2

$2/each

The price of a package of 4 pens is $8.00. The same pens are sold at $2.50 each. If Alex bought three packages of pens rather than buying 12 pens individually, the amount he saved on 12 pens is what percent of the amount he paid?

A) 12%

B) 20%

C) 25%

D) 30%

$3 \times 8 = 24$ total paid

individual price: $12 \times 2.5 = 30$

$30 - 24 = 6$ save

$\frac{6}{24} = \frac{1}{4} = 25\%$

5

There is a total of n pairs of shoes in a store, all of which are either black or brown. If there are m pairs of brown shoes in the store, then in terms of m and n, what percent of the shoes in the store are black?

total n pairs

m br $\quad$ $n-m$ black

A) $\frac{m}{n}\%$

B) $\frac{n-m}{n}\%$

C) $(1-\frac{100m}{n})\%$

D) $100(1-\frac{m}{n})\%$

$\frac{n-m}{n}$: prop of black

$\frac{n-m}{n}\cdot 100\% = 100\cdot\left(\frac{n-m}{n}\right)\%$

$= 100\left(\frac{n}{n}-\frac{m}{n}\right)\%$

$= 100\left(1-\frac{m}{n}\right)\%$

1

$$\frac{2^{(a+b)^2}}{2^{(a-b)^2}}$$

$= 2^{a^2+2ab+b^2} / 2^{a^2-2ab+b^2} = 2^{a^2+2ab+b^2-(a^2-2ab+b^2)}$

$= 2^{4ab}$

$= (2^4)^{ab}$

$= 16^{ab}$

Which of the following is equivalent to the expression shown above?

A) $8^{(a+b)}$

B) 8^{ab}

C) 16^{a+b}

D) 16^{ab}

$$(\frac{a+b}{2})^2-(\frac{a-b}{2})^2=$$

A) ab

B) $-ab$

C) $\frac{2ab+b^2}{2}$

D) $ab+b^2$

$(\frac{a+b}{2})^2=\frac{(a+b)^2}{4}=\frac{a^2+2ab+b^2}{4}$

$(\frac{a-b}{2})^2=\frac{a^2-2ab+b^2}{4}$

$\frac{\cancel{a^2}+2ab+\cancel{b^2}}{4}-\frac{\cancel{a^2}-2ab+\cancel{b^2}}{4}=\frac{4ab}{4}=ab.$

If $(x+\frac{1}{x})^2=9$, then $(x-\frac{1}{x})^2=$

$x^2+2+\frac{1}{x^2}=9$ $\qquad x^2-2+\frac{1}{x^2}=7-2=5.$

$x^2+\frac{1}{x^2}=7$

A) 3

B) 5

C) 7

D) 9

$(a+b)^2=a^2+2ab+b^2$

$(x+\frac{1}{x})^2=x^2+2\cdot\boxed{x\cdot\frac{1}{x}}+(\frac{1}{x})^2$

$=x^2+2+(\frac{1}{x})^2$

If $xy \neq 0$, then $\dfrac{(-2xy^2)^3}{4x^4y^5} =$

$$\frac{-8x^3y^6}{4x^4y^5}$$

A) $-\dfrac{xy}{2}$

$$= \frac{-8}{4} \cdot \frac{x^3}{x^4} \cdot \frac{y^6}{y^5}$$

B) $-\dfrac{2}{x}$

$$= -2 \cdot \frac{1}{x} \cdot y = \frac{-2y}{x}$$

C) $-\dfrac{2y}{x^2}$

D) $-\dfrac{2y}{x}$ ✓

The half-life of a radioactive substance is the amount of time it takes for half of the substance to decay. The table below shows the time (in years) and the amount of substance left for a certain radioactive substance.

Time (years)	Amount (grams)
0	1,200
14	850
28	600
42	425
56	300

28yr

$\times \frac{1}{2}$

28yrs

$\times \frac{1}{2}$

$1200 \times \left(\frac{1}{2}\right)^5 = 37.5$

How much of the original amount of the substance, to the nearest whole gram, will remain after 140 years?

