16/02/2024 — Demostraciones

- $Var\{x\} = \mathcal{E}\{(x-\mu_x)^2\} = \mathcal{E}\{x^2\} - \mathcal{E}^2\{x\}$

$$= \mathcal{E}\{x^2 - x\mu_x - x\mu_x + \mu_x^2\}$$
$$= \mathcal{E}\{x^2\} - \mathcal{E}\{2x\mu_x\} + \mathcal{E}\{\mu_x^2\}$$
$$= \mathcal{E}\{x^2\} - 2\mu_x\,\mathcal{E}\{x\} + \mu_x^2$$
$$= \mathcal{E}\{x^2\} - 2\mu_x^2 + \mu_x^2$$
$$= \mathcal{E}\{x^2\} - \mu_x^2$$
$$= \mathcal{E}\{x^2\} - (\mathcal{E}\{x\})^2 \;//$$

- $Cov\{x,y\} = \mathcal{E}_{x,y}\{(x-\mu_x)(y-\mu_y)\} = \mathcal{E}_{x,y}\{xy\} - \mathcal{E}\{x\}\,\mathcal{E}\{y\}$

$$= \mathcal{E}_{x,y}\{(xy - x\mu_y - y\mu_x + \mu_x\mu_y\}$$
$$= \mathcal{E}_{x,y}\{xy\} - \mu_y\,\mathcal{E}\{x\} - \mu_x\,\mathcal{E}\{y\} + \mu_x\mu_y$$
$$= \mathcal{E}_{x,y}\{xy\} - \mu_y\mu_x - \cancel{\mu_x\mu_y} + \cancel{\mu_x\mu_y}$$
$$= \mathcal{E}_{x,y}\{xy\} - \mu_x\mu_y$$
$$= \mathcal{E}_{x,y}\{xy\} - \mathcal{E}\{x\}\,\mathcal{E}\{y\} \;//$$

- $Cov(X,Y) = \mathcal{E}_{x,y}\{XY^T\} - \mathcal{E}\{X\}\,\mathcal{E}\{Y\}\;,\; X,Y \in \mathbb{R}^D$

$* \; Cov(X,X) = (X - \mathcal{E}\{X\})(X - \mathcal{E}\{X\})^T \;,\quad * \; \mathcal{E}\{\mathcal{E}\{X\}\} = \mathcal{E}\{X\}$

$$Cov(X,Y) = \mathcal{E}\{(X - E\{X\})(Y^T - \mathcal{E}\{Y\}^T)\}$$
$$= \mathcal{E}\{XY^T - X\,\mathcal{E}\{Y\}^T - Y^T\mathcal{E}\{X\} + \mathcal{E}\{X\}\,\mathcal{E}\{Y\}^T$$
$$= \mathcal{E}[XY^T\} - \mathcal{E}\{X\;\mathcal{E}\{Y^T\}\} - \mathcal{E}\{Y^T\mathcal{E}\{X\}\} + \mathcal{E}\{\mathcal{E}\{X\}\,\mathcal{E}\{Y\}^T\}$$
$$= \mathcal{E}[XY^T\} - \mathcal{E}[X\}\,\mathcal{E}\{Y\}^T - \mathcal{E}\{Y\}^T\mathcal{E}[X\} + \mathcal{E}[X\}\,\mathcal{E}\{Y\}^T$$
$$= \mathcal{E}[XY^T\} - \mathcal{E}\{Y\}^T\,\mathcal{E}[X\} \;//$$

19/02/2024 - Demostraciones

- $$\mathcal{E}\{x\} = \int_{-\infty}^{\infty} N(x|\mu\sigma^2)\, x\, dx = \mu$$

$$= \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{|x-\mu|^2}{2\sigma^2}\right) dx$$

* Quitar traslaciones - Sustitución $z = x - \mu$, $x = z + \mu$
$dz = dx$

$$= \int_{-\infty}^{\infty} z+\mu \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{|z|^2}{2\sigma^2}\right) dz$$

$$= \underbrace{\int_{-\infty}^{\infty} z \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{|z|^2}{2\sigma^2}\right) dz}_{0 \text{ Por Paridad}} + \int_{-\infty}^{\infty} \mu \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{|z|^2}{2\sigma^2}\right) dz$$

$$= \mu \underbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{|z|^2}{2\sigma^2}\right) dz}_{\to 1} = \mu //$$

