STA302 Final Project Part 1: Project Proposal

[Research Question]

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Introduction

When considering a diamond's value, many consumers rely on a grading system introduced by the Gemological Institute of America (n.d.) called the "4Cs": carat, cut, clarity, and colour. This system was designed to help consumers of the diamond market understand the value of their purchases based on the values assigned to each of these four traits. While information about factors that influence diamond pricing is publicly available, the numeric impact of each factor is unclear (Lee et al., 2014; Mamonov & Triantoro, 2018). For instance, two diamonds of the same weight can differ vastly in price after accounting for cut, clarity, and colour (Tiffany & Co., n.d.), but the explanation behind this variance has not been rigorously quantified based on our literature research. Thus, our research addresses the question: To what extent do carat, colour, clarity, table size, and length predict the price of a diamond? While previous studies have conducted similar analyses (Mamonov & Triantoro, 2018; Özmen, 2024), our goal is to later refine our model in hopes of explaining more of the observed price variation in our data. Quantifying the effects that different features have on diamond prices can better inform consumers' judgments on whether they are paying a fair price.

Multiple linear regression (MLR) is a suitable method to answer our question because we are exploring the association between price, a continuous response variable, with several explanatory variables simultaneously. This way, we can infer which diamond factors are the strongest predictors of price. MLR models the (estimated) average of the response versus given values of the predictors, which is relevant to our goal of helping consumers understand where the price of their diamond lies in relation to the estimated average price for their specific diamond profile. The focus of the model will be on interpretability since we are more interested in drawing conclusions about the average effects of our chosen diamond attributes on the price, rather than trying to predict exact prices of all diamonds.

Exploratory Data Analysis (EDA)

The selected dataset was retrieved and downloaded from Kaggle, a data science platform, and was uploaded by a university instructor for their students to practice data analysis skills. There was no explicit research question cited on the Kaggle posting, and our research question was constructed after a careful review of the dataset and the corresponding literature surrounding diamond pricing. Further investigation of the dataset revealed that the data was originally published in the ggplot2 R package. In particular, the diamond dataset was collected and curated by Wickham et al., the creators and contributors of ggplot2, using the Loose Diamond Search Engine, which contains historical data on diamond prices and descriptions of their various physical attributes.

Response Variable Summary

Table 1: Numerical Summary of Response Variable

Statistic	Value
Minimum	326.000
1st Quartile	950.000
Median	2401.000
Mode	605.000
Mean	3933.000
Standard Deviation	3989.338
3rd Quartile	5324.000
Maximum	18823.000

Predictor Variable Summaries

Table 2: Numerical Summary of Quantitative Predictor Variables

Statistic	Carat	Table	x
Minimum	0.2000	43.0000	0.0000
1st Quartile	0.4000	56.0000	4.7100
Median	0.7000	57.0000	5.7000
Mode	0.3000	56.0000	4.3700
Mean	0.7979	57.4573	5.7312
Standard Deviation	0.4740	2.2345	1.1217
3rd Quartile	1.0400	59.0000	6.5400
Maximum	5.0100	95.0000	10.7400