28×5

A) 85

B) 75

C) 53

D) 38

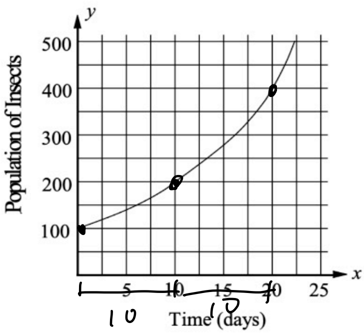
The price P, in dollars, of a truck t years after it was purchased is given by the function

$P(t) = 24,000(\frac{1}{2})^{\frac{t}{6}}$. To the nearest dollar, what is the price of the truck 9 years after it was purchased?

$t = 9$

$24000 \cdot \left(\frac{1}{2}\right)^{9/6}$

$24000 \cdot \left(\frac{1}{2}\right)^{\frac{3}{2}}$



The graph above shows the size of a certain insect population over 25 days. The population at time $t = 0$ was 100. A biologist used the equation

$f(t) = 100(2)^{\frac{t}{d}}$ to model the population.

initial 100
every time x increased by
d y doubled
d : 10 day

8

What is the value of d in the equation?

$d = 10$

9

What was the population of the insect after 15 days, to the nearest whole number?

$f(t) = 100 \cdot 2^{\frac{t}{d}} = 100 \cdot 2^{\frac{t}{10}}$

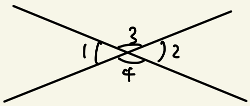
$t = 15$ $\quad f(15) = 10 \cdot 2^{\frac{15}{10}}$

$= 28\sim.$

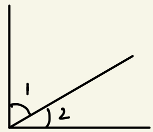
Geometry.

Angle:

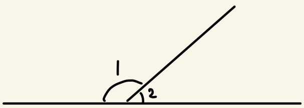
Angle chasing



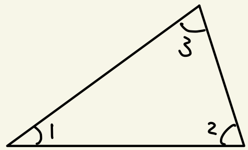
$\angle 1 = \angle 2$
$\angle 3 = \angle 4$