Función de densidad

- $$\mathcal{E}\{x\}^2 = \int_{-\infty}^{\infty} N(x|\mu,\sigma^2)\, x^2\, dx = \mu^2 + \sigma^2$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{|x-\mu|^2}{2\sigma^2}\right) x^2\, dx$$

* Cambio de variable $z = \frac{x-\mu}{\sqrt{2}\sigma}$; $dz = \frac{dx}{\sqrt{2}\sigma}$; $x = \mu + \sqrt{2}\sigma z$

$dx = \sqrt{2}\sigma\, dz$; $x^2 = \mu^2 + 2\sigma^2 z^2 + 2\mu\sqrt{2}\sigma z$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} e^{-z^2} (\mu^2 + 2\sigma^2 z^2 + 2\sqrt{2}\sigma z) \sqrt{2}\sigma\, dz$$

* Se expande la integral en 3 integrales

$$= \mu^2 \underbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} e^{-z^2} dz}_{\to 1} + 2\sigma^2 \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} e^{-z^2} z^2\, dz$$

Función densidad de probabilidad

$$+ 2\mu\sqrt{2}\sigma \underbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} e^{-z^2} z\, dz}_{\to 0 \;\therefore\; \text{Simetría}}$$

$$= \mu^2 + 2\sigma^2 \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} e^{-z^2} z^2 \, dz$$

$$= \mu^2 + \cancel{2}\sigma^2 \frac{1}{\sqrt{\pi}} \left(\frac{\sqrt{\pi}}{\cancel{2}} \right) = \mu^2 + \sigma^2 \; //$$

- $\text{Var}\{X\} = \sigma^2$

$$= \mathcal{E}\{x^2\} - \mathcal{E}^2\{X\} \quad ; \quad \mathcal{E}\{x^2\} = \mu^2 + \sigma^2 \quad ; \quad \mathcal{E}\{X\} = \mu$$

$$= \cancel{\mu^2} + \sigma^2 - \cancel{\mu^2}$$

$$= \sigma^2 \; //$$

21/02/24 — Demostraciones

- $\sigma^2_{ML} = \frac{1}{N} \sum_{n=1}^{N} (X_n - \mu_{ML})^2$

* Se calcula el logaritmo de la función para simplificar cálculos.

$$Log(\sigma^2_{ML}) = Log\left(\prod_{n=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2_{ML}}} \exp\left(-\frac{|X_n - \mu_{ML}|^2}{2\sigma^2} \right) \right)$$

$$= Log\left(\prod_{n=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2_{ML}}} \right) + log\left(\prod_{n=1}^{N} \exp\left(-\frac{|X_n - \mu_{ML}|^2}{2\sigma^2} \right) \right)$$

$$= log\left(\prod_{n=1}^{N} \frac{1}{(2\pi\sigma^2_{ML})^{\frac{1}{2}}} \right) + log\left(\exp\left(-\sum_{n=1}^{N} \frac{|X_n - \mu|^2}{2\sigma^2} \right) \right)$$

$$= -\frac{N}{2} log(2\pi\sigma^2_{ML}) - \sum_{n=1}^{N} \frac{|X_n - \mu|^2}{2\sigma^2}$$

$$Log(\sigma^2_{ML}) = -\frac{N}{2} log(2\pi) - \frac{N}{2} log(\sigma^2_{ML}) - \frac{1}{2\sigma^2} \sum_{n=1}^{N} |X_n - \mu_{ML}|^2$$

* Se deriva respecto a σ^2 y se iguala a cero

$$\frac{d}{d(\sigma^2)} \left(Log(\sigma^2_{ML}) \right) = \frac{d}{d(\sigma^2)} \left(\cancel{-\frac{N}{2} log(2\pi)}^{0} \right) \frac{d}{d(\sigma^2)} \left(-\frac{N}{2} log(\sigma^2_{ML}) \right) \frac{d}{d(\sigma^2)} \left(-\frac{1}{2\sigma^2} \sum_{n=1}^{N} |X_n - \mu_{ML}|^2 \right)$$

$$= -\frac{N}{2\sigma^2} + \frac{1}{2(\sigma^2)^2} \sum_{n=1}^{N} (X_n - \mu_{ML})^2 = 0$$

