Standard model of cosmology

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1 Introduction

Cosmology is the science of the universe as a whole. After the discovery of Hubble and Humason at Mount Palomar, the Universe itself has been understood as a dynamical entity, that one can predict the evolution. The further discovery of the cosmic microwave background (CMB) by Penizas and Wilson, allowed us to understand that this evolution had a beggining.

2 The standard models: gravity and high energy physics

2.1 General relativity

The principle of *general covariance* is at the heart of general relativity. While the cornerstone of special relativity is the *principle of relativity* asking for the invariance of the equations of physics under the *Poincaré group*, that is the space-time transformations going from an inertial frame to another, general relativity aim to generalize this group to diffeomophisms that is to all continuous cordinate transformations, even non linear. As a guiding principle:

> "All systems of reference are equivalent with respect to the formulation of the fundamental laws of physics."
> C. Møller The Theory of Relativity [1], p. 220

Following this principle will lead to the conclusion that space-time itself is a dynamical entity, and gravitation is the byproduct of its geometrical properties. The matter content will influence the geometry of space-time while the geometry will determine the particle trajectories, or as Wheeler famously puts it:

> "Spacetime tells matter how to move and matter tells spacetime how to curve"

In order to do so formally, space-time has to be promoted to a 4-D semi-riemanian smooth manifold, equiped with a rank-2 covariant metric tensor field $g \in \Gamma(T^0_2 M)$ of signature -1.

For a complete review of general relativity, see e.g. [2, 3, 4] and for an introduction to the framework of differential geometry in physics see e.g. [5, 6, 7].

The space-time manifold is *locally Lorentzian*, that is, at every point of space-time, it is possible to find a coordinate transformation in which the metric is the Minkowski metric η and the laws of physics are those of special relativity. This endows the Einstein Equivalence principle and the universality of gravity.

The test particles move on geodesics of the only torsionless [1]and metric preserving[2] affine connexion ∇_u on the tangeant bundle, with associated vector potential Γ called the *Levi-Civita* connexion. Intuitively γ connects the different tangent spaces to one another and allows to define a notion of *parallel transport* and differentiation (using ∇_u) between different points of M. It is the GR equivalent of the newtonian gravitational potential. In a given coordinate frame, one can express the components of Γ only with the first order partial space-time derivatives of the metric components.

The trajectories $\tau \to \gamma(\tau) : \mathbb{R} \to M$ of any free particle on the manifold will obey *parallel transport* with respect to ∇_u:

$$\nabla_{\gamma'}\gamma' = 0 \tag{1}$$

Where $\gamma' = \frac{d}{d\tau} \in TM$ is the 4-velocity vector of the particle tangent to the path.

The curvature of the manifold is given by the *Riemann tensor*:

$$\mathcal{R} = d\Gamma + \Gamma \wedge \Gamma \tag{2}$$

That can be expressed solely with second order derivatives of g. R quantifies the *holonomy* of a small closed path, that is the rotation induced to a vector that is parallel transported back to the same point with Γ allong that path. In term of physics, it encodes gravitational tidal forces. To every matter living in space-time, one can associate a stress-energy tensor T. The local conservation of mass is ensured by the continuity equation: [3][4]

$$\nabla_\mu T^{\mu\nu} = 0 \tag{3}$$

Looking for a field equation relating the geometry of space-time and its content that allows for the continuity equation to be satisfied aswell as

[1]$\forall u, v \in TM : [u, v] = \nabla_v u - \nabla_u v$. This hypothesis could drop when introducing spins in GR. Several extension of GR as Einstein-Cartan gravity drop this assumption.

[2]$\forall u, v, w \in TM : ug(v, w) = g(\nabla_u v, w) + g(v, \nabla_w v)$. Roughly speaking, this condition ensure the invariance of vector length.

