Hall Effect Guide

Principle:

In this experiment we will measure Hall voltage and resistivity of a semiconductor. From these measurements we will calculate the density of charge carriers in the semiconductor, their type, and mobility.

Theory:

We first discuss the Hall Effect in the framework of the Drude model (*https://en.wikipedia.org/wiki/Drude_model*). The Hall Effect, discovered by Edwin Hall in 1879, is the appearance of an electric field (the Hall field) in a current carrying conductor placed in an external magnetic field. This Hall field is perpendicular both to the current direction and the magnetic field direction.

Conductivity of a Semiconductor:

The conductivity of a semiconductor is due to movement of negatively charged electrons and/or positively "charged" holes (a hole is a "missing" electron in the semiconductor). In thermal equilibrium their densities obey the so-called mass action law:

$$(1) \qquad n \cdot p = {n_i}^2,$$

where n is the electrons' concentration, p is the holes' concentration, and n_i is the temperature-dependent intrinsic carrier density of the semiconductor. In an intrinsic (without dopants) semiconductor $n = p = n_i$. In a doped semiconductor at moderate temperatures above the dopant binding energy, but well below the forbidden gap energy, the density of the majority carriers is equal to the doping and the density of the minority ones can be derived from (1).

Example: The intrinsic density in silicon at room temperature is: $n_i = 1.4 \cdot 10^{10} cm^{-3}$. With donor doping of $N_d = 7 \cdot 10^{16} cm^{-3}$, the electron and hole densities are $n \approx N_d = 7 \cdot 10^{16} cm^{-3}$ and $p = \frac{{n_i}^2}{n} = 2800 cm^{-3}$.

In an intrinsic semiconductor the charge carriers appear due to excitation across the gap. This causes the density n_i to vary with the temperature roughly according to

$$(2) \qquad n_i \propto e^{-\frac{E_g}{2K_BT}},$$

where E_g is the energy gap between the conduction and the valence bands, K_B is the Bolzmann constant and T denotes the temperature.

The semiconductor's conductivity depends on the temperature. This dependency can be roughly separated into three regions: Freeze out (Ionization), Extrinsic and Intrinsic.

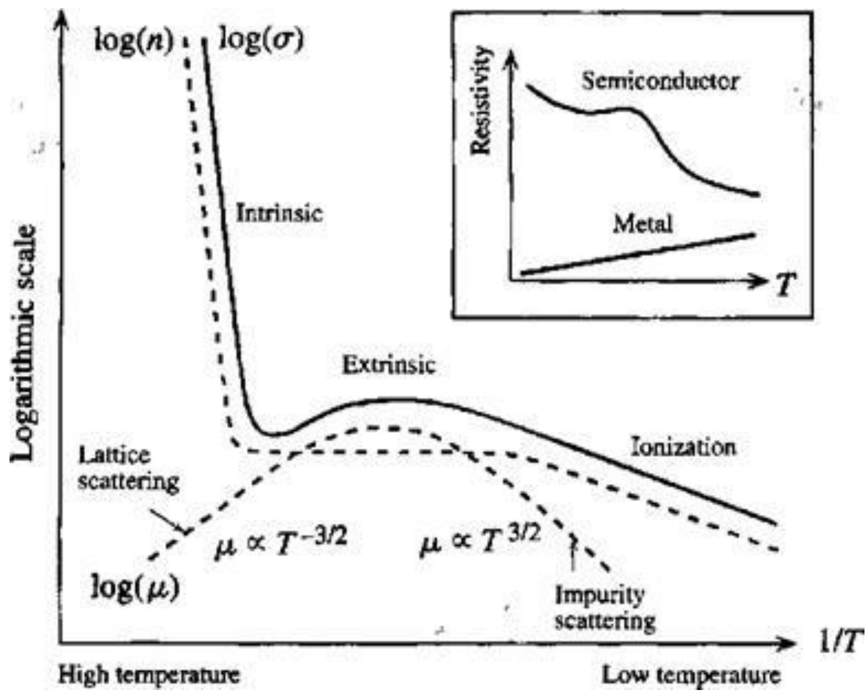


Fig. 1: Temperature dependence of conductivity, charge-carriers mobility and density in a semiconductor. Note that the temperature dependence of the mobility is just an illustration, and is different for electrons and holes, and for different semiconductors.