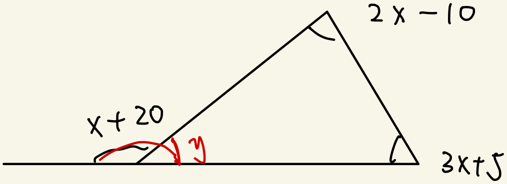
$\angle 1 + \angle 2 = 90°$



$\angle 1 + \angle 2 = 180°$



$\angle 1 + \angle 2 + \angle 3 = 180°$



what is x

$180 - y$

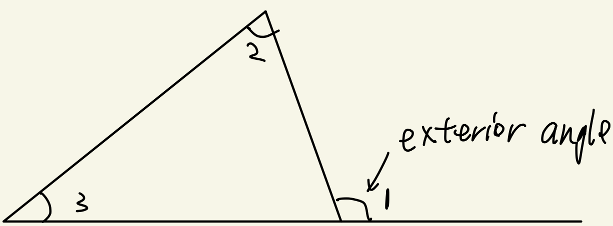
$\boxed{x+20} + y = 180$

$\boxed{2x-10 + 3x+5} + y = 180$

$x + 20 = 5x - 5$

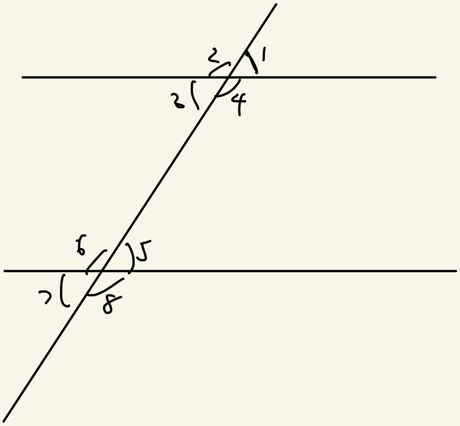
$4x = 25$

$x = 6.25$



$\angle 1 = \angle 2 + \angle 3$

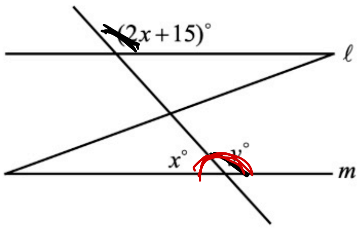
exterior angle = sum of 2 non adjacent interior angle parallel :



$\angle 1 = \angle 3 = \angle 5 = \angle 7$

$\angle 2 = \angle 4 = \angle 6 = \angle 8$

$\angle 1 + \angle 2 = 180°$



Note: Figure not drawn to scale.

In the figure above, $\ell \parallel m$. What is the value of y?

A) 120

B) 125

C) 130

D) 135

$2x + 15 = y$

$x + y = 180$

$x + 2x + 15 = 180$

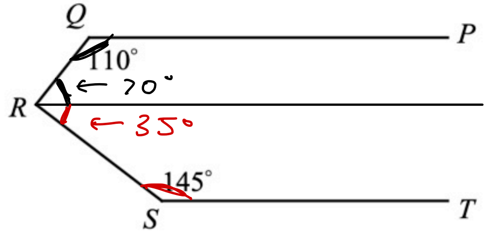
$3x + 15 = 180$

$3x = 165$

$x = 55$

$y = 2x + 15 = 2 \cdot 55 + 15 = 125$

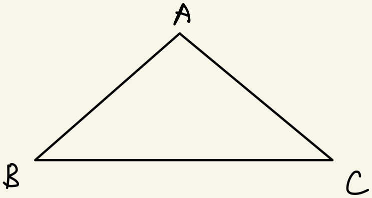
8



Note: Figure not drawn to scale.

In the figure above, $\overline{PQ}$ is parallel to $\overline{ST}$. What is the measure of $\angle QRS$?

$35° + 70° = 105°$

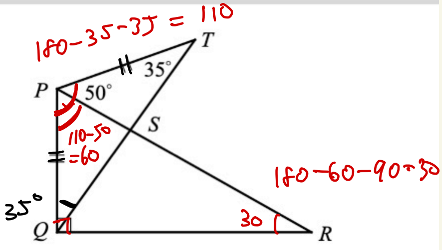


$AB = AC$

$\Downarrow$

$\angle B = \angle C$

2



In the figure above, $\overline{PQ} \perp \overline{QR}$ and $\overline{PQ} \cong \overline{PT}$.
What is the measure of $\angle R$?

A) 30

B) 35

C) 40

D) 45

polygon: n side polygon: sum of interior angle is $(n-2) \cdot 180°$

regular n side polygon: sum of interior: $(n-2) \cdot 180°$

each angle $\frac{(n-2) \cdot 180°}{n}$.

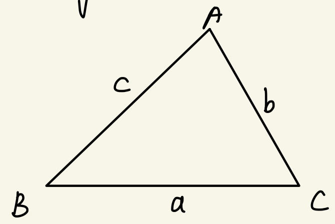
Question 35. *A polygon has exactly 87 sides. If the measure of each of the 87 interior angles of this polygon is $(180p)°$, what is the value of p?*

total $85 \cdot 180°$

each $\frac{85 \cdot 180°}{87} = \frac{85}{87} \cdot 180° = p \cdot 180$

$p = \frac{85}{87}$.

