ROTATIONAL MOTION

Rigid-Body

Angular Displacement (θ)

Angular Velocity (ω)

Angular acceleration ($\propto$)

Examples of rotational motion

ROTATIONAL MOTION

1) **Angular Displacement ($\boldsymbol{\theta}$):** It is the angle in radians through which a point or line has been rotated in a specified sense about a specified axis.

2) **Angular Velocity ($\boldsymbol{\omega}$):** It is the rate of change of angular displacement with time.

$$\boldsymbol{\omega} = \frac{\boldsymbol{d\theta}}{\boldsymbol{dt}}$$

3) **Angular acceleration ($\propto$):** It is the rate of change of angular velocity with time.

$$\propto = \frac{\boldsymbol{d\omega}}{\boldsymbol{dt}} = \frac{\boldsymbol{d^2\theta}}{\boldsymbol{dt^2}}$$

KINEMATIC ROTATION

Let's consider a body $\boldsymbol{p}$ rotating about an axis $\boldsymbol{o}$.

The angle $\boldsymbol{\theta}$ will continuously change relative to positive x-axis.

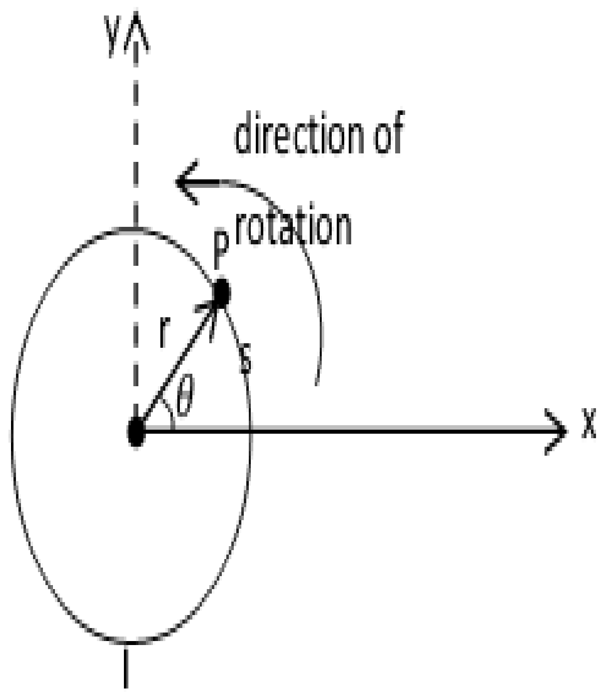
The radius $\boldsymbol{r}$ of the rotation remains the same.

θ is called: angular displacement and it is measured in radian.

$$\boldsymbol{\theta = \frac{s}{r}}$$

Where s = arc. It has the length as the radius.

$$\boldsymbol{s = r\theta}$$



TIPS

The circumference of a circle is $2\pi r$

1 complete cycle is = $2\pi\ radian$

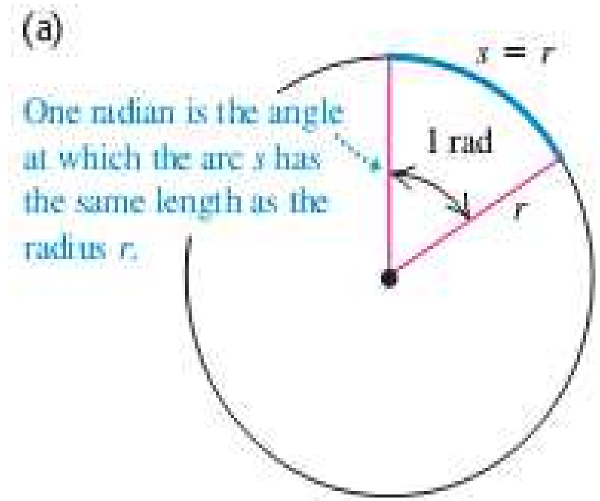
1 complete cycle is also one revolution.