* Se multiplica a ambos lados por $(\sigma^2)^2$

$$= -\frac{N}{2\sigma^2}(\sigma^2)^2 + \frac{1}{2(\sigma^2)^2}(\sigma^2)^2 \sum_{n=1}^{N} (x_n - \mu_{ML})^2 = 0\,(\sigma^2)^2$$

$$= -\frac{N}{2}\sigma_{ML}^2 + \frac{1}{2}\sum_{n=1}^{N}(x_n - \mu_{ML})^2 = 0$$

* Se despeja σ_{ML}^2

$$\sigma_{ML}^2 = \left(\frac{2}{N}\right)\frac{1}{2}\sum_{n=1}^{N}(x_n - \mu_{ML})^2 = \frac{1}{N}\sum_{n=1}^{N}(x_n - \mu_{ML})^2 \;//$$

- Ejercicio: determinar el estimador mas información

$$x[n] = A + W_n \;;\; A \in \mathbb{R}^+ \;\; y \;\; W_n \sim N(W_n \mid \mu, \sigma^2)$$

Sea: $\hat{A}_1 = \frac{1}{N}\sum_{n=1}^{N} x_n$ Estimador 1 ; $\hat{A}_2 = x_1$ Estimador 2

* Se evalua el Bias

Para $\hat{A}_1$

$$b(\hat{A}_1) = \mathcal{E}\{A_1\} - A$$

$$= \mathcal{E}\left\{\frac{1}{N}\sum_{n=1}^{N} x[n]\right\} - A$$

$$= \frac{1}{N}\sum_{n=1}^{N}\mathcal{E}\{A + w[n]\}$$

$$= \mathcal{E}\{A\} + \mathcal{E}\{w[n]\}$$

$$= \frac{1}{N}\sum_{n=1}^{N} A - A$$

$$= \frac{N}{N}A - A$$

$$b(\hat{A}_1) = 0$$

Para $\hat{A}_2$

$$b(A_2) = \mathcal{E}\{A_2\} - A$$

$$= \mathcal{E}\{x[1]\} - A$$

$$= \mathcal{E}\{A + w[1]\} - A$$

$$= A + \mathcal{E}\{w[1]\} - A$$

$$= A - A$$

$$b(A_2) = 0.$$

* No es posible concluir a partir del sesgo del estimador, Por lo tanto se calcula la varianza

Para $\hat{A}_1$

$$\mathrm{Var}\{A_1\} = \mathrm{Var}\left\{\frac{1}{N}\sum_{n=1}^{N} x[n]\right\}$$

$$= \mathrm{Var}\left\{\frac{1}{N}\sum_{n=1}^{N} A + w[n]\right\}$$

$$= \mathrm{Var}\left\{\frac{1}{N}\sum_{n=1}^{N} A + \sum_{n=1}^{N} w[n]\right\}$$

$$= Var\left\{ \frac{1}{N} \sum_{n=1}^{N} w[n] \right\}$$

$$= \sum_{n=1}^{N} Var\left\{ \frac{1}{N} w[n] \right\}$$

$$= \sum_{n=1}^{N} \frac{1}{N^2} \sigma^2 = \frac{N}{N^2} \sigma^2$$

$$Var\{A_1\} = \frac{\sigma^2}{N}$$

Para A_2

$$Var\{A_2\} = Var\{x[1]\}$$

$$= Var\{A + w[1]\}$$

$$= Var\{w[1]\}$$

$$Var\{A_2\} = \sigma^2$$

El estimador A_1 Contiene Mas información //

- Ejercicio: Demuestre que:

$$\mathcal{E}\{\mu_{ML}\} = \mu \quad ; \quad Con \quad x_n \sim N(x_n | \mu, \sigma^2)$$

$$\mathcal{E}\{\sigma^2_{ML}\} = \left(\frac{N-1}{N}\right)\sigma^2$$

$$\mathcal{E}\left\{\frac{1}{N}\sum_n (x_n - \mu_{ML})^2\right\} = \mathcal{E}\left\{\frac{1}{N}\sum_n \left(x_n^2 - 2x_n\mu_{ML} + \mu_{ML}^2\right)\right\}$$

$$= \mathcal{E}\left\{\frac{1}{N}\sum_n x_n^2 - 2\mu_{ML}\frac{1}{N}\sum_n x_n + \frac{1}{N}\sum_n \mu_{ML}^2\right\}$$

$$= \mathcal{E}\left\{\frac{1}{N}\sum_n x_n^2\right\} - 2\,\mathcal{E}\left\{\mu_{ML}\mu_{ML}\right\} + \mathcal{E}\left\{\frac{1}{N} N \mu_{ML}^2\right\}$$

