Determining the Characteristics & Behavior of a P-Type Germanium Semiconductor Using the Hall Effect

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Abstract

In this work, we used the Hall effect on a p-type Germanium (Ge) semiconductor in order to measure its electromagnetic properties, such as its Hall coefficient, resistivity, mobility of the dominant charge carriers, and energy gap between the conduction bands. By measuring various control variables, such as the sample current, magnetic field, and temperature, we are able to observe a wide variety of phenomena from the world of semiconductor physics, such as magneto-resistance and the Hall effect itself, to sufficient degrees of accuracy.

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1 Introduction

1.1 Experimental Setup

As seen in Figure (1), the experimental setup for this work relied on a magnetometer, central module, power supply, two coils with iron core, and two polar pieces.

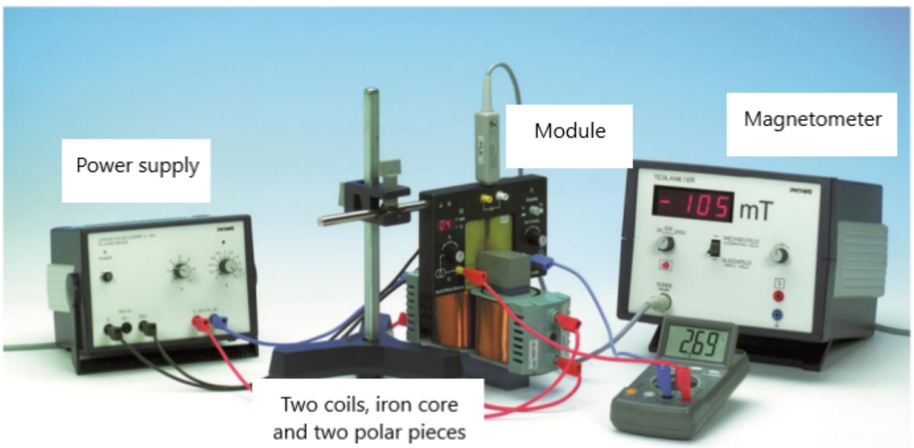

Figure 1: Experimental setup

1.2 The Classic Hall Effect

Measuring Hall voltage and resistivity of a semiconductor gives us the charge carriers their type and mobility. The conductivity of a semiconductor is due a movement of electrons and holes in it. We're dealing with intrinsic (pure) semiconductor, without any significant dopant species present. In which case we got:

$$n = p = n_i \tag{1}$$

where n and p are the densities of the electrons and the holes respectively and n_i is the temperature-dependent carrier density of the semiconductor. We put the conductor in a magnetic field of $\bar{B} = B\hat{z}$ and a current $\bar{I} = I\hat{x}$. The magnetic force is acting on the moving particles and thus causing them to go in the $-\hat{y}$ direction, creating a current. In equilibrium we get that $\pm\hat{y} = 0$ and there is accumulation of electrons and holes on the sides. Consequently there is a net force acting on the moving particles:

$$\bar{F}_{net} = -qvB - qE_H = 0 \Longrightarrow E_H = -vB$$

We also know that $I = nqvdW$ and $U_H = EW$, combining them all together will give us:

$$|U_H| = \frac{B}{nqd} \cdot I \tag{2}$$

where L, W and d are the length, width and thickness of the conductor. In parts 1 and 2 we assume there is only one type of carriers, either electrons or holes.

1.3 Generalization for Holes Current Density

In our experiment there's a direction to the resistant, and thus we use the resistivity tensor. Taking into account the conductivity of electrons and holes and the fact we're applying electric field:

$$\hat{\sigma} = \frac{\sigma_0}{1 + (\mu B)^2} \begin{pmatrix} 1 & -\frac{\mu B q}{|q|} \\ \frac{\mu B q}{|q|} & 1 \end{pmatrix}, \quad \sigma_0 = n\mu\,|q| \tag{3}$$

where μ is the mobility and $\hat{\sigma}$ is the conductivity tensor that satisfies $\bar{E} = \hat{\sigma}\bar{J}$ and $\hat{\sigma} = \hat{\rho}^{-1}$. The sum of the tensors is $\hat{\sigma} = \hat{\sigma_e} + \hat{\sigma_h}$. Looking at small magnetic fields $\mu B \ll 1$ we obtain the total resistivity in the xy direction:

