Measurement of the Coil Coefficient using the Electron Spin Resonance Method

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Abstract

In this work, we used ESR controller and oscillator to measure the k constant of the outer coil. This can be obtained using electron spin resonance method. In addition, we measured the spin-spin relaxation time for DPPH in a magnetic field.

Contents

1 Introduction

1.1 Experimental Setup

As seen in figure (1), the experimental setup for this work relied on oscilloscope, ESR controller and modulation current supply, AC ammeter to measure the current, DC power supply and ESR oscillator.

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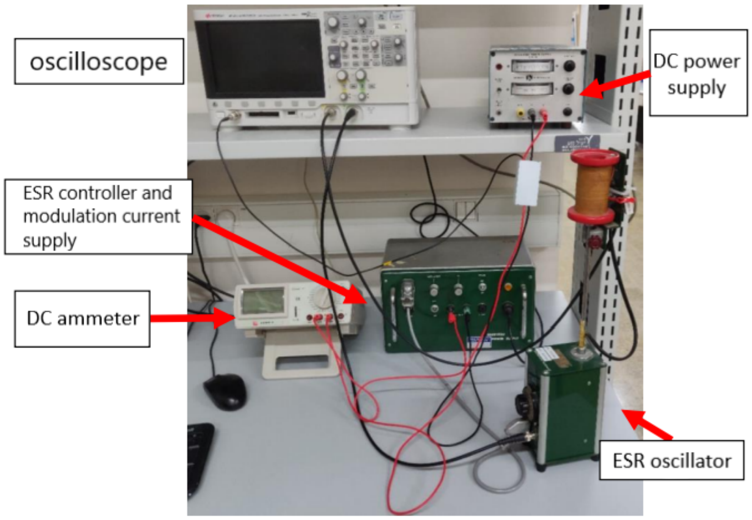

Figure 1: Picture of the experimental setup

In figure (2) we obtain the DPPH material, covered by a small coil and a large coil that wraps them.

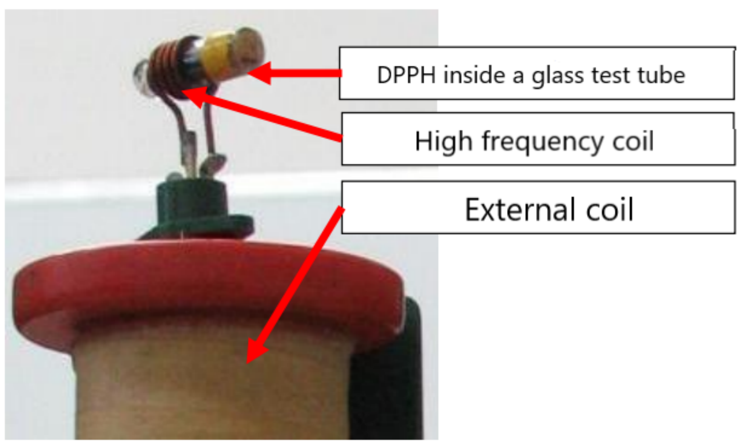

Figure 2: DPPH module

1.2 Measuring the Coil's constant using ESR method

In the first experiment we needed to measure the constant $k = \frac{H}{I}$ of the large coil. The DPPH is a paramagnetic material, it attracts to a magnetic field and have unpaired electrons (free radicals). We used AC current through the outer coil, which changes the direction and magnitude of the magnetic field inside. According to the Zeeman effect, the electrons will populate both up and down spins, in a way there are more down spins then up spins (Boltzmann's distribution). The energy of an electron in a magnetic field is

$$U_{\pm} = \pm\frac{1}{2}g\mu_B B \tag{1}$$

Where $g = 2.0023$ is the g-factor of an electron and mu_B is Bohr magneton. So we get that the absorption energy is given by

$$\Delta U = U_+ - U_- = g\mu_B B \tag{2}$$

In addition, the AC current in the small coil will supply photons that will hit the material all around. Once the energy of the photons will be equivalent to the energy gap of the electrons ΔU we will obtain resonance and see a pick in the signal graph.

$$h\nu_{RF} = g\mu_B B_{res} \tag{3}$$

When h is Planck's constant and ν_{RF} is the photon's frequency. We apply a time-varied magnetic field given by

$$B = \mu_0 k \left(I_0 + I_m sin\left(2\pi f t\right)\right) \tag{4}$$

1.3 Measuring spin-spin relaxation time

Spin-spin relaxation time donated by T_2 is the time constant indicating the time M_{xy} (transverse component of the magnetisation vector) exponentially decays towards its equilibrium value. Its relation to the absorption power is given by [1]

$$P\left(\nu_0\right) = \frac{A}{1 + T_2^2\left(\nu_0 - \nu_{RF}\right)^2} \tag{5}$$