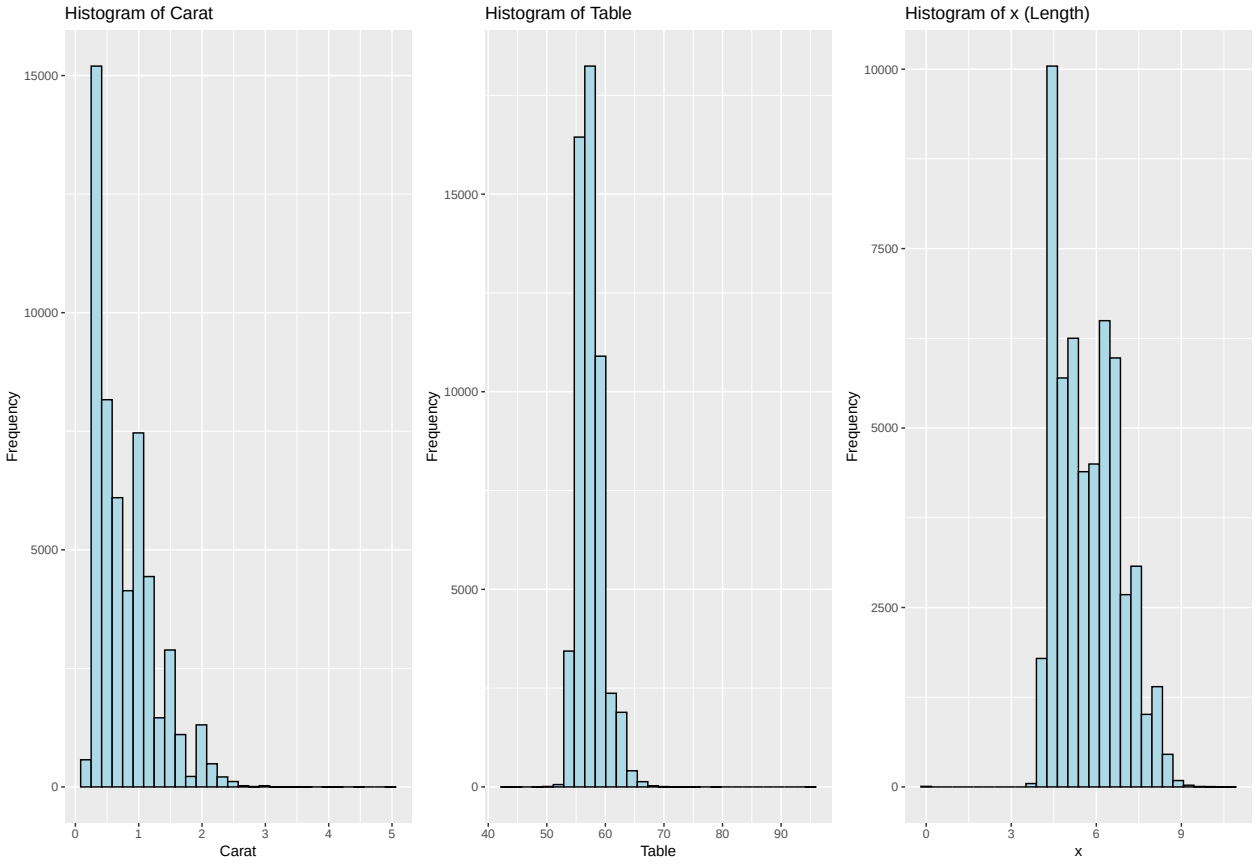
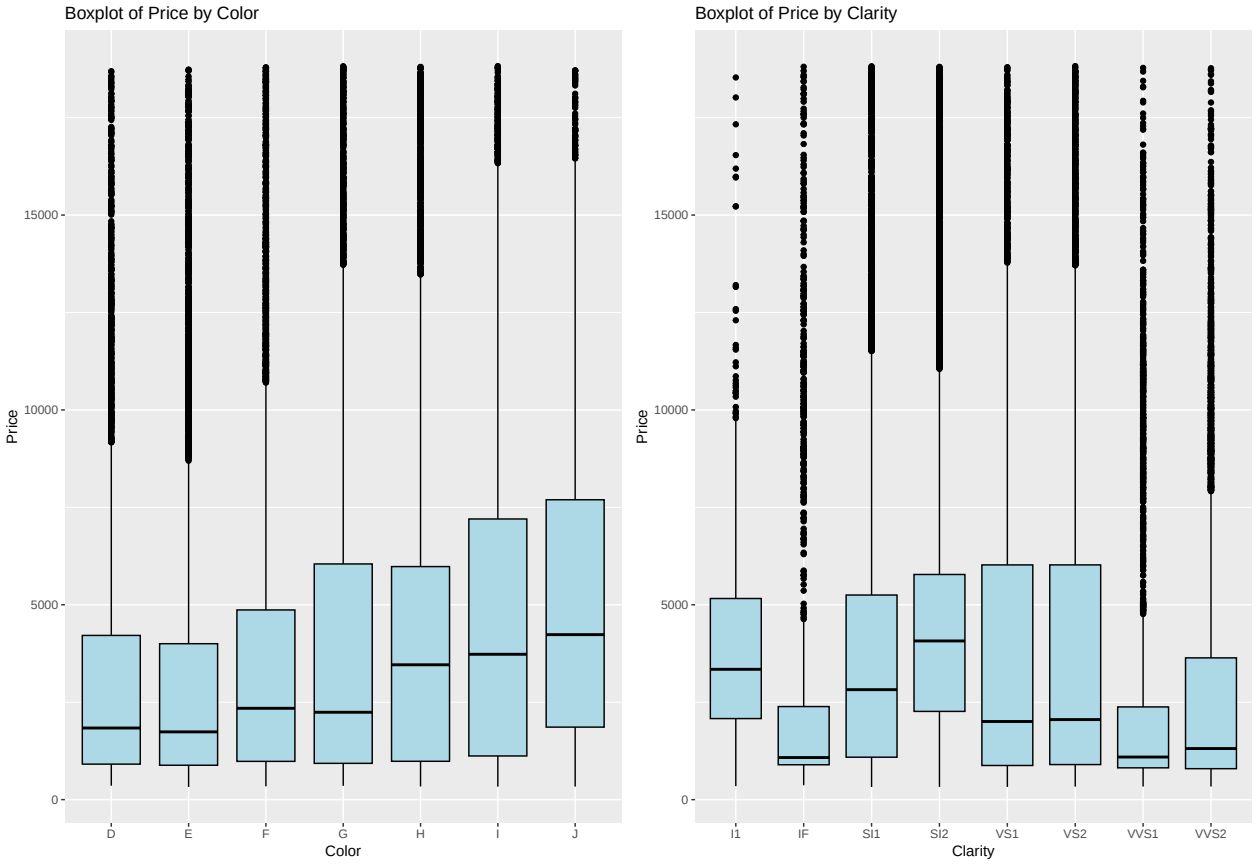


