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A PAN-LAKE EVAPORATION RELATIONSHIP

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Abstract: It is shown that lake evaporation over periods down to a month or even less may be estimated from nearby pan observations (U.S. Class A pan) by summing the daily estimates from

$$E_L = 1.50 \frac{e_L - e_4}{e_P - e_4} E_P .$$

This takes account of the different water surface temperatures of lake and pan, e_L and e_P being the corresponding saturation vapour pressures, while e_4 is the vapour pressure 4 m above the ground. e_L and e_4 are taken as afternoon average values and e_P as afternoon maximum, and the coefficient 1.50 is established from Lake Hefner data with an estimated standard error of 5 per cent. It is estimated that, provided observations are of reasonably high quality, this pan conversion method will indicate monthly total lake evaporation with a standard error of less than 10 per cent.

1. Introduction

The annual evaporation from lakes is commonly estimated by reference to nearby pan observations. A useful indication is obtained by multiplying the observed yearly total pan evaporation by an empirical "pan factor" which, for example, is approximately 0.7 in the case of the American Class A pan.

For shorter periods (a season or less), this simple method is, unfortunately, subject to large unpredictable errors, attributable mainly to the varying relationship between pan and lake water surface temperatures (Wisler and Brater[1]), Kohler[2]), Deacon *et al.*[3])). The lake has comparatively large thermal inertia, and its temperature varies only slowly, whereas the pan water temperature can vary greatly from day to day with changing meteorological conditions.

In this paper, the direct approach to dealing with the difficulty is discussed. This is to apply a simple conversion relationship to the daily pan readings, by assuming that the lake or pan evaporation is proportional to the vapour pressure difference between the respective water surface and some convenient observation height in the air. Since this entails measurement of the lake water surface temperature, it is apparent that it may be applied only to an existing lake, and not to the forecasting of future evaporation.

This type of approach has been discussed by Kohler[2], in connection with the well-known Lake Hefner investigation, but otherwise does not appear to have been exploited. A different type of approach which is more fundamental and more elaborate, incorporating heat budget considerations, is described by Kohler *et al.*[4]).

The present work also utilizes the Lake Hefner data (U.S. Geological Survey[5])). The pan conversion relationship is initially investigated, and its numerical coefficient evaluated, by making comparison with the primary determination of evaporation from the water budget, taking only those days which are rated as "class A" for the latter determination. An independent test of the method is then made by comparing its results with those from the heat budget method, over several short periods (8 to 14 days) for which the evaporation evaluated by the latter method has been reported (Anderson[6])).

2. The Pan-Lake relationship

Several investigations have shown that lake evaporation is well represented by the following form of the bulk aerodynamic (Dalton) relationship:

$$E = CU(e_s - e_a). \tag{1}$$

Here U represents wind speed; e_s and e_a represent partial pressure of water vapour, respectively at the water surface (taken as the saturation value corresponding to the water temperature) and at a convenient reference height above the surface; and C is a coefficient whose value (having some dependence on the heights at which U and e_a are measured) has been reasonably well established by experiment. Background discussions and references on the bulk aerodynamic formula are given by Marciano and Harbeck[7]), Kohler[2]), Deacon *et al.*[3]), Harbeck[8]), and Webb[9]).

Assuming that a relationship similar to Eq. (1) is applicable for pan evaporation E_P as well as for lake evaporation E_L, and taking the ratio of the two, we have

$$E_L = \text{const.} \times \frac{e_L - e_a}{e_P - e_a} E_P \tag{2}$$

where e_L and e_P represent the partial pressures of water vapour at the water surfaces of lake and pan, respectively. The numerical constant in Eq. (2) incorporates not only the ratio of the different values of C for pan and lake, but also the ratio of wind speeds, the wind being generally rather stronger over an expanse of water than over a nearby land surface.

In Eq. (2), no distinction is made between values of e_a over lake and pan, as these will be nearly the same if the lake is not too large and if the height of measurement is at least 4 m or so. In a practical case (Webb[10])), e_a at a height of 4 m was found to increase by only about $2\frac{1}{2}$ per cent of $(e_L - e_a)$ as the air traversed a 2 km path over the lake surface.