At **freeze-out** temperatures the carrier density drops with cooling, and eventually the semiconductor becomes an insulator.

As the temperature rises the dopants become ionized. Once they are all ionized the density of the majority carriers remains almost independent on temperature, while the density of the minority ones increases. This temperature range is called the **extrinsic** region.

As the temperature increases even more, as $n_i \propto \exp\left(-\frac{E_g}{2K_BT}\right)$ also increases and eventually becomes comparable or even larger than the doping concentration. Such temperature region is called the **intrinsic**. Note that there are no well-defined temperatures which separate the regions, rather this separation reflects approximations which are typically made to find n and p without solving nonlinear equations.

Temperature dependence of the conductivity at freezeout and the intrinsic regions is determined mostly by change in the carrier density. In contrast, in the extrinsic region the majority carrier density is approximately constant, and the change of the carriers' mobility determines the conductivity temperature dependence. At low temperatures, the carriers are scattered mostly by ionized dopants, while at high temperatures they are mostly scattered by phonons in the crystal. In the Drude model, the scattering is accounted for through relaxation time τ; the carriers' mobility is proportional to it.

The Hall Effect:

Consider a conductor in Fig. 2 in magnetic field $\vec{B} = B\hat{z}$. Let the conductor carry a current $\vec{I} = I\hat{x}$ in the direction $\hat{x}$. The Lorentz force acting on a moving charge carrier is

$$(3) \qquad \vec{F}_B = q\left(\vec{v} \times \vec{B}\right) = -qvB\hat{y},$$

where q is its electric charge and v is the velocity of the charged carrier. We used the fact that v is in $\hat{x}$ direction. **Both v and q can be positive or negative.**

Let us consider positively charged carriers as an example. In this case, $\vec{F}_B$ pushes the charges down. Charge accumulation, in turn, induces a Hall electric field $\vec{E}_H = E_H\hat{y}$. This electric field applies a force on the charge carriers in direction $\hat{y}$, so that in equilibrium there is no net current in $\hat{y}$ direction. The condition for this is (3)

$$(4) \qquad \vec{F}_B = -q\vec{E}_H$$

This field creates a potential difference between the points with the same x on the lower and upper edges of the sample (we assumed here and below that the sample is long and uniform, so the current flaws in $\hat{x}$ direction everywhere through the relevant cross section). This potential difference is called the Hall voltage U_H. This treatment is simplistic, and the velocity that appears in the Drude model is the average or the so-called *drift* velocity. But, since the Eq. 3 should hold on average, it gives the correct classical answer.

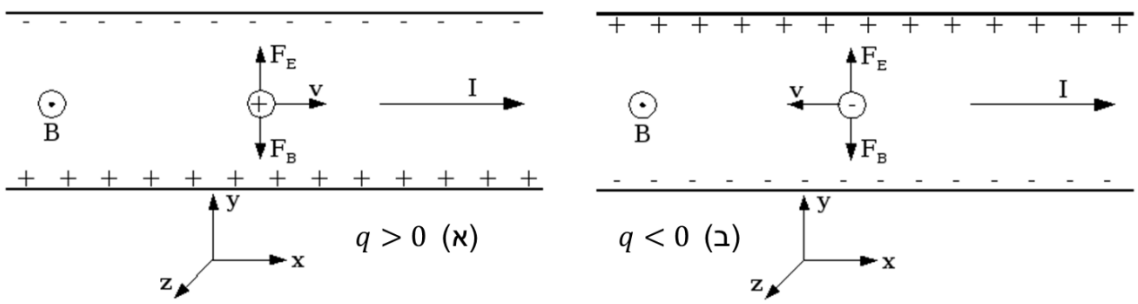


Fig. 2: Illustration of the Hall Effect for positive and negative charges. In the steady state the Lorentz force and the Hall electric field compensate each other and no additional charge is accumulated.

