Introduction

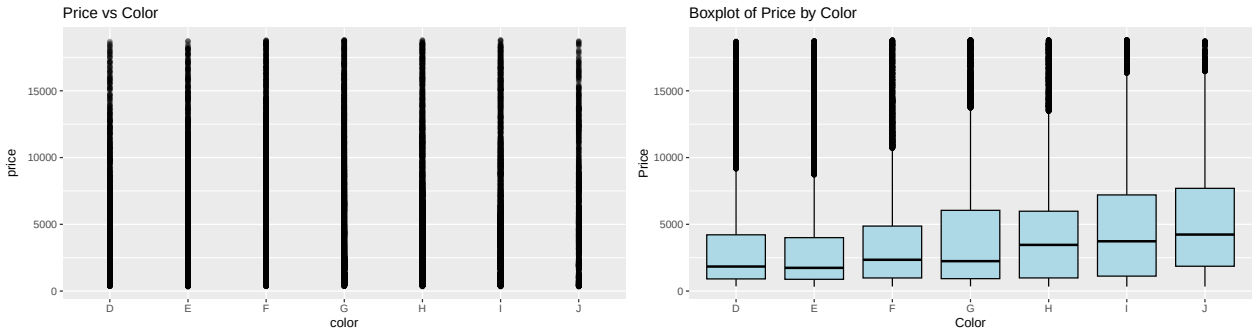
When considering a diamond's value, many consumers rely on a grading system introduced by the Gemological Institute of America (n.d.) called the "4Cs": carat, cut, clarity, and colour. This system was designed to help consumers of the diamond market understand the value of their purchases based on the values assigned to each of these four traits. While information about factors that influence diamond pricing are publicly available, the numeric impact of each factor is unclear (Lee et al., 2014; Mamonov & Triantoro, 2018). For instance, two diamonds of the same weight can differ vastly in price if they have different clarity levels (Tiffany & Co., n.d.), but the explanation behind this variance has not been rigorously quantified based on our literature research. Thus, our research addresses the question: To what extent do carat, colour, clarity, table size, and length predict the price of a diamond? While previous studies have conducted similar analyses (Mamonov & Triantoro, 2018; Özmen, 2024), our goal is to later refine our model in hopes of explaining more of the observed price variation in our data. Quantifying the effects that different features have on diamond prices can better inform consumers' judgments on whether they are paying a fair price.

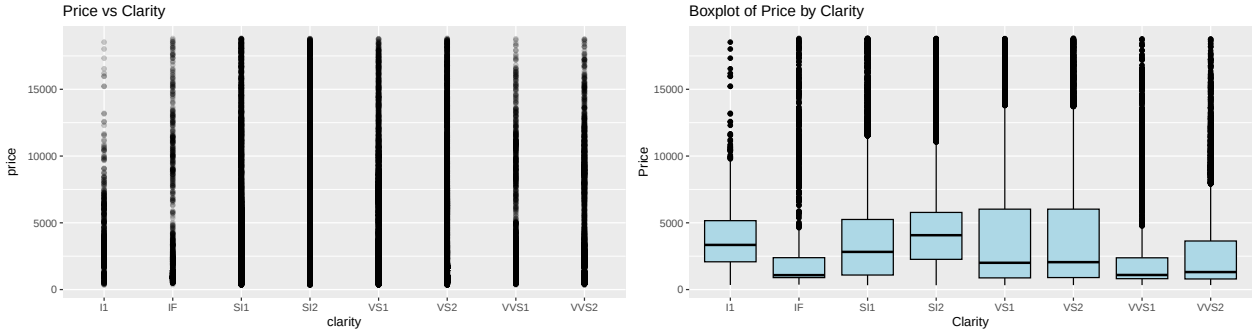
Multiple linear regression (MLR) is a suitable method to answer our question, because we wish to explore the association between price, a continuous response variable, with several explanatory variables simultaneously. This way, we can infer which factors are most influential on the response. MLR models the (estimated) average of the response versus given values of the predictors, which is relevant to our goal of helping consumers see where the price of their diamond lies in relation to the average price for their specific diamond profile. The focus of the model will be interpretability since our main objective is to draw conclusions about the main effects of our chosen explanatory variables, rather than stress predictive accuracy of the response.