[3]Which is nothing less than the familiar equation of fluid dynamics: $\frac{\partial \rho}{\partial t} = -\vec{\nabla}\vec{j}$

[4]We use the standard notation $\nabla_{\partial\mu} := \nabla_\mu$

general covariance, that is the invariance of the equations of motion under diffeomorphisms. The simplest expression one can derived is given by the *Einstein equation*:

$$G = 8\pi G T + \Lambda g \tag{4}$$

With the *Einstein tensor* defined as:

$$G = \mathfrak{R} + \frac{1}{2} g R \tag{5}$$

Where $\mathfrak{R}$ is the *Ricci tensor*: $\mathfrak{R}_{\mu\nu} = \mathcal{R}^{\delta}{}_{\mu\delta\nu}$ and R is the scalar curvature $R = g^{\mu\nu}\mathfrak{R}_{\mu\nu}$. The numerical factor of $8\pi G$ ensure that our theory gives back Newton's law of gravity in the low energy limit (here $(c = 1)$). The whole theory can be derived from action principles. Einstein's equation is derived from:

$$\mathcal{S}_{EH} = (8\pi G)^{-1} \int \sqrt{-g} \left(\frac{R}{2} - \Lambda \right) d^4x, \tag{6}$$

The *Einstein-Hilbert* action. On the other hand, the geodesic equation of moving bodies is recovered by extremization of the space-time interval or proper time times its mass (hence the name geodesics):

$$\mathcal{S}_p = -m \int ds \tag{7}$$

There exists also canonical (Hamiltonian) formulations of GR and more powerfull formalisms using tetrads fields instead of g, allowing to introduce spinors on curved space-time.

2.2 Particle physics and Gauge theories

In the standard model of particle physics, particles are understood as quantized excitations of fields over space-time. These fields really are sections of different associated bundles depending on the way they interact with each others. The fundamental forces they are witnessing are expressed by the fact that the bundle is curved (see e.g. [8]) and the invariance of coordinate transformations in the fibers is called gauge invariances, gauge fields being the connexion fields living in the adjoint bundles. In this sense, gauge theories are very similar to general relativity, diffeormorphism being a gauge group (see e.g. [9, 10, 11]).

Locally (i.e. in each tangent bundle), the particle types are constrained to be representations of the Poincaré group (See e.g. [12]). This group has two casimirs, the spin and the mass. Today, only scalar, spinors and vector fields has been observed.

The sections are constrained to follow equation of motions, obtained from a least action principle. Each corresponding lagrangian densities can be fully determined from symetry considerations. The coupled *Yang-Mills* equation of a fermion field ψ interacting with a gauge field in flat space-time is given by (for an introduction to geometrical approaches of the standard model see e.g. [5, 13, 6] and for a more complete review in [14]):

$$\mathcal{L}_{YM} = \bar{\psi}(\not{D} - m)\psi + \frac{1}{4}F \wedge *F \tag{8}$$

Where the covariant derivative $\not{D} = \gamma^\mu(\partial_\mu + qA_\mu)$ describe the motion of the field in the curved fiber bundle with the gauge fields A appearing as a connexion. q is the charge quantifying the coupling between the Dirac field and the gauge field. m is the mass of ψ, simply corresponding to a zero point energy. F is the curvature associated to the connexion D: $F = dA + A \wedge A$. The equation of motion resulting from this Lagrangian are the following:

$$\bar{\psi}(\not{D} - m)\psi = 0 \tag{9}$$
$$dF = 0 \tag{10}$$
$$\star d \star F = J \tag{11}$$

The first one is Dirac equation. The second one is simply the Biancci identity for curvature, equivalent to the two charge free Maxwell's equations if $G = U(1)$ while the third one describe the coupling between the spinor field and the gauge field, with the current-vector: $J^\mu = q\bar{\psi}\gamma^\mu\psi$. The other interactions have non-abelian gauge groups. For weak interaction, ψ is a weak doublet (of $U(1)$ singlets), A is the W-boson fied and gauge invariance is traduced by $G = SU(2)$. For strong force, ψ is a color triplet and $G = SU(3)$ (of quarks being themselves $SU(2)$ doublets). A are the 8 colored gluon field. Has for fermions, the Standard model contains three generations of quarks doublets and three generation of leptons doublets (electron and neutrons) of increasing masses. Explaining these three generations and the existence of the three gauge groups is one of its biggest challenge.

