Encontrar $\mu_{a|b}$ y $\Sigma_{a|b}$ completando cuadrados Para $P(x_a|x_b)$

—Apartir de la distribucion del termino x_b

$$-\frac{1}{2}(x_a-\mu_a,\ x_b-\mu_b)^T\,\Sigma^{-1}\,(x_a-\mu_a,\ x_b-\mu_b)$$ gaussiana cuadratica multivariada
Distancia "Nombre vard"

$$-\frac{1}{2}(x-\mu)^T\,\Sigma^{-1}\,(x-\mu)$$, sin descomponer x

Ahora, $P(x) = P([x_a, x_b])$; $P(x_a|x_b) = N(x_a|\mu_{a|b}, \Sigma_{a|b})$

Se toma x_b como una constante

Reescribiendo: distributiva

$$-\frac{1}{2}(x-\mu)^T\,\Sigma^{-1}\,(x-\mu) = -\frac{1}{2}\Big[(x_a-\mu_a)^T\Lambda_{aa}(x_a-\mu_a) + (x_b-\mu_b)^T\Lambda_{ba}(x_a-\mu_a) + (x_a-\mu_a)^T\Lambda_{ab}(x_b-\mu_b) + (x_b-\mu_b)^T\Lambda_{bb}(x_b-\mu_b)\Big]$$

$$= -\frac{1}{2}\Big[x_a^T\Lambda_{aa}x_a - x_a^T\Lambda_{aa}\mu_a \underline{-\mu_a^T\Lambda_{aa}x_a} + \mu_a^T\Lambda_{aa}\mu_a + x_b^T\Lambda_{ba}x_a - x_b^T\Lambda_{ba}\mu_a - \mu_b^T\Lambda_{ba}x_a + \mu_b^T\Lambda_{ba}\mu_a$$
$$+ x_a^T\Lambda_{ab}x_b - x_a^T\Lambda_{ab}\mu_b - \mu_a^T\Lambda_{ab}x_b + \mu_a^T\Lambda_{ab}\mu_b + x_b^T\Lambda_{bb}x_b - x_b^T\Lambda_{bb}\mu_b \underline{-\mu_b^T\Lambda_{bb}x_b} + \mu_b^T\Lambda_{bb}\mu_b\Big]$$

Se toman los valores que dependan de x_a y x_b con x_a

El termino cuadratico en x_a: $-\frac{1}{2}x_a^T\Lambda_{aa}x_a$ tiene la informacion de la matriz Σ de covarianza

Del termino cuadratico $\Sigma_{a|b} = \Lambda_{aa}^{-1}$

- Ahora buscamos los terminos lineales en x_a

$$\frac{1}{2}\left[\underset{1}{x_a^T\Lambda_{aa}\mu_a} - \underline{\mu_a^T\Lambda_{aa}x_a} + x_b^T\Lambda_{ba}x_a - \mu_b^T\Lambda_{ba}x_a + \underset{1}{x_a^T\Lambda_{ab}x_b} - \underset{1}{x_a^T\Lambda_{ab}\mu_b}\right]$$

se tiene, se pueden factorizar (factor comun es x_a^T

$$x_a^T\Lambda_{aa}\mu_a - x_a^T\Lambda_{ab}x_b + x_a^T\Lambda_{ab}\mu_b = x_a^T(\Lambda_{aa}\mu_a - \Lambda_{ab}x_b + \Lambda_{ab}\mu_b$$

se factoriza Λ_{ab} $\quad = x_a^T(\Lambda_{aa}\mu_a - \Lambda_{ab}(x_b + \mu_b))$

Se busca despejar el termino lineal en x donde $x^T\Sigma^{-1}\mu$

$$\underline{x_a^T\underbrace{\Sigma^{-1}_{a|b}\mu_{a|b}}_{\alpha} = x_a^T\underbrace{(\Lambda_{aa}\mu_a - \Lambda_{ab}(\mu_b - x_b))}_{\beta = \alpha}}$$

Como $\Sigma^{-1}_{a|b} = \Lambda_{aa}$, se multiplica a ambos lados por $\Sigma_{a|b}$

x_a^T se cancela (ambos lados) (No se cancela, se plantea igualdad) $A^{-1}A = I$

$$\underbrace{\Sigma_{a|b}\Sigma^{-1}_{a|b}}_{I}\mu_{a|b} = \underbrace{\Sigma_{a|b}\Lambda_{aa}}_{I}\mu_a - \Sigma_{a|b}\Lambda_{ab}(\mu_b - x_b) \qquad \Sigma^{-1}_{ab} = \Lambda_{aa}$$