When $\nu_0 = \frac{H_0 g\mu}{h}$, $\nu_{RF} = \frac{\Delta U}{h}$ and A is a constant. The derivative is given by

$$P'\left(\nu_0\right) = \frac{-2AT_2^2\left(\nu_0 - \nu_{RF}\right)}{\left(1 + T_2^2\left(\nu_0 - \nu_{RF}\right)^2\right)^2} \tag{6}$$

And that is what we can measure using the oscilloscope.

2 Results

2.1 Absorption Signal Calibration

To get the resonant frequency, we use the following linear interpolation:

$$\nu_{RF}[MHz] = 96.25 + \frac{95.4 - 96.25}{40 - 0} \cdot (x - 0)$$

We found $x = 44$, and thus:

$$\nu_{RF} = 96.25 + \frac{95.4 - 96.25}{40 - 0} \cdot (44 - 0) = 95.315[MHz] \pm 0.1[MHz]$$

From equation (3), we can now extract the resonant magnetic field:

$$B_{res} = \frac{h\nu_{RF}}{g\mu_B} = \frac{(6.626 \cdot 10^{-34}[J \cdot s])(95.315 \cdot 10^6[Hz])}{(2.0036)(9.274 \cdot 10^{-24}[\frac{J}{T}])} = 3.399 \cdot 10^{-3}[T] = 3.399 \pm 0.004[mT]$$

2.2 Measurment of k under Minimal Modulation for Resonance

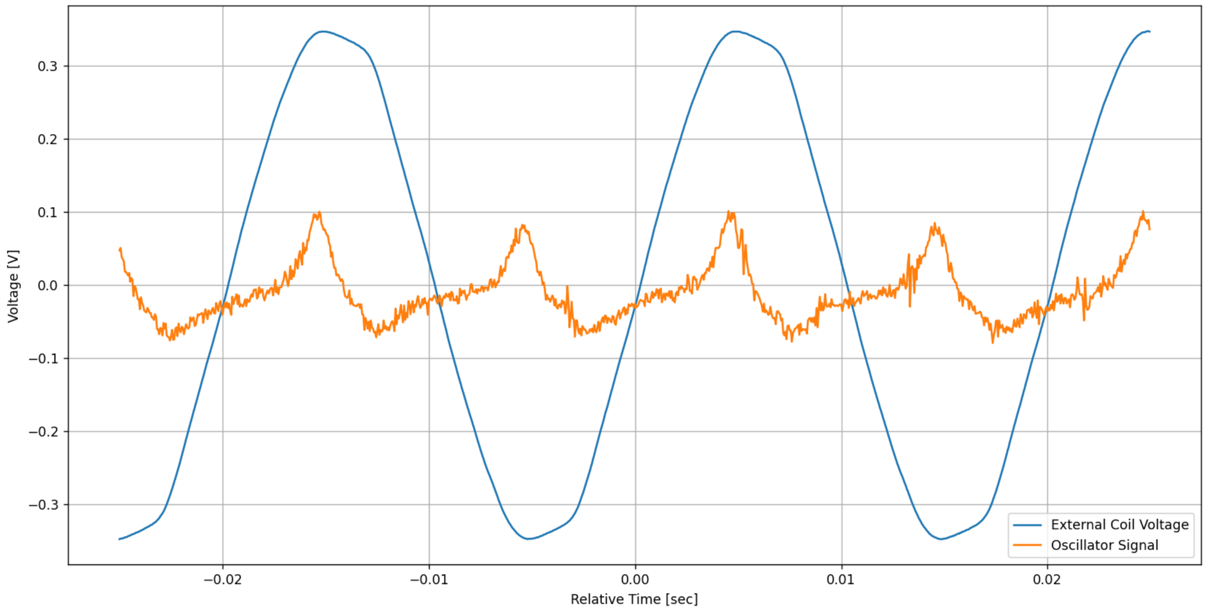

Figure 3: Signal measurements for the minimal modulation resonance method

$$V_{coil} = \frac{1}{2} \cdot V_{pp} = \frac{1}{2} \cdot (0.347 - (-0.347)) = 0.347 \pm 0.001[V] \tag{7}$$

and we are given that the resistance is $R = 0.82[\Omega]$, thus the current through the external coil in this step is:

$$I_{coil} = \frac{V_{coil}}{R} = \frac{0.347[V]}{0.82[\Omega]} = 0.423 \pm 0.050[A] \tag{8}$$