In examining the Lake Hefner data, e_a is taken at height 4 m and denoted by e_4. It is to be expected that pan evaporation will, in general, be greatest

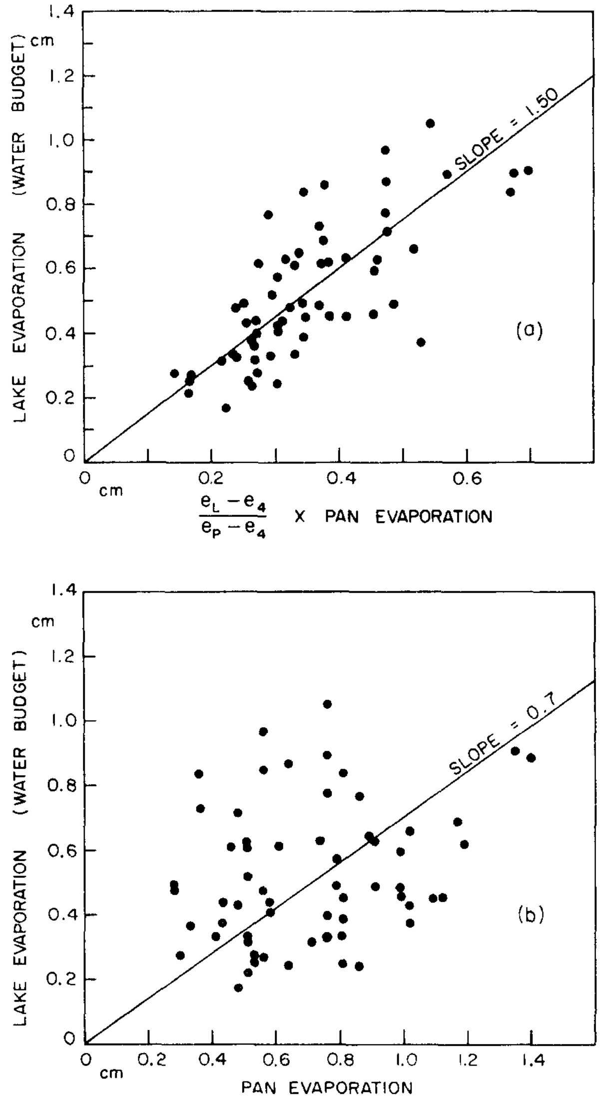

Fig. 1. Lake Hefner daily evaporation (from water budget) plotted against (a) Class A pan evaporation modified as in Eq. (2), and (b) raw Class A pan evaporation. In (a), the slope of the straight line represents the regression coefficient 1.50 as adopted in Eq. (3); in (b), the slore represents the application of a simple pan factor 0.7.

during the afternoon, when the pan water is warmest; therefore we take, as a representative and easily determined daily value of e_P, the saturation vapour pressure corresponding to the observed *daily maximum* pan water temperature. The values of e_L and e_4 are also taken in the afternoon; for convenience, they are determined here from the 12–18 h average values of the relevant temperatures, though in general these vary only slowly and the exact times or durations of the measurements should not be important.

Eq. (2) is examined in terms of Lake Hefner daily values in Fig. 1 (a), where the lake evaporation E_L evaluated from the water budget is plotted against $E_P \times (e_L - e_4)/(e_P - e_4)$. The pan observations utilized are those of the Class A pan at the principal (south) station. The values of e_L and e_4 are from measurements at the barge station near the centre of the lake (except in a few cases of missing data, when one of the other stations is substituted).

The pan observations were daily at 09 h – the analysis excludes the initial period when the pan observations were at 15 h. The reported water-budget E_L is for 00 to 24 h each day; therefore, to permit comparison with the pan observation at 09 h on the following day, the E_L values have been adjusted here by subtracting the bulk-aerodynamic estimate (Eq. (58) of Marciano and Harbeck[7])) for 00 to 09 h, and adding that for 00 to 09 h the following day.

In order to avoid data which are likely to be too inaccurate, the plotting in Fig. 1 (a) has been restricted by the following objective criteria

(i) It includes only days rated in the Lake Hefner report as "class A" for water budget accuracy.

(ii) It excludes days with more than 0.05 inch (0.13 cm) of rain recorded.

(iii) It excludes days with very small evaporation, i.e. 0.1 inch (0.25 cm) or less for the pan and 0.15 cm or less for the lake.

(iv) It excludes days when the accuracy of the pan reading at 09 h may have been impaired by strong wind – days with 8 m wind averaging greater than 18 knot during 06–09 h or 09–12 h.

By least-squares fitting to the results plotted in Fig. 1 (a), the constant in Eq. (2) is evaluated as 1.46 or 1.57, depending on whether the regression is of ordinate on abscissa or vice versa, with a standard error of 4 per cent in either case. The most appropriate value is probably near the mean, and we therefore adopt the value 1.50, and take 5 per cent as a reasonable estimate of its standard error. Thus, we adopt as the pan conversion formula

$$E_L = 1.50 \frac{e_L - e_4}{e_P - e_4} E_P, \tag{3}$$

where e_4 is the afternoon average vapour pressure at a height of 4 m above the surface, e_L corresponds to saturation at the afternoon average temperature of the lake water, and e_P to saturation at the maximum temperature of the pan water reached the same afternoon.

