{1,2,3,4\}$ such that aRb means $|a-b| \le 1$.

$$M_R = \begin{bmatrix} 1&1&0&0 \\ 1&1&1&0 \\ 0&1&1&1 \\ 0&0&1&1 \end{bmatrix} \qquad M_{R^2} = M_R \odot M_R = \begin{bmatrix} 1&1&1&0 \\ 1&1&1&1 \\ 1&1&1&1 \\ 0&1&1&1 \end{bmatrix}$$

c) $\overline{R}$, where R is the relation on $\{w,x,y,z\}$ such that $R = \{(w,w),(w,x),(x,w),(x,x),(x,z),(y,y),(z,y),(z,z)\}$.

$$M_R = \begin{bmatrix} 1&1&0&0 \\ 1&1&0&1 \\ 0&0&1&0 \\ 0&0&1&1 \end{bmatrix} \qquad M_{\overline{R}} = \begin{bmatrix} 0&0&1&1 \\ 0&0&1&0 \\ 1&1&0&1 \\ 1&1&0&0 \end{bmatrix}$$

9

5. [9 points] Let R be the relation on $A = \{1,2,3,4,5\}$ where $R = \{(1,1),(1,3),(1,4),(2,2),(2,1),(3,3),(3,4),(4,1),(4,3),(5,5)\}$.

(1) Find the reflexive closure of R.

$R = \{(1,1),(1,3),(1,4),(2,2),(2,1),(3,3),(3,4),(4,1),(4,3),(4,4),(5,5)\}$

(2) Find the symmetric closure of R.

$R = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(3,1),(3,3),(3,4),(4,1),(4,3),(5,5)\}$

(3) Use Warshall's algorithm to find the transitive closure of R.

$$M_R = \begin{bmatrix} 1&0&1&1&0 \\ 1&1&0&0&0 \\ 0&0&1&1&0 \\ 1&0&1&0&0 \\ 0&0&0&0&1 \end{bmatrix} = W_0 \qquad W_1 = \begin{bmatrix} 1&0&1&1&0 \\ 1&1&1&1&0 \\ 0&0&1&1&0 \\ 1&0&1&1&0 \\ 0&0&0&0&1 \end{bmatrix} = W_2 = W_3$$

$$W_4 = \begin{bmatrix} 1&0&1&1&0 \\ 1&1&1&1&0 \\ 1&0&1&1&0 \\ 1&0&1&1&0 \\ 0&0&0&0&1 \end{bmatrix} = W_5 \quad W_5 \text{ is the transitive closure of } R$$

4

6. [4 points] Let (G,*) be a group with G={1,2,3,4}. Here is an incomplete operation table for *:

*	1	2	3	4
1	1	2	3	?
2	2	1	?	?
3	3	?	1	?
4	?	?	?	1

Redraw this table and fill the missing entries.

*	1	2	3	4
1	1	2	3	4
2	2	1	4	3
3	3	4	1	2
4	4	3	2	1

7. [20 points] Let

$$H = \begin{bmatrix} 1&0&0 \\ 1&1&0 \\ 0&1&1 \\ 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{bmatrix}$$

be a parity check matrix.

(a) Determine the (3,6) group code e_H. (5 points)

e_H: $e(000) = 000000$
$e(001) = 001011$
$e(010) = 010110$
$e(011) = 011101$
$e(100) = 100100$
$e(101) = 101111$
$e(110) = 110010$
$e(111) = 111001$

(b) Determine the number of errors that e_H will detect and its associated decoding function will correct. (2 points)

the least weight of nonzero code word is 2.

e_H will detect 1 errors, its associated decoding function can not correct any error.

(c) Constructing a decoding table relative to a maximum likelihood decoding function associated with e_H. (5 points)

000000	001011	010110	011101	100100	101111	110010	111001
000001	001010	010111	011100	100101	101110	110011	111000
000010	001001	010100	011111	100110	101101	110000	111011
000100	001111	010010	011001	100000	101011	110110	111101
101000	000011	011110	010101	101100	100111	111010	110001
[illegible]10000	011011	000110	001101	110100	111111	100010	101001
[illegible]00101	001110	010011	011000	100001	101010	110111	111100
[illegible]01100	000111	011010	010001	101000	100011	111110	110101