For a long rectangular conductor carrying current due to only one type of charge carriers, one can get from Eqs. 3 and 3 the magnitude of the Hall voltage:

$$(5) \qquad U_H = \frac{IB}{nqd},$$

where n is the density of the charge carriers and d is the thickness of the conductor. The Hall voltage may be positive or negative depending on the sign of the carrier's charge.

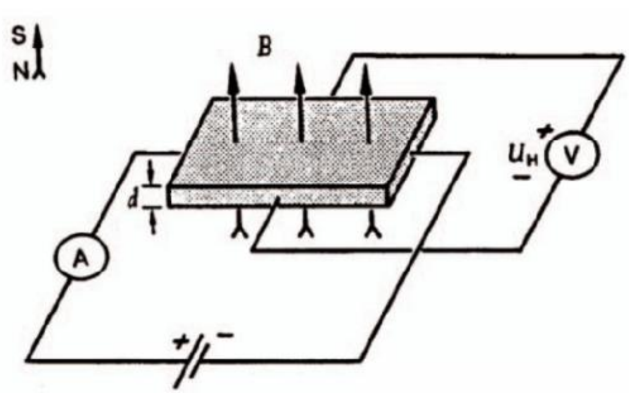


Fig. 3: The Hall Effect in a rectangular sample.

We define a parameter called the Hall coefficient R_H as:

$$(6) \qquad R_H = {}^{U_H d}\!/_{IB} = 1/nq$$

Thus, Hall coefficient in the simplest case is set by the charge density and sign.

Generalization of the Hall Effect for more than one type of carriers

At relatively high temperatures a semiconductor is in the intrinsic region, and the current is carried by both electrons and holes. This regime is relevant to the experiment you are going to preform; the Hall effect in this regime is more involved, and we discuss it below in some detail.

We shall start from expressing Hall effect in tensor form. For this we introduce resistivity tensor. We shall assume both current density $\vec{J}$ and electric field $\vec{E}$ to be in $x - y$ plane, and therefore we can express the relation between them as:

$$(7) \qquad \vec{E} = \hat{\rho}\,\vec{J}$$

with $\hat{\rho}$ being resistivity tensor:

$$(8) \qquad \hat{\rho} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix}$$

For materials with sufficiently high symmetry (and Ge you will be measuring does poses high enough symmetry) one can show that $\rho_{xx} = \rho_{yy}$ and $\rho_{yx} = -\rho_{xy}$.

In the simplest case of an isotropic material in zero magnetic field $\hat{\rho}$ is diagonal:

$$(1) \qquad \hat{\rho} = \rho_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \rho_0 = R_0 \cdot d \cdot W/L,$$

where W is the width, L the length, and d the thickness of the sample, and $R_0 = U_c/I$ is zero magnetic field two terminal resistance of the sample, determined from the voltage across the current contacts of the sample.

An important remark is in place here: due to particular geometry of the commercial samples, we are forced to determine two-terminal resistance from the voltage U_c across the current leads of the sample. It includes contact resistance that can be not small. One would prefer a more accurate 4-terminal measurement scheme, which requires both current leads and potential probs. As the result, we are bound to overestimate ρ_0, and thus underestimate carriers' mobility μ defined below.

Eqs. (7) and (8) are local expression for ohmic conductance and the Hall effect, with $\rho_{xy} = -\rho_{yx}$ being related to the Hall coefficient:

$$(2) \qquad \rho_{xy} = B/nq.$$

In multi-carrier case the carriers are subjected to the same electric field, and the resulting currents are summed up. To facilitate the treatment, we need to invert Eq. (7):

$$(3) \qquad \vec{J} = \hat{\sigma}\vec{E}, \quad \hat{\sigma} = \hat{\rho}^{-1}$$

We defined the conductivity tensor $\hat{\sigma}$. Pay attention for matrix inversion in (3).

