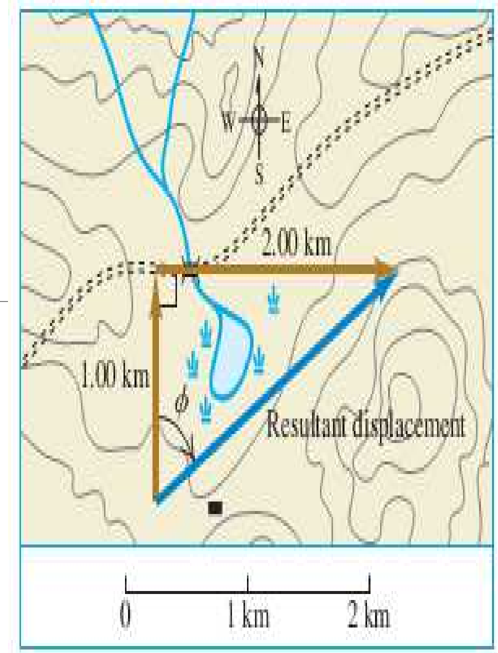
VECTORS



Let's assume a car is moving with a speed and towards North.

Magnitude and Direction are used to describe the **vector** quantity.

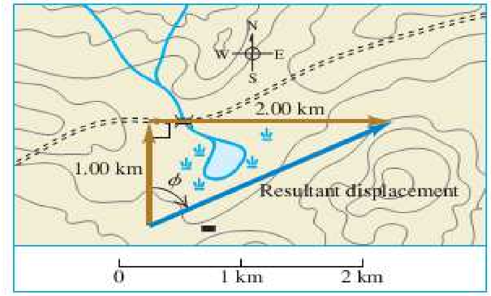
The motion of an airplane moving from Lagos to Aba is best described in terms of its magnitude (469km/h) and direction (towards east).

Examples of vector quantity: Velocity, Acceleration, Momentum, Force e.t.c

What is a Force?

Answer: It is a pull or a push. It is also the agency that tends to change the momentum of a body.

Vector and Scalar Quantities



Scalar Quantities

They are quantities that do not have direction and can be merely described by a single number.

They can be performed with the aid of normal arithmetic, multiplication or subtraction

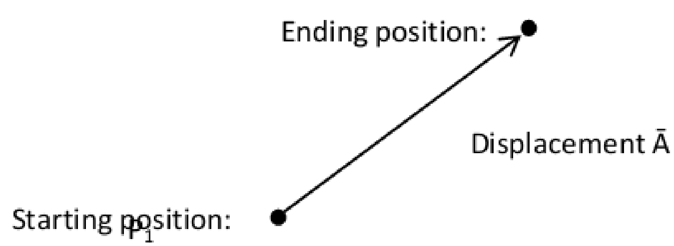
Examples: Mass, Length and Time

Vector Quantities

They are defined by both magnitude and directions.

Operations are performed geometrically with certain rules.

Example: Displacement



Vector and Scalar Quantities

Displacement is represented as $\vec{\boldsymbol{A}}$

If two vectors are in the same direction, they are called **parallel vectors.**

Vector $\vec{\boldsymbol{A}} = \overrightarrow{\boldsymbol{A^1}}$ because they same magnitude and direction.

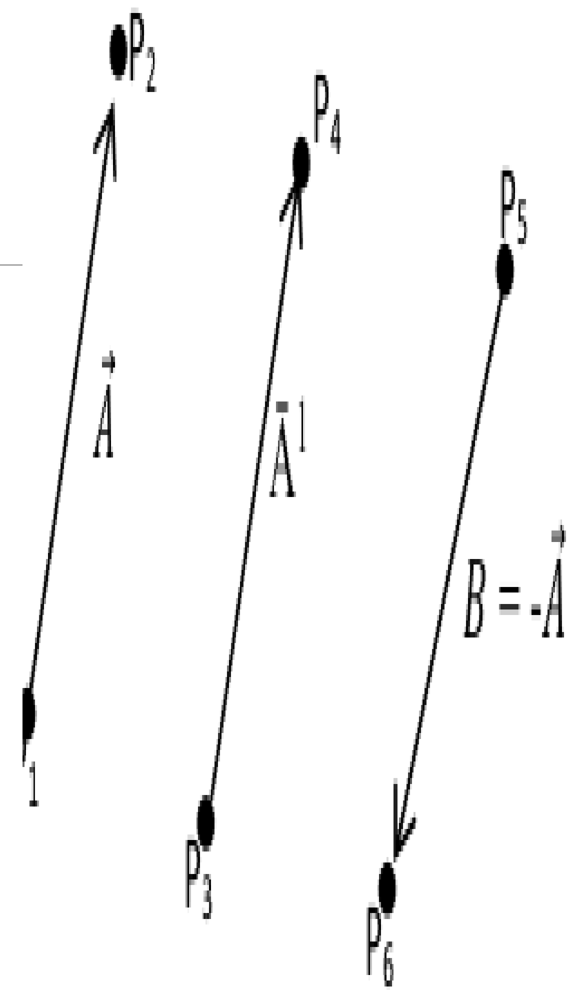
Vector $\vec{\boldsymbol{B}} \neq \vec{\boldsymbol{A}}$, hence it is a negative of vector $\vec{\boldsymbol{A}}$