$$= \frac{1}{N}\sum_n \mathcal{E}\left\{x_n^2\right\} - 2\,\mathcal{E}\left\{\mu_{ML}^2\right\} + \mathcal{E}\left\{\mu_{ML}^2\right\}$$

$$\mathcal{E}\left\{\frac{1}{N}\sum_n (x_n - \mu)^2\right\} = \frac{1}{N} N\, \mathcal{E}\{x_n^2\} - \mathcal{E}\left\{\mu_{ML}^2\right\} = \mathcal{E}\left\{x_n^2\right\} - \mathcal{E}\left\{\mu_{ML}^2\right\}$$

- $Var\{x\} = \mathcal{E}[x^2] - \mathcal{E}^2\{x\} \Rightarrow \mathcal{E}\{x_n^2\} = Var\{x_n\} + \mathcal{E}\{\mu_{ML}^2\}$
- $x_n \sim N(x_n|\mu, \sigma^2)$; $\mathcal{E}\{x_n^2\} = \sigma^2 + \mu^2$

$$\mathcal{E}\{x_{ML}^2\} = Var\{\mu_{ML}\} + \mathcal{E}^2\{\mu_{ML}\}$$

$$\mathcal{E}\{\mu_{ML}^2\} = \frac{\sigma^2}{N} + \mu^2$$

- $$\mathcal{E}[\mu_{ML}] = \mathcal{E}\left\{\frac{1}{N}\sum_n x_n\right\}$$

$$= \frac{1}{N}\sum_n \mathcal{E}\{x_n\} = \frac{1}{N}\sum_n \mu = \frac{1}{N} N\mu$$

$$\mathcal{E}\{\mu_{ML}\} = \mu \quad //$$

- $$Var\{\mu_{ML}\} = Var\left\{\frac{1}{N}\sum_n x_n\right\} = \frac{1}{N^2}\sum_n Var\{x_n\}$$

$$= \frac{1}{N^2} N\sigma^2 = \frac{\sigma^2}{N}$$

- $$\mathcal{E}\left\{\frac{1}{N}\sum_n (x_n - \mu_{ML})^2\right\} = \mathcal{E}\{x_n^2\} - \mathcal{E}\{\mu_{ML}^2\}$$

$$= \sigma^2 + \mu^2 - \left(\frac{\sigma^2}{N} + \mu^2\right)$$

$$= \sigma^2 - \frac{\sigma^2}{N} + \mu^2 - \mu^2$$

$$= \sigma^2\left(1 - \frac{1}{N}\right)$$

$$\mathcal{E}\left\{\frac{1}{N}\sum_n (x_n - \mu_{ML})^2\right\} = \sigma^2\frac{(N-1)}{N}$$

$$\sigma_{ML}^2 = \sigma^2\frac{(N-1)}{N} \quad //$$

¿Cual debería ser la correccion sobre σ_{ML}^2 para evitar el Bias?

$$\frac{N}{N-1}\sigma_{ML} = \sigma^2$$

$$\frac{N}{N-1}\frac{1}{N}\sum_n (x_n - \mu_{ML})^2 = \sigma^2$$

$$\frac{1}{N-1}\sum_n (x_n - \mu_{ML})^2 = \sigma^2 \quad //$$

26/02/24 - Demostración

$S^2 = \Delta$

* Eigenvalores : $X^T X = \Sigma = V \Delta V$

* Descomposición por valores singulares : $X = U S V^*$

$$(U S V^*)^T (U S V^*)^T = V \Delta V^*$$

$$(V^*)(U S)^T \; U S V^* =$$

$$V S^* U^* U \; S V^* = \qquad U^* U = I$$

$$V S^* S V^* = V \Delta V^*$$

$$V S^2 V^* = V \Delta V^* \quad //$$

28/02/24 - Ejercicio

$$ecmr(Y, \phi W) = E\{\|Y - \phi W\|_2^2\} + \lambda \|W\|_2^2$$

$$\|Y - \phi W\|_2^2 = [Y^T Y - 2 Y^T \phi W + W^T \phi^T \phi W) \frac{1}{N}$$

* $\langle Y - \phi W, \; Y - \phi W \rangle$; $\lambda \|W\|_2^2 = \lambda \langle W, W \rangle = \lambda = W^T W$

$$\frac{\partial}{\partial W} \|Y - \phi W\|_2^2 = [-2 \phi^T Y + 2 \phi^T \phi W] \frac{1}{N} \in \mathbb{R}^Q$$