In the [Drude theory]

$$(4) \qquad \hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} = \frac{\sigma_0}{1+(\mu B)^2}\begin{pmatrix} 1 & -\mu Bq/|q| \\ \mu Bq\,/\,|q| & 1 \end{pmatrix}, \ where\ \sigma_0 = n\mu|q|$$

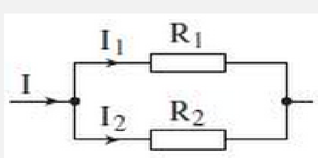
We introduced mobility μ, which is the proportionality between the electric field acting on carriers and their drift velocity. Its units are $m^2/V \cdot s$. *Mobility is usually defined as a positive parameter, and drift direction is determined by the sign of the charge carriers.*

To treat the case *of both electrons and holes* carrying current together, we shall introduce current densities $\vec{J}_{e,h}$, conductivity tensor $\hat{\sigma}_{e,h}$, resistivity $\hat{\rho}_{e,h}$, and carrier mobility $\mu_{e,h}$ for electrons and hales respectively.

The total current through the sample is:

$$(5) \qquad \hat{J} = \vec{J}_e + \vec{J}_h = \hat{\sigma}_e\vec{E} + \hat{\sigma}_h\vec{E} \ \rightarrow \ \hat{\sigma} = \hat{\sigma}_e + \hat{\sigma}_h.$$

We can think of these currents of these two carrier types as flowing in parallel, like a current flowing through parallel resistors.



Since in the experiment we fix a current and measure voltages, we must express the voltages through the resistivity $\hat{\rho} = \hat{\sigma}^{-1}$.

One can easily reverse the tensor $\hat{\sigma}$ for the case of just two types of the charge carriers. The resulting expression, however, is rather bulky. Since the experiment we are preforming involves rather small magnetic fields ($\mu B \ll 1$), we restrict ourselves to the lowest non-vanishing orders in B, which happen to be the first for the off-diagonal (Hall), and the second for the diagonal components (a question: explain qualitatively why it is the case). The off-diagonal component of ρ gives the Hall coefficient, which in two-carrier types case should *not be interpreted* as in Eq.(6). Instead:

$$(6) \qquad \rho_{yx} = -\rho_{xy} = B \cdot \frac{{\mu_h}^2 p - {\mu_e}^2 n}{e \cdot (\mu_e n + \mu_p p)^2} = B \cdot R_H.$$

The Hall coefficient might even turn to zero, which does not mean that the density is infinite.

Magnetic field also affects the diagonal component of ρ, and therefore the sample resistance; this effect is called **magnetoresistance**. Since we do not expect the diagonal component of the resistivity to be dependent on the magnetic field direction, ρ_{xx} should be an even function of B. For more rigorous explanation, you may look up Onsager relations. Indeed, up to the second order in B, the diagonal component of ρ is given by

$$(7) \qquad \rho_{xx} = \rho_{yy} = \frac{1}{e \cdot (\mu_e n + \mu_h p)} + B^2 \cdot \frac{\mu_e n \mu_h p (\mu_e + \mu_h)^2}{e \cdot (\mu_e n + \mu_p p)^3},$$

where $e = |e| \approx 1.6 \times 10^{-19}\mathrm{C}$ is the value of the electron charge.

The electron's mobility is greater than the mobility of holes since their effective mass is smaller. In the extrinsic region the minority carriers can be neglected. In contrast, deep in the intrinsic region the densities are approximately equal: $n = p = n_i$. Note that for a p-type semiconductor, the sign of R_H changes as the temperature rises and the semiconductor becomes intrinsic.

Magnetic field dependence of the sample resistance (magnetoresistance)

The presence of two carrier types lead to the sample resistance increase with magnetic field, according to formula (12). There is, however, another reason for the resistance increase, also in proportion to B^2, related to the finite sample length. Close to the sample edges, the current flows at an angle to the edges of the rectangular sample (see Fig. 4). To find the potential distribution in a sample, one need to solve the Laplace equation with appropriate boundary conditions. Although the procedure is straightforward, the solution cannot be presented in a simple analytical form. There is, however, a quite accurate approximate formula, which is sufficient in most of the cases. The details can be found in (Jensen, 1972).

