Compton Scattering

Purpose

In this experiment you will perform a Compton scattering experiment, where a photon is scattered off an electron. You will measure the energy shift of the scattered photon and deduce from it the electron rest mass. You will also estimate the dependence of the scattering cross section on the scattering angle and compare it to the Klein-Nishina formula, first derived in 1929.

Theory

Compton scattering is a relativistic effect that occurs when a high energy photon, in the X-ray or Gamma energy region, is scattered off a free electron. Even if the electron is bounded to an atom it is regarded as "free" if the binding energy is much smaller than the energy of the incoming photon. In the low energy limit of the photon (not X-ray), the scattering is called **Thomson scattering**.

Usually the mechanics of Compton scattering, the process $k + p \rightarrow k' + p'$ (see table of notation at the end), is analyzed in the lab frame where the electron (p) is initially at rest. Let the incoming photon (k) have momentum only in the $\hat{z}$ direction and the scattering plane is identified with the xz-plane. The initial 4-vector momentums for the electron and photon are:

$$p = (m_e c, 0{,}0{,}0)$$

$$k = \left(\frac{h}{\lambda}, 0{,}0, \frac{h}{\lambda}\right)$$

After the collision, the outgoing photon has momentum of

$$k' = \left(\frac{h}{\lambda'}, \frac{h}{\lambda'}\sin\theta\,, 0, \frac{h}{\lambda'}\cos\theta\right)$$

And the electron

$$p' = \left(\sqrt{(m_e c)^2 + p_x^2 + p_z^2}, p_x, 0, p_z\right)$$

Where we used the relation $\left(\frac{E}{c}\right)^2 = (mc)^2 + \boldsymbol{p}^2$. Demanding conservation of energy, we get

$$\frac{h}{\lambda} + m_e c = \frac{h}{\lambda'} + \sqrt{(m_e c)^2 + p_x^2 + p_z^2}$$

and from conservation of momentum:

$$k - k' = p - p'$$

Taking the square of this equation (under Minkowski metric) gives:

$$\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 - \left(\frac{h}{\lambda'}\sin\theta\right)^2 - \left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta\right)^2 = \left(m_e c - \sqrt{(m_e c)^2 + p_x^2 + p_z^2}\right)^2 - p_x^2 - p_z^2$$

and after some calculations, using the energy conservation equation, we get the **Compton formula**

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

for the frequency shift of the scattered photon as a function of the scattering angle. The coefficient $h/(m_e c) \equiv \lambda_e$ is called **"the Compton wavelength" of the electron**. Note that this formula is just a results of energy and momentum conservation. We did not need consider the actual scattering process in order to reach the Compton formula. This tells us that just measuring the wavelength shift of the photon does not give information on the details of the interactions governing the scattering process. It is the **differential cross section** that is a measure of these interactions.

Calculating the differential cross section using Quantum electrodynamics (QED), two Feynman diagrams are taken into consideration (see fig. 1). The left one describes the case where the electron first absorbs the incoming photon and then emits another photon. The right diagram shows the electron first emitting a photon and then absorbing the incoming photon. By summing the amplitudes of both diagrams we can calculate the differential cross section of Compton scattering.

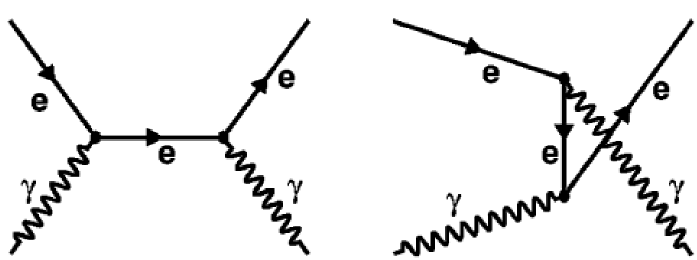

Figure 1: Feynman Diagrams for Compton scattering [1].