The magnetic field strength here is given by:

$$H = \frac{B_{res}}{\mu_0} = \frac{3.399 \cdot 10^{-3}[T]}{1.256 \cdot 10^{-6}[N/A^2]} = 2704.761 \pm 2.850[\frac{A}{m}] \tag{9}$$

From here, the coil coefficient k is directly calculated:

$$k = \frac{H}{I_{coil}} = \frac{2704.761[\frac{A}{m}]}{0.423[A]} = 6391.932 \pm 0.118[\frac{1}{m}] \tag{10}$$

2.3 Measurement of k with Direct Current

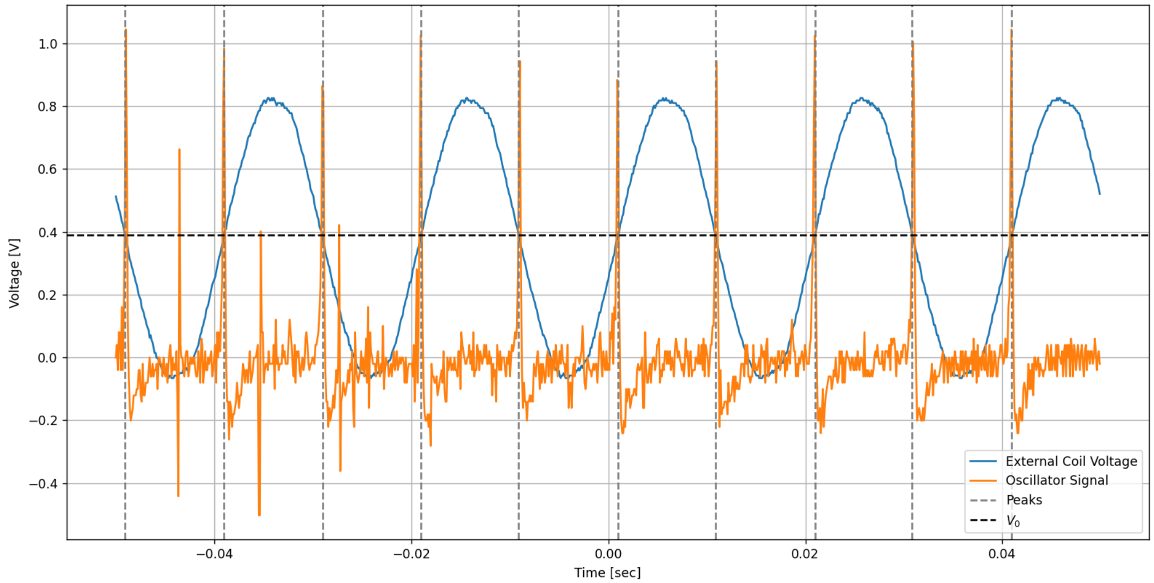

Figure 4: Signal measurements for the DC measurement method; the DC voltage value is indicated, and corresponds to the peaks of the oscillator signal

From the graph, we can see that $V_0 = 0.390 \pm 0.002[V]$, and so

$$I_0 = \frac{V_0}{R} = \frac{0.390[V]}{0.82[\Omega]} = 0.476[A]$$

Thus, the coil coefficient is just

$$k = \frac{H}{I_0} = \frac{2704.761[\frac{A}{m}]}{0.476[A]} = 5686.933[\frac{1}{m}]$$

2.4 Resonance Detection using the XY Method

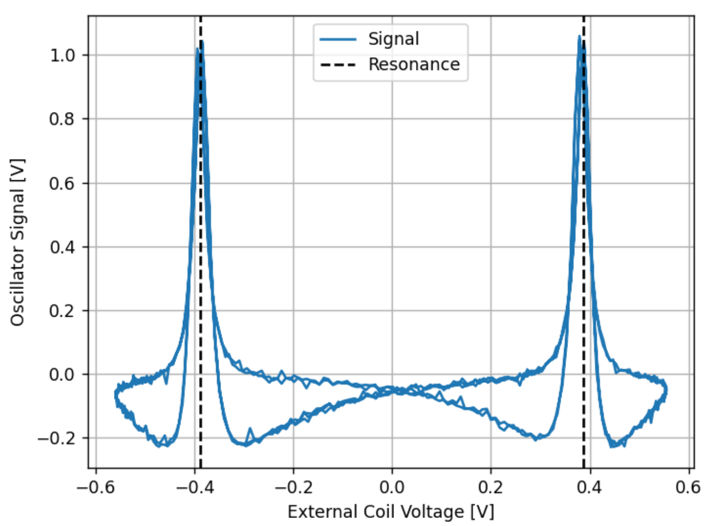

Figure 5: XY method signal representation; the symmetry of the graph indicates the presence of resonance, and the resonant voltage corresponds to the observed peaks

On the oscilloscope we plotted $Y(X)$. The resonance occurred when there was only one purple pick in the middle of the display.