For weak interaction, the theory can not allow gauge bosons to be massive without breaking gauge symetry. One has to add a new field, the Higgs boson h, spontaneously breaking the $SU(2) \times U(1)$ symetry and giving mass to the bosons and all the fermions through Yukawa couplings.

In curved space-time, the classical action of the standard model is given by:

$$\mathcal{L}_{SM} = \sqrt{-g}\Big(\frac{1}{8\pi G}\left(\frac{R}{2} - \Lambda\right) + \sum_\psi \bar{\psi}(\not{D} + \frac{1}{4}\not{\omega})\psi + \sum_F \frac{1}{4}F \wedge *F + \frac{1}{2}|Dh|^2 - V(h) + \sum_\psi \lambda\bar{\psi}h\psi\Big) \tag{12}$$

Where ω is the spin-connection. (see e.g. Chap. 19 of [7] or Chap. 9 and 10 of [15] for an approach based on Clifford algebras). The mass term is given by the Higgs boson h through Yukawa couplings λ. Λ could emerge directly from vaccum energy of the various fields but it is yet unclear how. All matter fields are *Unviersaly coupled* that is they are minimally coupled with the same and only metric g appearing in the lagragians. This embodies the Einstein equivalence principle and the geometric interpretation of gravitation. Also implying that all massless fields should propagate with the same velocity.

In quantum field theory (QFT), the fields ψ and A are promotted to operators at every space-time point. The quantum states are the occupation states of the Fock space, taking tensor products of single particle Hilbert-spaces is called the *second quantization*. The interaction probabilities are calculated in a perturbative fashion using Feynman graphs. For on QFT, more see e.g. [16]. It is yet unclear how gravity can take place in this picture.

2.3 Λ-CDM

The standard model of cosmology (see e.g. [17, 18]) is built on the three following assumptions:

- **i) cosmological principle**: on large scale the universe is isotrope (i.e. invariant under rotations) and homogeneous (i.e. invariant under translations). This principle seems to be largely verified with observation but is still questionned today. This strongly constrained the overall shape that the universe can have, but allows the existence of a constant curvature.

- **ii) content**: The universe contains the matter and gauge fields of the standard model of particle physics (2.2). They can be divided in two according to their collective behavior on large scales matter (pressurless massive particles) and radiation (relativistic light species). To account for the observations, one has to add an extra source of matter called *cold dark matter* (CDM).

- **iii) gravity**: The theory of gravity, acting on large scales is given by General relativity (2.1) with a non vanishing cosmological constant Λ, responsible for its late time accelerated expansion.

An additional assumption, inflation is nowadays added to the standard model of cosmology to account for very early times, but we will delay its introduction to Sec.X.

2.3.1 Cosmological principle and the FLRW metric (i)

Asking for **i)** strongly constraint the overall geometry of the universe and constrain the metric to be FLRW. Comoving frame centered at a given point of space-time (t, r, θ, φ) carried with the expansion (i.e. galaxies always have the same coordinates through cosmic evolution, but the coordinate frame expands or shrinks). We note $t_0$the comoving age of the universe i.e. today's value of t. a is the *scale factor*, $a \in [0, 1]$, quantifying the physical expansion, today $a(t = t_0) = a_0 := 1$, by definition.

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1 - \kappa r^2} + r^2 d\Omega^2\right) \tag{13}$$

t is called the *comoving time.* $d\Omega = \sin^2(\theta)d\varphi$ is the differential angular element. κ is the spatial curvature of the Universe $\kappa = +1$ spherical, $\kappa = -1$ hyperbolic and $\kappa = 0$ flat.

If $\kappa = 0$, assumption generally done since favored by observations, we simply have:

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right) \tag{14}$$

Where (t, x, y, z) is a cartesian comoving frame.

From a, one can define the redshift:

$$z = \frac{1}{a} - 1 \tag{15}$$

At $t = t_0$, $a = 1$ and $z = 0$. z can be related to the doppler shift of a frequency from an emitting galaxy carried by the expansion as:

$$z = \frac{\nu_e - \nu_r}{\nu_r} \tag{16}$$

ν_e: emitted frequency, ν_r: received frequency.