$$\rho_{xy} = \frac{\mu_h^2 p - \mu_e^2 n}{e \cdot (\mu_e n + \mu_p p)^2} = B \cdot R_H \tag{4}$$

where R_H is the Hall coefficient. Magnetic field also affect the diagonal components. There is a change in the electrical resistance due to the magnetic field, this is called magneto-resistance. The diagonal components of $\hat{\rho}$ are

$$\rho_{xx} = \rho_{yy} = \frac{1}{e(\mu_e n + \mu_h p)} + B^2 \cdot \frac{\mu_e n \mu_h p(\mu_e + \mu_h^2)}{e(\mu_e n + \mu_h p)^3} \tag{5}$$

In addition, the Hall voltage U_H and the longitudinal voltage U_p are given by:

$$U_p = \rho_{xx} \frac{L}{Wd} I_x \,,\; U_H = \rho_{yx} \frac{I_x}{d} \tag{6}$$

1.4 Temperature Dependence in a Semiconductor

A semiconductor's resistivity depends on the temperature of the material. In a pure semiconductor (intrinsic), the relation to the density is given by:

$$n_i \propto e^{-\frac{E_g}{2k_b T}} \tag{7}$$

where E_g is the energy gap between the conduction band and the valence band. We can make our semiconductor instrinsic by raising its temperature: at freeze-out temperature the semiconductor becomes insulator. The extrinsic region is where the temperature is higher. Thus, the heat starts to ionize the atoms and the carrier's density is approximately constant. At the last region of higher temperature, we get that n_i is now larger than the doping concentration, and Equation (7) becomes relevant.

2 Results

2.1 I-V Characteristics of the Sample

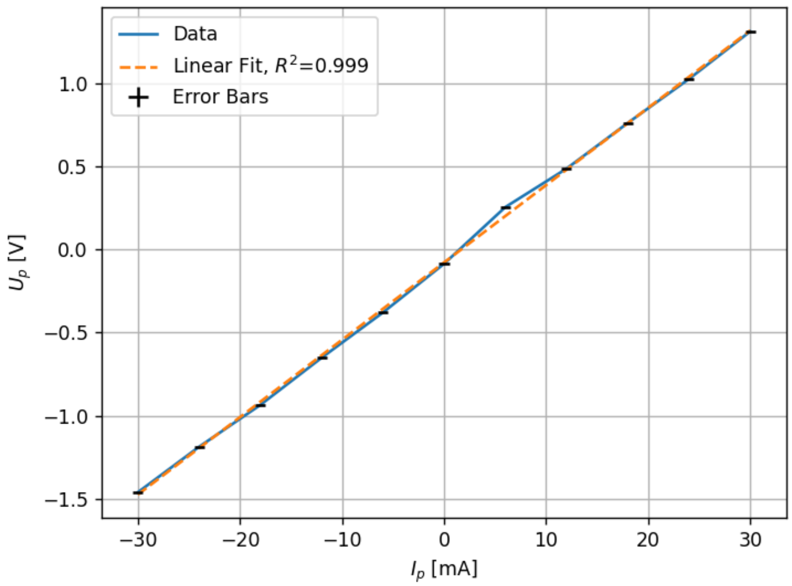

Figure 2: I-V curve

The resistance R_0 of the semiconductor is the slope of this curve, which was determined to be $46.579 \pm 0.357[\Omega]$ with an R^2 value of 0.999.