Triangle



$\angle A + \angle B + \angle C = 180°$

Sum of 2 sides > third sides
(Triangle inequality)

$a+b>c$
$b+c>a$
$c+a>b$

Question 29. *The triangle inequality theorem states that the sum of any two sides of a triangle must be greater than the length of the third side. If a triangle has side lengths of 8 and 13, which inequality represents the possible lengths, x, of the third side of the triangle?*

x

(A) $x < 21$

(B) $x > 21$

✓ (C) $5 < x < 21$

(D) $x < 5$ or $x > 21$

$8+13>x$
$8+x>13$
$13+x>8$

$x<21$
$x>5$
$x>-5$

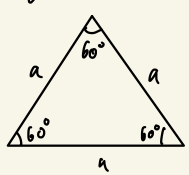
$5<x<21$

$x^2 = 100 \quad x = 10$

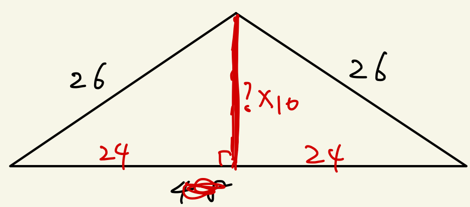
$x^2 + 576 = 676$

$x^2 + 24^2 = 26^2$

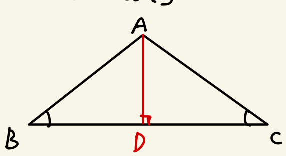
equilateral

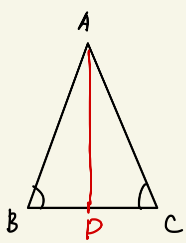


$\text{Area} = \frac{\sqrt{3}}{4} \cdot a^2$



Isosceles





Area of triangle

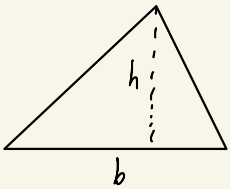
$\frac{\text{base} \cdot \text{height}}{2} = \frac{48 \cdot 10}{2}$

$= 240$

$AB = AC \Leftrightarrow \angle B = \angle C$

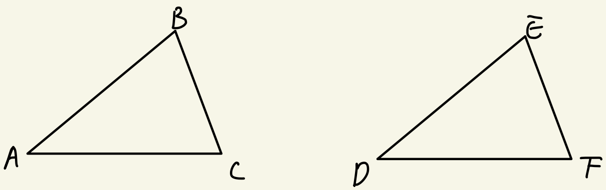
if $BD = DC \Rightarrow AD \perp BC$

if $AD \perp BC$, then $BD = DC$



Area $= \frac{b \cdot h}{2}$

Congruent & Similar triangle



$\triangle ABC \cong \triangle DEF$

$AB = DE$, $BC = EF$, $AC = DF$

$\angle A = \angle D$ $\angle B = \angle E$ $\angle C = \angle F$

Criteria for two triangle to be congruent

① SSS

② SAS

$AB = DE$

$AC = DF$ $\angle A = \angle D$

$\Downarrow$

$\triangle ABC \cong \triangle DEF$

$\triangle ABC$ $\triangle DEF$

$\angle A = 60^\circ$

$\angle B = 50^\circ$

$$\boxed{\begin{array}{c} AB = DE \\ BC = EF \\ AC = DF \end{array}}$$

$\hookrightarrow \triangle ABC \cong \triangle DEF$

What is angle F.

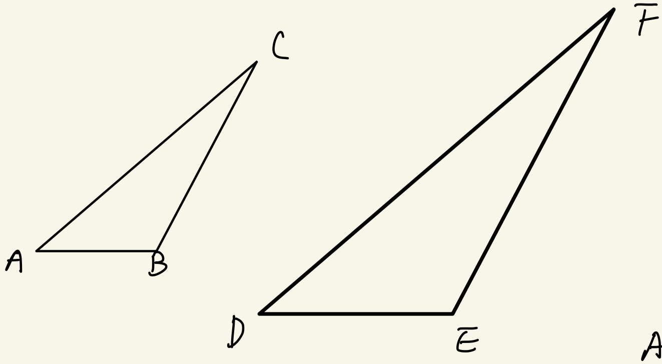
$\angle F = \angle C = 180^\circ - 50^\circ - 60^\circ = 70^\circ$