$$\Lambda_{aa}^{-1} = \Sigma_{ab}$$

$$\mu_{a|b} = \mu_a - \Lambda_{aa}^{-1}\Lambda_{ab}(\mu_b - x_b)$$

se deja todo en terminos de Λ

luego se reemplaza Λ por Σ

$$MM^{-1} = I = M^{-1}M$$

$$\Sigma^{-1} = \begin{bmatrix} \Sigma_{aa} & \Sigma_{ab} \\ \Sigma_{ba} & \Sigma_{bb} \end{bmatrix}^{-1} = \Lambda = \begin{bmatrix} \Lambda_{aa} & \Lambda_{ab} \\ \Lambda_{ba} & \Lambda_{bb} \end{bmatrix} \qquad \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} =$$

$$\Lambda = \Sigma^{-1} = \begin{bmatrix} (\Sigma_{aa} - \Sigma_{ab}\Sigma_{bb}^{-1}\Sigma_{ba})^{-1} & -(\Sigma_{aa} - \Sigma_{ab}\Sigma_{bb}^{-1}\Sigma_{ba})^{-1}\Sigma_{ab}\Sigma_{bb}^{-1} \\ -\Sigma_{bb}^{-1}\Sigma_{ba}(\Sigma_{aa} - \Sigma_{ab}\Sigma_{bb}^{-1}\Sigma_{ba})^{-1} & \Sigma_{bb}^{-1} + \Sigma_{bb}^{-1}\Sigma_{ba}(\Sigma_{aa} - \Sigma_{ab}\Sigma_{bb}^{-1}\Sigma_{ba})^{-1}\Sigma_{ab}\Sigma_{bb}^{-1} \end{bmatrix} = \begin{bmatrix} M & -MBD^{-1} \\ -D^{-1}CM & D^{-1} + D^{-1}CMBD^{-1} \end{bmatrix}$$

$$M = (A - BD^{-1}C)^{-1}$$

M^{-1} = complemento Schur

$$\mu_{a|b} = \mu_a - \Lambda_{aa}^{-1}\Lambda_{ab}(\mu_b - x_b)$$

$$= \mu_a - \underbrace{\Sigma_{aa} - \Sigma_{ab}\Sigma_{bb}^{-1}\Sigma_{ba}}_{M^{-1}}\left[-\underbrace{(\Sigma_{aa} - \Sigma_{ab}\Sigma_{bb}^{-1}\Sigma_{ba})^{-1}}_{M}\Sigma_{ab}\Sigma_{bb}^{-1}\right]$$

$$\boxed{\mu_{a|b} = \mu_a + \Sigma_{ab}\Sigma_{bb}^{-1}(\mu_b - x_b)}$$

$$\Sigma_{a|b} = \Lambda_{aa}^{-1}$$

$$\Sigma_{a|b}^{-1} = (\Sigma_{aa} - \Sigma_{ab}\Sigma_{bb}^{-1}\Sigma_{ba})^{-1}$$

$$\boxed{\Sigma_{a|b} = \Sigma_{aa} - \Sigma_{ab}\Sigma_{bb}^{-1}\Sigma_{ba}} = \Lambda_{aa}^{-1}$$

$P(x_b \mid x_a) = N(x_b \mid \mu_{b|a}, \Sigma_{b|a})$; $P(X) = P([X_a, X_b])$

Se distribuye la distancia de Mahalanobis (euclidea eliptica) $\frac{1}{2}(X-\mu)^T \Sigma^{-1}(x-\mu)$ y se sustituye $\Sigma^{-1} = \Lambda \rightarrow$ (precision)
↳ (covarianza)

$$-\frac{1}{2}\left[(x_a-\mu_a)^T \Lambda_{aa}(x_a-\mu_a) + (x_b-\mu_b)^T \Lambda_{ba}(x_a-\mu_a) + (x_a-\mu_a)^T \Lambda_{ab}(x_b-\mu_b) + (x_b-\mu_b)^T \Lambda_{bb}(x_b-\mu_b)\right]$$

$$=-\frac{1}{2}\left[(x_a^T \Lambda_{aa} x_a - x_a^T \Lambda_{aa}\mu_a - \mu_a^T \Lambda_{aa} x_a + \mu_a^T \Lambda_{aa}\mu_a + x_b^T \Lambda_{ba} x_a + x_b^T \Lambda_{ba}\mu_a - \mu_b^T \Lambda_{ba} x_a + \mu_b^T \Lambda_{ba}\mu_a + x_a^T \Lambda_{ab} x_b - x_a^T \Lambda_{ab}\mu_b - \mu_a^T \Lambda_{ab} x_b + \mu_a^T \Lambda_{ab}\mu_b + x_b^T \Lambda_{bb} x_b - x_b \Lambda_{bb}\mu_b - \underline{\mu_b^T \Lambda_{bb} x_b} + \mu_b^T \Lambda_{bb}\mu_b\right]$$