2.2 Hall Voltage vs. Control Current Experiment Results

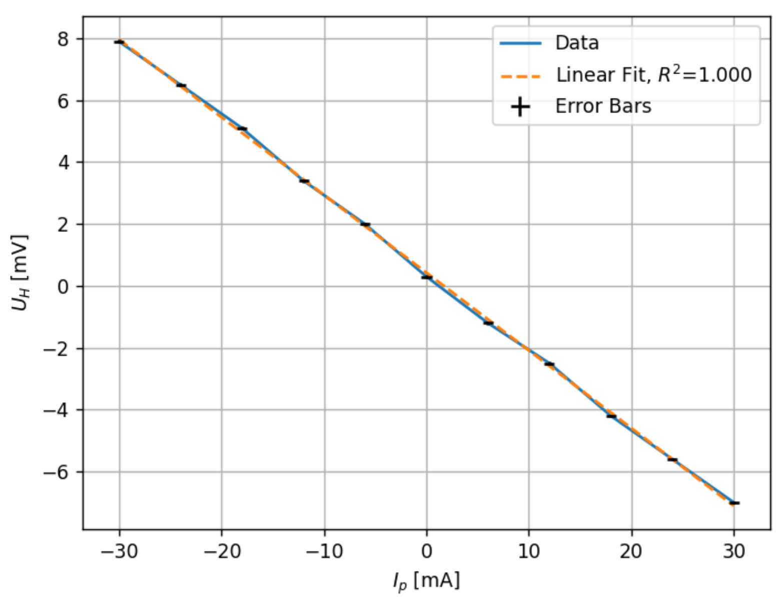

Figure 3: Hall Voltage vs. Control Current

We denote the slope of this curve to be X_1, and its value was determined to be $-0.251 \pm 0.002[\Omega]$ with an R^2 value of 1.000.

2.3 Hall Voltage vs. Magnetic Field Experiment Results

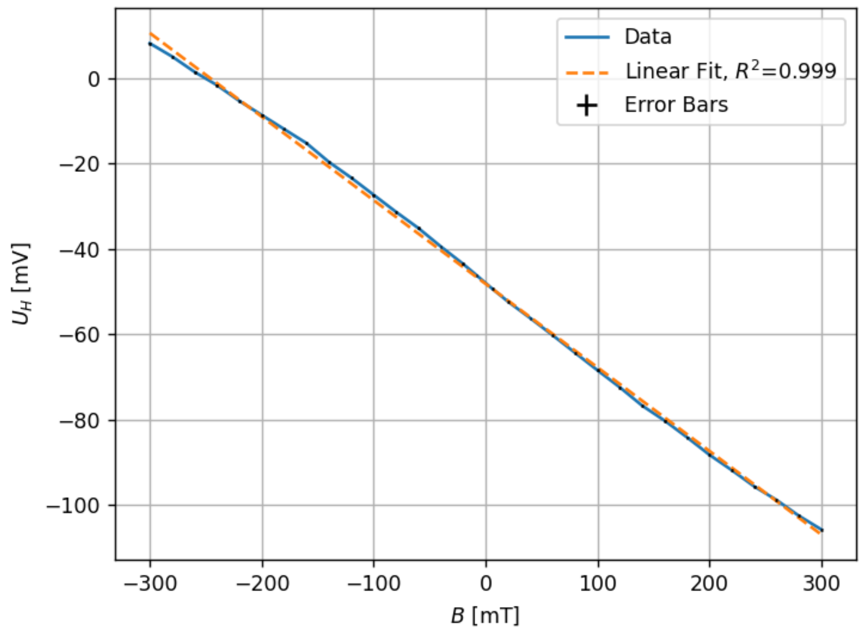

Figure 4: Hall Voltage vs. Magnetic Field

We denote the slope of this curve to be X_2, and its value was determined to be $-0.196 \pm 0.001[\frac{V}{T}]$ with an R^2 value of 0.999.

2.4 Sample Voltage vs. Magnetic Field Experiment Results

In this part of the experiment, we measured the sample voltage U_p while varying the magnetic field B.

