UNIVERSIDAD METROPOLITANA DE HONDURAS
VICERRECTORÍA DE PREGRADO
INGENIERÍA INFORMÁTICA

ASIGNATURA

Calculo II

TRABAJO

Guía 1 Parte A

NOMBRE DEL DOCENTE

Ing. Juan Ramón Zuniga Zuniga.

PRESENTADO POR:

Yoleth Carolina Gutiérrez Rivera 201901409

TOCOA, COLÓN **HONDURAS C.A**

OCTUBRE 2021

Guía de trabajo

Resuelva las siguientes integrales aplicando la técnica de

Integración apropiada.

$$\int xe^{3x}dx$$

$$\int (x+1)e^{2x}dx$$

$$\int e^{\sqrt{x}}dx$$

$$\int sen^3(x)dx$$

$$\int \cos^6(x)dx$$

$$\int x^2\sqrt{9-x^2}\,dx$$

7. $\int \frac{x^2}{\sqrt{9-x^2}}dx$

8. $\int \frac{dx}{(x-1)(x+3)}$

9. $\int \frac{dx}{x^4-1}$

10. $\int \frac{x^5}{(x^2+4)^2}dx$

Ejercicio # 1

$$\int xe^{3x}\,dx$$

$u = x \qquad \int dv = \int e^{3x}\,dx$

$du = dx \qquad v = 1/3\, e^{3x}$

$= uv - \int v\,du$

$= 1/3\, xe^{3x} - 1/3 \int e^{3x}\,dx$

$= 1/3\, xe^{3x} - 1/3\,(1/3\, e^{3x}) + c$

$= 1/3\, xe^{3x} - 1/9\, e^{3x} + c$

Ejercicio # 2

$$\int (x+1)\, e^{2x}\,dx$$

$u = x+1 \qquad \int dv = \int e^{2}x$

$du = dx \qquad v = 1/2\, e^{2}x$

$1/2\,(x+1)e^{2x} - 1/2 \int e^{2x}\,dx$

$= 1/2\,(x+1)\, e^{2x} - 1/2\,(1/2\, e^{2x}) + c$

$= 1/2\,(x+1)\, e^{2x} - 1/4\, e^{2x} + c$

Ejercicio #3

$\int e^{\sqrt{x}} dx$ $\quad u = t \quad \int dv = \int e^{t} dt$

$\int e^{x^{1/2}} dx$ $\quad du = dt \quad v = e^{t}$

$t = x^{1/2}$

$dt = \frac{1}{2} x^{-1/2} dx$

$\frac{dt}{dx} = \frac{1}{2x^{1/2}}$

$dx = 2x^{1/2} dt$

$dx = 2t\, dt$

$2\int t e^{t}$

$= 2te^{t} - 2\int e^{t} dt$

$= 2te^{t} - 2e^{t} + c$

$= 2x^{1/2} e^{x^{1/2}} - 2e^{x^{1/2}} + c$

$= 2\sqrt{x}\, e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$

Ejercicio # 4

$$\int sen^3(x)\,dx$$

$u = \cos x$

$du = -sen\,x$

$-du = sen\,x\,dx$

$$\int (sen^2 x)(sen\,x)\,dx$$

$$\int (1-\cos^2 x)\,sen\,x\,dx$$

$$\int sen\,x\,dx - \int \cos^2 x\,sen\,x\,dx$$

$$\int sen\,x\,dx + \int u^2\,du$$

$$-\cos x + 1/3\,u^3 + c$$

$$-\cos x + 1/3\,\cos^3 x + c$$

Ejercicio # 5

$$\int \cos^6(x)\,dx$$

$$\int \cos^6 x = \int (\cos^2 x)^3 \qquad \cos^2 x = \left(\frac{1+\cos 2x}{2}\right)$$

$$= \left(\frac{1+\cos 2x}{2}\right)^3$$

$$= \tfrac{1}{8}\int \left((1^3 + 3(1))^2 \cos 2x + 3(1)\cos^2 2x + \cos^3 2x \right) dx$$