2.3.2 The components of the universe (ii)

Let's now discuss more quantitavily the assumption **ii**). For the cosmological principle **i)** to hold, the fluid contains in the universe should not have any significant flux on large scales since it would select a prefered direction and break isotropy. We choose to model it by a *perfect fluid* filling it uniformly. A perfect fluid has an isotropic pressure, no shear, no viscosity, and do not conduct heat. Those assumptions implies that we neglect the interactions between the constituents (the galaxies) on the largest scales. It's stress energy tensor is diagonal and given by:

$$T_{\mu\nu} = \rho u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu) = \text{diag}(\rho, p, p, p) \tag{17}$$

where $u = (1, 0, 0, 0)$ is the 4-velocity of the fluid at rest in the comobile frame. Thermodynamicly, a perfect fluid has to obey an equation of state of the form:

$$p = w\rho \tag{18}$$

A classical example being given by the ideal gas who's interactions between molecules are neglected and $w = RT$.

The continuity equation is given by Eq. 3 and is expected to hold for every species independently if we neglect their interactions on large scales. Evaluating the component $\nabla_\nu T_0^\nu$ coupled with the equation of state given in Eq. 18, we obtain:

$$\frac{\partial \rho}{\partial t} + 3H(1+w)\rho = 0 \tag{19}$$

where we introduced the *Hubble parameter*:

$$H := \frac{\dot{a}}{a} \tag{20}$$

Where dotted quantities are derived with respect to comoving time.

Eq. 19 can be integrated to obtain $\rho(a)$:

$$\rho(a) = a^{-3(1+w)} \tag{21}$$

To determine the w of a given species, one has to use some consideration of thermodynamics/statistical physics.

matter and dark-matter

The energy density of matter is dominated by it's mass $\rho_m c^2 >> p_m \propto v_m^2$ .Baryonic and dark matter are assumed pressureless : $w_m = 0$. The largest amount of the mass is represented by dark matter Zwicky 1930, wimp, sterile neutrinos, primordial black holes, MOND, entropic gravity ...

$$\rho_m = \rho_h + \rho_{\rm cdm} \tag{22}$$

relativistic species: photons and neutrinos

photons and neutrinos $m_\nu \sim 0$. $w_r = 1/3$.

Dark energy

Since the observation of the accelerated expansion of the universe in X (ref), it is clear that some mechanism should be responsible for this acceleration. Such a general component is called *dark energy.* The equations of evolutions of the universe that we will soon derive (see Eq.27) impose that $w_{DE} < -1/3$ in order to allow $\ddot{a} > 0$. The cosmological constant appearing in the Einstein equation provides a natural candidate for dark energy. Indeed, by defining $T^\Lambda_{\mu\nu} = \Lambda g_{\mu\nu}$, we derive $w_\Lambda = -1$. When inserting this solution in Eq.21, we obtain $\rho_\Lambda(a) = cst$. It is then possible to treat the cosmological constant as an additional component of the universe behaving like a *vaccum energy* with constant density.

The associated density is:

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} = \Lambda M_{\text{Pl}}^2 \tag{23}$$

However, such an option appears unsatisfactory when we try to relate cosmology to high energy physics since renormalization of quantum field theories allows to calculate the vaccum energy of the fields and implies a discprency of $\sim 10^{120}$ with the measured value of Λ.

Alternatives as quintescence and modified gravity.

curvature

We can also introduce a $w_\kappa = -\frac{1}{3}$.

$$\rho_\kappa = -\frac{3\kappa}{8\pi G} = -3\kappa M_{\text{Pl}}^2 \tag{24}$$

2.3.3 Fundamental equations of cosmology (iii)

Friedmann-Lemaître equations

Let's now put all the pieces of the puzzle together. Inserting the expression for the stress-energy tensor (**iii)**) given in Eq.17 inside the Einstein equation (Eq. 4) describing gravity in vertue of **ii)**, rewritten as:

$$R^{\mu\nu} = 8\pi G\left(T^{\mu\nu} - \frac{1}{2}g^{\mu\nu}T^{\sigma}_{\ \sigma}\right), \tag{25}$$

and assuming a FLRW metric for g, (Eq.13) to ensure **i)**, one can derive the *Friedmann-Lemaitre* equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \tag{26a}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -8\pi G p \tag{26b}$$

where $\rho = \sum_i \rho_i$ and $p = \sum_i p_i$ are the sum over all the components in the universe.