(d) Decode the following words with the decoding table. (2 points)

a) 011001 b) 101011 c) 100101

a) $e(011) = 011101$ ~~the code word~~ $d(011001) = 011$

b) $e(101) = 101111$ $d(101011) = 101$

c) $e(100) = 100100$ $d(100101) = 100$

(e) Compute the syndrome for each coset leader found in (c). (3 points)

syndrome	coset leader
000	000000
001	~~001011~~ 000001
010	000010
100	000100
011	001000
110	010000
101	000101
111	001100

(f) Decode the following words with the syndromes of coset leader. (3 points)

a) 101001 b) 010011 c) 100101

a) the syndrome of 101001 is 110. code word is 111001
$d(101001) = 111$

b) the syndrome is 101 code word is 010110
$d(010011) = 010$

c) the syndrome is 001 code word is 100100
$d(100101) = 100$

8. [10 points]Let N be a normal subgroup of a group, and let R be the following relation on G:

aRb if and only if $a^{-1}b \in N$.

Prove that R is a congruence relation on G and N is the equivalence class [e] relative to R, where e is the identity of G.

① prove R is congruence relation :

1) $a^{-1}a = e$ ∵ N is a subgroup ∴ $a^{-1}a \in N$ reflexive

2) aRb $a^{-1}b \in N$ $b \in aN$ $\exists h \in N$ $b = ah$ $bN = ahN$ $aN = bN$ $a \in bN$ $b^{-1}a \in N$ bRa ∴ symmetric

3) aRb bRc $a^{-1}b \in N$ $b^{-1}c \in N$ N is a subgroup of group $a^{-1}c = a^{-1}bb^{-1}c$ ~~$a^{-1}c$~~ ∵ $(a^{-1}b)(b^{-1}c) \in N$ ∴ $a^{-1}c \in N$ ∴ aRc transitive

4) ∀ $a_1 R a_2$ $b_1 R b_2$ $a_1^{-1}a_2 \in N$ $b_1^{-1}b_2 \in N$ $a_1, a_2, b_1, b_2 \in G$

~~$(a_1b_1)^{-1}(a_2b_2) = a_1^{-1}a_1^{-1}b_1b_2$ ∵ $a_1^{-1}b_1$~~

$(a_1b_1)^{-1}(a_2b_2) = b_1^{-1}a_1^{-1}a_2b_2$ ∵ $a_1^{-1}a_2 \in N$ 设 $a_1^{-1}a_2 = n \in N$

∵ N is normal subgroup $b_1^{-1}nb_2 = (b_1^{-1}b_2)n$ $b_1^{-1}b_2 \in N$ $n \in N$ ∴ $b_1^{-1}nb_2 \in N$ ∴ $(a_1b_1)^{-1}(a_2b_2) \in N$ ∴ $a_1b_1 R a_2b_2$ ∴ congruence relation

② prove N is $[e]$

1) ∀ $x \in N$ $x^{-1} \in N$ because N is subgroup ∴ $x^{-1}e = x^{-1} \in N$ ∴ xRe $x \in [e]$ $N \subseteq [e]$

2) ∀ $x \in [e]$ xRe $x^{-1}e = x^{-1} \in N$ x is inverse of x^{-1} ∵ N is subgroup of a group ∴ $x \in N$ ∴ $[e] \subseteq N$

∵ $N \subseteq [e]$ and $[e] \subseteq N$ ∴ $N = [e]$

9. [10 points]Let R=[2×1 matrices, □], a,b,c,d are real number,where

$$\begin{bmatrix} a \\ b \end{bmatrix} \square \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} a+c-1 \\ b+d+1 \end{bmatrix}$$

Determine which of the following properties hold for this structure:

a) Close
b) Commutative
c) Associative
d) An identity element
e) An inverse for every element

a) ∀ $\begin{bmatrix} a \\ b \end{bmatrix}$, $\begin{bmatrix} c \\ d \end{bmatrix}$ are matrices