When vector $\vec{\boldsymbol{B}}$ and $\vec{\boldsymbol{A}}$ the same magnitude both opposite direction, $\vec{\boldsymbol{B}} = -\vec{\boldsymbol{A}}$

Therefore, when two vectors $\vec{\boldsymbol{B}}$ and $\vec{\boldsymbol{A}}$ have opposite directions, irrespective of whether they have the same magnitude or not, they are called **antiparallel vectors.**

The magnitude of a vector is represented as:

(Magnitude of $\vec{A}$) = A = $|\vec{A}|$



Vector Addition

Addition of vectors obey certain geometric rules:

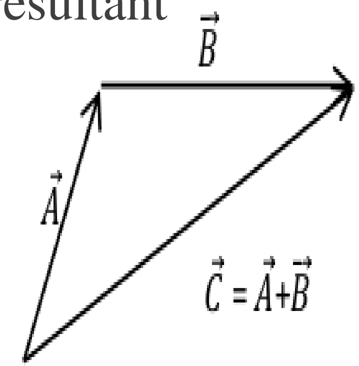
1) The vector sum or resultant

2) Commutative law

3) Associative law

4) Vector subtraction

1) The vector sum or resultant



$\vec{C} = \vec{A} + \vec{B}$

2) Commutative Law

When vectors are added in reverse orders, they give the same results.

$\vec{A} + \vec{B} = \vec{B} + \vec{A}$

3) Associative Law

$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + \overrightarrow{(B + C)}$

Vector Addition

4) Vector Subtraction

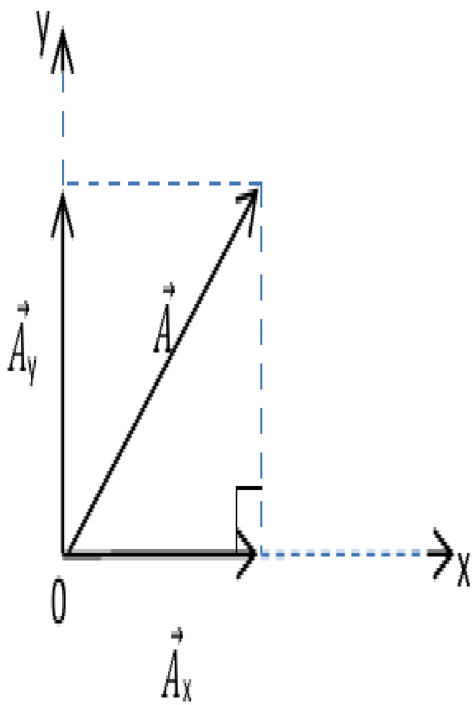
$\vec{A} - \vec{B} = \vec{A} + \overrightarrow{(-B)}$

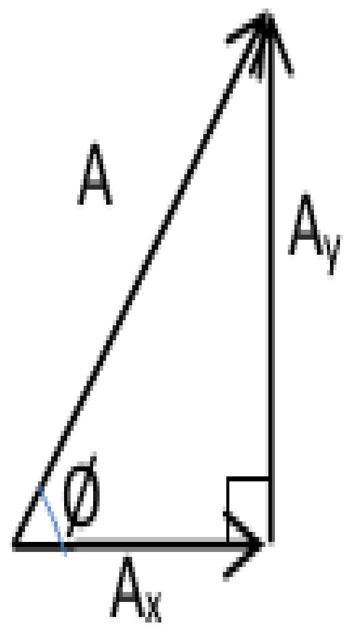
#ThursdayTrivia

1) An airplane flies 1.00km north and then 2.00km east on a fair weather. How far and in what direction is the airplane from the starting point?

Ans: Distance = 2.24km and Direction = 63.4°

Components of Vectors





A_x and A_y are called components of vectors.