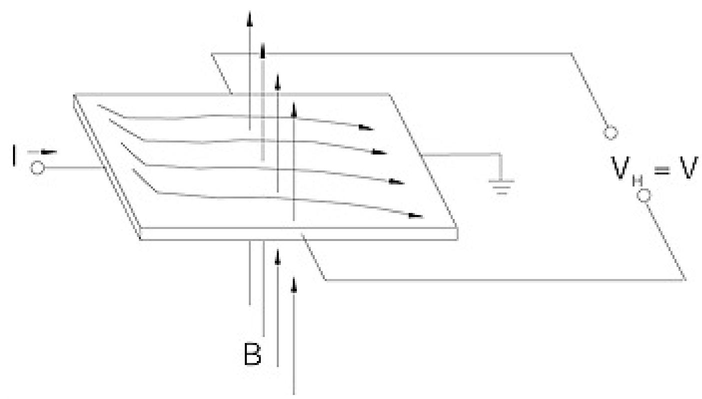


Fig. 4: The current flow in a rectangular sample. Note the deflection of the current direction in the vicinity of the sample edges.

This approximate expression is:

$$(8) \qquad R(B) - R(0) = \frac{4\rho_{xx}}{\pi d} \frac{\Theta^2}{\left(\frac{\pi}{2}\right)^2 - \Theta^2}$$

Here $\Theta = \rho_{xy}/\rho_{xx}$ is the Hall angle. You are asked to analyze and tell which contribution is the dominant one: from the two types of carriers or the geometrical one.

Quantities measured during the experiment

In the experiment we measure the voltages as a function of either B or I. In order to convert them into ρ_{xx} and ρ_{xy} we need to take into account the sample dimensions:

$$U_p = \rho_{xx} \frac{L}{Wd} I_x$$

$$U_H = \rho_{yx} \frac{I_x}{d}$$

Where L, W and d are the length, width and thickness of the conductor.

Questions:

1. Describe:
   a. U_p as a function I_x
   b. U_H as a function I_x
   c. U_H as a function B
   d. U_p as a function B
2. For a semiconductor of thickness $d = 1mm$, placed in a magnetic field $B_z = 300mT$, and carrying a current of $I_x = 30mA$ the Hall voltage of $U_H = -50mV$ between the upper and lower edges of the sample (see Fig. 3) was measured. What is the type and the density of the dominant carriers (in units of cm^{-3})?
3. What would be the magnetoresistance of a conductor with only one type of charge carriers (in absence of geometrical effects)?
4. For an intrinsic semiconductor resistance 37Ω was measured at the room temperature (20° Celsius). What would be the resistance at 140° Celsius, assuming the carrier mobility to be constant?

Setup of the Experiment:

In the experiment we create a magnetic field using two coils encircling an iron core and two polar pieces on top of them to direct the magnetic field towards the semiconductor. The semiconductor is attached to a module that is connected to a power supply.

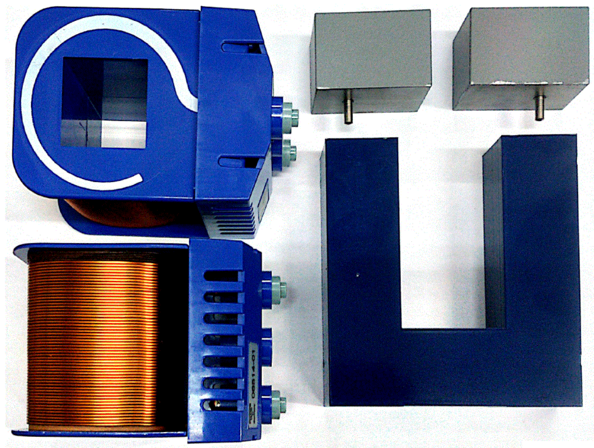


Fig. 5: Coils, an iron core and two polar pieces. The direction of the coils is indicated with the white strip (one of its ends is connected to the entrance and the other "disappears" into the coil).