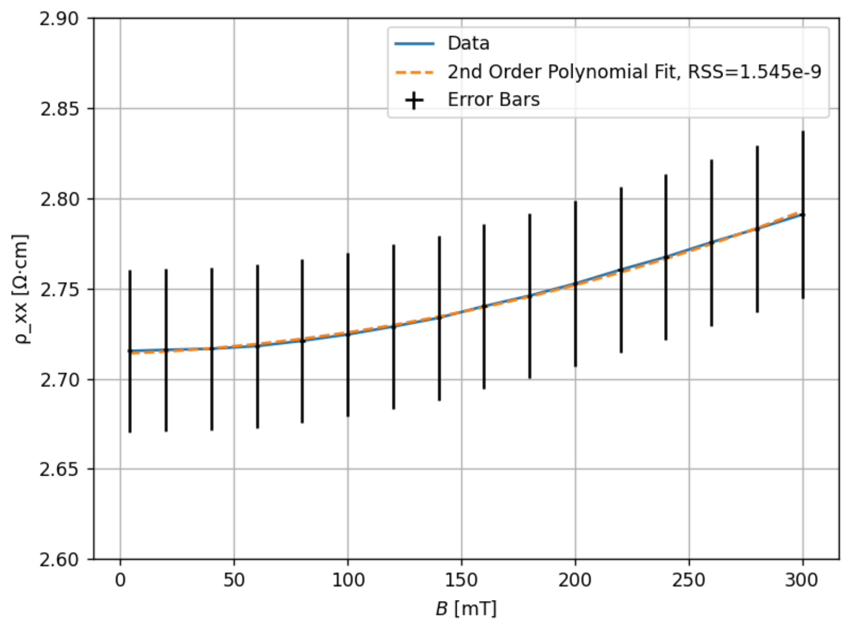

Figure 5: Resistivity vs. Magnetic Field

To find the resistivity in terms of the sample voltage, we can use the following expression:

$$\rho_{xx} = \frac{d \cdot W}{L} \cdot R = \frac{d \cdot W}{L} \cdot \frac{U_p}{I_p} \tag{8}$$

where the sample current was set to $I_p = 30 \pm 0.5[mA]$. Since we measured $U_p(B)$, we can thus use Equation (8) to obtain $\rho_{xx}(B)$, which we plot in Figure (5) and compare to Equation (5).

2.5 Sample Voltage vs. Temperature Experiment Results

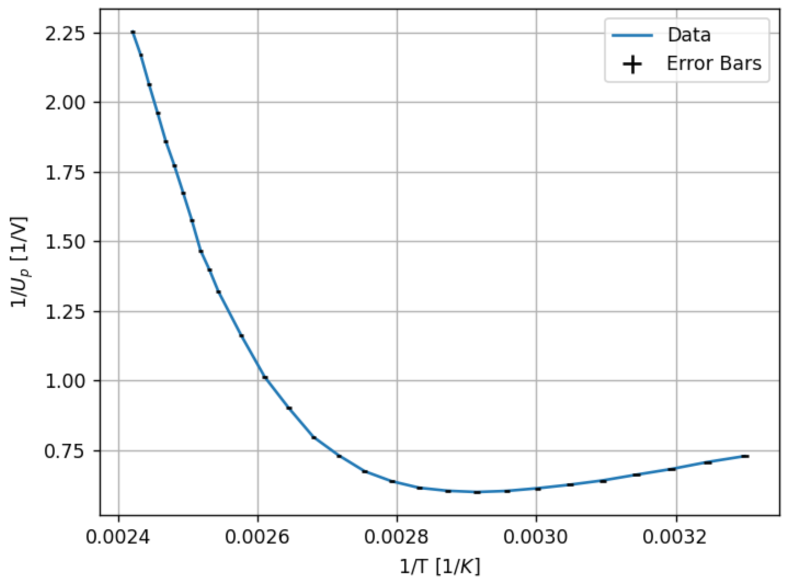

Figure 6: Inverse sample voltage vs. inverse temperature

Equation (11) gives us $ln(U_p)$ in terms of $\frac{1}{T}$, which we plot in the following figure (where the sample current was set to $I_p = 30 \pm 0.5[mA]$):

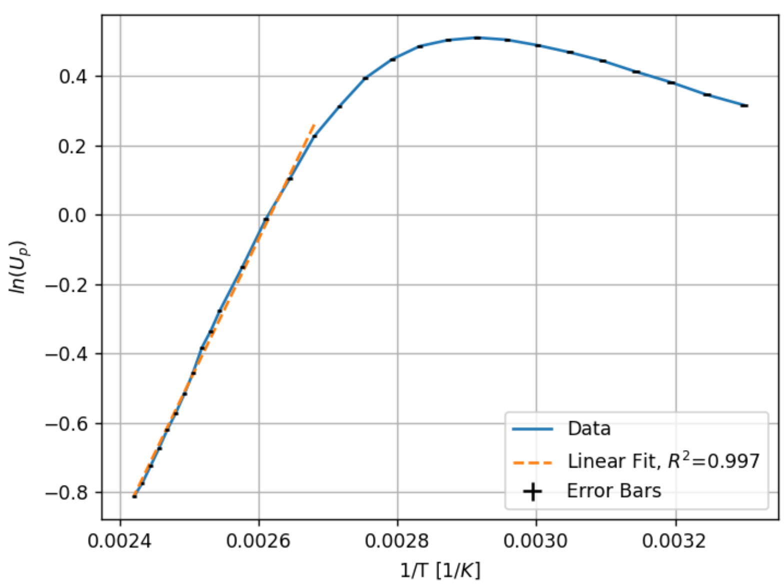

Figure 7: Natural logarithm of the sample voltage vs. inverse temperature

Conducting a linear regression on the intrinsic region ($T \geq 100°C$) in the graph, we get the slope of the fit (which we denote as X_3) to be $4126.799 \pm 59.870[K]$ with an R^2 value of 0.997.