③ ASA

④ AAS

⑤ SSA is not sufficient

Similar triangle (same shape, different size



$\triangle ABC \sim \triangle DEF$

$\angle A = \angle D$

$\angle B = \angle E$

$\angle C = \angle F$

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

AA $\angle A = \angle D$

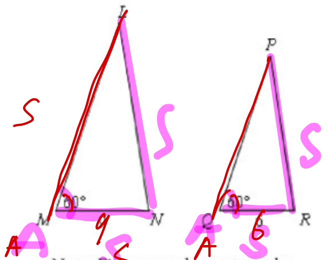
$\angle B = \angle E$

SAS $\frac{AC}{DF} = \frac{AB}{DE}$ & $\angle A = \angle D$

SSS $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

SSA not sufficient.

47. ΔLMN and ΔPQR each have an angle measuring 60° and a given side length, as shown.



Note: Figure not drawn to scale.

For ΔLMN and ΔPQR, which additional piece of information is sufficient to prove that the triangles are similar?

I. The length of line segment PQ is $\frac{2}{3}$ the length of line segment LM. SAS ✓

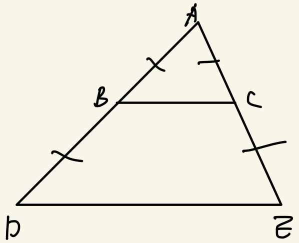
II. The length of line segment PR is $\frac{2}{3}$ the length of line segment LN. × SSA

✓ **(A)** I is sufficient, but II is not

(B) II is sufficient, but I is not

(C) I is sufficient and II is sufficient

(D) Neither I nor II is sufficient



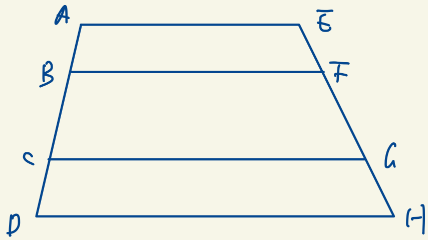
$BC \parallel DE$

$ABC \sim ADE$

$$\frac{AB}{AD} = \frac{AC}{AE}$$

$AB : AC = AD : AE$

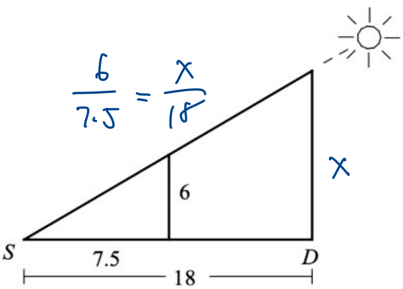
$= BD : CE$



$AE \parallel BF \parallel CG \parallel DH$

$AB/EF = BC/FG = CD/GH = AD/EH$

4



$$\frac{6}{7.5} = \frac{x}{18}$$

$$x = \frac{6 \cdot 18}{7.5}$$

$= 14.4$

Note: Figure not drawn to scale.

A person 6 feet tall stands so that the ends of his shadow and the shadow of the pole coincide. The length of the person's shadow was measured 7.5 feet and the length of the pole's shadow, SD, was measured 18 feet. How tall is the pole?

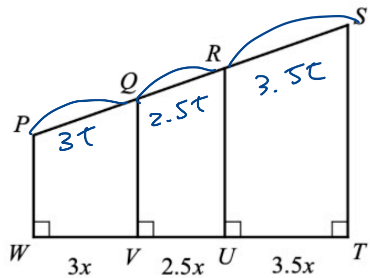
A) 12.8

B) 13.6

C) 14.4 ✓

D) 15.2

8



In the figure above, if $PS = 162$, what is the length of segment QR?

