Measurement of Excitations and Ionizations in Mercury Gas using the Franck-Hertz Experiment

Yotam Granov* and Ofir Shukrun†

April 2023

Abstract

In this work, we used a Franck-Hertz tube in order to measure the excitation energy and ionization voltage of mercury gas. This can be measured due to ionization of mercury atoms by electrons. We measured the voltage as a function of the current and compared to the spectroscopy of mercury.

Contents

1 Introduction

1.1 Experimental Setup

The experimental setup for this work relied on a Franck-Hertz tube filled with mercury gas, containing a cathode (K), anode (A), and electrode (M), connected in a circuit to a pico-ammeter (for

*ID: 211870241, Email: yotam.g@campus.technion.ac.il
†ID: 206228249, Email: ofir-s@campus.technion.ac.il

current measurement), a power supply for the acceleration voltage V_a (K-A) and braking voltage V_r (M), and a thermal heater with a sensor measuring the temperature of the mercury gas at any given time. The accuracies of the various instruments used here are described in Appendix A.

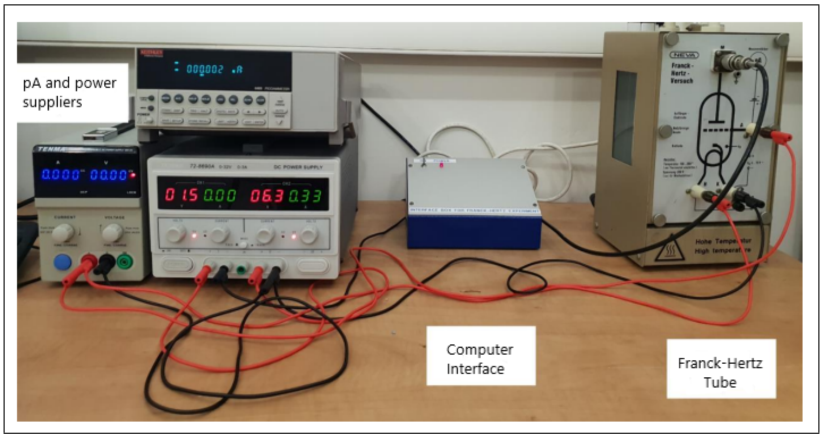

Figure 1: Picture of the experimental setup for Part 1

In Figure (1), we can see how our experimental setup looked for the first part of the experiment, which will be described in Section 1.2.

1.2 Measurement of the Excitation Energy and Contact Voltage of Mercury Gas

In this first part of the experiment, we sought to deduce the excitation energy and contact voltage of the mercury gas in our Franck-Hertz tube by measuring the I-V curve for varying V_r values (in the first half) and varying V_h values (in the second half). For every trial we conducted, we swept over the V_a values from 0 to $20[V]$ while measuring the current in the F-H tube using the pico-ammeter. A diagram of the setup for this part of the experiment can be seen in Figure (2).

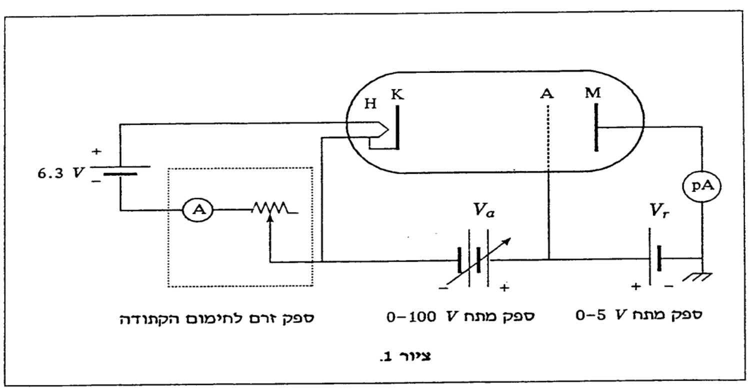

Figure 2: Schematic diagram for Part 1

What is this physics underlying this experiment? The electrons that enter the F-H tube received an energy of eV_a and moved through the tube, colliding with the mercury gas particles along the way. At the other end of the tube, there was a negative voltage (V_r) which slowed them down, causing them to move back and forth (depending on their energy). These electrons created a current that we measured at the anode A using the pico-ammeter. If the electrons were to ionize the mercury gas, we would have expected to see a non-linear change in the current when we increased the voltage.