Table 3: Numerical Summary of 'Color' Predictor

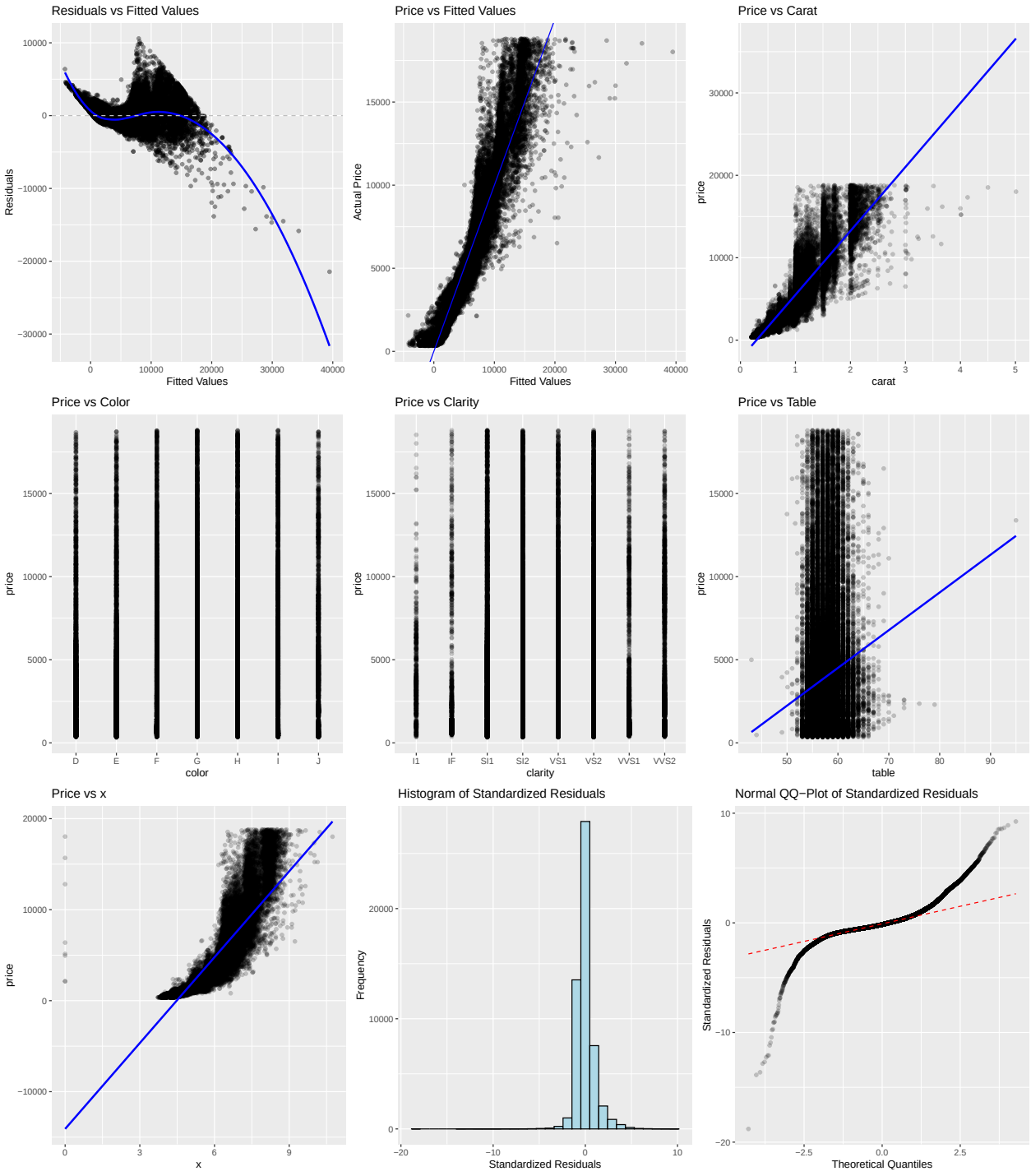
Color	Frequency	Percentage
D	6775	12.560
E	9799	18.165
F	9543	17.691
G	11292	20.933
H	8304	15.394
I	5422	10.051
J	2808	5.205