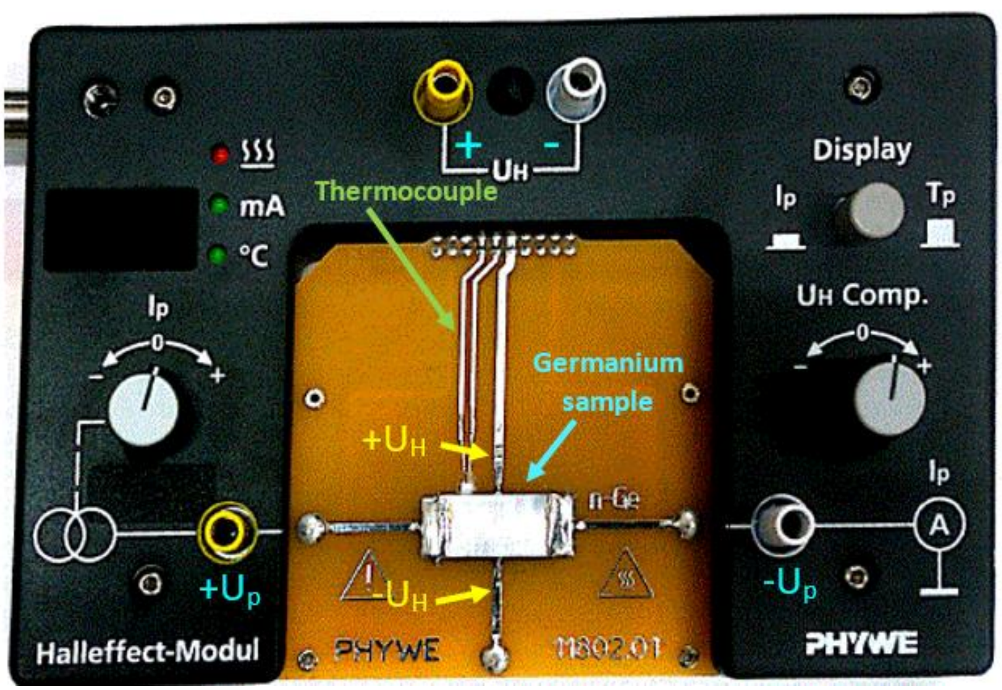


Fig. 6: The front side of the module.

Fig. 7: The back side of the module: the ac power supply connectors (12V) and a button to start and stop the sample heating.

Fig. 8: The experiment setup. When wires are connected as in the picture, the direction of the magnetic field is along the red arrow, and the magnetometer, if placed properly, gives positive reading.

Sample Dimensions:

Thickness d	1.00mm
Length L	16.00 mm (between the ohmic contacts)
Width W	10.00 mm

Setup Procedure:

1. Place the coils on the iron core and set it beneath the Hall Effect module.
2. Lower the module until it reaches the coils symmetrically. Pay attention to place the sample in the middle with its surface normal to the magnetic field.

3. Connect the blue and red current leads to the coils to build a positive magnetic field (that points out of the front of the module, as we defined in the introduction).
4. Connect the module to the power supply (12V AC) using the black leads, and the coil leads to the DC entrances. DON'T turn on the power supply.
5. Connect the probe to the Teslameter, take off its cape, and **accurately** insert it into the module through the hole on top. Make sure it faces the front. BE CAREFUL not to harm neither the semiconductor sample, nor the probe!
6. Add the polar pieces carefully.
7. Make sure the DC current of the power supply is set to zero and turn it on. **Because of the large induction of the coils, you must always set the current to zero before changing the connections of the coils. A sudden change of the current in the coils will create an "infinite" voltage that is very dangerous – it could give you an electric shock, or even worse -- to burn the devices.**
8. Turn on the Teslameter and zero it with the probe outside the module.

Experiment procedure:

Part 0: Measure I-V (current-voltage) characteristic of the sample.

1. Remove the pole pieces.
2. Connect the multimeter to the sockets of the sample voltage U_p (potential probes) on each side of the semiconductor on the module.
3. Set the display of the module to the "current mode".
4. Measure the sample voltage U_p for different sample currents I_p ranging from -30mA up to 30mA in steps of about 5mA. Get at least 10 measures.
5. Plot the sample voltage U_p as a function of the sample current I_p and calculate the resistance R_0 of the semiconductor.