Those are the fundamental equations of standard cosmology, that can be solved in specific scenarios to obtain the background evolution of the universe $a(t)$. Eq. 30 and Eq. 19 can be combined to obtain the *Raychaudhuri* or acceleration equation:

$$\frac{\ddot{a}}{a} = H^2 + \dot{H} = \frac{4\pi G}{3}(\rho + 3p) \tag{27}$$

density parameters

Introducing the critical density:

$$\rho_c := \frac{3H_0^2}{8\pi G} \tag{28}$$

We can define the *density* parameters:

$$\Omega_i := \frac{\rho_0^i}{\rho_c} \qquad i \in \{m, r, DE, \kappa\} \tag{29a}$$

$$\tag{29b}$$

Allowing us to simply rewrite the fundamental equations as:

$$\left(\frac{H}{H_0}\right)^2 = \sum_i \Omega_i a^{-3(1+w_i)} \tag{30}$$

$$\frac{\ddot{a}}{H_0^2} = -\frac{1}{2}\sum_i \Omega_i(1+3w_i) \tag{31}$$

Evaluating today ($a_0 = 1$), we get the closure equation:

$$\sum_i \Omega_i = 1 \tag{32}$$

If there is a single component of density Ω_i and equation of state w with $w \neq -1$, one can integrate Eq.30 to get the time between today and a given scale factor value a:

$$\left(\frac{da}{dt}\right)^2 = H_0^2 \Omega_i a^{-3(1+w)+2} \tag{33}$$

And so:

$$t = \frac{1}{\sqrt{\Omega_i} H_0} \int_0^a a^{\frac{3}{2}(1+w)-1} da = \frac{2a^{\frac{3}{2}(1+w)}}{3(1+w)\sqrt{\Omega_i} H_0} \tag{34}$$

From which we can derive the rate of evolution of the scale factor with time:

$$a(t) \propto t^{\frac{2}{3(1+w)}} \tag{35}$$

3 The cosmic history

3.1 The primordial singularity and the need for a quantum theory of gravity

As one can already see at the level of Friedmann and Raychaudhuri equations (Eq. 30 and 27), the Big-Bang cosmology becomes mathematically singular when $a \to 0$ [19]. This problems appears to be much more general and even true including inflation [20].

Formally, singularities in general relativity are points of space-time where all geodesics converges. Their most popular realization can be found at the center of black holes. They appears to be consequences of Einstein equation itself (Eq. 4) and not only properties of idealized solutions of it [21]. This is a clear indication of the breaking down of general relativity as a theory of gravitation in the very strong field limits and their existence has been discussed in great length in the past decades. In conditions of extreme density and energy, gravitation is expected to have a strength comparable to the other forces, leading requiring a quantum treatment of gravity or at least a classical but quantum compatible treatment of gravity.

The existence of primordial singularity towards $t \to 0$, actually translate our complete ignorance on the birth of our universe. Several solutions are proposed by quantum gravity models as string theory, loop quantum gravity etc.

3.2 The GUT phase and symetry breakings

It is a well known fact from statistical physics that highly energetic systems tends to be highly symetric and tends to loose spontaneously these symetries in phase transitions by cooling down.[5] Such a behavior is also expected at the universe's scale with gauge symetries and the cooling down due to expansion.