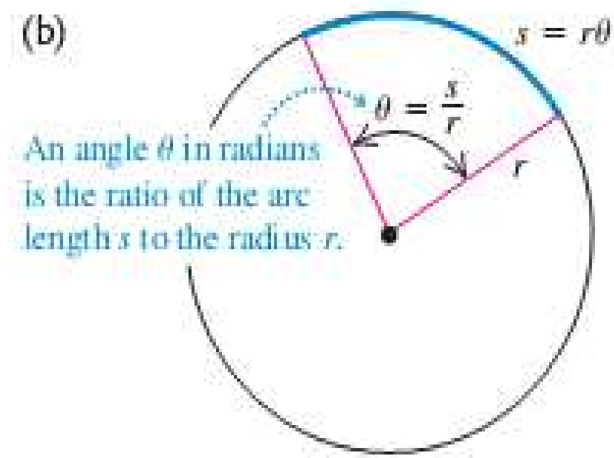
1 revolution is equivalent to 360°

$180° = \pi$

$90° = \frac{\pi}{2}$

1 rad = 57.3°





ANGULAR VELOCITY

When a body rotates, the θ changes over time.

The angular velocity is the rate of change of angular displacement over time.

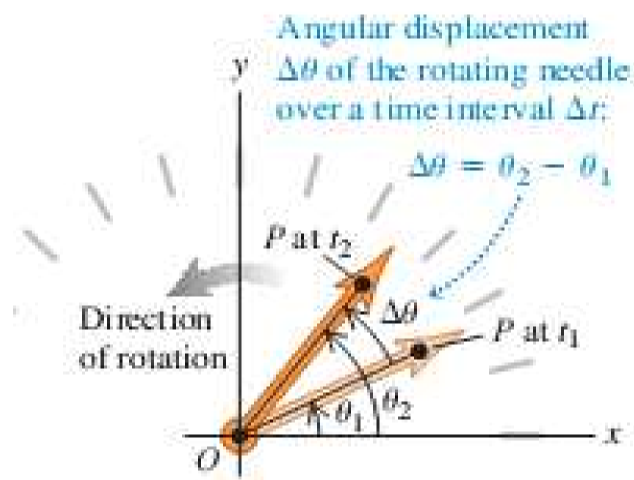
If the body moves from θ_1 to θ_2 over time interval t_1 to t_2; the angular velocity is:

$$\boldsymbol{\omega} = \frac{\boldsymbol{\theta_2 - \theta_1}}{\boldsymbol{t_2 - t_1}} = \frac{\boldsymbol{\Delta\theta}}{\boldsymbol{\Delta t}}$$

We can also find the instantaneous angular velocity:

$$\boldsymbol{\omega} = \lim_{\Delta t \to 0} \frac{\boldsymbol{\Delta\theta}}{\boldsymbol{\Delta t}} = \frac{\boldsymbol{d\theta}}{\boldsymbol{dt}}$$

Angular velocity is measured in radian per second (rad/s)



WORK-PROBLEM

1A) What angle in radians is subtended by an arc 1.50m long on the circumference of a circle of radius 2.5m? What is this angle in degrees?

1B) An arc 14.0cm long on the circumference of a circle subtends angle of 128°. What is the radius of the circle?

1C) The angle between two radii of a circle with radius 1.50m is 0.700 rad. What length of arc is intercepted on the circumference of the circle by the two radii?

Answer: 1a) 0.6 rad or 34.4° 1b) 0.063m 1c) 1.05m

ANGULAR ACCELERATION

When the angular velocity of a rigid body changes, then the rigid body is said to accelerate.

Angular acceleration (∝): It is the rate of change of angular velocity with time.

$$\propto = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

The instantaneous angular acceleration is defined as:

$$\propto = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

The S.I unit is rad/s^2.

Note: Both angular velocity and angular acceleration can be expressed in vector form.

Comparison between linear motion and rotational motion

Linear Motion	Rotational Motion
a = constant	$\propto = constant$
$v = v_0 + at$	$\omega = \omega_0 + \propto t$
$x = x_0 + v_0 t + \frac{1}{2} a t^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} a t^2$
$v^2 = u^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2 \propto (\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v - v_0)t$	$\theta - \theta_0 = \frac{1}{2}(\omega - \omega_0)t$

Relationship between Linear and Angular Motion

Recall:

$s = r\theta$ – [equ 1]

If we take time-derivatives of equ 1:

$$\frac{|ds|}{|dt|} = r\left|\frac{d\theta}{dt}\right|$$

$v = r\omega$ - [equ 2]

From linear motion: $a = \frac{dv}{dt}$

If we take time-derivatives of equ 2:

$$\frac{|dv|}{|dt|} = r\left|\frac{d\omega}{dt}\right|$$ - [equ 3]

Equation 3 turns into:

$a = r \propto$ - [equ 4]

This is the called the tangential acceleration of a point on a rotating body.