Table 4: Numerical Summary of 'Clarity' Predictor

Clarity	Frequency	Percentage
I1	741	1.374
IF	1790	3.318
SI1	13067	24.224
SI2	9194	17.044
VS1	8171	15.147
VS2	12259	22.726
VVS1	3655	6.776
VVS2	5066	9.391



Preliminary Model Results



$$
\begin{aligned}
y = \beta_0 &+ \beta_1 \cdot \text{carat} + \beta_2 \cdot I(\text{color} = E) + \beta_3 \cdot I(\text{color} = F) + \beta_4 \cdot I(\text{color} = G) + \beta_5 \cdot I(\text{color} = H) \\
&+ \beta_6 \cdot I(\text{color} = I) + \beta_7 \cdot I(\text{color} = J) + \beta_8 \cdot I(\text{clarity} = \text{IF}) + \beta_9 \cdot I(\text{clarity} = \text{SI1}) \\
&+ \beta_{10} \cdot I(\text{clarity} = \text{SI2}) + \beta_{11} \cdot I(\text{clarity} = \text{VS1}) + \beta_{12} \cdot I(\text{clarity} = \text{VS2}) \\
&+ \beta_{13} \cdot I(\text{clarity} = \text{VVS1}) + \beta_{14} \cdot I(\text{clarity} = \text{VVS2}) + \beta_{15} \cdot \text{table} + \beta_{16} \cdot x + \epsilon
\end{aligned}
$$

Assume $\mathbb{E}[\epsilon] = 0, \mathbb{V}[\epsilon] = \sigma^2$, and $\epsilon \overset{\text{i.i.d.}}{\sim} N(0, \sigma^2)$

$$
\begin{aligned}
\hat{E}[y] = \hat{y} = \ &\hat{\beta}_0 + \hat{\beta}_1 \cdot \text{carat} + \hat{\beta}_2 \cdot I(\text{color} = E) + \hat{\beta}_3 \cdot I(\text{color} = F) + \hat{\beta}_4 \cdot I(\text{color} = G) + \hat{\beta}_5 \cdot I(\text{color} = H) \\
&+ \hat{\beta}_6 \cdot I(\text{color} = I) + \hat{\beta}_7 \cdot I(\text{color} = J) + \hat{\beta}_8 \cdot I(\text{clarity} = \text{IF}) + \hat{\beta}_9 \cdot I(\text{clarity} = \text{SI1}) \\
&+ \hat{\beta}_{10} \cdot I(\text{clarity} = \text{SI2}) + \hat{\beta}_{11} \cdot I(\text{clarity} = \text{VS1}) + \hat{\beta}_{12} \cdot I(\text{clarity} = \text{VS2}) \\
&+ \hat{\beta}_{13} \cdot I(\text{clarity} = \text{VVS1}) + \hat{\beta}_{14} \cdot I(\text{clarity} = \text{VVS2}) + \hat{\beta}_{15} \cdot \text{table} + \hat{\beta}_{16} \cdot x
\end{aligned}
$$

Table 5: Numerical Summary of Coefficient Estimates

Term	Estimate
(Intercept)	-1388.839
carat	10945.695
colorE	-210.489
colorF	-286.752
colorG	-493.858
colorH	-993.809
colorI	-1474.570
colorJ	-2387.941
clarityIF	5665.892
claritySI1	3894.266
claritySI2	2926.092
clarityVS1	4845.059
clarityVS2	4528.148
clarityVVS1	5309.983
clarityVVS2	5234.228
table	-32.708
x	-900.270

Residual Analysis

Curvature in the **Residuals vs. Fitted Values** and **Price vs. Fitted Values** plots, and exponential trend in the **Price vs Carat** plot suggests non-linearity. Spread increasing with fitted values indicates heteroscedasticity.

color and **clarity** don't appear as strong predictors of **price** due to visible constant spread of points across levels. However, boxplots—shown in **EDA**—reveal stratified effects on price—confirming their contribution to **price** variation.