2.6 Hall Voltage vs. Temperature Experiment Results

Equation (13) gives us $ln(U_H)$ in terms of $\frac{1}{T}$, which we plot in the following figure (where the sample current was set to $I_p = 30 \pm 0.5[mA]$ and the magnetic field was set to $B = 250 \pm 1[mT]$):

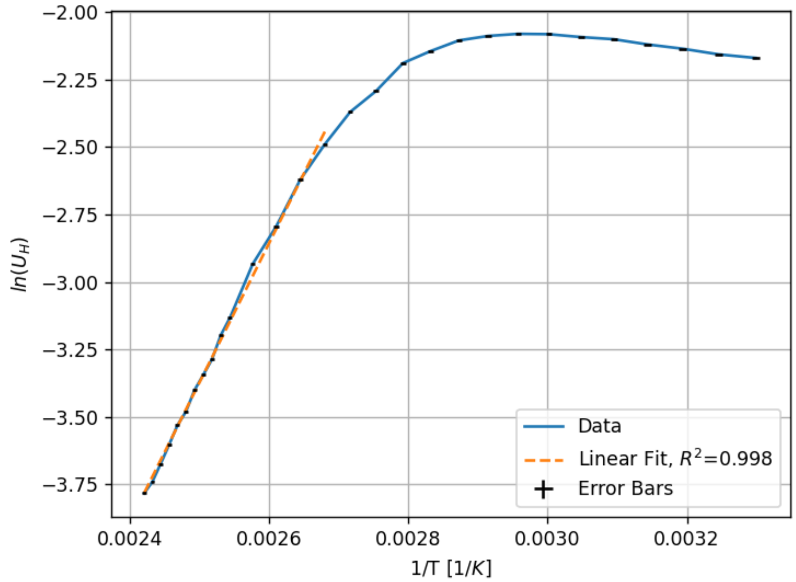

Figure 8: Natural logarithm of the Hall voltage vs. inverse temperature

Conducting a linear regression on the intrinsic region ($T \geq 100°C$) in the graph, we get the slope of the fit (which we denote as X_4) to be $5155.448 \pm 70.190[K]$ with an R^2 value of 0.998.

3 Analysis

The material is going through specific doping, which means that if we want to compare it to literature values, the best way is to use the official guide of the experiment. Thus, the theoretical values are those who were originally used by the creators.

3.1 Analysis of the Sample's I-V Characteristics

The results from this part are summarized in the following table and compared to their expected values [3]:

	Measured Value	Expected Value	χ^2
R_0	$46.579 \pm 0.357[\Omega]$	$50[\Omega]$	0.234

3.2 Analysis of the Hall Voltage vs. Control Current Experiment

The Hall coefficient of the semiconductor is:

$$R_H = \frac{d \cdot U_H}{B \cdot I_p} = \frac{d \cdot X_1}{B} = 1.001 \cdot 10^{-3}[\frac{\Omega \cdot m}{T}] = 1000.845 \pm 7.659[\frac{cm^3}{C}]$$

where $d = 1[mm]$ is the thickness of the sample and $B = -251 \pm 1[mT]$ was the measured magnetic field. We can relate the Hall coefficient to charge carrier density by:

$$R_H = \frac{1}{nq} \tag{9}$$

Since $n > 0$, and we obtained $R_H > 0$, it must be that $q > 0$ and hence our charge carriers are positively charged - the holes are the dominant charge carriers. This makes sense, since our semiconductor is p-type Germanium. The density of the majority charge carriers is then:

$$p = \frac{1}{q \cdot R_H} = 6.236 \cdot 10^{21}[m^{-3}] = (6.236 \pm 0.048) \cdot 10^{15}[cm^{-3}]$$