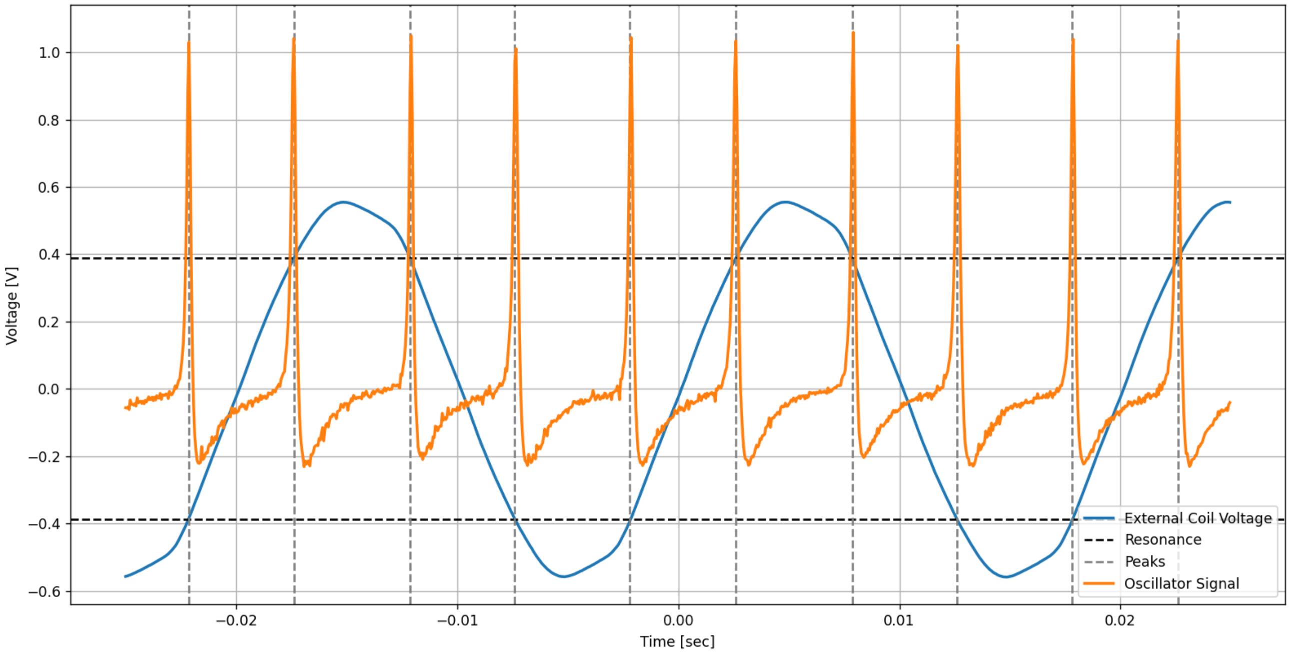

Figure 6: Signal measurements for the XY method; the resonant voltage value is indicated, and corresponds to the peaks of the oscillator signal

From the graph, we can see that $V_{res} = 0.388 \pm 0.002[V]$, and so

$$I = \frac{V_{res}}{R} = \frac{0.388[V]}{0.82[\Omega]} = 0.473[A]$$

Thus, the coil coefficient is just

$$k = \frac{H}{I} = \frac{2704.761[\frac{A}{m}]}{0.473[A]} = 5716.247[\frac{1}{m}]$$

2.5 Calculation of the Absorption Signal Derivative

On the oscilloscope, we measured each time the amplitude of the current, the amplitude of the signal and the phase sign $\pm$ between them. We then substitute them into the formula

$$P^{'}(\nu_0) = \frac{Amplitude\,(Y)}{Amplitude\,(X)} \cdot (Relative\,sign)\,|_{\nu_0} \tag{11}$$

We can see these measurements in figure (7) as well as the numerical integration used to create the absorption signal:

