

## BDA Asymptotics and BiBa Model - 2023/4

### List 4

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#### III. Random Walks and PageRank

1. Let  $G = (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}, \{1 \rightarrow 2, 2 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 3 \rightarrow 4, 4 \rightarrow 3, 4 \rightarrow 2, 4 \rightarrow 4, 4 \rightarrow 5, 5 \rightarrow 1, 1 \rightarrow 5, 9 \rightarrow 10, 8 \rightarrow 7, 7 \rightarrow 8, 4 \rightarrow 9, 10 \rightarrow 7, 8 \rightarrow 1\})$ . An agent is placed at in a vertex 1. In each step the agent chooses randomly out-going edge and goes to the vertex pointed by this edge. Find the distribution of the position of the agent after 2, 3, 4, 50 and 100 steps. HINT: Do this numerically. Remember that  $p^{(t+1)} = p^{(t)}\mathbf{P}$ .
2. Let  $G = (\{1, 2, 3, 4, 5, 6\}, \{1 \rightarrow 2, 2 \rightarrow 1, 2 \rightarrow 2, 2 \rightarrow 6, 2 \rightarrow 3, 3 \rightarrow 4, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 1, 6 \rightarrow 6\})$ . Consider a random walk on  $G$  and find numerically its ergodic probabilities.
3.  $G = (\{1, 2, 3, 4, 5, 6, 7, 8, 9\}, \{1 \rightarrow 2, 2 \rightarrow 1, 2 \rightarrow 2, 2 \rightarrow 6, 2 \rightarrow 3, 3 \rightarrow 4, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 1, 6 \rightarrow 6, 6 \rightarrow 7, 5 \rightarrow 8, 7 \rightarrow 9, 8 \rightarrow 9\})$ . Find PageRank value for each vertex representing a web-site. Beware that there are some some dead-ends. Assume taxation parameters  $\beta = 0$  and  $0.1$ .
4. Generate a directed graph  $G$  with  $n$  nodes in a following way. For each of  $n$  nodes choose  $l$  unique out-neighbours. For such randomly generated structure start a random walk of an agent in a randomly chosen vertex. Find the distribution after 1, 2, 10, 50, 100 and  $n$  steps for all configurations of parameters:
  - $n = 2, 10, 20, 1000$
  - $l = 1, 2, 5, 10, 20, 50$
  - $\beta = 0, 0.01, 0.05, 0.1, 0.2$