To find the mobility of the charge carriers, we must first obtain:

$$\rho_0 = \frac{R_0 \cdot d \cdot W}{L} = 2.911 \cdot 10^{-2}[\Omega \cdot m] = 2.911 \pm 0.022[\Omega \cdot cm]$$

where $W = 10.00 \cdot 10^{-3}[m]$ is the width and $L = 16.00 \cdot 10^{-3}[m]$ is the length of the sample. Then, the mobility is:

$$\mu = \frac{|R_H|}{\rho_0} = \frac{1.001 \cdot 10^{-3}[\frac{\Omega \cdot m}{T}]}{2.912 \cdot 10^{-2}[\Omega \cdot m]} = 3.438 \cdot 10^{-2}[\frac{1}{T}] = 343.794 \pm 3.722[\frac{cm^2}{V \cdot s}]$$

The results from this part are summarized in the following table and compared to their expected values [1][2][3]:

	Measured Value	Expected Value	χ^2
R_H	$1000.845 \pm 7.659 \left[\frac{cm^3}{C}\right]$	$4170 \left[\frac{cm^3}{C}\right]$	$2.409 \cdot 10^{-3}$
p	$(6.236 \pm 0.048) \cdot 10^{15} [cm^{-3}]$	$1.49 \cdot 10^{15} [cm^{-3}]$	$1.512 \cdot 10^{1}$
ρ_0	$2.911 \pm 0.022[\Omega \cdot cm]$	$1.750[\Omega \cdot cm]$	$7.705 \cdot 10^{-3}$
μ	$343.794 \pm 3.722 \left[\frac{cm^2}{V \cdot s}\right]$	$2380 \pm 5 \left[\frac{cm^2}{Vs}\right]$	$1.742 \cdot 10^{-1}$

We got a good result for ρ_0, R_H, and p (which are in the same order of magnitude), but μ is out of it. We assume that some of the results are not accurate due to the control current and the magnetic field since these values depend on them.

3.3 Analysis of the Hall Voltage vs. Magnetic Field Experiment

The Hall coefficient of the semiconductor is:

$$R_H = \frac{d \cdot U_H}{B \cdot I_p} = \frac{d \cdot X}{I_p} = 6.528 \cdot 10^{-3}[\frac{\Omega \cdot m}{T}] = 6528.068 \pm 113.835[\frac{cm^3}{C}]$$

where $d = 1.00 \cdot 10^{-3}[m]$ is the thickness of the sample and $I_p = -30 \pm 0.5[mA]$ was the sample current. We can relate the Hall coefficient to charge carrier density using Equation (9), and thus we obtain the density as follows:

$$p = \frac{1}{q \cdot R_H} = 9.561 \cdot 10^{20}[m^{-3}] = (9.561 \pm 0.167) \cdot 10^{14}[cm^{-3}]$$

Finally, the mobility of the charge carriers is:

$$\mu = \frac{|R_H|}{\rho_0} = \frac{6.528 \cdot 10^{-3}[\frac{\Omega \cdot m}{T}]}{29.112 \cdot 10^{-3}[\Omega \cdot m]} = 22.424 \cdot 10^{-2}[\frac{1}{T}] = 2242.417 \pm 42.706[\frac{cm^2}{V \cdot s}]$$

The results from this part are summarized in the following table and compared to their expected values [1][3]:

	Measured Value	Expected Value	χ^2
R_H	$6528.068 \pm 113.835[\frac{cm^3}{C}]$	$4170 \left[\frac{cm^3}{C}\right]$	$1.333 \cdot 10^{-3}$
p	$(9.561 \pm 0.167) \cdot 10^{14}[cm^{-3}]$	$14.9 \cdot 10^{14}\,[cm^{-3}]$	$1.913 \cdot 10^{-1}$
μ	$2242.417 \pm 42.706[\frac{cm^2}{V \cdot s}]$	$2380 \pm 5[\frac{cm^2}{V \cdot s}]$	$7.953 \cdot 10^{-4}$

We got good results for R_H, p, and μ, which are all in the same order of magnitude as their expected values.

3.4 Analysis of the Sample Voltage vs. Magnetic Field Experiment

The magneto-resistance is the tendency of a material to change its electrical resistance (ρ_{xx}, ρ_{yy}) due to a magnetic field. We assume that the resistance increases with the magnetic field due to the diamagnetism of the germanium.