The vector sum is represented as:

$$\vec{A} = A_x + A_y$$

Applying trigonometry rule:

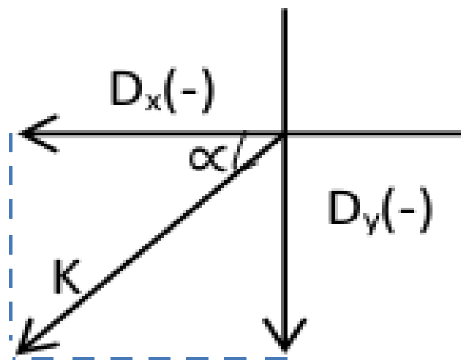
$$A_x = A\cos\emptyset$$

$$A_y = A\sin\emptyset$$

Components of Vectors

#ThursdayTrivia

2) What are the x and y components of vector $\vec{K}$ in the figure below. The magnitude of $\vec{K}$ in the figure below is 4.0m and the angle is $\propto = 45°$



Answer

$K_x = -2.83m$

$K_y = -2.83m$

The magnitude of $\vec{K} = 4.0\text{m}$

UNIT VECTORS

A unit vector is a vector that has a magnitude of 1, with no units.

It is used the describe the directions of vectors in space.

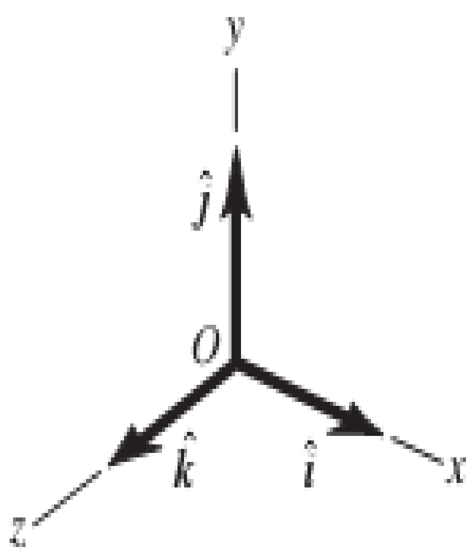
Unit vectors for x, y and z axes are denoted by: $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ respectively.

$\overrightarrow{A_x} = A_x\, \hat{\imath}$

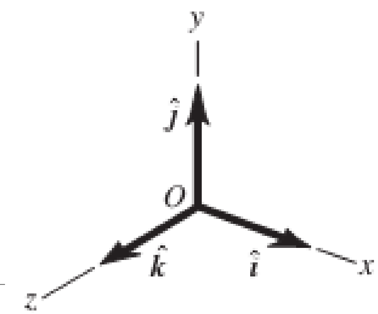
$\overrightarrow{A_y} = A_y\, \hat{\jmath}$

$\overrightarrow{A_z} = A_z\, \hat{k}$

$\vec{A} = A_x\, \hat{\imath} + A_y\, \hat{\jmath} + A_z\, \hat{k}$ (same for vector $\vec{B}$)



UNIT VECTORS



#ThursdayTrivia

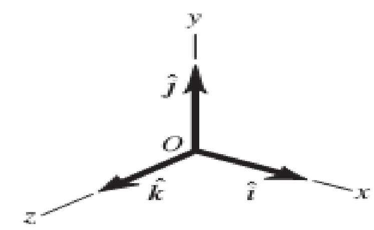
3) Given the vector notation of $\vec{A}$ *and* $\vec{B}$, add the two vectors together.

$$\vec{A} + \vec{B} = (A_x + B_x)\,\hat{\imath} + (A_y + B_y)\,\hat{\jmath} + (A_z + B_z)\,\hat{k}.$$

4) Given two vectors, $\vec{x} = (7\hat{\imath} + 2\hat{\jmath} - \hat{k})$ and $\vec{y} = (5\hat{\imath} - 6\hat{\jmath} + 9\hat{k})$. Find the magnitude for the vector $3\vec{x} - \vec{y}$.

Ans = $16\hat{\imath} + 12\hat{\jmath} - 12\hat{k}$ and 23.32

Products of Vectors



Note: Addition of vectors follows certain rules, so is vector multiplication too:

Vectors can be multiplied in three different ways:

1) Multiplication of a vector by a scalar.

2) Scalar Product

3) Vector Product

A) Multiplication of a vector by a scalar.

If we multiple a vector $\vec{A}$ by a scalar c, the product becomes $\vec{D} = c\vec{A}$

Let: c = 2 and $\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k}$. Find $\vec{D}$?