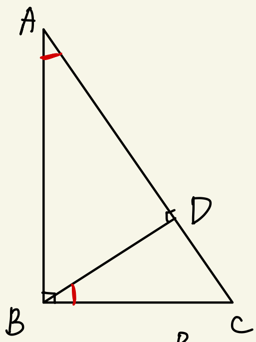
$3t + 2.5t + 3.5t = 162$

$9t = 162$

$t = 18$

$2.5t = QR$

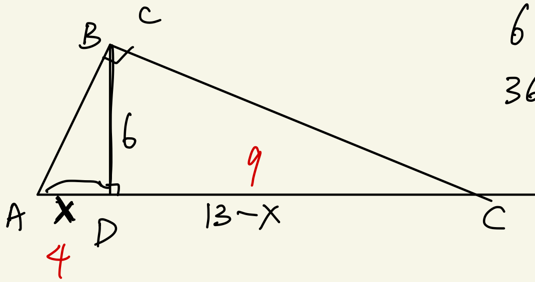
$= 18 \cdot 2.5 = 45$



$\triangle ABD \sim \triangle BCD \sim \triangle ACB$

$\underline{BD^2 = AD \cdot DC} \Leftarrow \frac{BD}{DC} = \frac{AD}{BD}$

$BD \cdot AC = AB \cdot BC = 2 \cdot \text{Area}$



$6^2 = x(13-x)$

$36 = 13x - x^2$

$x^2 - 13x + 36 = 0$

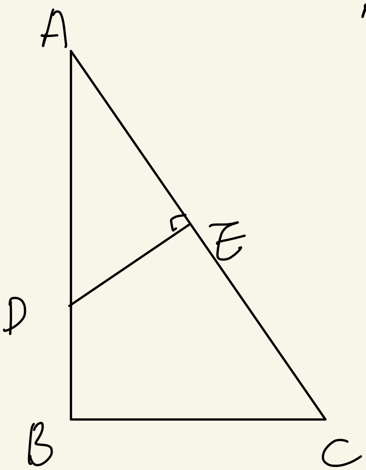
$(x-4)(x-9) = 0$

$x = 4$ or $x = 9$

$BD = 6$

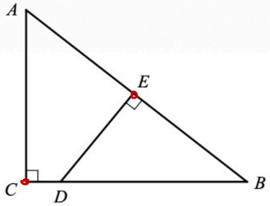
$AC = 13$

$\frac{DC}{AD} = ?\ \frac{9}{4}$



$ADE \sim ACB$.

12



$\triangle ABC \sim \triangle DBE$

$\frac{AB}{DB} = \frac{BC}{BE} = \frac{AC}{DE}$ 15 12 8

$BE = \frac{8 \cdot 15}{12} = 10$

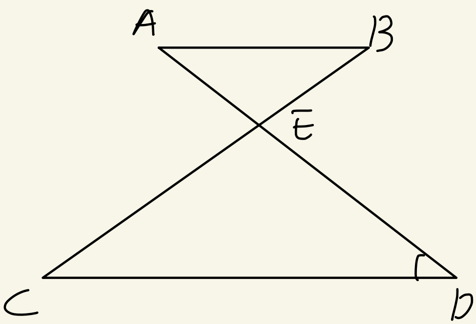
In the figure above, ΔABC and ΔDBE are right triangles. If $AC = 12$, $BC = 15$, and $DE = 8$, what is the length of BE?

