BDA Asymptotics and BiBa Model - 2023/4

List 3 for 6th or 7th December

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III. Asymptotics and BiBa model

1. In a single plot draw graphs of functions
   (a) $g(x) = x, h(x) = 100\sqrt{x}$ for $x \in [0, 500]$
   (b) $g(x) = x, h(x) = 100\sqrt{x}$ for $x \in [0, 10000000]$
   (c) $g(x) = x, h(x) = \ln(x), l(x) = \sqrt{x}$ for $x \in [1, 10]$
   (d) $g(x) = x, h(x) = \ln(x), l(x) = \sqrt{x}$ for $x \in [1, 200]$
   (e) $g(x) = x\sqrt{x}, h(x) = x \ln x, l(x) = x^2$ for $x \in [1, 50]$
   (f) $g(x) = \frac{x}{\ln(\ln(x))}, h(x) = \frac{x}{\ln x}$ for $x \in [1, 2000]$
   (g) $g(x) = \frac{\ln x}{\ln(\ln(x))}, h(x) = \frac{\ln \ln x}{\ln 3}$ for $x \in [1, 50000]$
2. We throw independently balls into n bins until all bins are non-empty. Let C be the number of thrown balls.
   - Use numerical experiments to find an approximation of PDF for C (i.e. a histogram). Present results for $n = 10$, $n = 100$ and $n = 1000$.
   - For $n \in [10, 50000]$ find an approximation of $E(C)$. Compare this with the theoretical results presented during the lecture.
3. We throw independently balls into n bins until there are two balls in a single bin (a conflict). Let B be the number of thrown balls.
   - Use numerical experiments to find an approximation of PDF for B (i.e. a histogram). Present results for $n = 10$, $n = 100$ and $n = 1000$.
   - For $n \in [10, 50000]$ find an approximation of $E(B)$. Compare this with the theoretical results presented during the lecture.
4. We throw independently m balls into n bins.
   - Repeat this experiment reasonable number of times for $m = n = 1000$. Then find the average number of balls in the most loaded bin.
   - Repeat this experiment reasonable number of times for $m = 100, n = 1000$. Then find the average number of balls in the most loaded bin.
   - Repeat this experiment reasonable number of times for $m = 2000, n = 100$. Then find the average number of balls in the most loaded bin.
   - Repeat this experiment reasonable number of times for $m = n = 1000$ and find the average number of empty bins.
5. "Power of two choices". Instead of throwing balls independently into bins, for each ball we choose randomly d bins and check the number of balls inside. Then we place the new ball in the bin (one of bins) with the smallest balls in the given moment. Check experimentally, how parameter d influences random variables C, B and the maximal load. Consider different d staring from 1. Note that the case $d = 1$ is just the regular BiBa model.