

6. One quarter of the five-element subsets of  $\{1, 2, 3, \dots, n\}$  contains the element 7. Determine  $n$  ( $n \geq 5$ )

(在  $\{1, 2, 3, \dots, n\}$  的 5 元子集中有四分之一的子集具有元素 7。确定  $n$  ( $n \geq 5$ ) 的值)

7. Prove that for any positive integer  $n$  ( $F(n)$  表示第  $n$  个斐波那契数,  $F(0)=0$ ,  $F(1)=1$ )

$$\sum_{i=1}^n \frac{F(i-1)}{2^i} = 1 - \frac{F(n+2)}{2^{n+1}}$$

8. For  $n=3$  let  $X_3 = \{1, 2, 3\}$ ,  $s_3 = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{1 \times 2} + \frac{1}{1 \times 3} + \frac{1}{1 \times 2 \times 3} = \sum_{\emptyset \neq a \subseteq X_3} \frac{1}{P_a}$ , where  $P_a$  denotes the product of all elements in a nonempty subset  $a$  of  $X_3$ .

(其中  $P_a$  表示  $X_3$  的非空子集  $a$  中所有元素的乘积)

a) Calculate  $s_2, s_4$

b) Conjecture the general result suggested by the calculations from part a). Prove your conjecture using the Principle of Mathematical Induction.

(由 1) 中的计算结果猜测出一个一般结论, 利用数学归纳法原理证明你的猜测)

9. Prove that if  $n \in \mathbb{Z}^+$  and  $n \geq 2$  then

$$\prod_{i=2}^n \left(1 - \frac{1}{i^2}\right) = \frac{n+1}{2n}$$

10. Let  $A = \{1, 2, 3, 4\}$  and  $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$ . (a) How many functions are there from  $A$  to  $B$ ? How many of these are one-to-one (一对一的)? How many are onto (到上的)? (b) How many functions are there from  $B$  to  $A$ ? How many of these are one-to-one? How many are onto?