$$= \tfrac{1}{8}\int (1 + 3\cos 2x + 3\cos^2 2x + \cos^3 2x)\,dx$$

$$= \tfrac{1}{8}\int dx + \tfrac{3}{8}\int \cos 2x\,dx + \tfrac{3}{8}\int \cos^2 2x\,dx + \tfrac{1}{8}\int \cos^3 2x\,dx$$

$$= \tfrac{1}{8}\int dx + \tfrac{3}{8}\int \cos 2x\,dx + \tfrac{3}{8}\int \left(\frac{1+\cos 4x}{2}\right) dx + \tfrac{1}{8}\int \cos 2x \cos^2 2x\,dx$$

$$\tfrac{1}{8}\int dx + \tfrac{3}{8}\int \cos 2x\,dx + \tfrac{3}{16}\int dx + \tfrac{3}{16}\int \cos 4x\,dx + \tfrac{1}{8}\int \cos 2x(1 - \operatorname{sen}^2 2x)\,dx$$

$$\tfrac{5}{16}\int dx + \tfrac{3}{8}\int \cos 2x\,dx + \tfrac{3}{16}\int \cos 4x\,dx\ \tfrac{1}{8}\int \cos 2x\,dx - \tfrac{1}{8}\int \operatorname{sen}^2 2x \cos 2x\,dx$$

$$\tfrac{5}{16}\int dx + \tfrac{1}{4}\int \cos u\,du + \tfrac{3}{64}\int \cos z\,dz - \tfrac{1}{16}\int \operatorname{sen}^2 u \cos u\,du$$

$$\tfrac{5}{16}x + \tfrac{1}{4}\operatorname{sen} u + \tfrac{3}{64}\operatorname{sen} z - \tfrac{1}{6}\int v^2\,dv$$

$$\tfrac{5}{16}x + \tfrac{1}{4}\operatorname{sen} u + \tfrac{3}{64}\operatorname{sen} z - \tfrac{1}{48}v^3 + C$$

$$\tfrac{5}{16}x + \tfrac{1}{4}\operatorname{sen} 2x + \tfrac{3}{64}\operatorname{sen} 4x - \tfrac{1}{48}\operatorname{sen}^3 2x + C$$

$u = 2x \qquad z = 4x$

$du = 2\,dx \qquad dz = 4\,dx \qquad v = \operatorname{sen} u$

$\frac{du}{2} = dx \qquad \frac{dz}{4} = dx \qquad dv = \cos u\,du$

Ejercicio # 6

$$\int x^2 \sqrt{9-x^2}\,dx$$

$sen^2 t = 1 - cos^2 t/2$ sen

$cos^2 = 1 + cos\,2\,t/2$

$sen^2 t = 1 - cos^2 t$

$x = 3\,sen\,t$

$dx = 3\cos t\,dt$

$$\int (3\,sen\,t)^2 \sqrt{9-(3\,sen\,t)^2}\;3\cos t\,dt$$

$$\int 9\,sen^2 t\sqrt{9-9\,sen^2 t}\,(3\cos t\,dt)$$

$$\int 27\,sen^2 t\sqrt{9(1-sen^2 t)}\,\cos t\,dt)$$

$$27\int sen^2 t\cos t\sqrt{9\cos^2 t}\,dt$$

$$27\int sen^2 t\cos t\,(3\cos)\,dt$$

$$81\int sen^2 t\cos^2 t\,dt$$

$$81\int (1-\cos 2t/2)(1+\cos 2t/2)$$

$$81\int \frac{1-\cos^2 2t\,dt)}{4}$$

$$\frac{81}{4}\int (1-\cos^2 2t)\,dt$$

$$81/4\int sen^2\,2t\,dt$$

$$81/4\int \left(\frac{1-\cos 4t}{2}\right)dt$$

$$81/8\int dt - 81/8\int \cos 4t\,dt$$

$$81/8\,t - 1/4\left(\frac{81}{8}\right) sen\,4t + c = 81/8\,t - 81/32\,sen\,4t + c$$

$$81/8\,sen^{-1}\left(\frac{x}{3}\right) - \frac{81}{32}\,sen\left(4\,sen^{-1}\left(\frac{x}{3}\right)\right) + c$$