A) 8.5

B) 9

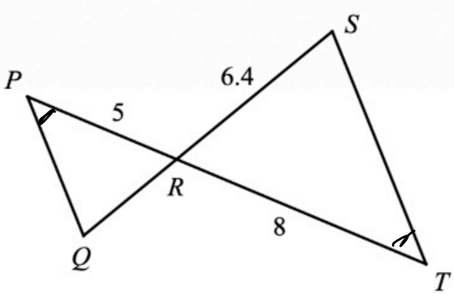
C) 9.5

D) 10 ✓



$\triangle ABE \sim \triangle DCE$

7



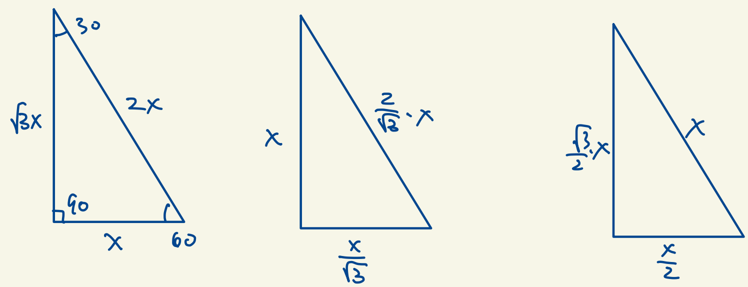
$\frac{5}{8} = \frac{QR}{6.4}$

$QR = \frac{5 \cdot 6.4}{8} = 4$

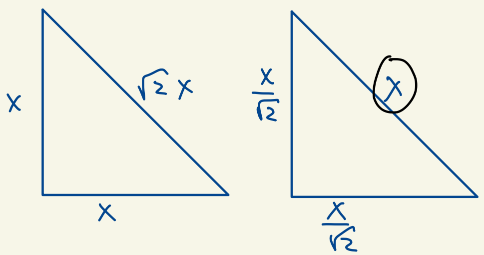
$QS = 4 + 6.4 = 10.4$

In the figure above, $\overline{PQ} \parallel \overline{ST}$ and segment PT intersects segment QS at R. What is the length of segment QS?

30 − 60 − 90



45−45−90 (isosceles right triangle



perimeter $x+\frac{x}{\sqrt{2}}+\frac{x}{\sqrt{2}}$

$$x+\frac{2}{\sqrt{2}}\cdot x = x(1+\sqrt{2})$$

$$=26+26\sqrt{2}$$

Suppose an isosceles right triangle have perimeter $26+26\sqrt{2}$. What is the hypotenuse.

$$x=\frac{26\sqrt{2}+26}{\sqrt{2}+1}\cdot\frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$=\frac{52-26+26\sqrt{2}-26\sqrt{2}}{1}$$

$$=26$$

Solid Geometry

Similar shape

2D similar ~~triangle~~ or similar rectangle.

Ratio of Area = (Ratio of sides)^2

$\triangle ABC \sim \triangle DEF$

Area of $\triangle ABC = 180$

$AB : DE = 3 : 2$

Ratio of Area $= (3:2)^2 = 3^2 : 2^2 = 9 : 4$

180 $\quad 180 \cdot \frac{4}{9} = 80$

Similar

3D: Ratio of Volume = (Ratio of sides)$^{\boxed{3}}$

Ratio of Surface Area = (Ratio of sides)$^{\boxed{2}}$

Question 32. *Right rectangular prism X is similar to right rectangular prism Y. The surface area of right rectangular prism X is 52 square centimeters (cm^2), and the surface area of right rectangular prism Y is 1,872 cm^2. The volume of right rectangular prism X is 10.5 cubic centimeters (cm^3). What is the volume, in cm^3, of right rectangular prism Y?* 2268

	X	Y
S·A	52	1872

$SA_X : SA_Y = 52 : 1872 = 1 : 36$ (side ratio)^2

$S_X : S_Y = 1 : 6$

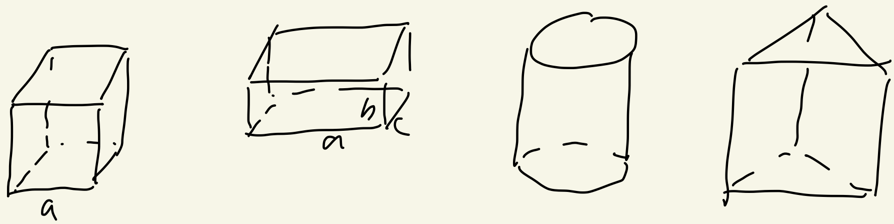
$V_X : V_Y = (1:6)^3 = 1^3 : 6^3 = 1 : 216$