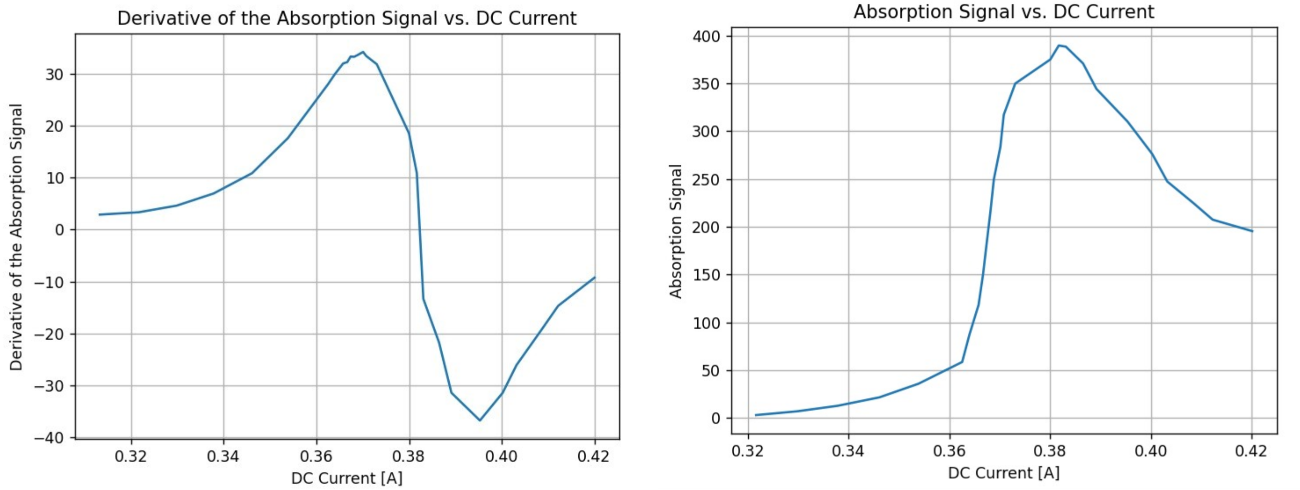

Figure 7: Absorption signal derivative and the absorption signal itself

From Kittel, we know that the free precession frequency is $\omega = \gamma B$ [1]. To convert from I to ω:

$$\omega = \gamma B = \gamma \mu_0 H = \gamma \mu_0 k I$$

where $\gamma = \frac{g\mu_B}{\hbar}$ is the gyromagnetic ratio of the electron. Thus:

$$\omega = \frac{g\mu_B \mu_0 k}{\hbar} I = 2\pi \cdot \frac{g\mu_B \mu_0 k}{h} I \ (= 2\pi \cdot \nu)$$

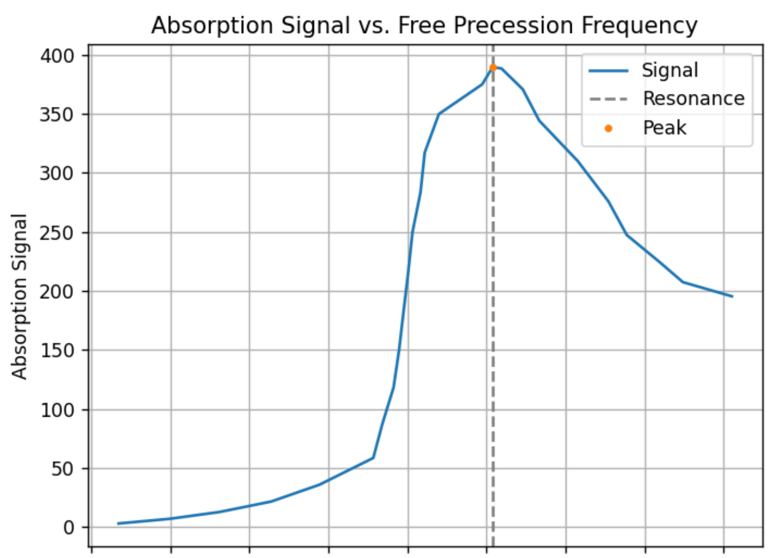

Figure 8: Absorption Signal

From this graph, we see that $\omega_0 = 1.504[Grad/s]$. Thus, the spin-spin relaxation time is:

$$T_2 = \frac{2\pi[rad]}{\omega_0} = \frac{2\pi[rad]}{1.504[Grad/s]} = 4.178[ns]$$

3 Analysis

3.1 Measurement of k under Minimal Modulation for Resonance

In this graph we obtained the resonance at the extrema, which is reasonable since we fitted the graph in a way that the extrema touch the resonance levels.