$\begin{bmatrix} a \\ b \end{bmatrix} \square \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} a+c-1 \\ b+d+1 \end{bmatrix}$ $a+c-1$ and $b+d+1$ are still real numbers

so $\begin{bmatrix} a+c-1 \\ b+d+1 \end{bmatrix} \in$ 2×1 matrices

close property holds for this

b) $\begin{bmatrix} a \\ b \end{bmatrix} \square \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} a+c-1 \\ b+d+1 \end{bmatrix} = \begin{bmatrix} c+a-1 \\ d+b+1 \end{bmatrix} = \begin{bmatrix} c \\ d \end{bmatrix} \square \begin{bmatrix} a \\ b \end{bmatrix}$

commutative property holds for this

c) $\left(\begin{bmatrix} a \\ b \end{bmatrix} \square \begin{bmatrix} c \\ d \end{bmatrix}\right) \square \begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} (a+c-1)+e-1 \\ b+d+1+f+1 \end{bmatrix} = \begin{bmatrix} a+c+e-1-1 \\ b+d+f+1+1 \end{bmatrix}$

$\begin{bmatrix} a \\ b \end{bmatrix} \square \left(\begin{bmatrix} c \\ d \end{bmatrix} \square \begin{bmatrix} e \\ f \end{bmatrix}\right) = \begin{bmatrix} a \\ b \end{bmatrix} \square \begin{bmatrix} c+e-1 \\ d+f+1 \end{bmatrix} = \begin{bmatrix} a+c+e-1-1 \\ b+d+f+1+1 \end{bmatrix}$

associative holds for this.

d) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is the identity element.

e) ∀ $\begin{bmatrix} a \\ b \end{bmatrix} \in$ matrices.

$\begin{bmatrix} 2-a \\ -2-b \end{bmatrix}$ is its inverse

10. [6 points] Given (Z,+) is an Abelian group. Prove that (Z ×Z,*) is also an Abelian group. * is defined as $(a_1,b_1)*(a_2,b_2)=(a_1+a_2,b_1+b_2)$.

① closed : ∀ $(a_1,b_1), (a_2,b_2) \in (Z \times Z, *)$

$(a_1,b_1) * (a_2,b_2) = (a_1+a_2, b_1+b_2)$ $a_1+a_2 \in Z$ ~~and~~ $b_1+b_2 \in Z$ ~~are both Z~~.

so $(a_1+a_2, b_1+b_2) \in (Z \times Z, *)$

② ~~com~~associative : ∀ $(a_1,a_2), (b_1,b_2), (c_1,c_2) \in (Z \times Z, *)$

$((a_1,a_2) * (b_1,b_2)) * (c_1,c_2) = (a_1+b_1+c_1, a_2+b_2+c_2) = (a_1,a_2) * ((b_1,b_2) * (c_1,c_2))$

③ $(0,0)$ is the identity element

④ ∀ $(a_1,b_1) \in (Z \times Z, *)$ $(-a_1,-b_1) \in (Z \times Z, *)$ $(-a_1,-b_1)$ is its inverse.

⑤ commutative

∀ $(a_1,b_1), (a_2,b_2) \in (Z \times Z, *)$

$(a_1,b_1) * (a_2,b_2) = (a_1+a_2, b_1+b_2) = (a_2+a_1, b_2+b_1)$

$= (a_2,b_2) * (a_1,b_1)$

so $(Z \times Z, *)$ is an abelian group

8

11. [10 points]Consider a group Z_6, the operation table shown in following figure.

⊕	[0]	[1]	[2]	[3]	[4]	[5]
[0]	[0]	[1]	[2]	[3]	[4]	[5]
[1]	[1]	[2]	[3]	[4]	[5]	[0]
[2]	[2]	[3]	[4]	[5]	[0]	[1]
[3]	[3]	[4]	[5]	[0]	[1]	[2]
[4]	[4]	[5]	[0]	[1]	[2]	[3]
[5]	[5]	[0]	[1]	[2]	[3]	[4]

a) Find all of the normal subgroups of Z_6 (4 points)

4

because. Z_6 is abelian group. all subgroups of Z_6 is

$\{[0]\}$ $\{[0],[3]\}$ $\{[0],[1],[3]\}$ $\{[0],[2],[4]\}$

$\{[0],[1],[2],[3],[4],[5]\}$ are normal subgroups of Z_6.