In the beginning of this experiment the electrons didn't have enough energy to ionize the atoms, and as a result of the relation $\frac{m_e}{m_{atom}} \approx 10^{-5}$ they reached M with their full kinetic energy. Eventually, we got to the point where some of the electrons had enough energy to ionize the mercury atoms once. Consequently, the braking voltage prevented their passage through A, causing a decrease in the current. Due to the continuous increase in voltage, the electrons will have had enough energy to ionize the atoms twice. Thus, the energy gap between two maxima in the I-V curve should've allowed us to derive the energy of the first atomic excitation level. The contact voltage between the anode and the cathode is given by the expression

$$\phi_A - \phi_K = V_{a,1} - \frac{E_{kin}}{e} \tag{1}$$

where $V_{a,1}[V]$ is the acceleration voltage at which the first current peak is obtained, $V_{exc} \triangleq \frac{E_{kin}}{e}[V]$ is the difference between any two peaks of V_a (known as the excitation voltage), and ϕ_i are the work functions of the electrodes. The contact voltage $V_{contact} \triangleq \phi_A - \phi_K$ and the excitation energy eV_{exc} were two of the quantities we sought to measure in this work.

1.3 Measurement of the Ionization Voltage of Mercury Gas

The setup for the second part of the experiment is similar to that of the first part, except for that V_r is connected in parallel to V_a. The diagram of this part can be seen in Figure (3).

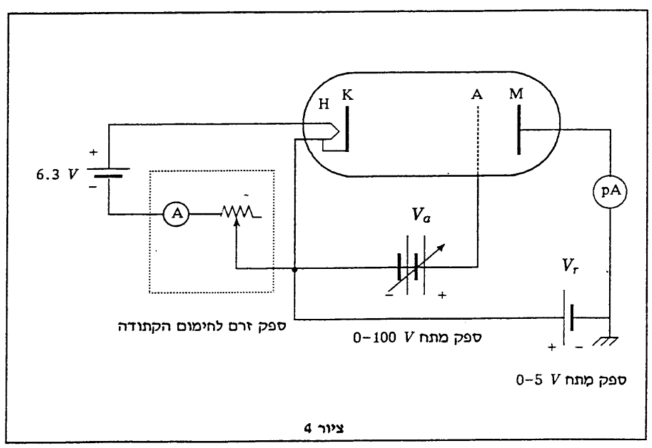

Figure 3: Schematic diagram for Part 2

Here, we measured the ionization voltage of the mercury gas in the F-H tube, which is the voltage we need to invest in order to ionize the mercury atoms. We connected the voltage supplies in a way such that the cathode will have a lower potential than the anode. The braking voltage is set to $1.5[V]$, so M would have a lower potential and thus the electrons wouldn't reach M. That way, once we created ions, they started to move towards M and thus creating a current (we expected the opposite sign for the current than that of the first part of the experiment, since ions have opposite charges to electrons). We created a graph of $I(V)$, and determined that the point where we started to see a nonzero current would indicate that there are indeed ions present in the F-H tube. Thus, to find the ionization voltage of mercury we'd need to find the voltage where the I-V curve starts to increase.

2 Results

2.1 Excitation Energy and Contact Voltage Measurements

In this experiment the thermal heater was set to $170°C$. We then measured the current as a function of V_a while changing the values of V_r from $1.1[V]$ to $1.8[V]$ in increments of $0.1[V]$. After that, we measured the current as a function of V_a where $V_r = 1.5[V]$ was fixed and V_a was changed from $4.9[V]$ to $5.7[V]$ in increments of $0.2[V]$. The results can be seen below in Figures (4) and (5).