$x = 3\,sen\,t$ $\qquad t = sen^{-1}\left(\frac{x}{3}\right)$

$\frac{x}{3} = sen\,t$

Ejercicio # 7

$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx \qquad \begin{aligned} x &= 3\,\text{sen}\,t \\ dx &= 3\cos t\,dt \end{aligned}$$

$$\int \frac{(3\,\text{sen}\,t)^2\;3\cos t\,dt}{\sqrt{9-(3\,\text{sen}\,t)^2}}$$

$$\int \frac{9\,\text{sen}^2 t\;3\cos t\,dt}{\sqrt{9-9\,\text{sen}^2 t}}$$

$$\int \frac{27\,\text{sen}^2 t\,\cos t\,dt}{\sqrt{9(1-\text{sen}^2 t)}}$$

$$\int \frac{27\,\text{sen}^2 t\,\cos t\,dt}{\sqrt{9\cos^2 t}}$$

$$\int \frac{27\,\text{sen}^2 t\,\cancel{\cos t}\,dt}{3\,\cancel{\cos t}}$$

$$\int 9\,\text{sen}^2 t\,dt$$

$$9\int \left(\frac{1-\cos 2t}{2}\right) dt$$

$$9/2\int dt - 9/2\int \cos 2t\,dt$$

$$9/2\,t - 9/2\,(1/2\,\text{sen}\,2t) + c$$

$$9/2\,t - 9/4\,\text{sen}\,2t + c$$

$$9/2\,\text{sen}^{-1}(x/3) - 9/4\,\text{sen}\,(2\,\text{sen}^{-1}\,x/3) + c$$

$$9/2\,\text{sen}^{-1}(x/3) - 9/4\left(\frac{2x}{3}\,\frac{\sqrt{9-x^2}}{7}\right) + c$$

$$9/2\,\text{sen}^{-1}(x/3) - 3/2\,x\sqrt{9-x^2} + c$$

$$9/2\,\text{sen}^{-1}(x/3) - 1/2\,x\sqrt{9-x^2} + c'$$

$$\text{sen}^2 t = \left(\frac{1-\cos 2t}{2}\right) \qquad \begin{aligned} x &= 3\,\text{sen}\,t \\ x/3 &= \text{sen}\,t \\ t &= \text{sen}^{-1}(x/3) \end{aligned}$$

Ejercicio # 8

$$\int \frac{dx}{(x-1)(x+3)} \qquad \begin{matrix} u = x-1 & z = x+3 \\ du = dx & dz = dx \end{matrix}$$

$$\frac{A}{x-1} + \frac{B}{x+3} = \frac{1}{(x-1)(x+3)}$$

$$\frac{A(x+3)+B(x-1)}{(x-1)(x+3)} = \frac{1}{(x-1)(x+3)}$$

$$Ax + 3A + Bx - B = \frac{\cancel{(x-1)}\cancel{(x+3)}}{\cancel{(x-1)}\cancel{(x+3)}}$$

$$Ax + Bx + 3A - B = 1$$

$$x(A+B) + 3A - B = 1$$

$$A + B = 0 \qquad 3A - B = 1$$

$$A = -B \qquad 3(-B) - B = 1$$

$$A = -(-1/4) \qquad -3B - B = 1$$

$$A = 1/4 \qquad -4B = 1$$

$$\int \frac{1}{\cancel{4}/x-1} dx + \int \frac{-1/4 \, dx}{x+3}$$

$$\frac{1}{4}\int \frac{dx}{x-1} - \frac{1}{4}\int \frac{dx}{x+3}$$

$$1/4 \int \frac{du}{u} \qquad \frac{1}{4}\int \frac{dz}{z}$$

$$\frac{1}{4}\ln u - \frac{1}{4}\ln z + C = \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+3| + C$$