2

b) Describe a congruence relation R on Z_6 and find a corresponding normal subgroup from (a). (3 points)

~~a R b if and only if the remainder of a divides six is the same as the remainder of b divides six.~~

∀ $[a], [b] \in Z_6$. $[a] R [b]$ if and only if ~~a+b=~~ $a = b$

2

c) For this congruence relation R in (b),Write the operation table of quotient group Z_6/R. (3 points)

~~⊗ [[0]] ... G~~

~~$Z_6/R = \{A, B, C\}$~~

~~$A = \{[a]\}$ a~~

$Z_6/R = \{\{[0]\}, \{[1]\}, \{[2]\}, \{[3]\}, \{[4]\}, \{[5]\}\}$

⊗	{[0]}	{[1]}	{[2]}	{[3]}	{[4]}	{[5]}
{[0]}	{[0]}	{[1]}	{[2]}	{[3]}	{[4]}	{[5]}
{[1]}	{[1]}	{[2]}	{[3]}	{[4]}	{[5]}	{[0]}
{[2]}	{[2]}	{[3]}	{[4]}	{[5]}	{[0]}	{[1]}
{[3]}	{[3]}	{[4]}	{[5]}	{[0]}	{[1]}	{[2]}
{[4]}	{[4]}	{[5]}	{[0]}	{[1]}	{[2]}	{[3]}
{[5]}	{[5]}	{[0]}	{[1]}	{[2]}	{[3]}	{[4]}

~~isumorphism to~~

isomorphism to Z_6

北京邮电大学 2017—2018 学年第一学期

Discrete Mathematics – Midterm Test

考试注意事项	
考试注意事项	一、学生参加考试须带学生证或学院证明，未带者不准进入考场。学生必须按照监考教师指定座位就坐。 二、书本、参考资料、书包等与考试无关的东西一律放到考场指定位置。 三、学生不得另行携带、使用稿纸，要遵守《北京邮电大学考场规则》，有考场违纪或作弊行为者，按相应规定严肃处理。 四、学生必须将答题内容做在试题答卷上，做在草稿纸上一律无效。

考试课程	离散数学				考试时间				2017 年 11 月				
题号	一	二	三	四	五	六	七	八	九	十	十一		总分
满分	4	10	8	9	9	4	20	10	10	6	10		
得分	4	7	8	9	9	4	20	10	10	6	8		95
阅卷教师													

4 1. [4 points]Give examples of relations on {1,2,3,4} having the properties specified.

a) Reflexive, symmetric, and not transitive.

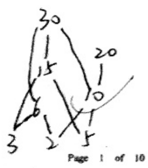
b) Not reflexive, not symmetric and transitive.

a) use Matrix to represent the order is the same as given. $M_R = \begin{bmatrix} 1&1&0&0 \\ 1&1&0&1 \\ 0&0&1&1 \\ 0&1&1&1 \end{bmatrix}$ $(4,2)\in R$ $(2,1)\in R$ $(4,1)\notin R$ not transitive.

b) $M_R = \begin{bmatrix} 0&0&1&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&1&0 \end{bmatrix}$ $R=\{(1,3),(4,3)\}$

7 2. [10 points] Suppose A = {2,3,5,6,10,15,20,30} and R is the partial order relation defined on A where xRy means x is a divisor of y.

(1) Draw the Hasse diagram for R.



(2) Find all maximal elements.

20, 30

(3) Find all minimal elements.

2, 3, 5

(4) Find all upper bounds for 3,5.

15, 30

(5) Find lub({5,10}).

10

(6) Find glb({6,15}).

3

(7) Is the poset(A,R) a lattice? Explain your answer.

−3

Yes, every two elements of (A,R). such as a, b

their glb({a,b}) = the greatest common divisor of a a

their lub({a,b}) = the least common mutiple of a an

8 3. [8 points] Let B={1,2,3,4,5}, A=B×B, and define R on A as follows: $(a,b)R(c,d)$ if and only if $a - b = c - d$.

(1) Prove that R is an equivalence relation.

① reflexive. $\forall (a,b)\in A$ $a-b=a-b$ $\therefore (a,b)R(a,b)$

② symmetric: $\forall (a,b),(c,d)\in A$ $(a,b)R(c,d)$ $a-b=c-d$

$\Rightarrow c-d=a-b$ $\therefore (c,d)R(a,b)$

③ transitive: $\forall (a,b)R(c,d)$ $(c,d)R(e,f)$

$a-b=c-d$ $c-d=e-f$ $a-b=e-f$

$\therefore (a,b)R(e,f)$

$\therefore$ R is an equivalence relation.