The standard deviation of the experimental spread of the ordinates from the regression line is found to be 28 per cent, from which, assuming the daily errors to be random, the estimated standard error of weekly total evaporation is 11 per cent, and of monthly total is 5 per cent. Thus, it can be said that in general the standard error of monthly total estimated evaporation is expected to be less than 10 per cent, provided the observations are of reasonably high quality.

The same data as in Fig. 1 (a) are again the basis of Fig. 1 (b), where E_L is plotted directly against the pan evaporation E_P, and the straight line is drawn to represent the simple application of a pan factor 0.7. The smaller scatter of the points in the former Figure than in the latter illustrates the advantage of the pan conversion formula over the simple pan factor.

In the analysis by Kohler[2]), e_P was determined from the daily average pan water temperature, rather than from the daily maximum as in the present work. Therefore, it is to be expected that the coefficient in Eq. (3) which arises from Kohler's analysis should have a lower value than 1.50 as found here. In fact, his value, as indicated by the slope of the line in his Fig. 101, is 0.70; (this coefficient is, of course, not to be confused with the simple pan factor having a similar value).

3. Test of the pan conversion method

As an independent test, the pan conversion method is now applied over several short periods (8 to 14 days) for which estimates of the Lake Hefner evaporation by the heat budget method are available for comparison (Anderson[6]), Table 20). Four winter-spring periods are excluded here, as some pan data are missing and freezing is sometimes encountered.

For each period, the pan conversion method using Eq. (3) is applied to the individual days, and the total gives the estimated evaporation for the whole period. Details of the data used are as before, except that, to simulate the most convenient operational arrangement, e_4 is now evaluated at the pan site rather than over the lake, and all days are included (i.e. none are eliminated for reasons such as (i) to (iv) above).

In Table 1, the pan conversion results, and also the estimates obtained by applying a simple pan factor 0.7, are compared with the heat budget results; these comparisons are also reproduced in Fig. 2 (a, b). Most of the nine periods contain no more than one day which had already been used in the original analysis displayed in Fig. 1, and they therefore provide a virtually independent test.

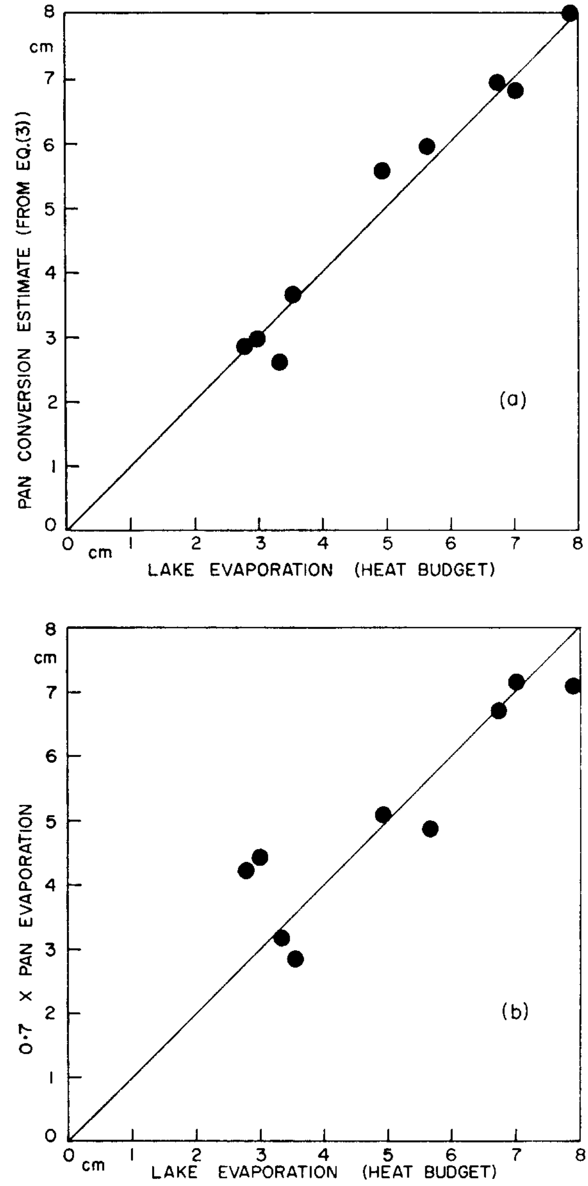

Fig. 2. Lake Hefner evaporation estimated over heat budget periods ranging from 8 to 14 days. In (a), the pan conversion estimates using Eq. (3) are plotted against the heat budget determinations, and in (b) the simple pan factor estimates are plotted against the heat budget determinations.