* $\frac{\partial}{\partial W} \|W\|_2^2 = 2 \lambda W$

$$[-\frac{2}{N} \phi^T Y + \frac{2}{N} \phi^T \phi W + 2 \lambda W] \frac{N}{2} = 0 \left[\frac{N}{2}\right]$$

$$\phi^T Y + \phi^T \phi W + \lambda N W = 0$$

$$(\phi^T \phi + \lambda N I)^{-1} (\phi^T \phi + \lambda N I) W = (\phi^T \phi + \lambda N I)^{-1} \phi^T Y$$

$$\hat{W} = (\phi^T \phi + \lambda N I)^{-1} \phi^T Y \quad //$$

01/03/24 - Ejercicio

Asumiendo conjunto datos i.i.d $D = \{X_n \in \mathbb{R}^p, Y_n \in \mathbb{R}\}_{N=1}^{N}$, Encontrar W y σ^2 que Maximizan la log verosimilitud

$* \; F(x \mid W) = \phi W$

$$\frac{d}{dW} \log \left(P(Y \mid F(x \mid W), \sigma_n^2) \right) \in \mathbb{B}^Q$$

$$= \log \left(\prod_{n=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\| Y_n - f(x \mid W) \|^2}{2\sigma^2} \right) \right)$$

$$= \log \left(\prod_{n=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\| Y_n - \phi W \|^2}{2\sigma^2} \right) \right)$$

$$= \log \left(\prod_{n=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \right) + \log \left(\prod_{n=1}^{N} \exp\left(-\frac{\| x_n - \phi W \|^2}{2\sigma^2} \right) \right)$$

$$= \log_n \left(\frac{1}{(2\pi\sigma^2)^{N/2}} \right) + \log \left(\exp\left(-\sum_{n=1}^{N} \frac{\| Y_n - \phi W \|^2}{2\sigma^2} \right) \right)$$

$$= \log_n \left((2\pi\sigma_n^2)^{-\frac{N}{2}} \right) - \sum_{n=1}^{N} \frac{\| Y_n - \phi W \|^2}{2\sigma_n^2}$$

$$= -\frac{N}{2} \log(2\pi\sigma_n^2) - \frac{1}{2\sigma_n^2} \| Y - \phi W \|_2^2$$

$$= -\frac{N}{2} \left\{ \log(2\pi) + \log(\sigma_n^2) \right\} - \frac{1}{2\sigma_n^2} \| Y - \phi W \|_2^2$$

$$= \frac{d}{dW} \left[-\frac{N}{2} \left(\log(2\pi) + \log(\sigma_n^2) \right) \right] - \frac{d}{dW} \left[\frac{1}{2\sigma_n^2} \| Y - \phi W \|_2^2 \right]$$

$$= 0 - \frac{1}{2\sigma_n^2} \left[-2\phi^T Y + 2\phi^T \phi W \right]$$

$$= -\frac{1}{2\sigma_n^2} \left[-2\phi^T Y + 2\phi^T \phi W \right]$$

$$\hat{W}_{ML} = (\phi^T \phi)^{-1} \phi^T Y \quad //$$

* Para σ^2 se deriva respecto a σ^2

$$\frac{d}{d\sigma_n^2}\left[-\frac{N}{2}\left(\log(2\pi)+\log(\sigma_n^2)\right)\right]-\frac{d}{d\sigma_n^2}\left[\frac{1}{2\sigma_n^2}\|\mathbb{Y}-\phi\mathbb{W}\|_2^2\right]$$

$$=\frac{d}{d\sigma_n^2}\left[\cancel{-\frac{N}{2}\left(\log(2\pi)\right)}^{0}\right]+\frac{d}{d\sigma_n^2}\left[-\frac{N}{2}\log(\sigma_n^2)\right]-\frac{d}{d\sigma_n^2}\left[\frac{1}{2\sigma_n^2}\|\mathbb{Y}-\phi\mathbb{W}\|_2^2\right]$$

$$=-\frac{N}{2\sigma^2}+\frac{1}{2(\sigma^2)^2}\|\mathbb{Y}-\phi\mathbb{W}\|_2^2=0$$