In the experimental system we do not control the polarizations of the initial electron and photon and we cannot measure the polarizations of the outgoing particles. Taking this into consideration, we need to average over the initial polarizations and sum over the final polarizations in order to get the differential cross section that we measure in the lab. The result of this calculation is

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_e^2 \left(\frac{\lambda}{\lambda'}\right)^2 \left(\frac{\lambda}{\lambda'} + \frac{\lambda'}{\lambda} - \sin^2 \theta\right)$$

where r_e in the **classical electron radius**. This formula is called the Klein-Nishina formula.

Experimental system

This experiment is performed with an X-ray tube unit (fig. 2). The X-ray photons emitted from the X-ray tube hit the quasi-free electrons of a Plexiglass cuboid and the scattered photons are detected with an energy detector. In order to measure single-wavelength photons, the anode of the X-ray tube is made of Molybdenum and we measure the scattering of the K_α characteristic line, which is easily detected in the tube's spectrum.

For details regarding the X-ray unit and the energy detector, please refer to the X-ray experiment guide.

Figure 2: PHYWE X-ray unit with an energy detector and a Plexiglass cuboid mounted on the goniometer.

Preparation Questions

1. What are the elements that consist the Descloizite mineral and what are their characteristic lines in the range of 3-30 keV?
2. What are the characteristic lines of Molybdenum in the range 3-30 keV? What is the maximal energy shift for each line, according to the Compton formula?
3. What is the literature value of the electron Compton wavelength?
4. What is the maximal binding energy of an electron in Plexiglass (look up the **k-edge** energies of its elements)? Why do we regard the electrons as quasi-free?
5. Calculate the energy shift of the K_α line of Molybdenum scattered at $90°$.
6. Plot a numeric theoretical curve of the differential cross section vs. scattering angle. What is the maximal relative change in scattering amplitude?

Experiment procedure

At the beginning of the experiment you will measure the direct spectrum of the X-ray tube. Then you will perform a calibration of the energy detector by measuring the spectrum emitted from a **Descloizite** sample and fitting the measured channels with the known lines of the elements composing the Descloizite mineral.

Then you will measure the spectrum scattered off the Plexiglass at different angles. You will extract the energy shift in the K_α line of the Molybdenum and fit it to the Compton formula to calculate the electron Compton wavelength. Also, you will compare the amplitude of the scattered K_α photons to the Klein-Nishina formula.

Energy Detector setup and direct spectrum measurement

*The MCA needs to be turned on about 15 minutes before taking measurements in order to get time-consistent values.

1. Make sure the Molybdenum (Mo) tube is inside the X-ray device and that no diaphragm is inserted.
2. Set the goniometer to the farthest horizontal position from the hole.
3. Insert a 1mm diaphragm.
4. Close and lock the door of the X-ray unit.

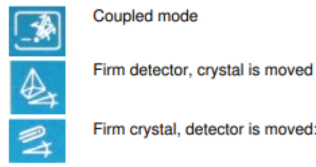

Fig. 5: Goniometer modes.

5. In the control panel on the X-ray unit, click “menu”, “Goniometer”, “parameters”, “modify”, and choose in scan type to move only the detector.
6. Set the detector angle to 3 degrees with the up/down arrows on the control panel.
7. Press “menu” and choose in the X-ray tube parameters: U=35kV, I=0.08mA.
8. Open the program “Measure”.
9. Click “Gauge” and choose “Multi Channel Analyzer”, then click “Spectra Recording” and “continue”.
10. In the MCA parameters (on the right side of the graph) set the minimal offset that cancels the false readings. Then, clear the graph by clicking “reset”.
11. Turn on the X-ray. Lower the detector angle until the counting rate is ~200 counts/sec.
12. Set the gain so that the measured spectrum is “spread” over most of the channels, without being truncated by the highest channel. As you change the gain, you might need to adjust the offset again. Check that by turning off the X-ray after you choose the proper gain.
13. Write down the gain and offset you chose in your log. You will not change them anymore during the experiment.
14. Click “reset” and then “start” to measure the incoming spectrum of the Molybdenum tube for about 5 minutes, until the spectrum is clear. When the measurement is done, **turn off the X-ray.**
15. Save the spectrum by clicking “accept data”, “measurement”, “Export Data”. Save your measurement to file as numbers. Close the figure after you saved your data.