3.5 Analysis of the Sample Voltage vs. Temperature Experiment

Figure (6) shows that the sample voltage decreases with increasing temperature for our semiconductor. Since the experiment was performed with a constant current, it can be assumed that the increase of charge carriers (transition from extrinsic to intrinsic conduction) with the associated reduction of the drift velocity is responsible for this (the same current for a higher number of charge carriers means a lower drift velocity). The drift velocity is in turn related to the sample voltage by the Lorentz force.

From Figure (7), we can find the energy gap E_g:

$$X_3 = \frac{E_g}{2k_B} \rightarrow E_g = 2k_B X_3 = 2 \cdot (1.381 \cdot 10^{-23}[\frac{m^2 kg}{s^2 K}]) \cdot 4126.799[K] = 1.140 \cdot 10^{-19}[\frac{m^2 kg}{s^2}]$$
$$\rightarrow E_g = 1.140 \cdot 10^{-19}[J] = 0.711 \pm 0.010[eV]$$

The results from this part are summarized in the following table and compared to their expected values [1]:

	Measured Value	Expected Value	χ^2
E_g	$0.711 \pm 0.010[eV]$	$0.72 \pm 0.03[eV]$	$1.066 \cdot 10^{-4}$

3.6 Analysis of the Hall Voltage vs. Temperature Experiment

We first remark that our values for U_H were all negative, and thus $ln(U_H)$ by itself is undefined. Looking back to our expression for $n_i(U_H)$ in Equation (12), we see that the fact that $B, I_p, d, n_i > 0$ necessitates $q \cdot U_H > 0$, and so negative values for U_H demand that q be negative as well, meaning that electrons should now be the dominant charge carriers! This is what we'd expect, actually, since we know that p-type semiconductors experience a change in the sign of their Hall coefficient

R_H (i.e. the dominant charge carriers change) as the temperature rises and the semiconductor becomes intrinsic. Thus, our real expression for $ln(U_H)$ as a function of $\frac{1}{T}$ is:

$$ln(|U_H|) = ln(\frac{B \cdot I_p}{d \cdot e \cdot n_0}) + \frac{E_g}{2k_B} \cdot \frac{1}{T}$$

where the sample current was set to $I_p = 30 \pm 0.5[mA]$ and the magnetic field was set to $B = 250 \pm 1[mT]$. From Figure (8), we can find the energy gap E_g:

$$X_4 = \frac{E_g}{2k_B} \rightarrow E_g = 2k_B X_4 = 2 \cdot (1.381 \cdot 10^{-23}[\frac{m^2 kg}{s^2 K}]) \cdot 5155.448[K] = 1.424 \cdot 10^{-19}[\frac{m^2 kg}{s^2}]$$
$$\rightarrow E_g = 1.424 \cdot 10^{-19}[J] = 0.889 \pm 0.012[eV]$$

This is higher (and farther from the expected value) than the result we obtained in the previous part, using U_p (and ρ). The results from this part are summarized in the following table and compared to their expected values [1]:

	Measured Value	Expected Value	χ^2
E_g	$0.889 \pm 0.012[eV]$	$0.72 \pm 0.03[eV]$	$3.945 \cdot 10^{-2}$

4 Conclusion

In this work, we measured Hall's coefficient, density of the charge carriers and the energy gap between the valence and conduction bands. It thought us about the characteristics of the p-type germanium, values that we got when doping the semiconductor. We saw that the behavior of the semiconductor change due to the temperature: the densities of the charge carriers are approximately the same for high enough temperatures. We obtained not accurate results for all of the values that were a function of $\bar{E}$ and $\bar{B}$. On the other hand, the measurement of E_g using the temperature change was fit. Consequently, we assume that the non accurate values were a result of the electromagnetic system, while the temperature was fine.

References

[1] PHYWE. Hall effect in n- and p-germanium (teslameter). https://www.phywe.com/experiments-sets/university-experiments/hall-effect-in-n-and-p-germanium-teslameter_11053/.

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[3] PHYWE. Phywe hall-effect unit hu 2. https://www.phywe.com/physics/modern-physics/atomic-molecular-physics/phywe-hall-effect-unit-hu-2_1917/.

[4] PHYWE. Phywe teslameter, digital. https://www.phywe.com/equipment-accessories/measurement-devices/oscilloscopes/phywe-teslameter-digital_2108/.

