Transient Heat Transfer in Multilayer Human Skin During Cryotherapy: A 3D Numerical Study

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Introduction

Cryotherapy removes heat from the skin surface to reduce pain and inflammation, but its effectiveness depends on how rapidly and how deeply cooling penetrates into the tissue. In this task, transient heat conduction in a three-layer skin model (epidermis, dermis, and subcutaneous fat) is simulated under surface cooling for a 10-minute period. The report discusses temperature variation with time and depth and analyzes the effect of the convective heat transfer coefficient on cooling rate and penetration depth.

Model Description and Numerical Setup

Geometry and Layered Structure

A three-dimensional rectangular domain was used to represent a simplified multilayer skin–tissue system subjected to external cooling. The model dimensions were W = 40 cm, H = 20 cm, and total thickness L = 20 mm. The thickness was partitioned into three layers: epidermis (1 mm), dermis (2 mm), and subcutaneous fat (17 mm). The lateral dimensions were selected sufficiently large to ensure that the dominant heat transfer occurs in the through-thickness direction and to reduce boundary edge effects in the region of interest.

Material Properties

Each layer was assigned distinct thermophysical properties—thermal conductivity k, density ρ, and specific heat capacity c—in addition to a volumetric metabolic heat generation term $\dot{q}_{\text{met}}$. The values used for all layers were taken directly from the task statement to maintain consistency across the simulated cases.

Governing Physics

Transient heat transfer in the tissue was modeled using heat conduction with metabolic heat generation (bioheat model without perfusion):

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T + \dot{q}_{\text{met}}\,, \tag{1}$$

Blood perfusion was neglected in accordance with the provided problem formulation.

Initial and Boundary Conditions

The initial condition was a uniform tissue temperature:

$$T(\mathbf{x}, 0) = 37^\circ\text{C}. \tag{2}$$

At the top surface, an external cooling pack was modeled using a convective boundary condition with $T_{\text{ice}} = 0^\circ$C:

$$-k \frac{\partial T}{\partial n} = h\,(T_s - T_{\text{ice}})\,, \tag{3}$$

where h is the convective heat transfer coefficient. Three convective cases were simulated: h = 20, 50, and 100 W/m^{2}K. The bottom surface was maintained at core body temperature:

$$T(L) = 37^\circ\text{C}. \tag{4}$$

Numerical Discretization and Simulation Time

The computational domain was discretized using a mesh comprising 2592 cells and 13,077 nodes. All simulations were performed for a total duration of 600 s (10 min) to capture the transient cooling response of the multilayer tissue system.

Results and Discussion

Baseline Cooling Response

Figures 1, 2, and 3 present the transient thermal response of the multilayer skin model for the baseline convective coefficient (h = 50 W/m^{2}K). These baseline results are reported as representative behavior; the remaining cases follow similar numerical procedures with only the convective coefficient varied.

Figure 1 illustrates the temperature distribution through the skin thickness at selected times (0, 60, 300, and 600 s). Initially, the temperature is uniform at body temperature. As cooling progresses, a sharp temperature gradient develops near the skin surface, indicating rapid heat extraction in the epidermal layer. With increasing time, the cooling front advances into the dermis; however, temperatures recover quickly toward the interior, indicating limited thermal penetration into the deeper subcutaneous region over the 10-minute exposure.

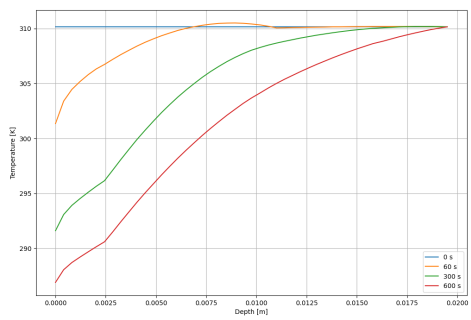

Figure 1: Baseline case: temperature distribution through thickness at selected times (0, 60, 300, 600 s), illustrating transient cooling penetration into the multilayer tissue.

Figure 2 shows the temperature evolution at depths of 1, 3, and 17 mm, representing superficial (epidermal), intermediate (dermal), and deep subcutaneous locations within the 20 mm model thickness. The superficial region exhibits the fastest temperature drop, while deeper locations cool more gradually and remain considerably warmer. This behavior indicates that, under moderate cooling conditions, cryotherapy primarily affects superficial tissues within the simulated time window.

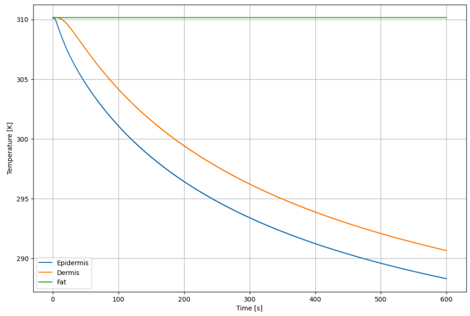

Figure 2: Baseline case: temperature evolution at selected depths (1, 3, and 17 mm), demonstrating rapid superficial cooling and limited deep-tissue temperature change over 600 s.

