UNIT 8: Limit theorems and classical statistics — Summary

- Markov inequality: If $X \geq 0$ and $a > 0$, then $\mathbf{P}(X \geq a) \leq \dfrac{\mathbf{E}[X]}{a}$

- Chebyshev inequality: If $c > 0$, then $\mathbf{P}\Big(|X - \mathbf{E}[X]| \geq c\Big) \leq \dfrac{\text{var}(X)}{c^2}$

- Convergence in probability: For every $\epsilon > 0$, $\mathbf{P}\Big(|X_n - a| \geq \epsilon\Big) \to 0$

- Weak law of large numbers: X_i : i.i.d. : $M_n = \dfrac{X_1 + \cdots + X_n}{n} \to \mathbf{E}[X]$

- Central limit theorem, X_i : i.i.d.:

$$\text{CDF of} \quad \frac{X_1 + \cdots + X_n - n\mathbf{E}[X]}{\sqrt{n}\,\sigma_X} \to \text{standard normal CDF}$$

  - pretend $X_1 + \cdots + X_n$ is normal
  - "1/2-correction" for integer r.v.'s

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- Unknown constant θ — not a r.v.; model $p_X(x;\theta)$, $f_X(x;\theta)$
- Use sample means to estimate expectations:
  - If $\theta = \mathbf{E}[X]$, $\widehat{\Theta} = \frac{1}{n}\sum_{i=1}^{n} X_i$
  - If $\theta = \mathbf{E}\big[g(X)\big]$, $\widehat{\Theta} = \frac{1}{n}\sum_{i=1}^{n} g(X_i)$
- Confidence interval $[\widehat{\Theta}^-, \widehat{\Theta}^+]$: $\mathbf{P}(\widehat{\Theta}^- \leq \theta \leq \widehat{\Theta}^+) \geq 0.95$ (or 0.99, etc.)
  - often need the variance of estimator: estimated using "sample variance"
- Maximum Likelihood: $\max_\theta p_X(x;\theta)$