Reminders of analytical mechanics*

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Abstract

I. LAGRANGIAN FORMULATION

$$S = \int L dt \tag{1}$$

$$\delta S = 0 \tag{2}$$

$$L = T - V \tag{3}$$

$$p = \frac{\partial L}{\partial \dot{q}} \tag{4}$$

$$\frac{\partial p}{\partial t} - \frac{\partial L}{\partial q} = 0 \tag{5}$$

A. A word on DG

tangent bundle

B. connection to QM

path integrals

II. CANONICAL FORMULATION

$$p = \frac{\partial L}{\partial \dot{q}} \tag{6}$$

Legendre transformation:

$$H = L - p\dot{q} \tag{7}$$

$$\{F, G\} = \frac{\partial}{\partial} \tag{8}$$

A word on legendre transformations

The canonical variable satisfy:

$$\{t, H\} = 1 \tag{9}$$

$$\{f, H\} = \frac{\partial f}{\partial t} \tag{10}$$

As such, $\{f, H\} = 0$ will signify the conservation of f through the time translation evolution of the system generated by H.

$$\{., H\} = \frac{\partial}{\partial t} \tag{11}$$

The canonical variable has to satisfy:

$$\{q, p\} = 1 \tag{12}$$

$$\{f, p\} = \frac{\partial f}{\partial q} \tag{13}$$

As such, $\{f, p\} = 0$ will signify the conservation of f through the space translation evolution of the system generated by p.

$$\{., p\} = \frac{\partial}{\partial q} \tag{14}$$

The same way

$$\{\theta, L_\theta\} = 1 \tag{15}$$

$$\{., L_\theta\} = \frac{\partial}{\partial \theta} \tag{16}$$

A. A word on DG

symplectic geometry

* A footnote to the article title

B. connection to QM

In order to preserve the probability interpretation of the state vectors $\langle\psi|\psi\rangle = 1$, one will ask that any operator $\hat{U}$ acting on $|\psi\rangle$ and describing its evolution under a given, transformation $|\psi\rangle \to |\psi'\rangle$ has to be unitary, that is $U^\dagger U = \mathrm{I}$. As such, the dot product and thus the probabilty densities will be conserved under such a transformation:

$$\langle\psi_1'|\psi_2'\rangle = \langle\psi_1'|\, U^\dagger U \,|\psi_2'\rangle = \langle\psi_1|\psi_2\rangle \tag{17}$$

$$\frac{d\langle\hat{A}\rangle}{dt} = \langle\frac{\partial\hat{A}}{\partial t}\rangle + \frac{1}{i\hbar}\langle[\hat{A},\hat{H}]\rangle \tag{18}$$

using $\hat{A} = \hat{P}$ or $\hat{A} = \hat{Q}$, one finds back the Hamilton equations of motion:

$$\langle\frac{\partial\hat{P}}{\partial t}\rangle = -\langle\frac{\partial H}{\partial\hat{Q}}\rangle \tag{19}$$

$$\langle\frac{\partial\hat{Q}}{\partial t}\rangle = \langle\frac{\partial H}{\partial P}\rangle \tag{20}$$

$$\tag{21}$$

Where we used the following relations:

$$[f(\hat{A},\hat{B}),\hat{B}] = [\hat{A},\hat{B}]\frac{\partial f}{\partial\hat{A}} \tag{22}$$

Since the demonstration of this formula is a bit long and tedious, we will admit it.

III. HAMILTON-JACOBI FORMULATION

A. Canonical transformations

B. connection to QM

Schrodinger equation + Bohm

IV. FIELD THEORY

A. Link with QM and QFT

Connection between klein-gordon and Schrodinger equations