From centripetal acceleration: $a_c = \frac{v^2}{r}$

From rotational motion: $a_c = r\omega^2$

WORK-PROBLEM

2) A ceiling fan has an initial angular velocity of 1.60 rad/s.

(a) If its angular acceleration is constant and equal to 0.320 rad/s^2, what is its angular velocity at t = 2.4s?

(b) Through what angle has the fan turned between t = 0 and t = 2.4s?

ROTATIONAL KINETIC ENERGY OF RIGID BODIES

When a rigid body rotates, all the system of particles in it experience a motion and the kinetic energy of the rotating body is the cumulative effect of the individual particles.

Assume that a rotating body consists of masses $m_1,\ m_2\ , \ldots m_i$ at distances $r_1,\ r_2\ , \ldots r_i$ to the axis of rotation of the body.

The angular velocity acquired by the individual particle (i) is expressed as:

$$v_i = r_i \omega_i$$

Hence the kinetic energy of each particle can be expressed as:

$$\frac{1}{2} m_i {v^2}_i = \frac{1}{2} m_i {r^2}_i {\omega^2}_i$$

ROTATIONAL KINETIC ENERGY OF RIGID BODIES

The total kinetic energy of the rigid body is the sum of the kinetic energy of the individual particles.

$$K = \frac{1}{2}m_1r_1^2w_1^2 + \frac{1}{2}m_2r_2^2w_2^2 + \cdots = \sum \frac{1}{2}m_ir_i^2w_i^2$$

By Factorizing:

$$K = \frac{1}{2}(m_1r_1^2 + m_2r_2^2 \pm - - - -)\omega^2 = \frac{1}{2}(\sum m_ir_i^2)\omega^2$$

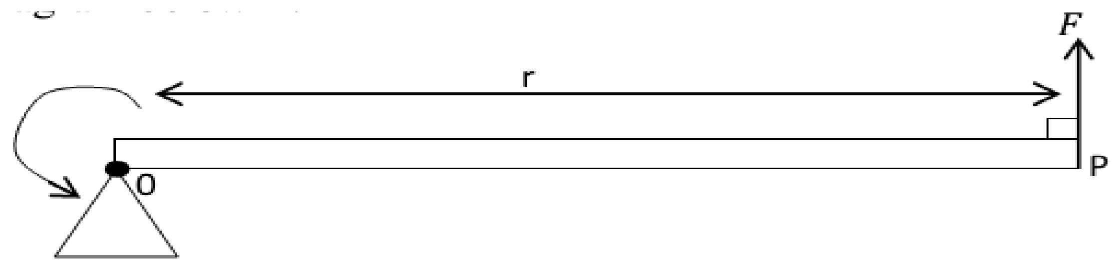
The moment of inertial (I) is expressed as: $I = \sum_i m_i r^2{}_i$

Rotational Inertia (k):

It determines the amount of kinetic energy that a rotating rigid body will have.

$$k = \frac{1}{2} I\omega^2$$

Translational Concept		Rotational Concept
Displacement	S	θ
Velocity	v	ω
Acceleration	a	$\propto$
Mass	m	I
Force	ma	τ
Momentum	mv	Iω
K.E	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$
Impulse	Ft	τt
Work	Fs	$\tau\theta$
Power	Fv	$\tau\omega$



TORQUE

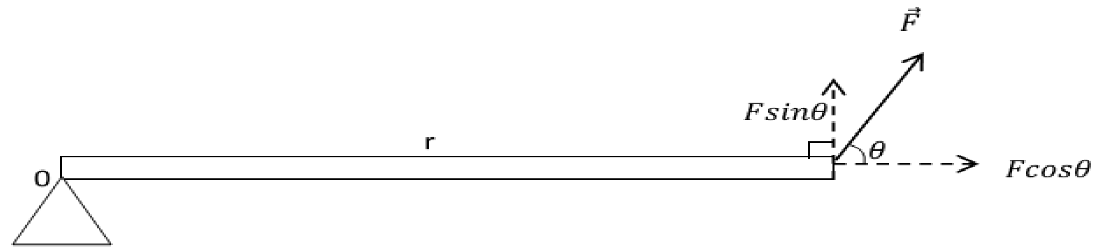
Moment: It is the measure of the turning effect produced by a force about an axis.