A spontaneous gauge symetry breaking is already attested within the standard model of particle physics in the framework of Weinberg-Glashow-Salam electroweak unification [22, 23, 24]. At high energy, electromagnetism and weak force are unfied within a single gauge group $SU(2) \times U(1)_Y$, mediated by three W_i and a B bosons. All particles are massless. For a temperature lower than ~ 160 GeV, the Higgs boson's potential aquire a "mexican hat" shape, allowing the scalar field to vaccum decay and roll down sponteneously to a false vaccum, breaking the gauge group to $U(1)_{\mathrm{em}}$ and leading to the apparition of two charged massive bosons $W^{\pm}$ and two neutral ones Z^0 and the photon γ.

There are strong indications that the strong force could also be unfied to the other two forces at higher energy in a larger gauge group [25, 26]. For exemple, all the coupling constants seems to run with energy towards a similar value. A theory unifying the three forces is called a Grand Unification Theory (GUT). $SU(5)$ was favored as a gauge group but was excluded by proton decay experiments. Other groups has $SO(10)$ are still tested. Full unification model including gravity seems to require larger gauge groups as e.g. $SO(32)$ or $E_8 \times E_8$ for superstring theories.

[5] One can think of magnetic moments in ferromagnetic materials ligning up with the earth magnetic field when cooling.

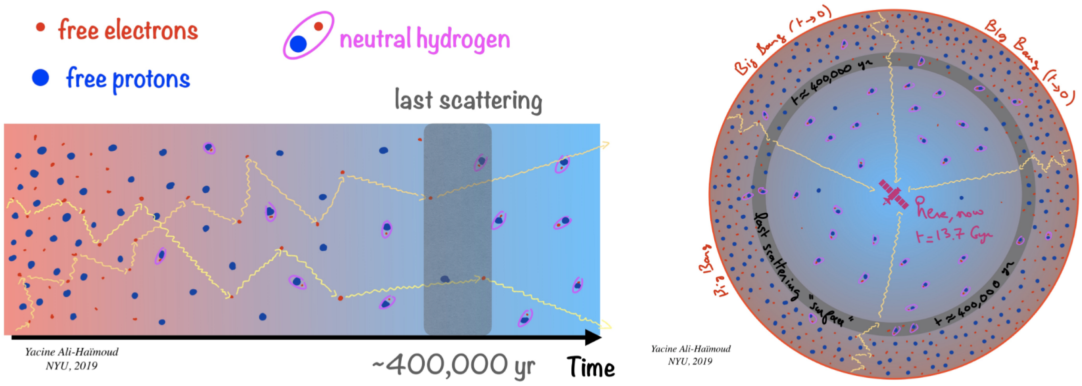

Figure 1:

When a gauge group is spontaneously broken, the breaking is expected to be different at different point of space-time from which it will propagates at the speed of light. Topological defects types: Domain walls are two-dimensional objects (in other words, two-branes) which form when a phase transition breaks discrete symmetry. Cosmic strings are one-dimensional or line-like (one-brane) objects which form when a phase transition breaks an axial or cylindrical symmetry. Monopoles are zero-dimensional point-like (zero-brane) objects which form when a phase transition breaks a spherical symmetry. [27] [28]

Domain walls: repulsive gravity Monopoles: unavoidable in GUTs, very massive and bearing a single magnetic charge. Cosmic string: present in all phase transitions models including electroweak. Very dense and thin, rotating fast. Form loops and decay. No gravitionial pull but sources of gravitational waves and lensing.

3.3 Big Bang Nucleosynthesis

Lithium problem.

3.4 Perturbation theory

3.5 Radiation domination, recombination and the CMB

$a(t) \propto \sqrt{t}$

3.6 Reionization, structure formation and matter domination

The photons travels

[29].

From Eq.35:$a(t) \propto t^{\frac{2}{3}}$

3.7 Dark energy domination and modern times

Having both a contribution from both Ω_m and Ω_Λ, one can derive:

$$a(t) = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{\frac{1}{3}} \left[\sinh\left(\frac{3}{2}\sqrt{\Omega_\Lambda} H_0 t\right)\right]^{\frac{2}{3}} \tag{36}$$

Having Ω_Λ for only contribution in the Friedmann equation Eq.30, we get:

$$\dot{a} = H_0 a \sqrt{\Omega_\Lambda} \tag{37}$$

Implying an exponential growth of the scale factor:

$$a(t) \propto e^{H_0 \sqrt{\Omega_\Lambda} t} \tag{38}$$

+ coincidence problem.