3.2 Measurement of k with Direct Current

Here we raised the graph by DC current until we found peaks in the signals that are in equal distances from each other. This was what we've expected since the resonance voltage is now right between the extrema.

3.3 Resonance Detection using the XY Method

Using the XY method we saw two picks in the signal around $V_{1,2} = \pm 0.4[V]$ which states that this is the resonance voltage. We then plotted the second graph to double check the results and saw that the voltage matches our expectations.

3.4 Calculation of the Absorption Signal Derivative

We obtained a Gaussian-like graph from the integration and got one peak that indicates the current in which we have resonance, as expected. In this case, we see a maximum of absorption signal. In literature $T_2 = 62[ns]$ which is far from our experimental value.

4 Conclusion

We found k in 4 different ways and they all agreed with each other. In addition, the relaxation time is between the correct values as found from the literature.

Appendix: Error Analysis

The accuracies and errors present in our experiment were as follows:

Resistor Value: $\delta R = 5\% \cdot 0.82[\Omega] = 0.041[\Omega]$

g-Factor: $\delta g = 0.0002$

Resonant Frequency: $\delta \nu_{RF} = 0.1[MHz]$

KEYSIGHT InfiniiVision DSOX2002A Oscilloscope (Voltage Measurements): $\delta V = 0.002[V]$ [2]

For part 2.1, we need to analyze the error on equation (3) to find the error on the resonant magnetic field calculation:

$$\delta B_{res} = \delta(\frac{h\nu_{RF}}{g\mu_B}) = \frac{h}{\mu_B} \cdot \delta(\frac{\nu_{RF}}{g}) = \underbrace{\frac{h}{\mu_B} \cdot \frac{\nu_{RF}}{g}}_{B_{res}} \sqrt{(\frac{\partial \nu_{RF}}{\nu_{RF}})^2 + (\frac{\partial g}{g})^2}$$

$$= (3.399[mT])\sqrt{(\frac{0.1[MHz]}{95.315[MHz]})^2 + (\frac{0.0002}{2.0036})^2} = 0.00358[mT] \approx 0.004[mT]$$

The second equality is due to the fact that we take h and μ_B to have negligible errors. For part 2.2, we can analyze the error on equation (7) to find the error on the coil voltage calculation:

$$\partial V_{coil} = \partial(\frac{1}{2} \cdot (V_{max} - V_{min})) = \frac{1}{2} \cdot \partial(V_{max} - V_{min}) = \frac{1}{2} \cdot \sqrt{(\delta V_{max})^2 + (\delta V_{min})^2} = \frac{1}{2} \cdot \sqrt{2(\delta V)^2}$$
$$= \frac{\sqrt{2}}{2} \cdot \delta V = 0.0014[V]$$

Then, the error on the current in equation (8) is:

$$\partial I_{coil} = \partial(\frac{V_{coil}}{R}) = \sqrt{(\frac{\partial V_{coil}}{V_{coil}})^2 + (\frac{\partial R}{R})^2} = \sqrt{(\frac{0.0014[V]}{0.347[V]})^2 + (\frac{0.041[\Omega]}{0.82[\Omega]})^2} = 0.050[A]$$

The error on the magnetic field strength in equation (9) is:

$$\partial H = \partial(\frac{B_{res}}{\mu_0}) = \frac{1}{\mu_0}\partial(B_{res}) = \frac{1}{1.256 \cdot 10^{-6}[N/A^2]} \cdot 0.00358[mT] = 2.850[\frac{A}{m}]$$

And finally, error on the coil coefficient k from equation (10) is:

$$\partial k = \partial(\frac{H}{I_{coil}}) = \sqrt{(\frac{\partial H}{H})^2 + (\frac{\partial I_{coil}}{I_{coil}})^2} = \sqrt{(\frac{2.850[\frac{A}{m}]}{2704.761[\frac{A}{m}]})^2 + (\frac{0.050[A]}{0.423[A]})^2} = 0.118[\frac{1}{m}]$$

References

[1] Charles Kittel and Paul McEuen. *Introduction to solid state physics*. John Wiley & Sons, 2018.

[2] KEYSIGHT Technologies. Keysight technologies infiniivision 2000 x-series oscilloscopes datasheet, 2022. https://www.keysight.com/il/en/assets/7018-02733/data-sheets/5990-6618.pdf.