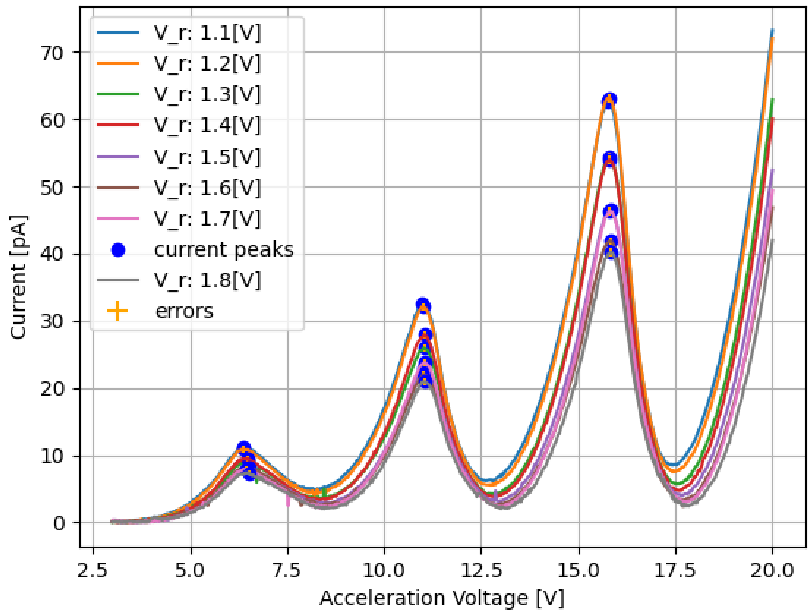

Figure 4: I-V curve for Part 1.1, where $E_{exc} = 4.680 \pm 0.020[eV]$ *and* $V_{contact} = 1.778 \pm 0.028[V]$

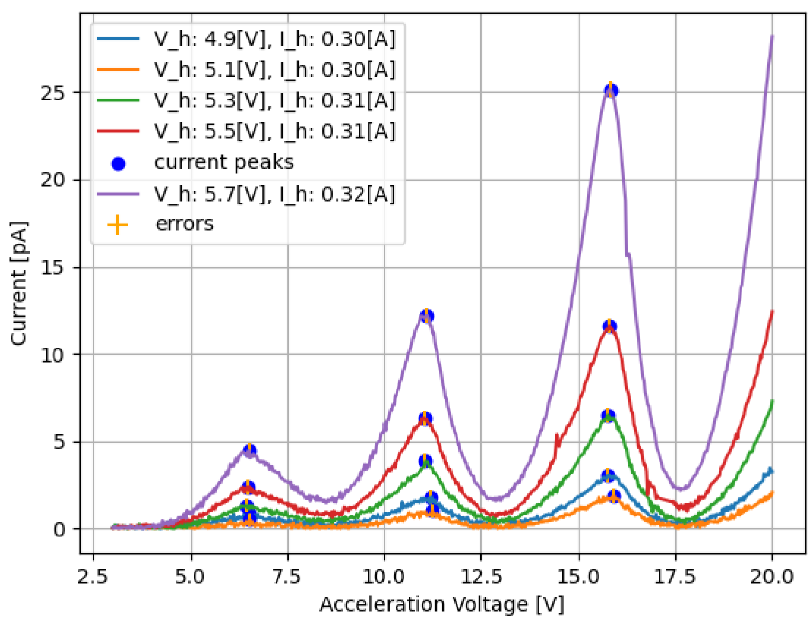

Figure 5: I-V curve for Part 1.2, where $E_{exc} = 4.665 \pm 0.020[eV]$ *and* $V_{contact} = 1.833 \pm 0.028[V]$

2.2 Ionization Voltage Measurement

In this experiment the thermal heater was set to $110°C$, which is the heat needed to ionize mercury and prevent electrons from reaching M. We created a graph of $I(V)$, which is seen in Figure (6).

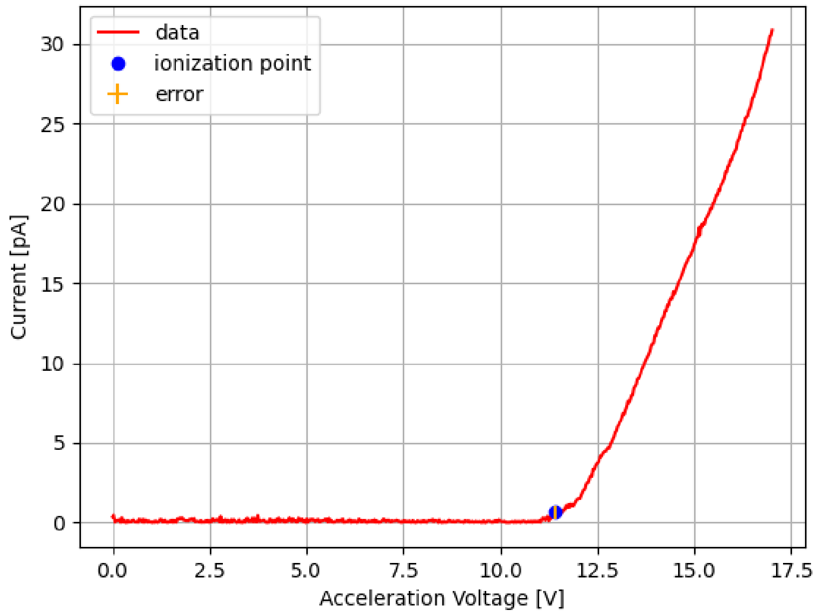

Figure 6: I-V curve for Part 2.1, where the ionization voltage is $V_{ion} = 9.623 \pm 0.030[V]$

2.3 Remark on Error Bars

We note that the measurement errors on the I and V_a values were relatively small enough such that the error bars in Figures (4), (5), and (6) are not clearly visible - though they're indeed there.