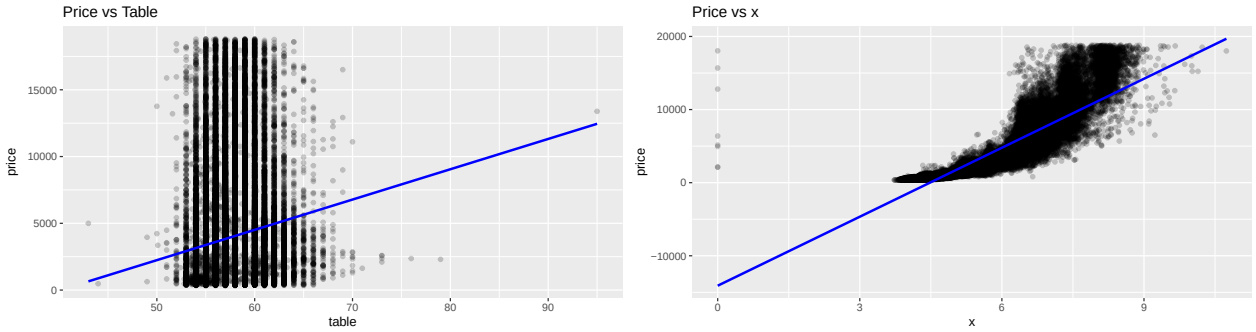
Data Description

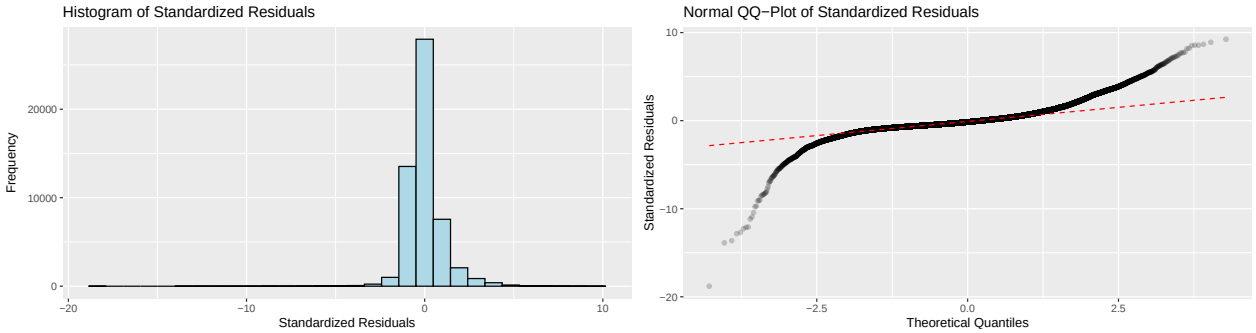
summarize numerically or graphically (in a single figure/table) each predictor in your dataset that will be used in the preliminary model, and interpret the descrip8ve sta8s8cs in the context of what the predictors measure and how it relates to the research ques8on


Residuals vs Fitted Values
Residuals
Fitted Values
Price vs Fitted Values
Actual Price
Fitted Values
Price vs Carat
price
carat


Price vs Color
price
color
Boxplot of Price by Color
Price
Color


Price vs Clarity
price
clarity
Boxplot of Price by Clarity
Price
Clarity


Price vs Table
price
table
Price vs x
price
x


Histogram of Standardized Residuals
Frequency
Standardized Residuals
Normal QQ−Plot of Standardized Residuals
Standardized Residuals
Theoretical Quantiles

Preliminary Model Results

$$
\begin{aligned}
y = \beta_0 &+ \beta_1 \cdot \text{carat} + \beta_2 \cdot I(\text{color} = E) + \beta_3 \cdot I(\text{color} = F) + \beta_4 \cdot I(\text{color} = G) + \beta_5 \cdot I(\text{color} = H) \\
&+ \beta_6 \cdot I(\text{color} = I) + \beta_7 \cdot I(\text{color} = J) + \beta_8 \cdot I(\text{clarity} = \text{IF}) + \beta_9 \cdot I(\text{clarity} = \text{SI1}) \\
&+ \beta_{10} \cdot I(\text{clarity} = \text{SI2}) + \beta_{11} \cdot I(\text{clarity} = \text{VS1}) + \beta_{12} \cdot I(\text{clarity} = \text{VS2}) \\
&+ \beta_{13} \cdot I(\text{clarity} = \text{VVS1}) + \beta_{14} \cdot I(\text{clarity} = \text{VVS2}) + \beta_{15} \cdot \text{table} + \beta_{16} \cdot x + \epsilon
\end{aligned}
$$

$$
\text{Assume } \mathbb{E}[\epsilon] = 0, \mathbb{V}[\epsilon] = \sigma^2, \text{and } \epsilon \overset{\text{i.i.d.}}{\sim} N(0, \sigma^2)
$$