Part 1: Measure Hall voltage vs control current

1. Connect the multimeter to the sockets of the Hall voltage U_H on the front side of the module.
2. Place the pole pieces.

3. Compensate the offset of the Teslameter to show zero magnetic field when the Hall probe is outside the module, then insert the Hall probe into the module. Be careful not to harm the semiconductor sample.
4. Set the magnetic field to a value to approx. 250mT by changing the voltage and current on the power supply and note the value and precision. Pay attention to the polarity of the field!
5. Use the "U_H Comp." button to zero the offset of the Hall voltage U_H for zero sample current (I_p = 0). **If you cannot set offset to zero, inform the instructor.**
6. Measure the Hall voltage U_H for different sample currents I_p ranging from -30mA up to 30mA in steps of about 5mA.
7. Determine the type of the charge carriers in the semiconductor sample.
8. Plot the Hall voltage U_H as a function of the sample current I_p and calculate the Hall coefficient: $R_H = U_H \cdot d\ /B \cdot I$
9. Calculate the density of the majority charge carriers (the doping) using: $R_H = 1/nq$ and assuming that one type of carriers dominates (n here can be density of either electrons or holes, depending on sign of q.
10. Calculate mobility of the majority charge carriers using: $\mu = |R_H| \cdot \sigma_0 = |R_H|/\rho_0$, with ρ_0 calculated using Eq.(1)

Part 2: Measure Hall voltage vs magnetic field

1. Set the sample current I to 30mA and note the value and precision.
2. Set the current through the coils to zero.
3. Use the "U_H Comp." button to zero the offset of the Hall voltage U_H for zero magnetic field. **If you cannot set offset to zero, inform the instructor.**
4. Measure the Hall voltage U_H as a function of the magnetic field for B values ranging from -300mT up to 300mT in steps of approximately 20mT.

   To obtain negative magnetic field, swap the current leads on the power supply.
   <u>Important! Make sure to TURN OFF the current through the coils before disconnecting the current leads!</u>
5. Plot the Hall voltage U_H as a function of the magnetic field B and calculate the Hall coefficient R_H.
6. Calculate the density of the majority charge carriers.

7. Calculate the mobility of the majority charge carriers.
8. Compare the results.

Part 3: Measure sample voltage vs magnetic field

1. Make sure the offset of the Teslameter is zero outside the module.
2. Connect the multimeter to the sockets on each side of the semiconductor to measure the sample voltage U_p.
3. Set the control current I_p to 30mA. Note the value and precision.
4. Measure the sample voltage U_p for different magnetic fields, from 0 up to 300mT with the highest precision possible.
5. At the end TURN OFF the magnetic field before disconnecting the current leads!
6. Plot the resistance of the semiconductor as a function of the magnetic field.
7. Check if the magnetoresistance can be due to the current being carried by both electrons and holes, as per Eq. (7). Does the resistance decrease or increase with magnetic field? Can Eq. (7) explain the value of the change?
8. Look at the effect of the finite sample length on the magnetoresistance. Which of the two explanations discussed above better agree with the experiment?
   One of the carrier's mobilities, either electron or hole, you have found experimentally. For the other one, take a reference value. At T=300 K, the electron mobility $\mu_e \approx 0.39\frac{m^2}{Vs}$, and the hole mobility, $\mu_h \approx \ 0.19\frac{m^2}{Vs}$. These values are for reference only, since mobilities depend on doping of the samples. See references in http://www.ioffe.ru/SVA/NSM/Semicond/Ge/electric.html#Hall.

Part 4: Measure sample voltage vs temperature

1. Take the Hall probe OUT of the module!
2. Set the sample current I_p to 30mA and the magnetic field to zero.
3. Remove the pole pieces.
4. Connect the multimeter to measure the sample voltage U_p.
5. Change the display on the module to show the temperature T, using the display button.
6. Place the multimeter next to the module and make sure you can film both display screens. Prepare a filming device (smartphone, camera).