3 Analysis

In the first graph (Fig. 4), increasing the braking voltage caused a decrease in the current. In the second graph (Fig. 5), increasing the acceleration voltage caused an increase in the current but didn't change the distance between the maxima. In the third graph (Fig. 6), there was a clear point at which the acceleration voltage caused an increase in the current. These behaviors agree with the theory: the first and second graph shows that less or more electrons got to M and the third indicates the occurrence of ionization.

3.1 Comparison to Literature

To assess our results, we compare the relevant calculated values to those that we found in the literature. The excitation energy of mercury gas is known to be $4.88[eV]$ [4], while its ionization voltage is known to be $10.39[V]$ [2].

In our results, we got that the excitation energy was $4.680 \pm 0.020[eV]$ for part 1.1 and $4.665 \pm 0.020[eV]$ for part 1.2. In part 2.1, we got that the ionization voltage was $9.623 \pm 0.030[V]$. That gives us a chi-squared values of $\chi^2_{ion} = 0.0567$ and $\chi^2_{exc} = 0.0177$. These are sufficiently low, what makes our results satisfactorily accurate.

3.2 Bonus: The Effect of Gas Temperature on the Franck-Hertz Graph

What will happen to the graphs at higher temperatures of the gas? For higher temperature of the mercury gas we get more collisions of electrons with the atoms since the atoms has higher energy. Thus, the current that we measure will be higher, so both part 1.1 and 1.2 graphs will go up. As for the graph of part 2.1 we assume that we will need a lower voltage in order to get the current, since the atoms are more energetic.

4 Conclusion

In this work, we successfully measured and derived the excitation energy and ionization voltage of mercury gas using a Franck-Hertz tube (whose contact voltage we also found). We obtained values of $4.680 \pm 0.020[eV]$ and $4.665 \pm 0.020[eV]$ ($\chi^2_{exc} = 0.0177$) for the excitation energy, and values of $1.778 \pm 0.028[V]$ and $1.833 \pm 0.028[V]$ for the F-H tube's contact voltage. We also obtained a value of $9.623 \pm 0.030[V]$ for the ionization voltage of mercury ($\chi^2_{ion} = 0.0567$).

From a physical standpoint, we saw that electrons can indeed ionize mercury atoms under certain conditions. One time we measured their current after ionizing the atoms and in the other we measured the current of the ions created inside. We see that the chi-squared values of our results are persistently low ($\chi^2 < 0.06$) and therefore the measurements fit the theory sufficiently.

Appendix A: Error Analysis

The instrument accuracies present in our experiment were as follows:

TENMA Model 72-2715 DC Power Supply (Acceleration Voltage): $\delta V_a = 10.[mV] = 0.010[V]$ [3]

Keithley Instruments Model 6485 Picoammeter (Current): $\delta I = 0.004 \cdot I + 0.400[pA]$ [1]

For parts 1.1 and 1.2, we need to analyze the error on equation (2) to find the error on the excitation energy calculation:

$$\delta E_{exc} = \delta(eV_{exc}) = e \cdot \delta V_{exc} = e \cdot \delta(\sum_{j=1}^{M=8} \frac{V_{peak;N_j,j} - V_{peak;1,j}}{N_j - 1}) = \frac{e}{3-1}\sqrt{\sum_{j=1}^{8}((\delta V_{peak,N})^2 + (\delta V_{peak,1})^2)}$$

$$= \frac{e}{2}\sqrt{\sum_{j=1}^{8}((\delta V_a)^2 + (\delta V_a)^2)} = \frac{e\sqrt{16 \cdot \delta V_a^2}}{2} = \frac{4e \cdot \delta V_a}{2} = 2e \cdot \delta V_a$$

We note here that $\delta e = 0$. Plugging in our numbers, we get:

$$\delta E_{exc} = 2e \cdot 0.010 = 0.020[eV]$$

We also want to analyze the error on equation (3) to find the error on the contact voltage calculation:

$$\delta V_{contact} = \delta(\sum_{j=1}^{M=8} \frac{\delta V_{peak;1,j}}{N_j - 1} - V_{exc}) = \sqrt{\frac{1}{3-1} \cdot \sum_{j=1}^{8}(\delta V_{peak;1,j})^2 + (\delta V_{exc})^2} = \sqrt{\frac{8}{2} \cdot (\delta V_a)^2 + (\delta V_{exc})^2}$$