9. $\int \frac{dx}{x^4-1}$ $\quad \int \frac{dx}{(x^2-1)(x^2+1)} = \int \frac{dx}{(x+1)(x-1)(x^2+1)}$

$$\frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+d}{x^2+1} = \frac{1}{x^4-1}$$

$$\frac{A(x+1)(x^2+1) + B(x-1)(x^2+1) + (cx+d)(x^2-1)}{(x^2+1)(x-1)(x+1)} = \frac{1}{x^4-1}$$

$A(x^3+x+x^2+1) + B(x^3+x-x^2-1) + Cx^3 - cx + dx^2 - d = \cancel{\frac{x^4-1}{x^4-1}}$

$Ax^3 + Ax^2 + Ax + A + Bx^3 - Bx^3 - Bx^2 + Bx - B + cx^3 - cx + dx^2 - d = 1$

$Ax^3 + Bx^3 + cx^3 + Ax^2 - Bx^2 + dx^2 + Ax + Bx - cx + A - B - d = 1$

$x^3(A+B+c) + x^2(A-B+d) + x(A+B-c) + A-B-d = 1$

$A+B+c=0 \quad A-B+d=0 \quad A+B-c=0 \quad A-B-d=1$

$A=-B-c \quad -B-c-B+d=0 \quad -\cancel{B}-c+\cancel{B}-c=0 \quad -B-B-2B=1$

$A=-B \quad -2B-c+d=0 \quad -2c=0 \quad -4B=1$

$A=-\left(\frac{-1}{4}\right) \quad -2B-0+d=0 \quad c=0/-2=\emptyset \quad B=-1/4$

$A=1/4 \quad -2B+d=0$

$d=2B$

$d=2(-1/4) \Rightarrow d=1/2$

$$\int \frac{1/4\,dx}{x-1} + \int \frac{-1/4\,dx}{x+1} + \int \frac{0-1/2\,dx}{x^2+1}$$

$$1/4 \int \frac{dx}{x-1} - 1/4 \int \frac{dx}{x+1} - 1/2 \int \frac{dx}{x^2+1}$$

$$1/4 \int \frac{du}{u} - \frac{1}{4}\int \frac{dv}{v} - \frac{1}{2}\tan^{-1} x + c$$

$$\frac{1}{4}\ln u - \frac{1}{4}\ln v - \frac{1}{2}\tan^{-1} x + c$$

$$1/4 \ln|x-1| - 1/4 \ln|x+1| - 1/2 \tan^{-1}(x) + c$$

$$\int \frac{x^5}{(x^2+4)^2}\,dx$$

$$u = x^2+4 = x^2 = u-4$$
$$du = 2x\,dx$$
$$\frac{du}{2} = x\,dx$$

$$\int \frac{x^2 x^2 x\,dx}{(x^2+4)^2}$$

$$\frac{1}{2}\int \frac{(u-4)(u-4)\,du}{u^2}$$

$$1/2\int \frac{u^2-8u+16\,du}{u^2}$$

$$1/2\int \frac{u^2}{u^2}\,du - \frac{8}{2}\int \frac{u}{u^2}\,du + \frac{16}{2}\int \frac{du}{u^2}$$

$$1/2\int du - 4\int \frac{du}{u} + 8\int u^{-2}\,du$$

$$1/2\,u - 4\ln u + 8/-1\,u^{-1} + c$$

$$1/2\,(x^2+4) - 4\ln|x^2+4| - 8/u + c$$

$$1/2\,(x^2+4) - 4\ln|x^2+4| - \frac{8}{x^2+4} + c$$

$$\frac{1}{2}x^2 - 4\ln|x^2+4| - \frac{8}{x^2+4} + c$$