TABLE 1

Estimated evaporation over Lake Hefner heat budget periods

Commencing date	No. of days	No. of days also included in Fig. 1	Lake evaporation: Evaluated by heat budget (a) cm	Lake evaporation: Estimated by pan conversion Eq. (3) (b) cm	Lake evaporation: Estimated by applying simple pan factor 0.7 (c) cm	Relative departures: Pan conversion $\frac{(b)-(a)}{(a)}$ per cent	Relative departures: Pan factor $\frac{(c)-(a)}{(a)}$ per cent
14.ix.50	9	3	3.35	2.59	3.16	− 23	− 5.7
8.x.50	10	5	4.93	5.55	5.09	+ 13	+ 3.2
17.x.50	9	3	3.56	3.64	2.84	+ 2.2	− 20
3.iv.51	14	1	6.73	6.93	6.70	+ 3.0	− 0.4
16.iv.51	8	0	2.99	2.98	4.42	− 0.3	+ 48
12.vi.51	8	1	2.79	2.83	4.23	+ 1.4	+ 52
2.viii.51	9	1	7.90	8.01	7.08	+ 1.4	− 10
15.viii.51	8	0	5.66	5.94	4.87	+ 4.9	− 14
22.viii.51	9	1	7.02	6.81	7.14	− 3.0	+ 1.7
Total	84		44.93	45.28	45.53	+ 0.8	+ 1.3

The agreement between the pan conversion and the heat budget results is within 5 per cent for all except two of the periods, being consistent with the standard errors as estimated near the end of the preceding Section. By comparison, the pan factor estimates are in poorer agreement.

Over the whole 84 days, the total evaporation given by the pan conversion method differs by only 0.8 per cent from that evaluated by the heat budget. Thus, no further adjustment of the coefficient 1.50 in Eq. (3) is indicated.

4. Possible refinements

From the smallness of the scatter in Fig. 2 (a), it would seem unlikely that further useful refinement could be made at this stage. Several possibilities have been examined, and the findings are, in fact, essentially negative. Brief details are given below. In each case, the possibility of a systematic contribution to the scatter in Fig. 1 (a) has been examined by plotting the ratio E_{WB}/E_{PC} against the quantity concerned. Here E_{WB} and E_{PC} denote, respectively, the daily water-budget evaluation and the pan-conversion estimate (Eq. (3)) of the lake evaporation.

4.1. DAILY MEAN WIND

Some relationship of E_{WB}/E_{PC} with wind strength might be anticipated, since it is possible that in Eq. (1) the dependence on wind is incompletely represented by the simple proportionality, particularly in the case of pan evaporation. Taking U_4 as the daily average 09–09 h at the barge station, there is found to be, though with considerable scatter, some degree of association which can be approximated by

$$\frac{E_{WB}}{E_{PC}} = \frac{1.57\,U_4}{U_4 + 2.4} \tag{4}$$

where U_4 is in m sec^{-1}.*

When the comparison is repeated with U_4 now taken as the daily maximum of the 3-h averages, the relationship is somewhat weakened. When U_4 is taken as the daily mean over 00–24 h instead of 09–09 h, relationship is again somewhat weakened, and this latter suggests that any implied departure from the simple proportionality with U in Eq. (1) is for pan evaporation (which was measured over 09–09 h) rather than for lake evaporation (which was measured over 00–24 h). On the contrary, however, it is found that when U_4 is taken as the afternoon average (noon to 18 h) at the pan site, the relationship, far from being enhanced as one might then expect, is practically extinguished.

Moreover, when the influence of wind according to Eq. (4) is incorporated into the test reproduced in Fig. 2(a), the scatter there becomes somewhat greater, rather than less. From all these considerations, it is doubtful whether a relationship like Eq. (4) is of appreciable significance, though it is possible that more extensive investigation could prove worthwhile.