10.5 $\quad 10.5 \times 216 = 2268$

Sphere: Volume $= \frac{4}{3} \cdot \pi \cdot r^3$ r : radius

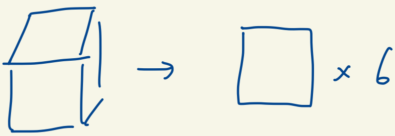
hemisphere (half) $= \frac{1}{2} \cdot \frac{4}{3} \cdot \pi \cdot r^3$

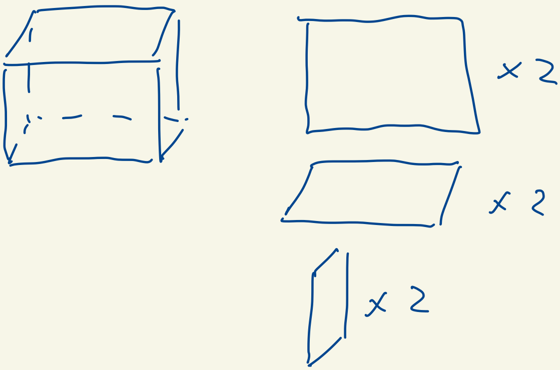
prisms.

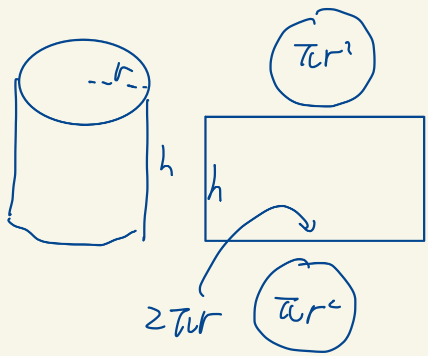


Volume = base area · height

a^3 abc $\pi r^2 h$

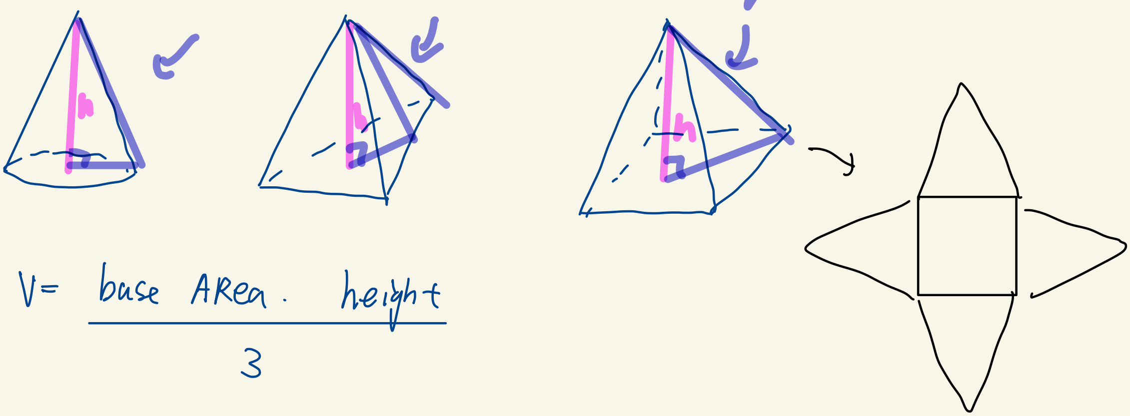






$2\pi r^2 + 2\pi r h$.

Cone   pyramid   Slant height

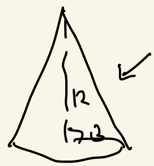


$$V = \frac{\text{base Area} \cdot \text{height}}{3}$$

Cone radius 3. Volume = 36π, slant height ?

$$\frac{\cancel{\pi} \cdot 3^2 \cdot h}{3} = 36\cancel{\pi}$$

$$3h = 36 \qquad h = 12$$



$\sqrt{12^2+3^2} = \sqrt{153}$