北京邮电大学 2023—2024 学年第一学期

《离散数学(下)》期末考试试题（A 卷）

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考试注意事项	一、学生参加考试须带学生证或学院证明，未带者不准进入考场。学生必须按照监考教师指定座位就坐。 二、书本、参考资料、书包等物品一律放到考场指定位置。 三、学生不得另行携带、使用稿纸，要遵守《北京邮电大学考场规则》，有考场违纪或作弊行为者，按相应规定严肃处理。 四、学生必须将答题内容做在试题答卷上，做在草稿纸上一律无效。

一、suppose that A={2,3,6,9,10,12,14,18,20} and R is the partial order relation defined on A where xRy means x is a divisor of y.

a) Draw the Hasse diagram for R.

b) Find all maximal elements.

c) Find all minimal elements.

d) Find lub({3,10}).

e) Find glb({14,10}).

二、

（1）Let (G,*) be a group and define $f: G \to G$ by $f(a) = a^{-1}$. Is f an isomorphism, justify your answer.

（2）Find all of the normal subgroups of $Z2 \times Z3$.

三、Let $H = \begin{matrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$ be a parity check matrix；

a) Determine the (3,6) group code eH: $B^3 \to B^6$.

b) Find the minimal distance of eH.

c) How many errors will eH detect?

d) Suppose x_t = 000111, compute the syndrome of x_t.

四、

a) When graph G is an undirected graph without self-loops, with |V|=6 and |E|=16, then graph G is an __ graph.

b) the total degree of the W_8 is ___.

c) There is a circuit with a length of ___ in bipartite graphs $K_{3,3}$.

d) There are _____ connected components in the above graph.

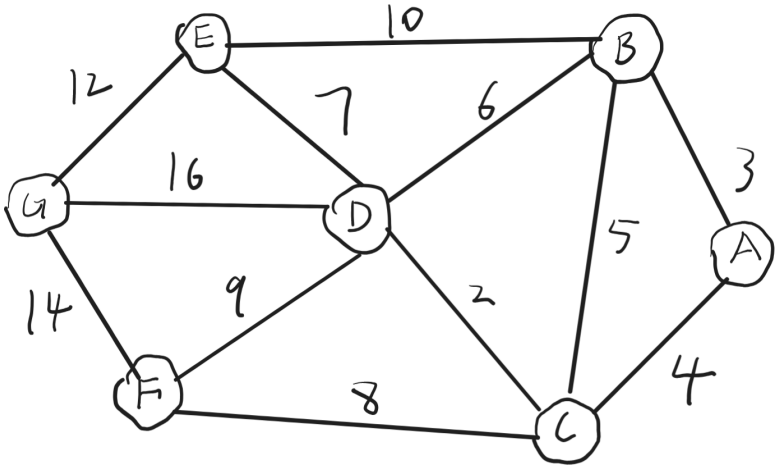
e) It is known that an undirected graph G contains 16 edges, with 3 vertices having a degree of 4 and 4 vertices having a degree of 3. The degree of all other vertices are less then 3. The number of vertices contained in Figure G is at least_____.

五、

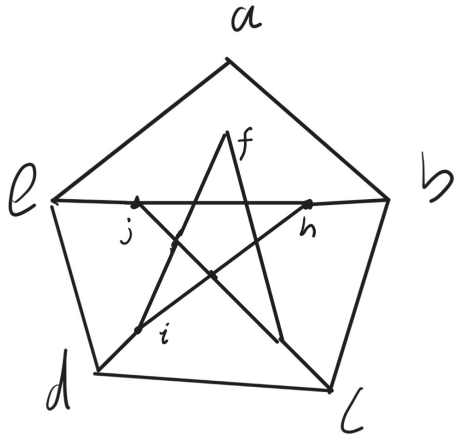
a) Draw an Euler diagram with odd edges and even vertices.

b) Does the graph have a Hamilton circuit? If so, find such a path. If not, give an argument to show why no such path exists.

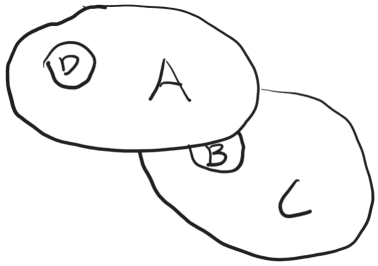
六、Use Dijkstra's Algorithm to find the shortest path length between the vertices A and G in this weighted graph (show process).



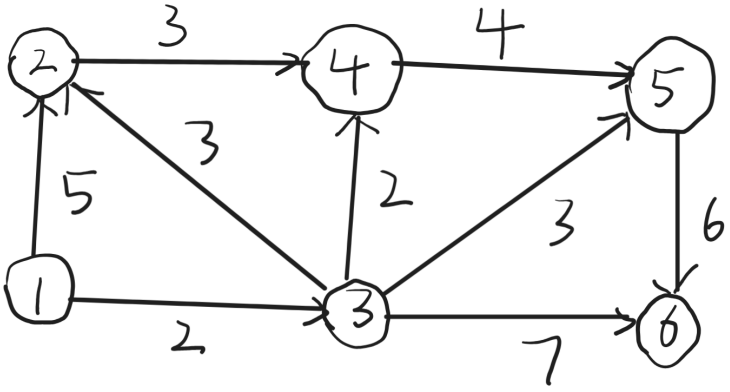
七、Use Kuratowski's Theorem to determine whether the given graph is planar.



八、Construct the dual graph for the map shown : Each region of the map is represented by a vertex, Edges connect two vertices if the regions represented by these vertices have a common border. Then find the chromatic polynomial P_G for the map shown and use PG to find $x_{(G)}$.



九、Find the maximal flow for the network N given in following figure. And find a minimun cut of this network (Draw the initial situation labeled situation of each flow).



十、Find the solution to the recurrence relation $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$ for $n \geq 3$ with initial conditions $a_0 = 1$, $a_1 = 0$, $a_2 = 2$.

十一、Use generating functions to solve the recurrence relation $a_k = 3a_{k-1} + 4^{k-1}$, $k \geq 0$,with the initial condition $a_0 = 2$.

十二、Use Prime's algorithm to design a minimum cost communications network connecting all the computers represented by the graph in the next figure.