7. Start filming and press the <u>on/off button</u> on the backside of the module to activate the heating coil. You can stop the heating by pressing the <u>on/off button</u> again.
8. Film until the temperature drops back to 30˚ Celsius (about 7 minutes).
9. Measure the sample voltage U_p as a function of the temperature T. Make sure you have at least 10 measurements for 120˚-140˚ Celsius and at least 15 measurements for the rest of the temperatures.
10. Plot U_p^{-1} as a function of T^{-1}. Explain the graph.
11. Plot $ln(U_p)$ as a function of $1/T$. Find the linear region (and therefore close to the intrinsic one) and calculate the energy gap E_g using: $\sigma = \sigma_0 * e^{-\frac{E_g}{2K_BT}}$

Part 5: Measure the Hall voltage vs temperature.

These measurements are similar to **Part 4** with one important check that stems from a problem with Hall contacts with part of the samples. You may look at is as a window in the real world of experimentalists, in which not everything is perfect, and debugging is an essential part of the job.

You will need first to check that Hall contacts are good enough for measurements at elevated temperature. Bad contacts would affect your measurements at the room temperature as well: you might not be able to compensate for zero magnetic field Hall voltage, as was discussed in **Part 2**. But at elevated temperatures a thermopower far exceeding expected Hall voltage can develop across the Hall probes. To make sure that it is not the case do the following:

1. Set the current through the sample to zero. You may also set at this stage the magnetic field to 250-300mT to save time.
2. Connect the multimeter to measure the Hall voltage U_H.
3. Set the current to zero.
4. Set the offset of the Hall voltage U_H to zero (the magnetic field should be zero at this stage).

   It is very important for this particular experiment, since you don't have the luxury to subtract offset using linear fit, as in the case of $\boldsymbol{U_H(I)}$ or $\boldsymbol{U_H(B)}$.

If you cannot compensate offset, and this happens sometimes due to bad sample contacts, try to measure at both positive and negative magnetic field, and calculate $(U_H(B) - U_H(-B))/2$. **Report the problem to the instructor**.

5. Heat the sample and watch Hall voltage. It should be small compared to the Hall voltage you measured in **Part 2**. If it is comparable, or even worse – much larger, then you have a contact problem, and you won't be able to continue measurements with this sample. Then, contact the lab instructor or engineer.
   If the sample passed the test above, proceed as follows:
6. Set the control current I_p to 30mA.
7. Try to remember the position of the knob that sets magnetic field, and reduce the field to zero. Your system should be still relatively hot. If is had already cooled down – reheat.
8. Try to compensate U_H as in Part 2. If you are able – great. If there is still a small offset — write it down and proceed. If the offset is big -- contact the lab instructor or engineer.
9. Return the magnetic field knob to the previous position. You should get approximately the same magnetic field. You will need to check it in the end when the system is cold to know the value better.
10. Reheat the sample.
11. Measure the Hall voltage U_H as a function of the temperature *T* by filming the system during cooldown, as in **Part 4.**

12. Plot logarithm of the Hall voltage $\ln(U_H)$ as a function of $1/T$ and explain the graph.
13. Do you think it is possible to estimate the energy gap E_g using: $n_i \propto e^{-\frac{E_g}{2K_B T}}$? If you think it is, then choose a proper temperature range, and try to do it. Dos the result agree with the one from $\rho_{xx}(T)$?
14. Compare the results to the literature and explain the difference, if any.

where μ (in units of $\frac{m^2}{\mathrm{V}\cdot sec}$) is the mobility of the charge carriers, defined as

$$(1) \qquad v = \mu E = \frac{\sigma}{nq} E \rightarrow \mu = \frac{\sigma}{nq},$$

where σ (in units of $\frac{1}{\Omega \cdot m}$) is the conductivity.

Mobility is usually defined as a positive parameter, and drift direction is determined by the sign of the charge carriers.

References

Jensen, H. H. (1972). Geometrical effects in measurements of magnetoresistance. *Journal of Physics C: Solid State Physics*, 2867.