Question: How can force be made to generate the turning of rigid bodies?

Torque ($\vec{\tau}$): It is the turning effect of a force. It is defined as the product of the force and the perpendicular distance from the pivot (or hinge) about which the body turns or rotates.

Torque is simply expressed as "force x distance r": $\boldsymbol{\tau = rF}$

TORQUE



If the force **F** is applied at a particular angle θ to the distance that comes from the pivot, then torque is deduced by finding the component of that force that acts perpendicularly to the distance from the pivot.

$\boldsymbol{Fsin\theta}$ is the component of the force **F** that produces the rotation about **o**, and therefore by definition:

$$\boldsymbol{\tau = rFsin\theta}$$

In vector notation form, torque is expressed in vector product form:

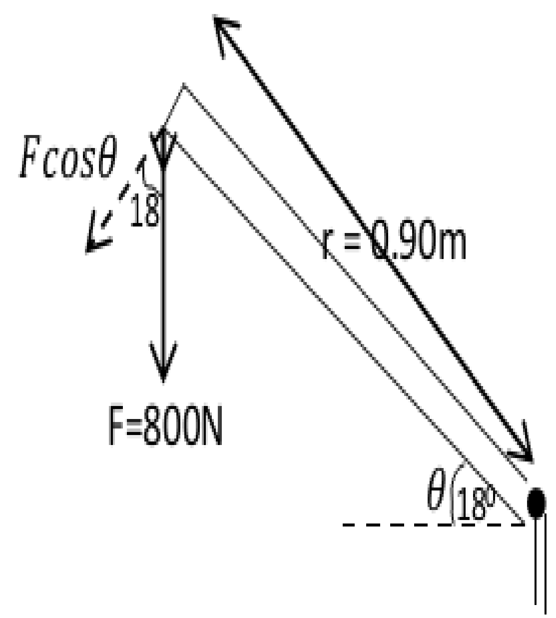
$$\boldsymbol{\tau = r \times F}$$

Torque can be also be called moment.

It is measured in Nm has the same dimension as work and energy but, it is neither of the two.

WORK-PROBLEM!

3) A man exert a downward force of 800N on the arm of a long wrench inclined at angle 18° to the horizontal. If the arm of the wrench is 0.90m, find the torque he applies about the center of the bolt fitting.



PRACTICE QUESTIONS

1) The flywheel of an engine has moment of inertia 5.0 kgm^2 about its rotation axis. What constant torque is required to bring it to an angular speed of 500 rev/min in 10.00s, starting from rest?

2) An engine delivers 180hp to an airplane propeller at 2200rev/min.
a) how much torque does the aircraft engine provide.

b) How much work does the engine do in one revolution of the propeller?

3) The rotor of an airplane rotates initially at 300rev/min and it later changed to 225rev/min in one minute.

a) Find the average angular acceleration during the interval?

b) What is the total angular displacement of the rotor within the interval of the time?

PRACTICE QUESTIONS

4) Calculate the torque of a disk rotating at 90rev/min if it makes only 3 revolution before stopping. The rotational inertia of the disc is 320 kgm^2.

5) A bicycle wheel has initial angular velocity of 1.60 rad/s.

a) If its angular acceleration is constant and equal to 0.250 rad/s^2, what is its angular velocity at t = 2.00s?

b) Through what angle has the wheel turned between t = 0 and t = 2.00s.

6) A ceiling fan rotates with a constant 2.25 rad/s^2 angular acceleration. After 3.00s it has rotated through an angle 65rad. What was the angular velocity of the wheel at the beginning of the 3.00s interval?

BIBLIOGRAPHY

1) Sears and Zemansky's University Physics with Modern Physics. 14^{th} Edition: Hugh D. Young. Roger A. Freedman

2) Schaum's Outlines College Physics. 11^{th} Edition: Eugene Hecht.

3) University Physics. Volume 1. Poh Liong Young, M.W. Anyakoha, P.N. Okeke

4) The Physics for University and Colleges. Volume 1. Kehinde Daniel and Opadele Abayomi

QUESTION TIME????????????????

About Lecturer:

Opadele A.E is a physics enthusiast with special interest in Medical Physics. He loves to present the complex theories in physics in seemingly simple approach for effectual understanding.

opadelea@babcock.edu.ng

abayomi_opadele

@abayomiopadele

Opadele Abayomi