Appendix A: Error Analysis

A.1 Measurement Errors

The instrument accuracies present in our experiment were as follows:

Sample Current Error: $\delta I_p = 0.5[mA] = 5 \cdot 10^{-4}[A]$

Sample & Hall Voltage Error (Voltmeter): $\delta U = 5 \cdot 10^{-5}[V]$

Magnetic Field Error (Teslameter): $\delta B = 1[mT] = 10^{-3}[T]$ [4]

Temperature Error (PT-100 Sensor): $\delta T = 0.5[C^\circ]$

A.2 Error Propagation for Products & Quotients

For $F = x \cdot y$ or $F = \frac{x}{y}$:

$$\delta F = F \cdot \sqrt{(\frac{\delta x}{x})^2 + (\frac{\delta y}{y})^2}$$

A.3 Standard Error of Linear Regression Slope

$$\sigma_\beta = \sqrt{\frac{1}{n-2} \cdot \frac{\Sigma(y_i - \hat{y}_i)^2}{\Sigma(x_i - \bar{x})^2}}$$

A.4 χ^2 Metric

$$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

Appendix B: Calculations

B.1 Calculations for the Sample Voltage vs. Temperature Experiment

The conductivity of the semiconductor should change with temperature according to the following relation:

$$\sigma = \sigma_0 \cdot e^{-\frac{E_g}{2k_B T}} \tag{10}$$

where we seek to determine the value of E_g. In this part of the experiment, we measured the sample voltage U_p while varying the temperature T. To find the energy gap E_g, we first need to obtain σ in terms of U_p:

$$\sigma = \frac{1}{\rho} = \frac{L}{d \cdot W \cdot R} = \frac{L}{d \cdot W \cdot \frac{U_p}{I_p}} = \frac{L \cdot I_p}{d \cdot W \cdot U_p}$$

Taking the natural logarithm of both sides in Equation (10), we see that:

$$ln(\sigma) = ln(\sigma_0 \cdot e^{-\frac{E_g}{2k_B T}}) \rightarrow ln(\frac{L \cdot I_p}{d \cdot W \cdot U_p}) = ln(\sigma_0) + ln(e^{-\frac{E_g}{2k_B T}}) \rightarrow ln(\frac{L \cdot I_p}{d \cdot W}) - ln(U_p) = ln(\sigma_0) - \frac{E_g}{2k_B T}$$

and thus we obtain $ln(U_p)$ as a linear function of $\frac{1}{T}$:

$$ln(U_p) = ln(\frac{L \cdot I_p}{d \cdot W \cdot \sigma_0}) + \frac{E_g}{2k_B} \cdot \frac{1}{T} \tag{11}$$

B.2 Calculations for the Hall Voltage vs. Temperature Experiment

The density of the dominant charge carriers should change with temperature according to the following relation in Equation (7), where we again seek to determine the value of E_g. In this part of the experiment, we measured the Hall voltage U_H while varying the temperature T. To find the energy gap E_g, we first need to obtain n_i in terms of U_H:

$$\rho_{xy} = \frac{B}{n_i q},\ U_H = \frac{\rho_{xy} I_p}{d} \rightarrow n_i = \frac{B}{\rho_{xy} q} = \frac{B}{\frac{d \cdot U_H}{I_p} q} = \frac{B \cdot I_p}{d \cdot q \cdot U_H} \tag{12}$$

Taking the natural logarithm of both sides in Equation (7), we see that:

$$n_i = n_0 \cdot e^{-\frac{E_g}{2k_B T}} \rightarrow ln(n_i) = ln(n_0 \cdot e^{-\frac{E_g}{2k_B T}}) \rightarrow ln(\frac{B \cdot I_p}{d \cdot q \cdot U_H}) = ln(n_0) + ln(e^{-\frac{E_g}{2k_B T}})$$
$$\rightarrow ln(\frac{B \cdot I_p}{d \cdot q}) - ln(U_H) = ln(n_0) - \frac{E_g}{2k_B T}$$

and thus we obtain $ln(U_H)$ as a linear function of $\frac{1}{T}$:

$$ln(U_H) = ln(\frac{B \cdot I_p}{d \cdot q \cdot n_0}) + \frac{E_g}{2k_B T} \tag{13}$$