*) If the lake evaporation is assumed to be given by the Lake Hefner bulk formula (Eq. (58) of Marciano and Harbeck[7]), which, converted for 4 m height, is

$$E_L = 1.39 \times 10^{-3}\, U_4\, (e_L - e_4),$$

then Eq. (4) is equivalent to expressing the pan evaporation by

$$E_P = 0.59 \times 10^{-3}\, (U_4 + 2.4)\, (e_P - e_4).$$

Here the units are E in cm per 3 h, U in m sec^{-1}, and e in mB. In the same units, the class A pan formula given by Kohler[2]) (his Table 26) can be written approximately (basing the height conversion on an assumed roughness parameter of 1 cm for the grass surface surrounding the pan – *cf.* Kohler's Fig. 94) $E_P \approx 1.19 \times 10^{-3}\, (U_4 + 0.88)\, (e_P - e_4)$. It is apparent that the multiplier of $(e_P - e_4)$ is, at all ordinary wind speeds, greater in the latter formula for E_P than in the former; this is reasonable, since e_P is the daily average in Kohler's formula, rather than the daily maximum as in the present paper.

4.2. THERMAL STABILITY OF THE ATMOSPHERE

As discussed by Deacon and Webb[11]), a suitable measure of thermal stability is the "bulk Richardson number" Rb, which is proportional to $(T_4—T_L)/U_4^2$ over the lake or to $(T_4—T_P)/U_4^2$ over the pan; here T denotes temperature and the subscripts have the same meaning as before. When the daily values of E_{WB}/E_{PC} are compared with afternoon values of Rb for the lake and Rb for the pan, a small degree of association is evident in both cases, in the sense as if lake evaporation tends to be smaller with strongly stable stratification over the lake and pan evaporation tends to be larger with unstable stratification over the pan. However, when either one or both of these associations are incorporated into the test represented in Fig. 2(a), the scatter there again increases rather than decreases.

If Rb for the lake is taken not in the afternoon, but at the time of maximum wind strength each day (when the lake evaporation is likely to be greatest), the association disappears. When the test of Fig. 2(a) is repeated incorporating the relationship with wind (Eq. (4)), as well as that with the afternoon value of Rb for the lake, or Rb for the pan, or both, then again the scatter increases.

It is probably not worthwhile to pursue thermal stability relationships any further. Apart from the considerations outlined above, the application of any comprehensive relationship with thermal stability would require its measurement several times throughout the daily cycle, and this would be far too elaborate as an adjunct to pan observations.

4.3. MAXIMUM PAN WATER TEMPERATURE

In Eq. (3), e_P is determined from only the maximum temperature of the pan water; therefore, a factor which could possibly be relevant is the magnitude of the pan water temperature rise during the afternoon, which may vary from day to day, depending on the solar radiation and the wind strength. However, no significant relationship is found when the daily E_{WB}/E_{PC} is plotted against either $(T_P—T_D)$, $(T_P—T_W)$, or $(T_P—T_{min})$; here T_D and T_W are the afternoon average 4 m dry- and wet-bulb temperatures, and T_{min} is the minimum temperature of the pan water the same morning.

4.4. SOLAR RADIATION

From a similar point of view, E_{WB}/E_{PC} has been compared with the daily total solar radiation. There is no apparent relationship, irrespective of whether or not U_4 (average noon to 18 h at the pan site) is also taken into account.

5. Concluding remarks

It is concluded that the pan conversion method, using Eq. (3), can provide an estimate of daily lake evaporation from nearby Class A pan observations. With observations of reasonably high quality, the standard error of monthly total estimated evaporation is expected to be less than 10 per cent.

The coefficient 1.50 is established from Lake Hefner data, within an estimated standard error of 5 per cent. In principle, the same value should remain valid for most lakes, irrespective of altitude or ambient temperature, if a Class A pan is used in an exposure comparable with that at Lake Hefner. However, in the case of appreciably smaller lakes, some increase may be anticipated, in accordance with the trend of the bulk aerodynamic coefficient as found by Harbeck[8]). In the case of large lakes (width greater than 5 km say), some increase in the coefficient of Eq. (3) may again be anticipated, on account of the increased wind strength over the lake; this effect would, however, be reduced by the rise in vapour pressure e_4 over the lake, assuming the measured values of e_4 at the pan site to be unaffected by the lake.

Only simple measurements are needed. In addition to the morning pan observation, including maximum water temperature, the afternoon averages of vapour pressure at a height of about 4 m above the ground, and of lake water temperature, are required.

For the measurement of vapour pressure, a thermohygrograph of high quality would be suitable. Alternatively, even an Assman or whirling psychrometer observation once or twice each afternoon should suffice. Again, for lake water temperature it should suffice to make one or two measurements each afternoon with a thermometer held in a dip bucket. The lake measurement should, of course, be made well out from the shore, particularly if on the windward side where upwelling may bring colder water to the surface.

No useful further refinements to the pan conversion relationship have been found here, though it is possible that an association with daily mean wind speed could be worth further examination.

Acknowledgments

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