Products of Vectors

SCALAR PRODUCT

The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is given as $\vec{A}.\vec{B}$.

It is also known as the **dot product.**

The dot product of $\vec{A}.\vec{B}$ is expressed as:

$\vec{A}.\vec{B} = AB\ cos\theta = |\vec{A}||\vec{B}|\ cos\theta$

$\vec{B}.\vec{A} = BA\ cos\theta = |\vec{B}||\vec{A}|\ cos\theta$

Scalar product obeys commutative law:

$\vec{A}.\vec{B} = \vec{B}.\vec{A}$

Products of Vectors

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = (1)(1)\cos 0^\circ = 1$$

$$\hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = (1)(1)\cos 90^\circ = 0$$

<u>SCALAR PRODUCT</u>

The scalar product is a scalar quantity.

It can be +ve, -ve or zero. It is always zero when the vectors are perpendicular.

If $\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k}$ and $\vec{B} = B_x\,\hat{\imath} + B_y\,\hat{\jmath} + B_z\,\hat{k}$. Determine $\vec{A}.\vec{B}$

$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$

Hence, the scalar product of two vectors is the sum of the products of their respective components.

Products of Vectors

Class Exercise:

1) Using dot product, find the angle between the two vectors below.

$\vec{A} = (\ 2\hat{\imath} + 3\hat{\jmath} + \ \hat{k})$

$\vec{B} = (\ -4\hat{\imath} + 2\hat{\jmath} - \ \hat{k})$

ANSWER: $\boldsymbol{\vec{A}.\vec{B}}$ **= -3 A = 3.74 B = 4.58** $\boldsymbol{\theta = 100.07°}$

Products of Vectors

VECTOR PRODUCT

The vector product of two vectors $\vec{A}$ and $\vec{B}$ is also called the **cross product.**

Denoted as: $\vec{A} \times \vec{B}$

It is a vector quantity.

For cross product: $\vec{A} \times \vec{B} = AB\ sin\theta$

Vector product is not commutative, hence $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

When the angle is either 0° or 180°, vector product is always zero.

Products of Vectors:
Calculating vector products

1) Distributive Law (long multiplication)

$\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k}$

$\vec{B} = B_x\,\hat{\imath} + B_y\,\hat{\jmath} + B_z\,\hat{k}$

Find $\vec{A} \times \vec{B}$

$\hat{\imath}\mathrm{x}\hat{\imath} = 0,\ \hat{\imath}\mathrm{x}\hat{\jmath} = \mathrm{k},\ \hat{\imath}\mathrm{x}\hat{k} = \hat{\jmath},\ \hat{\jmath}\mathrm{x}\hat{\imath} = -\hat{k},\ \hat{\jmath}\mathrm{x}\hat{\jmath} = 0$

$\hat{\jmath}\mathrm{x}\hat{k} = \hat{\imath},\ \hat{k}\mathrm{x}\hat{\imath} = -\hat{\jmath},\ \hat{k}\mathrm{x}\hat{\jmath} = -\hat{\imath},\ \hat{k}\mathrm{x}\hat{k} = 0$

2) Determinant method

$$\vec{A}\mathrm{x}\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Products of Vectors: Calculating vector products

<u>Class Exercise:</u>

2) The angular momentum $\vec{L}$ of a particle is given by the vector product of its linear momentum *p* and position vector *v*.

If $\vec{P} = (\ 9\hat{\imath} + 10\hat{\jmath} +\ \widehat{15k})$

If $\vec{V} = (\ 2\hat{\imath} + 3\hat{\jmath} +\ \widehat{5k})$

Find $\vec{L}$

Answer: $\vec{\boldsymbol{L}} = (\ \boldsymbol{5\hat{\imath} - 15\hat{\jmath} + 7\hat{k}})$

ASSIGNMENT

1) An airplane leaves the airport in Lagos and flies 170 km at 68° east of north and then changes direction to fly 230 km at 48° south of east, after which it makes an immediate emergency landing in a form. When the airport sends out a rescue crew, in which direction and how far should this crew fly to go directly to this plane?

2) Given two vectors: $\vec{A} = (4.0\hat{\imath} + 3.0\hat{\jmath})$ and $\vec{B} = (5\hat{\imath} - 2\hat{\jmath})$.

A) find the magnitude of each vector?

B) write an expression for the vector difference A-B

C) find the magnitude and direction of the vector difference $\vec{A}$ - $\vec{B}$

QUESTION TIME????????????????