In addition, the cross-sectional temperature contour at $t = 600$ s (Figure 3) confirms that temperature gradients are almost aligned with the through-thickness direction, with negligible lateral variation. For more clarification, the plot shows that cooling occurs mainly through the skin thickness, with minimal variation across the surface.

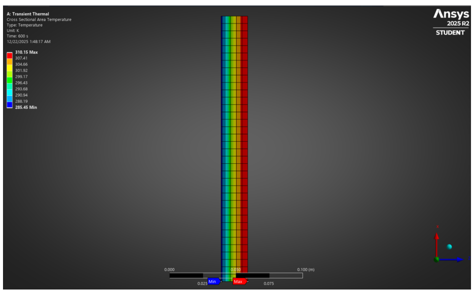

Figure 3: Baseline case: cross-sectional temperature contour at $t = 600$ s, showing predominantly through-thickness temperature gradients with minimal lateral variation.

Comparison Across Convective Coefficients

To quantify the sensitivity of cooling to convective heat transfer strength, simulations were repeated for $h = 20$, 50, and 100 W/m^{2}K, while keeping the geometry, material properties, and exposure time fixed. Quantitative comparisons are summarized in Tables 1, 2, and 3.

Table 1: Minimum temperature attained in each layer over the 600 s exposure for different convective coefficients.

Layer	$h = 20$ **(K)**	$h = 50$ **(K)**	$h = 100$ **(K)**
Epidermis	297.07	288.31	282.85
Dermis	298.65	290.66	285.51
Fat	310.15	310.15	310.15

As the convective coefficient increases, the minimum temperatures attained within the epidermis and dermis decrease markedly, reflecting enhanced surface heat removal and stronger driving temperature gradients. For light cooling ($h = 20$ W/m^{2}K), no layer reaches 15°C within 600 s, indicating shallow and limited cooling. The baseline case ($h = 50$ W/m^{2}K) achieves therapeutic-level cooling only in the epidermis near the end of exposure. Under aggressive cooling ($h = 100$ W/m^{2}K), the epidermis reaches 10°C within the simulated duration and the dermis reaches 15°C, indicating increased cooling penetration.

Table 2: Time to reach temperature thresholds (15°C and 10°C) for each case. "NR" indicates not reached within 600 s.

Case	**Epidermis**		**Dermis**		**Fat**	
	$t_{15°C}$ **(s)**	$t_{10°C}$ **(s)**	$t_{15°C}$ **(s)**	$t_{10°C}$ **(s)**	$t_{15°C}$ **(s)**	$t_{10°C}$ **(s)**
$h = 20$	NR	NR	NR	NR	NR	NR
$h = 50$	600	NR	NR†	NR	NR	NR
$h = 100$	274.74	583	424.54	NR	NR	NR

†Reported as > 650 s (not reached within the simulated 600 s). (NR stands for not reached)

Table 3: Comparison of cooling effectiveness across convective coefficients, including penetration depth below 15°C and reported cooling-rate ranges.

Metric	$h = 20$	$h = 50$	$h = 100$
Depth where $T < 15°$C (mm)	-	0.4	3.25
Epidermis cooling rate (K/s)	0.03 to 5.55×10^{-3}	0.10 to 6.66×10^{-3}	2.0 to 0.0167
Dermis cooling rate (K/s)	0.025 to 5.88×10^{-3}	0.033 to 7.69×10^{-3}	0.03 to 0.02
Fat cooling rate (K/s)	0	0	0

Epidermis reaches approximately 15°C at $t = 600$ s in the baseline case; deeper layers remain above 15°C.

Cooling penetration depth increases with h, consistent with stronger convective heat extraction at the surface. While no penetration below 15°C is observed for $h = 20$ W/m^{2}K, the baseline case exhibits penetration limited to the superficial layer. For $h = 100$ W/m^{2}K, the temperature drops below 15°C to a depth of approximately 3.25 mm, demonstrating a substantial increase in the effective cooling zone compared to the baseline.

Physiological Interpretation of Cooling Behavior

The epidermis cools rapidly primarily because it is directly exposed to the imposed convective boundary at the skin surface, leading to a high initial heat flux. Its small thickness and relatively low thermal mass enable a fast transient response. By contrast, **deeper tissues cool more slowly** due to the finite rate of heat diffusion through the layered structure and the presence of internal metabolic heat generation (particularly within the dermis and fat), which partially offsets external cooling. Moreover, the fixed-temperature boundary at the body core continuously supplies heat from below, further limiting temperature reduction at larger depths over the 10-minute exposure.

Effect of Fat as a Thermal Insulator

Subcutaneous fat acts as a strong thermal insulator, substantially limiting heat transfer to deeper tissue. This behavior is explained by its comparatively low thermal conductivity and its large thickness in the model. As a result, the temperature decrease within fat remains minimal across all convective conditions, indicating that cooling is largely confined to the superficial layers. This insulating effect helps protect deeper tissues from excessive cooling, even when the surface convection is increased.