Se toma el termino cuadratico de x_b

$-\frac{1}{2}\left[x_b^T \Lambda_{bb} x_b\right]$, $\Rightarrow$ Λ_{bb} tiene la informacion de la covarianza $\Sigma_{b|a}$

$$\boxed{\Sigma_{b|a} = \Lambda_{bb}^{-1}}$$

Para los terminos lineales de x_b

$$-\frac{1}{2}\left[-x_b^T \Lambda_{ba} x_a + x_b^T \Lambda_{ba}\mu_a - x_b^T \Lambda_{bb}\mu_b - \mu_b^T \Lambda_{bb} x_b + \cancel{x_a^T \Lambda_{ab} x_b} - \mu_a^T \Lambda_{ab} x_b\right]$$

$- x_a^T \Lambda_{aa}\mu_a - \mu_a^T \Lambda_{aa} x_a$

x_b, x_a = Vector

Se agrupan con x_b^T

$$-x_b^T \Lambda_{ba} x_a + x_b^T \Lambda_{ba} \mu_a + x_b^T \Lambda_{bb} \mu_b = x_b^T(-\Lambda_{ba} x_a + \Lambda_{ba}\mu_a + \Lambda_{bb}\mu_b)$$

$$= x_b^T(\Lambda_{bb}\mu_b + \Lambda_{ba}(\mu_a - x_a))$$

Con $x^T \Sigma^{-1} \mu$; $\Sigma_{b|a}^{-1} = \Lambda_{bb}$

$$x_b^T \underbrace{\Sigma_{b|a}^{-1} \mu_{b|a}}_{\alpha} = x_b^T \underbrace{(\Lambda_{bb}\mu_b + \Lambda_{ba}(\mu_a - x_a))}_{\beta} \qquad \beta = \alpha$$

Se multiplica a ambos lados por $\Sigma_{b|a}$ y se "cancela" x_b^T

$$\underbrace{\Sigma_{b|a}\Sigma_{b|a}^{-1}}_{I} \mu_{b|a} = \Sigma_{b|a}(\Lambda_{bb}\mu_b + \Lambda_{ba}(\mu_a - x_a))$$

$$\mu_{b|a} = \underbrace{\Sigma_{b|a}\Lambda_{bb}}_{I}\mu_b + \Sigma_{b|a}\Lambda_{ba}(\mu_a - x_a) \qquad (\Lambda_{ba} = \Sigma_{ba}^{-1})$$

$$\mu_{b|a} = \mu_b + \Sigma_{b|a}\Lambda_{ba}(\mu_a - x_a)$$

$$\mu_{b|a} = \mu_b + \Sigma_{b|a}\Sigma_{ba}^{-1}(\mu_a - x_a)$$

$$\boxed{\mu_{b|a} = \mu_b + \Lambda_{bb}^{-1}\Lambda_{ba}(\mu_a - x_a)}$$

$$\Lambda_{ba} = -\Sigma_{bb}^{-1}\Sigma_{ba}(\Sigma_{aa} - \Sigma_{ab}\Sigma_{bb}^{-1}\Sigma_{ba})^{-1}$$

$$\Sigma_{b|a} = \Lambda_{bb}^{-1}$$

$$\Lambda_{bb} = \underbrace{\Sigma_{bb}^{-1}}_{D^{-1}} + \underbrace{\Sigma_{bb}^{-1}}_{D^{-1}} \underbrace{\Sigma_{ba}}_{C} \underbrace{(\Sigma_{aa} - \Sigma_{ab} \Sigma_{bb}^{-1} \Sigma_{ba})^{-1}}_{M} \underbrace{\Sigma_{ab}}_{B} \underbrace{\Sigma_{bb}^{-1}}_{D^{-1}}$$

$$\boxed{\Sigma_{b|a} = \left(\Sigma_{bb}^{-1} + \Sigma_{bb}^{-1} \Sigma_{ba} (\Sigma_{aa} - \Sigma_{ab} \Sigma_{bb}^{-1} \Sigma_{ba})^{-1} \Sigma_{ab} \Sigma_{bb}^{-1}\right)^{-1}}$$

$$M|A = D - CA^{-1}B$$

$$(A-D)^{-1} = A^{-1} + A^{-1}B(A-B)^{-1}$$

$$\mu_{b|a} = \mu_b + \Lambda_{bb}^{-1} \Lambda_{ba} (\mu_a - x_a)$$

$$\boxed{\mu_{b|a} = \mu_b + \left(\Sigma_{bb}^{-1} + \Sigma_{bb}^{-1} \Sigma_{ba} \left(\Sigma_{aa} - \Sigma_{ab} \Sigma_{bb}^{-1} \Sigma_{ba}\right)^{-1} \Sigma_{ab} \Sigma_{bb}^{-1}\right)^{-1} \left(- \Sigma_{bb}^{-1} \Sigma_{ba} \left(\Sigma_{aa} - \Sigma_{ab} \Sigma_{bb}^{-1} \Sigma_{ba}\right)^{-1}\right) \left(\mu_a - x_a\right)}$$