$$
\begin{aligned}
\hat{E}[y] = \hat{y} = \ &\hat{\beta}_0 + \hat{\beta}_1 \cdot \text{carat} + \hat{\beta}_2 \cdot I(\text{color} = E) + \hat{\beta}_3 \cdot I(\text{color} = F) + \hat{\beta}_4 \cdot I(\text{color} = G) + \hat{\beta}_5 \cdot I(\text{color} = H) \\
&+ \hat{\beta}_6 \cdot I(\text{color} = I) + \hat{\beta}_7 \cdot I(\text{color} = J) + \hat{\beta}_8 \cdot I(\text{clarity} = \text{IF}) + \hat{\beta}_9 \cdot I(\text{clarity} = \text{SI1}) \\
&+ \hat{\beta}_{10} \cdot I(\text{clarity} = \text{SI2}) + \hat{\beta}_{11} \cdot I(\text{clarity} = \text{VS1}) + \hat{\beta}_{12} \cdot I(\text{clarity} = \text{VS2}) \\
&+ \hat{\beta}_{13} \cdot I(\text{clarity} = \text{VVS1}) + \hat{\beta}_{14} \cdot I(\text{clarity} = \text{VVS2}) + \hat{\beta}_{15} \cdot \text{table} + \hat{\beta}_{16} \cdot x
\end{aligned}
$$

Table 1: Numerical Summary of Coefficient Estimates

Term	Estimate
(Intercept)	-1388.839
carat	10945.695
colorE	-210.489
colorF	-286.752
colorG	-493.858
colorH	-993.809
colorI	-1474.570
colorJ	-2387.941
clarityIF	5665.892
claritySI1	3894.266
claritySI2	2926.092
clarityVS1	4845.059
clarityVS2	4528.148
clarityVVS1	5309.983
clarityVVS2	5234.228
table	-32.708
x	-900.270

The residual analysis revealed significant violations of core regression model assumptions: linearity, constant error variance (homoscedasticity), and normality of errors. The **Residuals vs Fitted Values** plot showed a clear, curved LOESS trendline with increasing spread as fitted values increased, suggesting non-linearity and heteroscedasticity.

Similarly, the **Price vs Fitted Values** plot showed a generally strong positive relationship, but with slight curvature and growing variance at higher price levels, this causes concern about unequal error variance—indicating potential non-linearity and heteroscedasticity. The **Price vs Carat** scatterplot suggested an exponential relationship that a smile linear model fails to capture reasonably.

At first glance, categorical variables **color** and **clarity** don't appear to be strong predictors of price due to seemingly constant spread of points across levels. However, the boxplots reveal significant stratified effects on **price**, confirming that these variables do show variation.

For **color**, although the maximum prices are similar across all levels, the median prices differ, with diamonds in **color** J having higher median prices, and D showing more high outliers.

Diamonds with **clarity** I1 show outliers and higher median prices, whereas those with IF appear to have more outliers and lower medians. This stratification reinforces the idea that both are valid categorical predictors for diamond price modeling.

However, the **Price vs Table** plot showed a weak and noisy association, with data points highly concentrated in a narrow range, suggesting that the table predictor may not be a strong predictor to predict price. The **Price vs x** plot also showed curvature, suggesting non-linearity.

Furthermore, the **histogram of standardized residuals** sharply peaked around 0 with heavy tails, and the **normal QQ-plot** confirmed it as it showed deviations at both tails, indicating violations of the normality assumption.

From model estimates, a diamond with **color** D, **clarity** I1, and 0 values for **table**, **x**, and **carat**, is predicted to have an average price of -$1,388.389. Though this scenario isn't practically meaningful, it serves as the model's (baseline) reference level. When the diamond's **color** is E and **clarity** is IF, with the same 0 values for the continuous variables, the (estimated) average price increases to $4,066.564, reflecting the strong positive contribution of high **clarity** IF to the overall **price**.

Moreover, a 1-gram increase in **carat**—with the same values as the previous example—raises the predicted average price by $10,945.695 per gram of **carat**, confirming that **carat** is the most influential predictor.

Interestingly, **table** and **x** are both negatively associated with **price**, with coefficients of −$32.71 and −$900.27, respectively. This is counterintuitive as larger dimensions are generally more desirable. **x** likely reflects multicollinearity with **carat**, since both capture size-related information. This redundancy suggests the need to reconsider variable inclusion—possibility removing **x** or replacing with a composite variable like volume.

Bibliography