4 Inflation

4.0.1 Puzzles of the Big-Bang model and early acceleration phase

Puzzles:

- **Flatness problem (fine tuning):** Introducing:

  $$\tilde{\Omega}(a) = \sum_{i \neq \kappa} \frac{8\pi G \rho_i(a)}{3H(a)} \tag{39}$$

  The Friedmann equation we can be rewriten as:

  $$1 - \tilde{\Omega}(a) = -\frac{\kappa}{\dot{a}^2} \tag{40}$$

  Showing that the quantity $1 - \tilde{\Omega}(a)$ quantifies deviation to flatness. Recent observations constrains $|1 - \tilde{\Omega}(a_0)| < 5 \times 10^{-3}$ [30].

  Derivating Eq.40 with respect to time gives:

  $$\frac{d}{dt}(1 - \tilde{\Omega}) = -2\frac{\ddot{a}}{\dot{a}}(1 - \tilde{\Omega}) \tag{41}$$

  As we show in previous sections, untill very recent times the universe only knew phases of decelerated ($\ddot{a} < 0$) expansions ($\dot{a} > 0$). In this evolution, the point $1 - \tilde{\Omega} = 0$ is an unstable point and any small deviation to zero would be dragged away. We can clearly now see that having a flat universe $\kappa \sim 0$ today requires an incredible fine tuning of the early universe, requiring $|\tilde{\Omega} - 1| \leq 10^{-5}$ at recombination, $|\tilde{\Omega} - 1| \leq 10^{-16}$ during BBN and $|\tilde{\Omega} - 1| \leq 10^{-61}$ at the Planck scale.

- **Horizon problem (fine tuning)**: The CMB is extremley homogeneous $\Delta T/T_{\text{CMB}}\ 1\times10^{-5}\,\text{K}$. The CMB can be splitted in $\sim 10^4$ causaly independant patches. From Eq.33, we can show:

$$\frac{d}{dt}\left((aH)^{-1}\right) = \frac{d}{dt}\left(\frac{1}{\dot{a}}\right) \propto 1+3w \tag{42}$$

  For all standard fields dominating the cosmological evolution untill now $1+3\omega \geq 0$ (except dark energy that strikes at very late times). Thus the Hubble horizon $(aH)^{-1}$ can only have shrinked with time.

- **Low entropy (fine tuning)**:

- **Cosmic topological defects**:

- **The primordial seed problem**:

All of this can be solve by a brutal acceleration phase in the early universe with $\frac{\ddot{a}}{a} > 0$ which would require $w \leq -\frac{1}{3}$ [31, 32]:

- In, Eq.41 the factor of $1-\tilde{\Omega}$ is negative, making the flat universe $1-\tilde{\Omega}=0$ a attractor point.

- Would create a decreasing of the Hubble radius Eq.42.

4.0.2 The inflaton

A solution is provided by introducing a scalar field
$\rho_\phi = \dot{\phi}^2 + V(\phi)$ $P_\phi = \dot{\phi}^2 - V(\phi)$. $w = P/\rho$
Reheating.
[33]. Starobinzki inflation + α-attractors;

4.0.3 Predictions

Predictions:

- formation of structures from primordial fluctuations

- Nearly gaussian perturbations and nearly scale invariant primordial power spectra (success)

- Gravitational wave background and B modes (under investigations)

- Small primordial non gaussianities (under investigations)

4.0.4 Alternatives to inflation

CCC, Ekpyrotics

5 Λ-CDM: 6 parameters to describe the Universe.

5.1 Cosmological probes

5.1.1 CMB

5.1.2 Strong vs weak Lensing

5.1.3 Cosmic clock

5.1.4 BAO and LSS

Euclid

5.1.5 Supernovae

5.2 best constraints

Planck 2018

Best constraints:

5.3 Tensions and unanswered questions

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