Although maximum prices are similar across levels of **color**, median prices differ. **color** J shows higher medians while D shows higher outliers.

Diamonds with **clarity** I1 show outliers and higher median prices, whereas **clarity** IF shows more outliers and lower medians. This stratification supports both being valid diamond price predictors.

The **Price vs Table** plot showed a weak, noisy association with data points concentrated in a narrow range, suggesting **table** size may not strongly predict price. The **Price vs x** plot also curves, suggesting non-linearity.

The **histogram of standardized residuals** sharply peaked around 0 with heavy tails, and **normal QQ-plot** showed deviations at both tails—violating normality of errors.

From the model estimates, a diamond with **color** D, **clarity** I1, and 0 for **table**, **x**, and **carat**, predicted an average **price** of -$1,388.389. Though not practically meaningful, it serves as model's baseline.
Keeping the same values for continuous predictors with **color** E and **clarity** IF, the estimated average **price** increases to $4,066.564, reflecting strong contribution of high **clarity**.

With same values, a 1-gram increase in **carat** raises predicted average price by $10,945.695/**carat**, confirming **carat** as the strongest predictor.

Interestingly, **table** and **x** are negatively associated with **price**: coefficients −$32.71 and −$900.27, respectively. This is counterintuitive as larger dimensions are generally desirable. **x** likely reflects multicollinearity with **carat**; both capture size-related information, suggesting we can remove/replace **x**.

Bibliography

Al Aswad, Ms. N. (2022, July 9). *Diamonds Prices*. Kaggle. https://www.kaggle.com/datasets/nancyalaswad90/diamonds-prices

Diamond Color Scale | Color Chart, Scale, & Grading Guide. Brilliant Earth. (n.d.). https://www.brilliantearth.com/en-ca/diamond/buying-guide/color/?utm_source=google&utm_medium=cpc&utm_campaign=SEM_Search_CA001_Bridal_Conversion&gad_campaignid=14386530662

Diamond Clarity Guide | Clarity Chart, scale, & best grades. Brilliant Earth. (n.d.). https://www.brilliantearth.com/en-ca/diamond/buying-guide/clarity/

Diamond Prices Comparison. Loose Diamonds Search Engine. (n.d.). https://www.diamondse.info/diamond-prices.asp

Gemological Institute of America. (n.d.). *4Cs of diamond quality*. Retrieved May 20, 2025, from https://4cs.gia.edu/en-us/4cs-of-diamond-quality/

Lee, J. A., Caudill, S. B., & Mixon, F. G. (2014). Shine bright like a diamond: A hedonic model of grading and pricing an experience good. *Applied Economics, 46*(16), 1829–1838. https://doi.org/10.1080/00036846.2014.884707

Mamonov, S., & Triantoro, T. (2018). Subjectivity of diamond prices in online retail: Insights from a data mining study. *Journal of Theoretical and Applied Electronic Commerce Research, 13*(2), 15–28. https://doi.org/10.4067/s0718-18762018000200103

Özmen, G. (2024, August 12). *More than carat: A comprehensive regression analysis for diamond price prediction*. Medium. https://medium.com/@gizemzmen/more-than-carat-a-comprehensive-regression-analysis-for-diamond-price-prediction-b9222f337154

Tiffany & Co. (n.d.). *What Is A Diamond Carat?* Retrieved May 20, 2025, from https://www.tiffany.ca/engagement/the-tiffany-guide-to-diamonds/carat/

Wickham, H., van den Brand, T., Dunnington, D., Yutani, H., Woo, K., Wilke, C., Takahashi, K., Pedersen, T. L., Henry, L., & Chang, W. (n.d.). *Prices of Over 50,000 Round Cut Diamonds*. ggplot2. https://ggplot2.tidyverse.org/reference/diamonds.html#ref-usage