Energy Detector Calibration from Descloizite lines

16. In the device panel, set the detector angle to about 90 degrees.
17. Unlock the door and take out the diaphragm.
18. Use the crystal holder and a rubber band to set a Descloizite (#10 - $PbZn(OH)VO_4$) in the crystal position on the goniometer.
19. Close and lock the door. Go back to the “spectra recording” module in the program.
20. Turn on the X-ray. Set the current so that the impulse rate is less than 300#/sec (you should aim for ~200#/sec). Then press reset and let the histogram accumulate.

21. When the histogram is clear turn off the X-ray, Press “Accept Data” and save it (Measurement=>Export Data=>save to file as numbers). Close the figure after you saved the data.

Scattering Measurements

22. Take out the Descloizite and return it to the box.
23. Insert the plexiglass cuboid on the goniometer, and a diaphragm of 5mm. Close and lock the door.
24. In the control panel, click “menu”, “Goniometer”, “parameters”, “modify”, and choose in scan type to move only the crystal. Then, set the angle of the plexiglass to 10 degrees.
25. Similarly, set the detector angle to 20 degrees.
26. Set the tube parameters to U=35kV, I=0.20mA. These are to be kept the same in all the scattering measurements, so you’ll be able to compare the amplitudes.
27. Set recording time to 5 minutes in the program.
28. Measure the spectra of each **detector** angle from 20 to 150, in increments of 10 degrees. Every measurement should take precisely 5 minutes. **<u>Set an alarm in your phone to notify you when a measurement is almost done.</u>** After each measurement is saved, close its figure (to prevent mixing up the measurements when saving).

While waiting for the measurements, do the followings:

Channels-to-Energy Calibration

29. Look up the strong lines of the elements in Descloizite and Mo, and write their energies in MATLAB.
30. Import the spectra you measured (Mo and Descloizite) to MATLAB using importdata().
31. Find the peaks in the spectra and identify to which line each peak belong.
32. Plot the energies versus the channels you found. Is the graph linear? Correct your vectors if not.
33. Find the linear coefficients of the graph using a linear regression (fitlm()).
34. Make a function that converts channels into energies.

Energy shift evaluation

35. Plot the Mo spectrum, where the x axis is energy.
36. Write a function that finds the energy of the K_α line and its amplitude from the data.
37. Load the scattered spectra you measured so far and use your function to extract the energy of the K_α line and its amplitude for each scattering angle.
38. Plot a graph for the Compton formula from your measured data.

39. Calculate the electron Compton wavelength from your data.
40. Plot a graph of the amplitude vs. scattering angle and add a theoretical curve from the Klein-Nishina formula.
41. Verify whether there is a systematic error in the angle. If there is, try to correct for it.

Table of Notation

Notation	Description
k	Momentum of the incoming photon
p	Momentum of the incoming electron
k'	Momentum of the outgoing photon
p'	Momentum of the outgoing electron
m_e	Electron's rest mass
c	Speed of light
h	Planck constant
λ	Incoming photon wavelength
λ'	Outgoing photon wavelength
θ	Scattering angle
p_x	Outgoing electron momentum in the x direction
p_z	Outgoing electron momentum in the z direction
E	Energy
λ_e	Electron Compton wavelength
$\frac{d\sigma}{d\Omega}$	Differential cross section
r_e	Classical electron radius

References

[1] P. e. a. Ambrozewicz, ""High precision measurement of Compton scattering in the 5 GeV region."," *Physics Letters B 797,* 2019.