Plugging in our numbers, we get:

$$\delta V_{contact} = \sqrt{4(0.010)^2 + (0.020)^2} = 0.028[V]$$

Finally, we want to analyze the error on equation (4) to find the error on the ionization voltage calculation:

$$\delta V_{ion} = \delta(V_a^* - V_{contact}) = \sqrt{(\delta V_a^*)^2 + (\delta V_{contact})^2} = \sqrt{(\delta V_a)^2 + (\delta V_{contact})^2}$$

Plugging in our numbers, we get:

$$\delta V_{ion} = \sqrt{(0.010)^2 + (0.028)^2} = 0.030[V]$$

Comparing to theoretical value, the chi square is given by:

$$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

Excitation energy:

$$\chi^2_{exc} = \frac{(4.68 - 4.88)^2}{4.88} + \frac{(4.665 - 4.88)^2}{4.88} = 0.0177$$

Ionization voltage:

$$\chi^2_{ion} = \frac{(9.623 - 10.39)^2}{10.39} = 0.0567$$

Appendix B: Calculations

The calculation of the excitation energy is conducted as follows:

$$V_{exc} = \sum_{j=1}^{M} \sum_{i=1}^{N_j - 1} \left(\frac{V_{peak;i+1,j} - V_{peak;i,j}}{N_j - 1}\right)$$

$$= \sum_{j=1}^{M} \frac{1}{N_j - 1}(V_{peak;N_j,j} - V_{peak;N_j-1,j} + V_{peak;N_j-1,j} - V_{peak;N_j-2,j} + \cdots + V_{peak;3,j} - V_{peak;2,j} + V_{peak;2,j} - V_{peak;1,j})$$

$$\rightarrow V_{exc} = \sum_{j=1}^{M} \frac{V_{peak;N_j,j} - V_{peak;1,j}}{N_j - 1} \rightarrow E_{exc} = eV_{exc} \qquad (2)$$

where $V_{peak;i,j}$ is the acceleration voltage at current peak i in the Franck-Hertz graph for trial j, and N_j is the number of peaks in the Franck-Hertz graph for that trial; in our case, $N_j = 3 \forall j$ and $M = 8$.

The calculation of the contact voltage is then conducted as follows:

$$V_{contact} = \sum_{j=1}^{M} \frac{V_{peak;1,j} - V_{exc,j}}{N_j - 1} = \sum_{j=1}^{M} \frac{V_{peak;1,j}}{N_j - 1} - \underbrace{\sum_{j=1}^{M} \frac{V_{exc,j}}{N_j - 1}}_{V_{exc}} = \sum_{j=1}^{M} \frac{V_{peak;1,j}}{N_j - 1} - V_{exc} \qquad (3)$$

where $V_{peak,1}$ is the acceleration voltage at the first current peak in the Franck-Hertz graph.

Lastly, the calculation of the ionization voltage is conducted as follows:

$$V_{ion} = V_a^* - V_{contact} \tag{4}$$

where V_a^* is the smallest acceleration voltage such that the current measurement reads higher than some threshold $\varepsilon \geq 0$. Ideally, we'd want to first the smallest V_a where the current become nonzero, however due to measurement noise we see nonzero current values for unreasonably low V_a values. By letting $\varepsilon = 0.5[pA]$ (a value we chose empirically based on the data we obtained), we obtain a reasonable value of $V_a^* = 11.40[V]$. Letting $V_{contact} = 1.78[V]$ (the result from our first experiment), we see that $V_{ion} = 9.623 \pm \delta V_{ion}[V]$, where $\delta V_{ion} = 0.030[V]$ (see Appendix A).

References

[1] Keithley Instruments. Keithley instruments model 6485 picoammeter datasheet. https://www.testequipmentdepot.com/media/akeneo_connector/asset_files/6/4/6485_datasheet_b275.pdf.

[2] Dr. Sushil Kumar. Ionization potential of mercury experiment viva, 2019. https://apniphysics.com/viva/ionization-experiment-physics/.

[3] TENMA. Tenma model 72-2715 dc power supply datasheet, 2017. https://www.farnell.com/datasheets/2207205.pdf.

[4] S.T. Thornton and A. Rex. *Modern Physics for Scientists and Engineers*. Cengage Learning, 2012.