* Se multiplica por σ^2 y se despeja

$$=-\frac{N}{2\cancel{\sigma^2}}(\sigma_n^2)^2+\frac{1}{2\cancel{(\sigma^2)^2}}\cancel{(\sigma^2)^2}\|\mathbb{Y}-\phi\mathbb{W}\|_2^2=0\,(\sigma^2)^2$$

$$=-\frac{N}{2}\sigma_n^2+\frac{1}{2}\left[-2\phi^T\mathbb{Y}+2\phi^T\phi\mathbb{W}\right]=0$$

$$\sigma_n^2=\frac{1}{N}\left[-2\phi^T\mathbb{Y}+2\phi^T\phi\mathbb{W}\right]\quad //$$

Ejercicio

Encontrar los $\mathbb{W}$ que maximizan el log-MAP

$$\log(P(\mathbb{W})\mathbb{Y},\phi,\sigma_n^2))=\log\left(\prod_{n=1}^{N}(\mathbb{Y}_n|\phi(\mathbb{X}_n)\mathbb{W},\sigma_n^2)\prod_{q=1}^{Q}N(\mathbb{W}_q|0,\sigma$$

$$=\underbrace{\left\{-\frac{N}{2}\left[\log(2\pi)+\log(\sigma_n^2)\right]-\frac{1}{2\sigma_n^2}\|\mathbb{Y}-\phi\mathbb{W}\|_2^2\right\}}_{A}$$

$$+\log\left(\prod_{q=1}^{Q}\left(\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\|\mathbb{W}_q-0\|_2^2}{2\sigma_w^2}\right)\right)\right)$$

$$=A+\log\left(\prod_{q=1}^{Q}\frac{1}{(2\pi\sigma_w^2)^{1/2}}\right)+\log\left(\prod_{q=1}^{Q}\exp\left(\frac{|\mathbb{W}_q|^2}{2\sigma_w^2}\right)\right)$$

$$=A+\log\left(\frac{1}{(2\pi\sigma_w^2)^{Q/2}}\right)+\log\left(\exp\left(-\sum_{q=1}^{Q}\frac{|\mathbb{W}_q|^2}{2\sigma_w^2}\right)\right)$$

$$=A-\frac{Q}{2}\left[\log(2\pi)+\log(\sigma_w^2)\right]-\sum_{q=1}^{Q}|\mathbb{W}_q|^2$$

* $\|\mathbb{W}\|_2^2=\left(\sqrt{\sum_{q=1}^{Q}|\mathbb{W}_q|^2}\right)^2=\sum_{q=1}^{Q}|\mathbb{W}_q|^2$

$$= -\frac{N}{2}\left[\log(2\pi)+\log(\sigma_n^2)\right] - \frac{1}{2\sigma_n^2}\|Y-\phi W\|_2^2 - \frac{Q}{2}\left[\log(2\pi)+\log(\sigma_w^2)\right]$$

$$-\frac{1}{2\sigma_w^2}\|W\|_2^2$$

$$= Max - \left[\frac{2\sigma_n^2}{2\sigma_n^2}\|Y-\phi W\|_2^2 + \frac{2\sigma_n^2}{2\sigma_w^2}\|W\|_2^2\right] + cte$$

$$= Min\ [Y-\phi W\|_2^2 + \frac{\sigma_n^2}{\sigma_w^2}\|W\|_2^2 + cte$$

$$= \hat{W}_{MAD} = \left(\phi^T\phi + \frac{\sigma_n^2}{\sigma_w^2} I\right)^{-1}\phi^T Y \rightarrow W_{MCRU} \quad //$$

* MAP tiene la misma estructura que MCRU, En donde
$$\lambda = \frac{\sigma_n^2}{\sigma_w^2} \quad //$$

Exprese la Solucion de MAP mediante descomposición espectral

- $$\hat{W}_{MAD} = \left(\phi^T\phi + \frac{\sigma_n^2}{\sigma_w^2} I\right)^{-1}\phi^T Y$$

* $X = USV^*$; $X = \phi$

$$\left(((SV^T)^T U^T)(USV^T) + \frac{\sigma_n^2}{\sigma_w^2} I\right)^{-1} (SV^T)^T U^T Y$$

$$\left((VS^2U^T)(USV^T) + \frac{\sigma_n^2}{\sigma_w^2} I\right)^{-1} (VS^2)U^T Y$$

$$(VS^TV^T) + \frac{\sigma_n^2}{\sigma_w^2} I)^{-1} (VS^2U^T Y)$$